# Transverse Hadron Structure from Lattice QCD

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– QCDSF Collaboration –

Special mention:

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## Outline

# Lattice Simulation

## Action

$N_f = 2$

$$S = S_G + S_F$$

$$S_G = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr}\,U_{\mu\nu}(x)\right)$$

$$S_F = \sum_x \Big\{ \bar\psi(x)\psi(x) - \kappa\,\bar\psi(x)U^\dagger_\mu(x-\hat\mu)[1+\gamma_\mu]\psi(x-\hat\mu) - \kappa\,\bar\psi(x)U_\mu(x)[1-\gamma_\mu]\psi(x+\hat\mu) - \frac{1}{2}\kappa\, c_{SW}\, g\,\bar\psi(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x) \Big\}$$

$\Updownarrow$

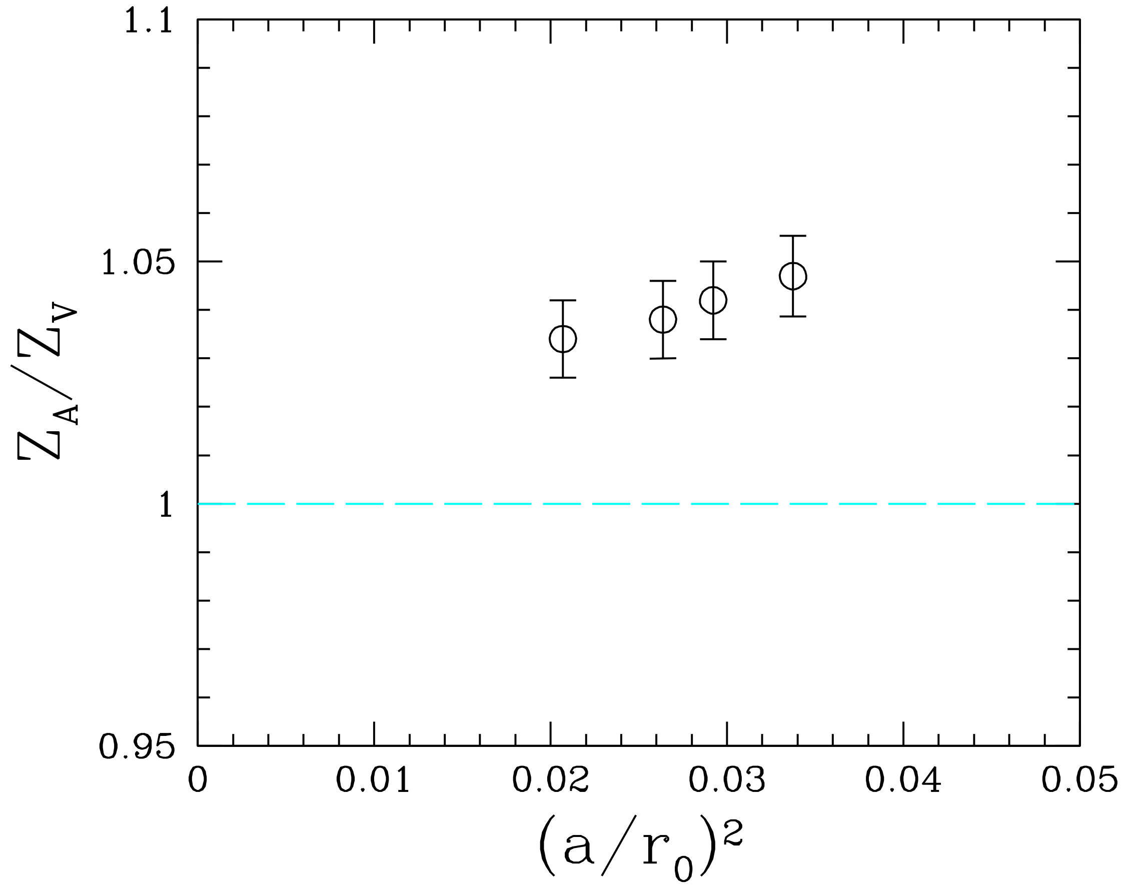
$\partial_\mu A_\mu^{\mathrm{imp}} = 2m_q P$

Clover Fermions

## Advantages

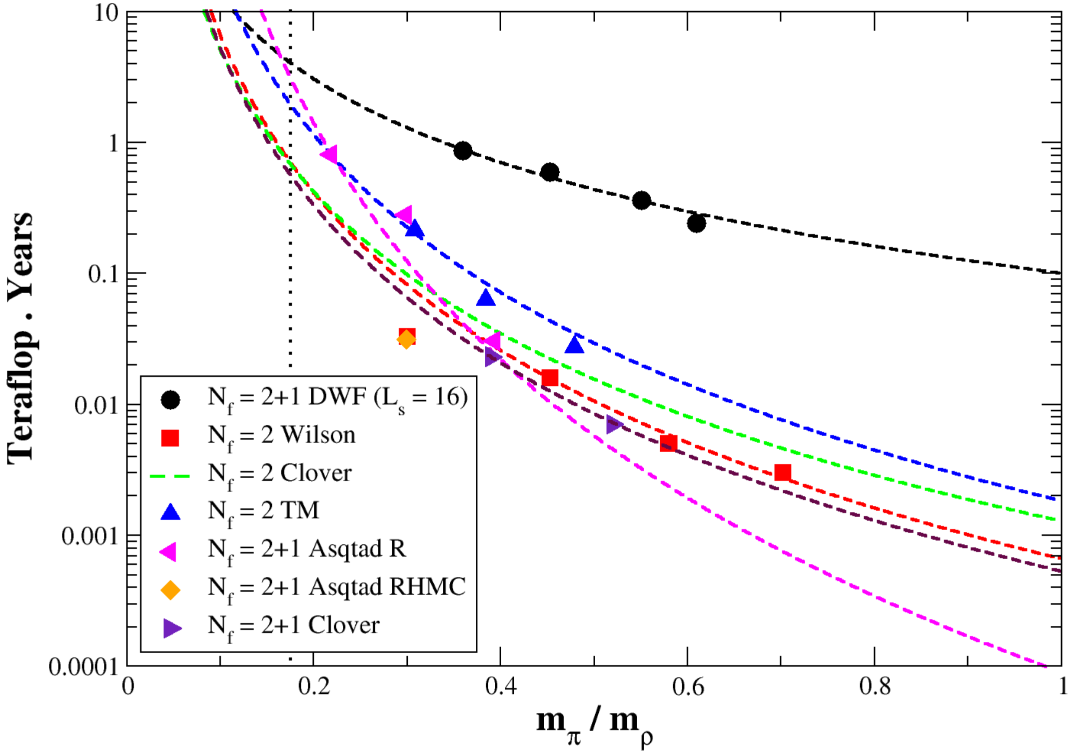
- Local
- Transfer matrix
- $O(a)$ improved
- Flavor symmetry — Prerequisite to making contact with $SU(2)$ ChPT
  - Finite size corrections
  - Chiral extrapolation
  - Determination of low-energy constants
- Fast to simulate

# Chiral Symmetry ?




↑
NPRen

# Cost of Simulation




Clark

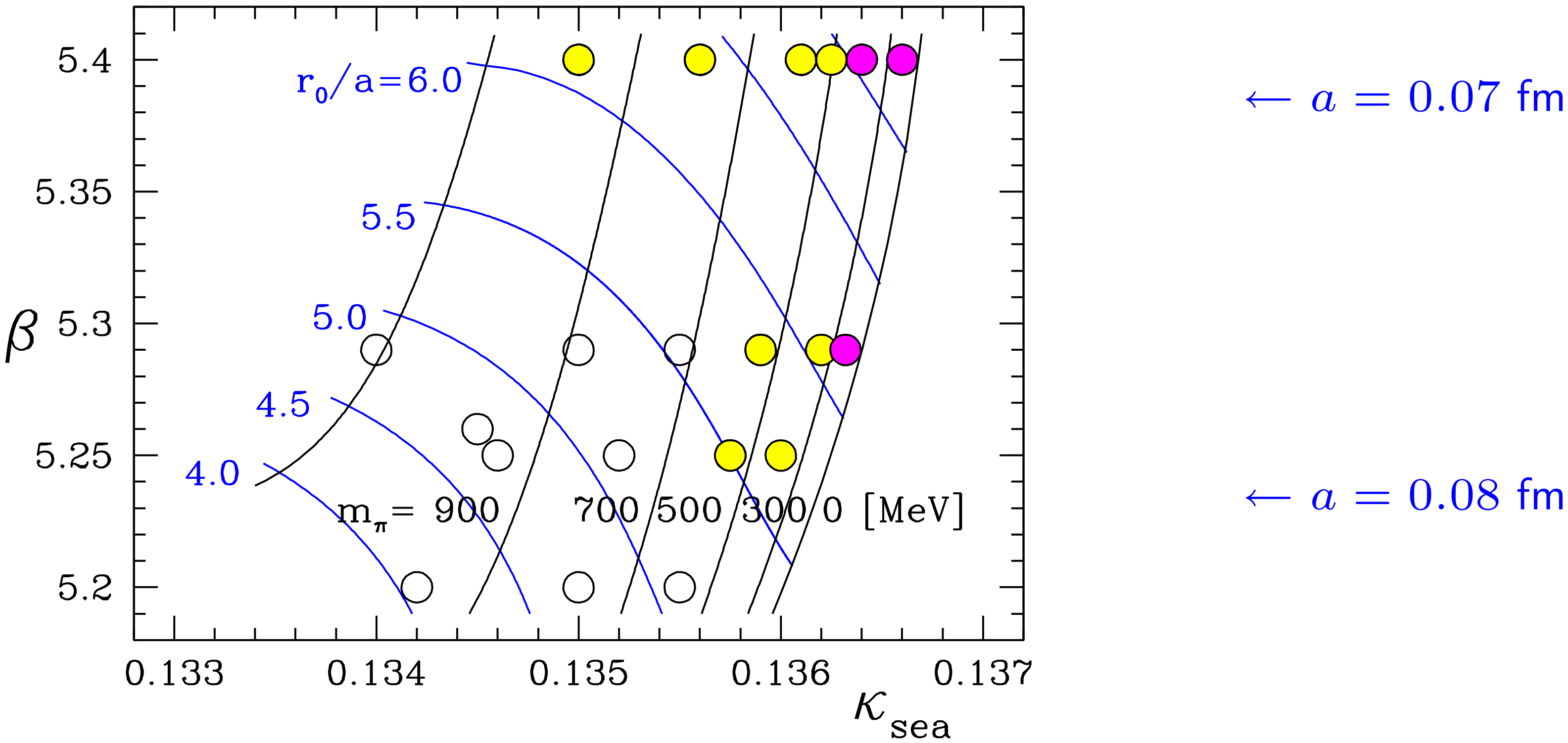
# Lattices

| $\beta$ | $\kappa_{\text{sea}}$ | Volume | $a$ [fm] | $m_{PS}$ [MeV] |
|---|---|---|---|---|
| 5.20 | 0.13420 | $16^3 \times 32$ | 0.11 | 1010 |
| 5.20 | 0.13500 | $16^3 \times 32$ | 0.10 | 830 |
| 5.20 | 0.13550 | $16^3 \times 32$ | 0.09 | 620 |
| 5.25 | 0.13460 | $16^3 \times 32$ | 0.10 | 990 |
| 5.25 | 0.13520 | $16^3 \times 32$ | 0.09 | 830 |
| 5.25 | 0.13575 | $24^3 \times 48$ | 0.08 | 600 |
| 5.25 | 0.13600 | $24^3 \times 48$ | 0.08 | 450 |
| 5.26 | 0.13450 | $16^3 \times 32$ | 0.10 | 1010 |
| 5.29 | 0.13400 | $16^3 \times 32$ | 0.10 | 1170 |
| 5.29 | 0.13500 | $16^3 \times 32$ | 0.09 | 930 |
| 5.29 | 0.13550 | $24^3 \times 48$ | 0.08 | 770 |
| 5.29 | 0.13550 | $16^3 \times 32$ | | 780 |
| 5.29 | 0.13550 | $12^3 \times 32$ | | 880 |
| 5.29 | 0.13590 | $24^3 \times 48$ | 0.08 | 590 |
| 5.29 | 0.13590 | $16^3 \times 32$ | | 630 |
| 5.29 | 0.13590 | $12^3 \times 32$ | | 870 |
| 5.29 | 0.13620 | $24^3 \times 48$ | 0.08 | 400 |
| 5.29 | 0.13632 | $32^3 \times 64$ | 0.08 | 340 |
| 5.29 | 0.13632 | $40^3 \times 64$ | 0.08 | 290 |
| 5.40 | 0.13500 | $24^3 \times 48$ | 0.08 | 1040 |
| 5.40 | 0.13560 | $24^3 \times 48$ | 0.07 | 840 |
| 5.40 | 0.13610 | $24^3 \times 48$ | 0.07 | 630 |
| 5.40 | 0.13625 | $24^3 \times 48$ | 0.07 | 530 |
| 5.40 | 0.13640 | $24^3 \times 48$ | 0.07 | 440 |
| 5.40 | 0.13640 | $32^3 \times 64$ | 0.07 | 440 |
| 5.40 | 0.13660 | $32^3 \times 64$ | 0.07 | 280 |

Not yet analyzed $\longrightarrow$ (5.29, 0.13632, $40^3 \times 64$)

Not yet analyzed $\longrightarrow$ (5.40, 0.13660, $32^3 \times 64$)

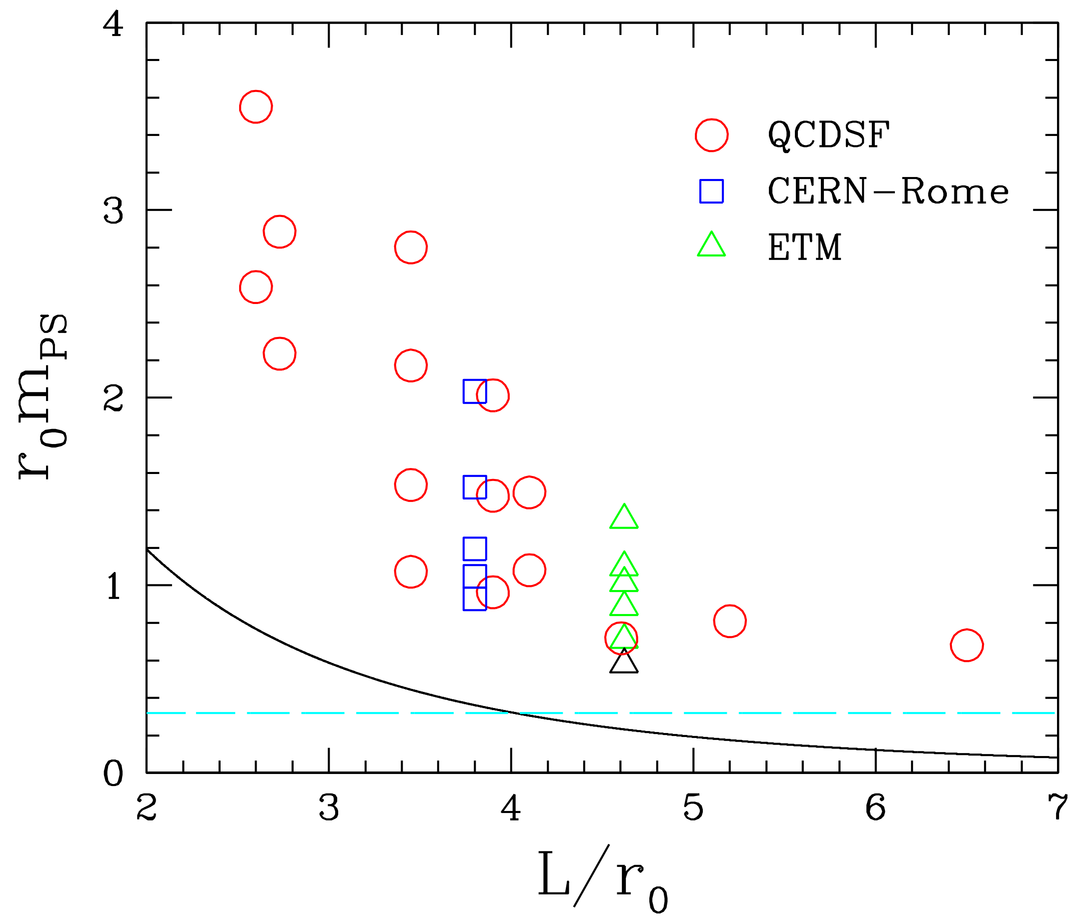
# Coverage




$24^3\,48$ $\quad$ $32^3\,64 + 40^3\,64$

For gauge field sampling we use 'ordinary' HMC algorithm with Hasenbusch integration + 3 time scales

## Landscape




Minimal pion mass : $$m_{PS}(L) = \frac{3}{2f_0^2 L^3}\left(1 + \frac{2}{4\pi f_0^2 L^2}\,2.837\right)^{-1}$$

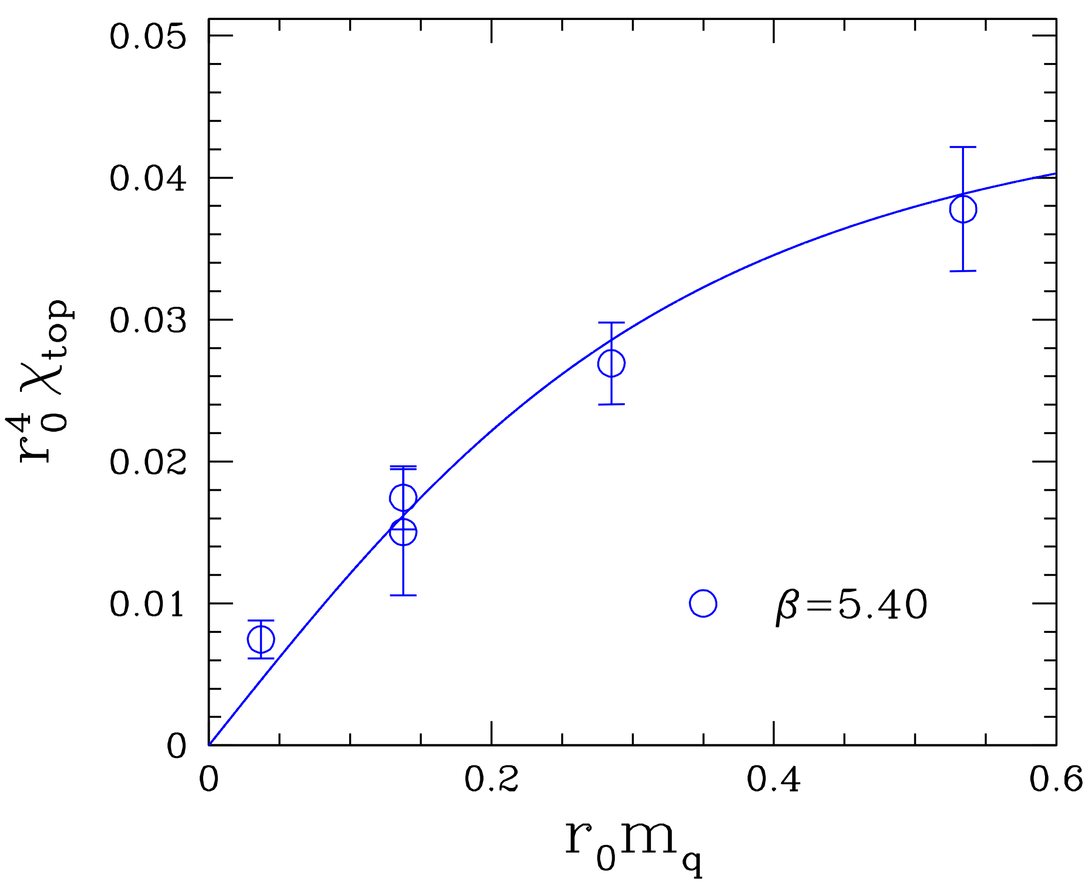
Leutwyler

Hasenfratz & Niedermayer

## Effect of Unquenching ?

$$\chi_{\rm top} \equiv \frac{\langle Q^2 \rangle}{V} = \frac{\Sigma\, m_q}{2}$$

Vector Ward Identity √

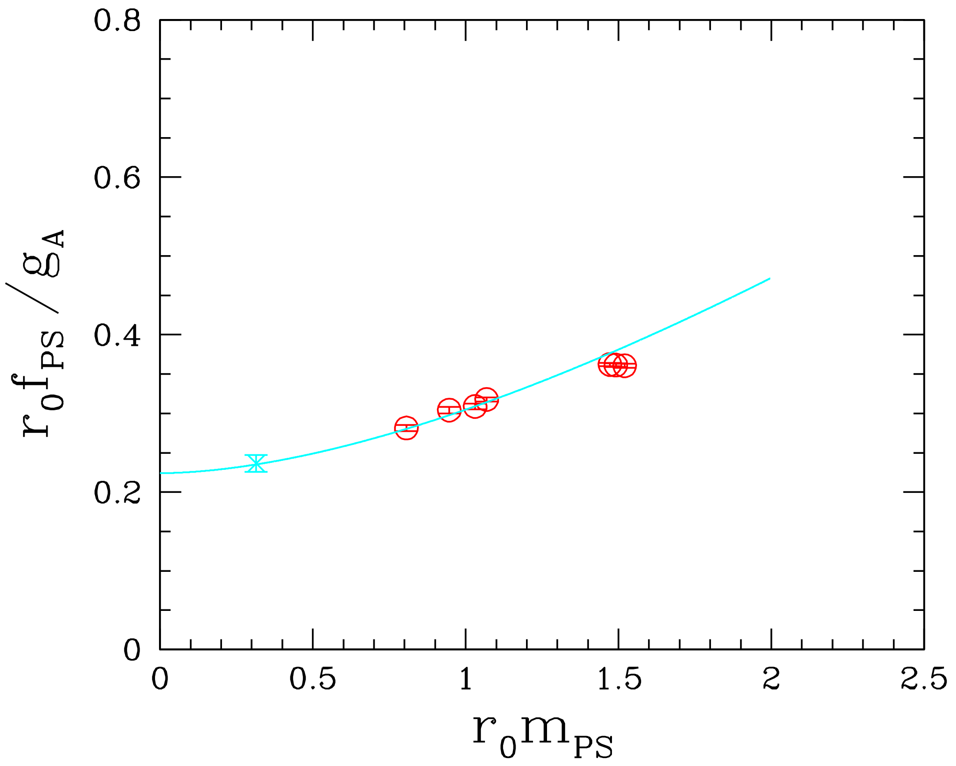



$$\left(\frac{1}{\chi_{\rm top}}\right)^2 = \left(\frac{2}{\Sigma\, m_q}\right)^2 + \left(\frac{1}{\chi_{\rm top}^\infty}\right)^2$$

Dürr

$\Sigma^{\overline{MS}}(2\ \text{GeV}) = [276(12)\ \text{MeV}]^3$

## Determination of Scale

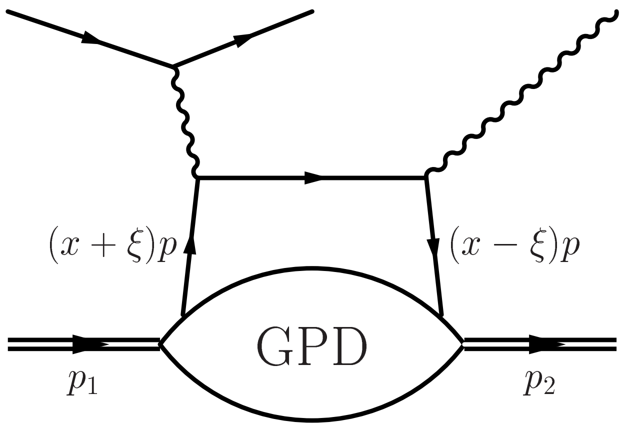



$Z_A$ cancels, FS effects & leading log's largely cancel

$r_0 = 0.45(1)$ fm

# Basics

## OPE




$p = \frac{1}{2}(p_1+p_2)$, $\Delta = p_2 - p_1$, $q = \frac{1}{2}(q_1+q_2)$

$\xi = 0$: Momentum transfer of the struck parton purely transverse, i.e. $\Delta = \boldsymbol{\Delta}_\perp$

$\Uparrow$

Of interest to us here only

$\Updownarrow$

## Matrix elements of local operators

$$\mathcal{O}^{q}_{\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{q}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n}q$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n} \bar{q}\gamma_{\sigma}\gamma_5\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n}q$$

$$\mathcal{O}^{Tq}_{\mu\nu\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n} \bar{q}\sigma_{\mu\nu}\gamma_5\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n}q$$

## Nucleon

$$\langle p_1, s|\, \mathcal{O}^q_{\{\mu_1\cdots\mu_n\}}\, |p_2, s\rangle = \bar{u}(p_1, s)\Big[A^q_n(\Delta^2)\ \gamma_{\{\mu_1}$$

$$+ B^q_n(\Delta^2)\ \frac{\mathrm{i}\Delta^\alpha}{2m_N}\sigma_{\alpha\{\mu_1}\Big]p_{\mu_2}\cdots p_{\mu_n\}}\, u(p_2, s)\ +\ \cdots$$

$$\langle p_1, s|\mathcal{O}^{5q}_{\{\mu\mu_1\cdots\mu_n\}}|p_2, s\rangle = \bar{u}(p_1, s)\Big[\tilde{A}^q_{n+1}(\Delta^2)\ \gamma_{\{\mu}\gamma_5 p_{\mu_1}\cdots p_{\mu_n\}}\Big]u(p_2, s)\ +\ \cdots$$

$$\langle p_1, s|\mathcal{O}^{Tq}_{\mu\{\nu\mu_1\cdots\mu_n\}}|p_2, s\rangle = \bar{u}(p_1, s)\Big[A^{Tq}_{n+1}(\Delta^2)\ \sigma_{\mu\{\nu}\gamma_5 - \tilde{A}^{Tq}_{n+1}(\Delta^2)\big(\frac{\Delta^2}{2m_N^2}\sigma_{\mu\{\nu} - \frac{\Delta_\mu\Delta_\alpha}{2m_N^2}\sigma_{\alpha\{\nu}\big)\gamma_5$$

$$+ \bar{B}^{Tq}_{n+1}(\Delta^2)\ \epsilon_{\alpha\beta\mu\{\nu}\frac{\Delta_\alpha\gamma_\beta}{2m_N}\Big]\ p_{\mu_1}\cdot\cdot p_{\mu_n\}}u(p_2, s)\ +\ \cdots$$

$$A_n^q(\Delta^2) = \int_0^1 dx\, x^{n-1} H^q(x, \Delta^2)$$

$$H^q(x, 0) = q(x)$$

$$B_n^q(\Delta^2) = \int_0^1 dx\, x^{n-1} E^q(x, \Delta^2)$$

$$\tilde{A}_n^q(\Delta^2) = \int_0^1 dx\, x^{n-1} \tilde{H}^q(x, \Delta^2)$$

$$\tilde{H}^q(x, 0) = \Delta q(x)$$

$$A_n^{Tq}(\Delta^2) = \int_0^1 dx\, x^{n-1} H^{Tq}(x, \Delta^2)$$

$$H^{Tq}(x, 0) = \delta q(x)$$

↑ GFFs

↑ GPDs

$$\boxed{\frac{1}{2}\left(A_2^q(0) + B_2^q(0)\right) = J^q}$$

Ji

$$A_1^q\ (\Delta^2) = F_1^q(\Delta^2)$$

$$B_1^q\ (\Delta^2) = F_2^q(\Delta^2)$$

$$\tilde{A}_1^q\ (\Delta^2) = g_A^q\ (\Delta^2)$$

$$A_1^{Tq}(\Delta^2) = g_T^q\ (\Delta^2)$$

$$\Delta^2 = t = -Q^2$$

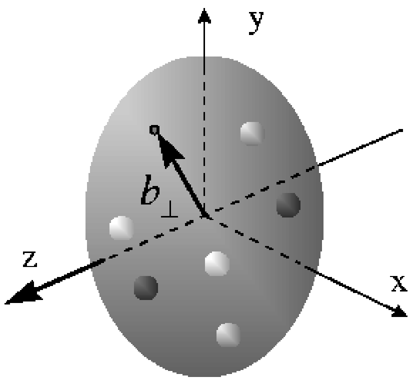
## Impact Parameter Space

Generically

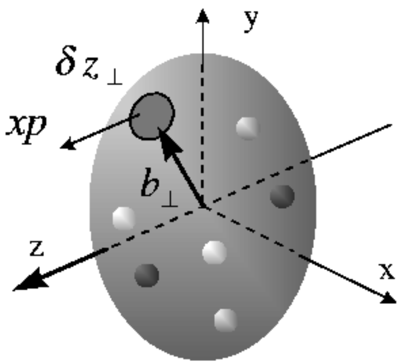
$$A_n^q(\mathbf{b}_\perp^2) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} \mathrm{e}^{\,\mathrm{i}\,\mathbf{b}_\perp \mathbf{\Delta}_\perp} A_n^q(\mathbf{\Delta}_\perp^2) \qquad \Longleftrightarrow \qquad \boxed{\langle p_+, s|\bar{q}(\mathbf{b}_\perp)\cdots q(\mathbf{b}_\perp)|p_+, s\rangle}$$

$$|p_+, s\rangle = \mathcal{N} \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} |p_+, \mathbf{p}_\perp, s\rangle$$

$$H^q(x, \mathbf{b}_\perp^2) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} \mathrm{e}^{\,\mathrm{i}\,\mathbf{b}_\perp \mathbf{\Delta}_\perp} H^q(x, \mathbf{\Delta}_\perp^2)$$




$F_1(\mathbf{b}_\perp^2) \equiv A_1(\mathbf{b}_\perp^2)$




$H(x, \mathbf{b}_\perp^2)$

Probability interpretation

Burkardt

$$H^q(x, \Delta^2) = \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}, \Delta^2\right) q(y)$$

Similarly for $\tilde{H}^q$ and $H^{T\,q}$

$$\underbrace{\int_0^1 dx\, x^n C(x, \Delta^2)} = \frac{A_{n+1}(\Delta^2)}{A_{n+1}(0)} = \frac{1}{(1-\Delta^2/M_n^2)^2}$$

By inverse Mellin transform

$$H^q(x, \mathbf{b}_\perp^2) = \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}, \mathbf{b}_\perp^2\right) q(y)$$

$$C(x, \mathbf{b}_\perp^2) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{i\,\mathbf{b}_\perp \boldsymbol{\Delta}_\perp} C(x, \boldsymbol{\Delta}_\perp^2)$$

# Nucleon Structure

$N_f = 2$ ‘Valence’ quark distributions

## Form Factors

Sachs form factors

$$F_1(\Delta^2) = A_1(\Delta^2)$$

$$F_2(\Delta^2) = B_1(\Delta^2)$$

$$G_e(\Delta^2) = F_1(\Delta^2) + \frac{\Delta^2}{4m_N^2} F_2(\Delta^2)$$

$$G_m(\Delta^2) = F_1(\Delta^2) + F_2(\Delta^2)$$

$$F_1(0) = e^N$$

$$F_2(0) = \mu^N - e^N = \kappa^N$$

$$G_e(0) = e^N$$

$$G_m(0) = \mu^N = 1 + \kappa^N$$

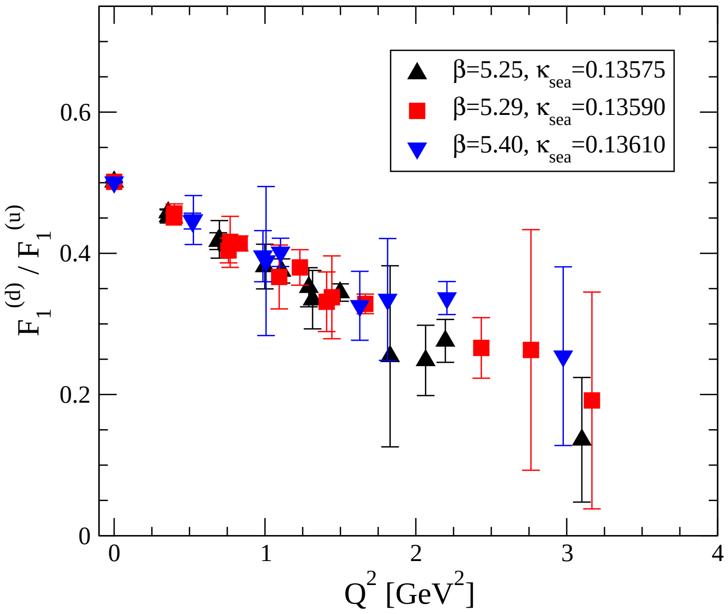
Benchmark calculation

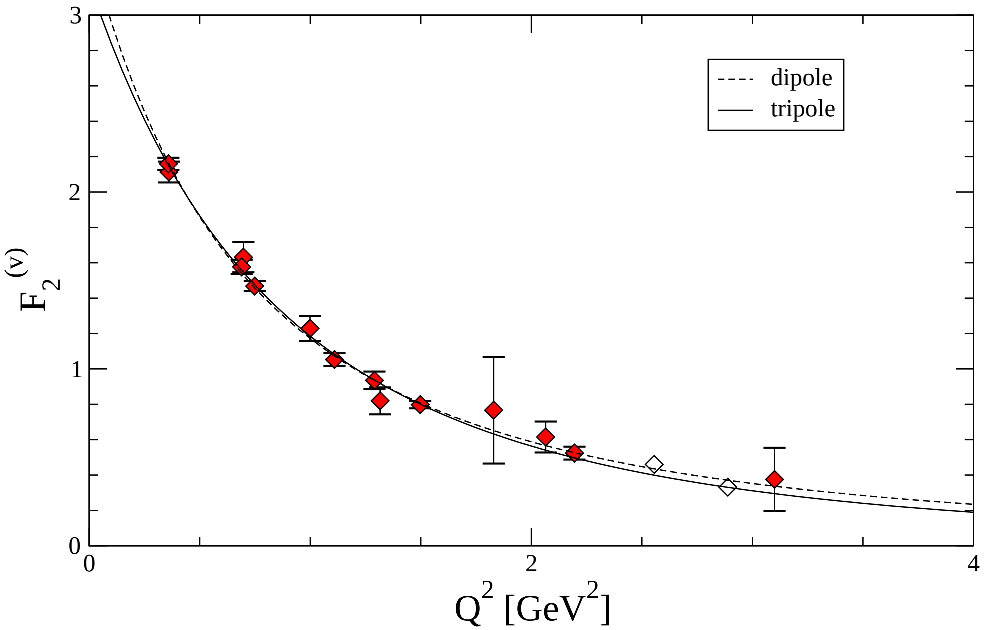
Expect (dimensional counting)

$$F_1(Q^2) \propto \frac{1}{(Q^2)^2}$$

$$F_2(Q^2) \propto \frac{1}{(Q^2)^3}$$

$$Q^2 = -\Delta^2$$




$$F^p = \frac{2}{3}F^u - \frac{1}{3}F^d$$

$$F^n = -\frac{1}{3}F^u + \frac{2}{3}F^d$$

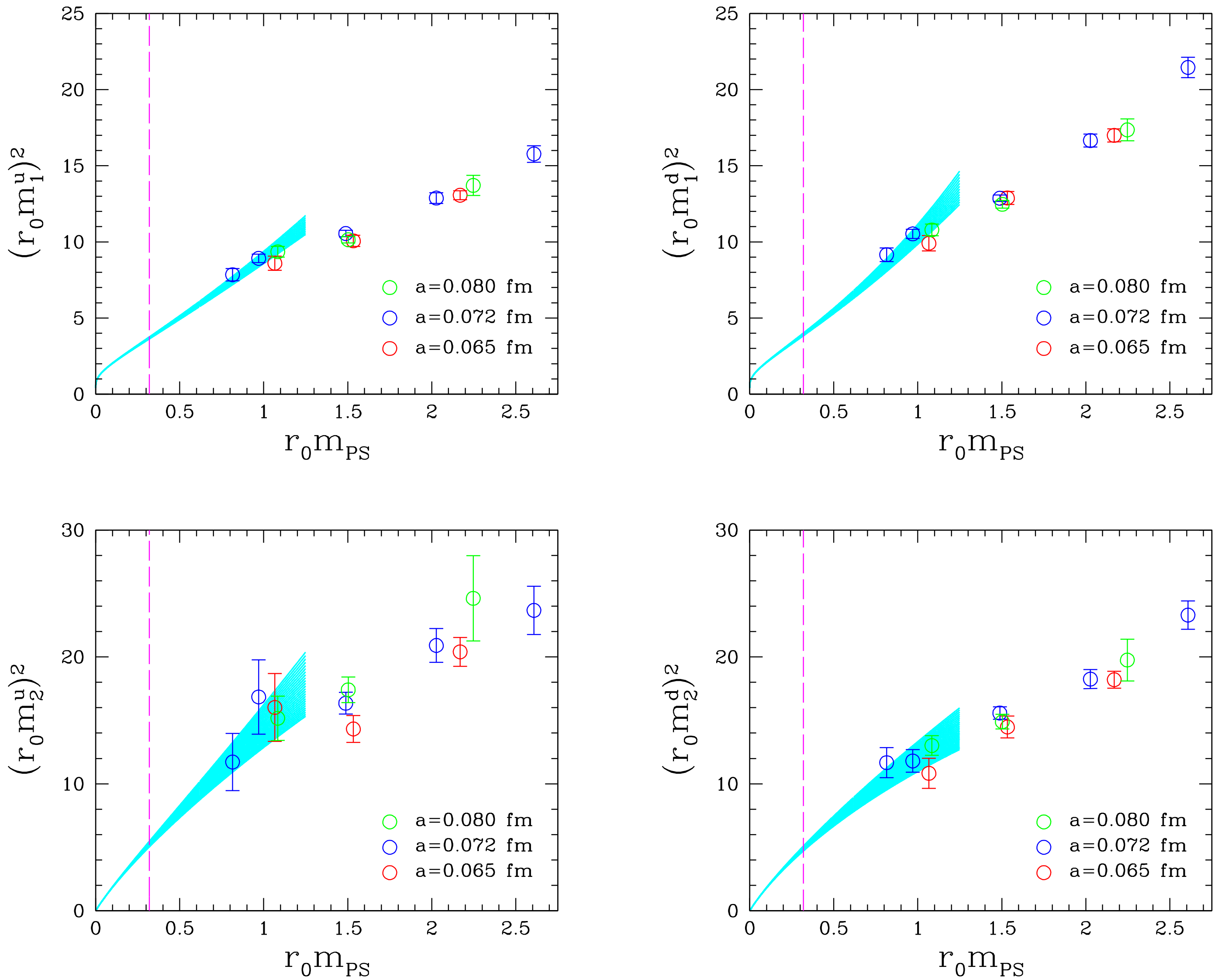
$$F^v = F^u - F^d$$

$$F(Q^2) = F(0)\left(1 + Q^2/m^2\right)^{-n}$$

$F_1^u$ $n = 2$

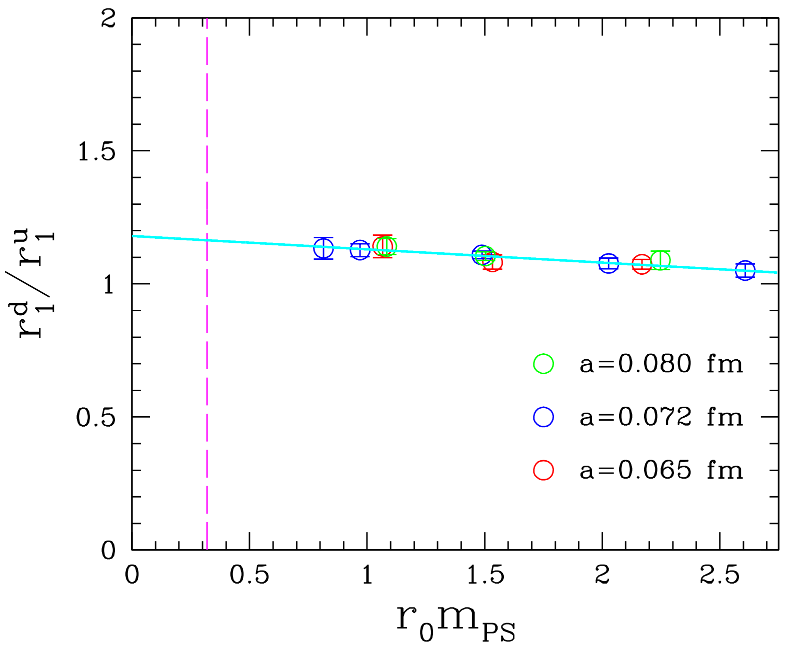
$\left.\begin{matrix} F_1^d \\ F_2^{u,d} \end{matrix}\right\}$ $n = 3$

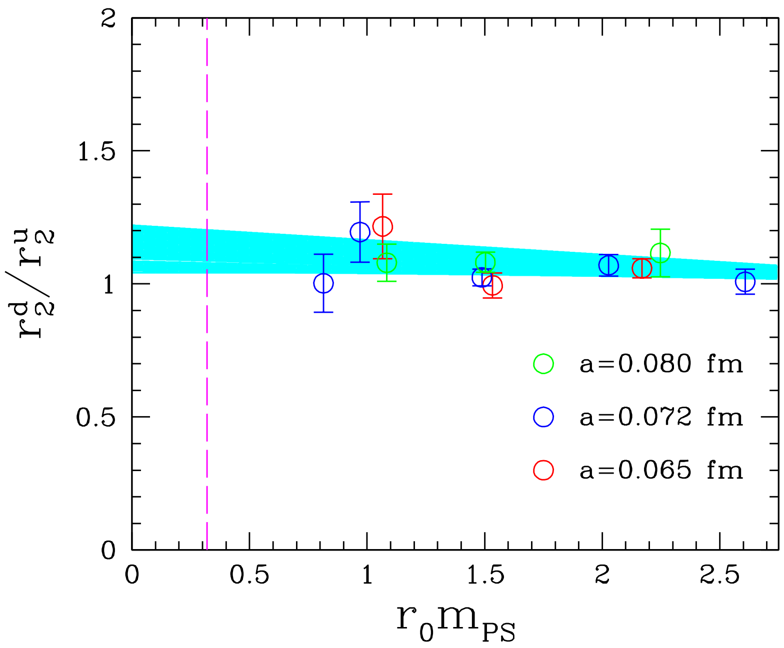
# Chiral extrapolation



$$F_i(Q^2) = F_i(0)\left(1 - \frac{1}{6} r_i^2 Q^2 + O(Q^4)\right)$$

$$r_i^2 = 6\, n / m_i^2$$




$$r^d_{1,2} > r^u_{1,2}$$

ChPT

$$r_1^2 = -\frac{1}{(4\pi F_\pi)^2}\left\{1 + 7g_A^2 + (10g_A^2 + 2)\log\left[\frac{m_\pi}{\lambda}\right]\right\} - \frac{12B_{10}^{(r)}(\lambda)}{(4\pi F_\pi)^2}$$
$$+ \frac{c_A^2}{54\pi^2 F_\pi^2}\left\{26 + 30\log\left[\frac{m_\pi}{\lambda}\right] + 30\frac{\Delta}{\sqrt{\Delta^2 - m_\pi^2}}\log R(m_\pi)\right\}$$

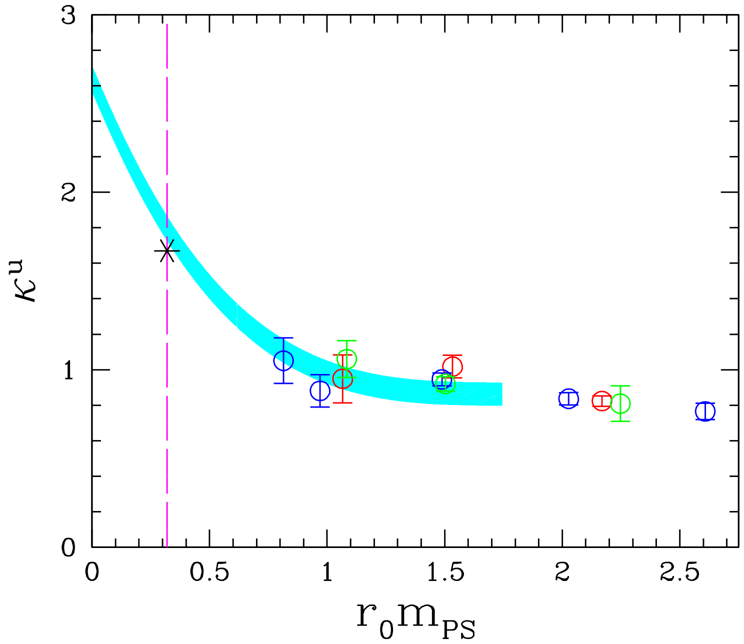
$$r_2^2 = \frac{g_A^2 m_N}{8F_\pi^2 \kappa\pi m_\pi} + \frac{c_A^2 m_N}{9F_\pi^2\kappa\pi^2\sqrt{\Delta^2 - m_\pi^2}}\log R(m_\pi) + \frac{24m_N}{\kappa}B_{c2}$$

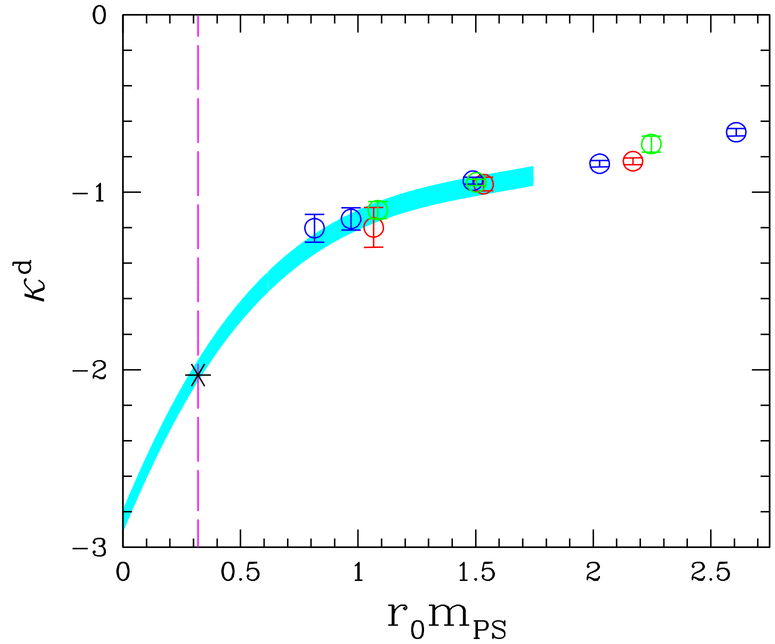
$$R(m) = \frac{\Delta}{m} + \sqrt{\frac{\Delta^2}{m^2} - 1}$$

Radii

| [fm$^2$] | Lattice | Experiment | ChPT |
|---|---|---|---|
| $(r_1^u)^2$ | 0.67(3) | | |
| $(r_1^d)^2$ | 0.93(4) | | |
| $(r_1^v)^2$ | 0.41(5) | 0.58 | 0.71 |
| $(r_2^u)^2$ | 0.69(3) | | |
| $(r_2^d)^2$ | 0.74(5) | | |
| $(r_2^v)^2$ | 0.72(6) | 0.80 | 0.60 |

Chiral extrapolation (ctd.)




$$\kappa^p = \frac{2}{3}\kappa^u - \frac{1}{3}\kappa^d$$

$$\kappa^n = -\frac{1}{3}\kappa^u + \frac{2}{3}\kappa^d$$

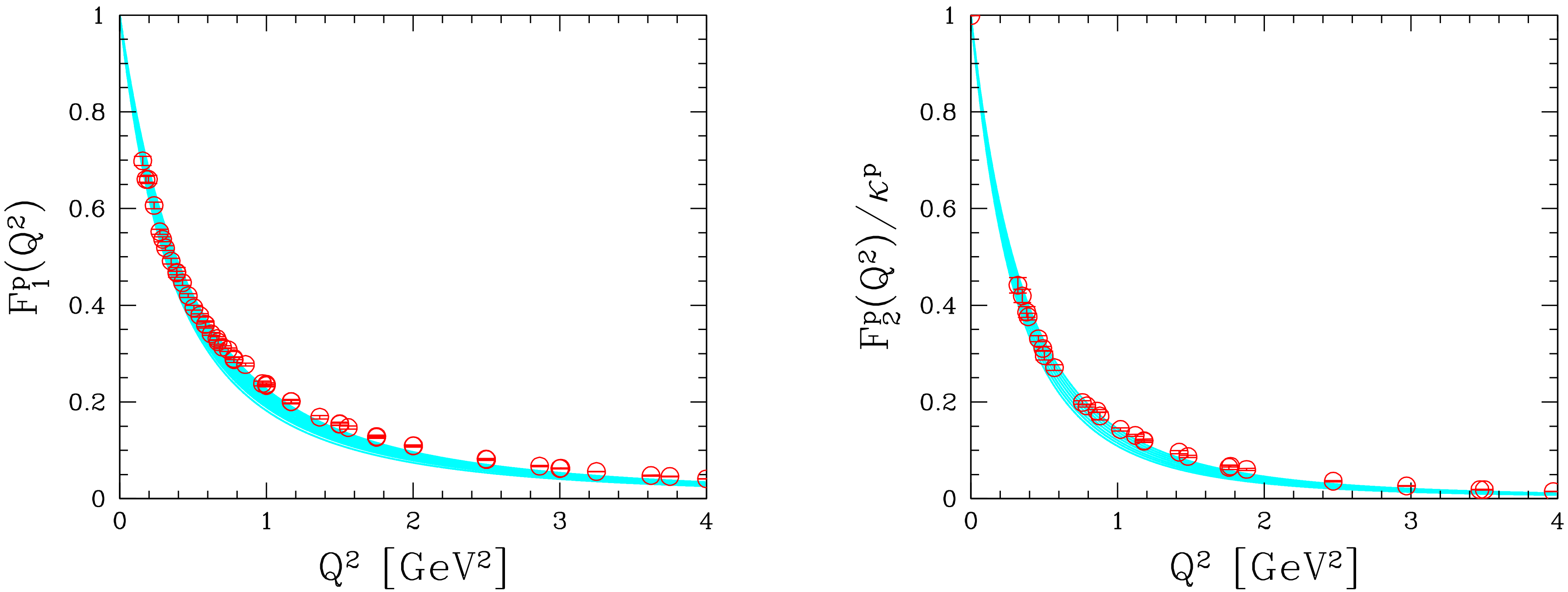
# ChPT

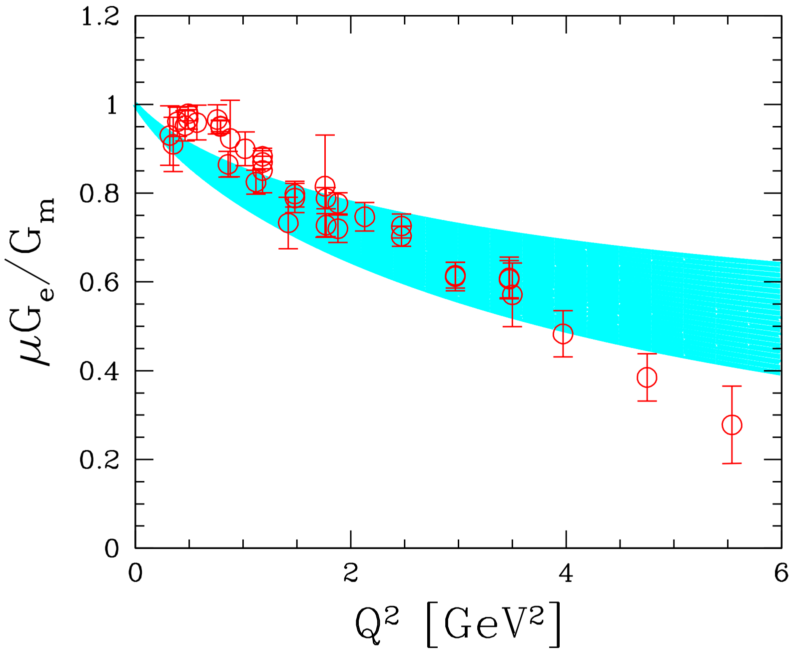
$$\kappa(m_\pi) = \kappa_v^0 - \frac{g_A^2\, m_\pi M_N}{4\pi F_\pi^2} + \frac{2c_A^2 \Delta M_N}{9\pi^2 F_\pi^2}\left\{\sqrt{1-\frac{m_\pi^2}{\Delta^2}}\log R(m_\pi) + \log\left[\frac{m_\pi}{2\Delta}\right]\right\}$$

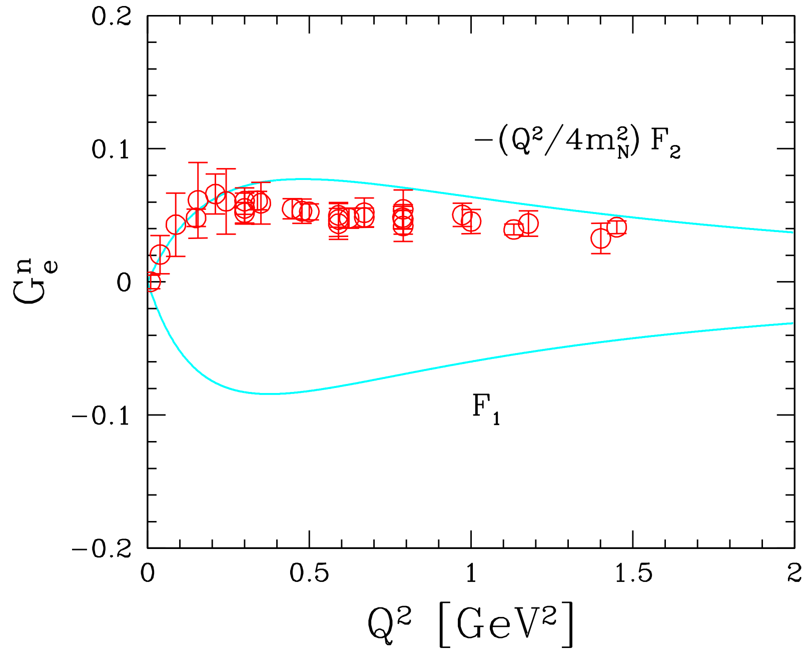
$$- 8E_1^{(r)}(\lambda)M_N m_\pi^2 + \frac{4c_A c_V g_A M_N m_\pi^2}{9\pi^2 F_\pi^2}\log\left[\frac{2\Delta}{\lambda}\right] + \frac{4c_A c_V g_A M_N m_\pi^3}{27\pi F_\pi^2 \Delta}$$

$$- \frac{8c_A c_V g_A \Delta^2 M_N}{27\pi^2 F_\pi^2}\left\{\left(1-\frac{m_\pi^2}{\Delta^2}\right)^{3/2}\log R(m_\pi) + \left(1-\frac{3m_\pi^2}{2\Delta^2}\right)\log\left[\frac{m_\pi}{2\Delta}\right]\right\}$$

# Finally






Very preliminary

# Spin Asymmetries

Transverse spin density

$\Downarrow$

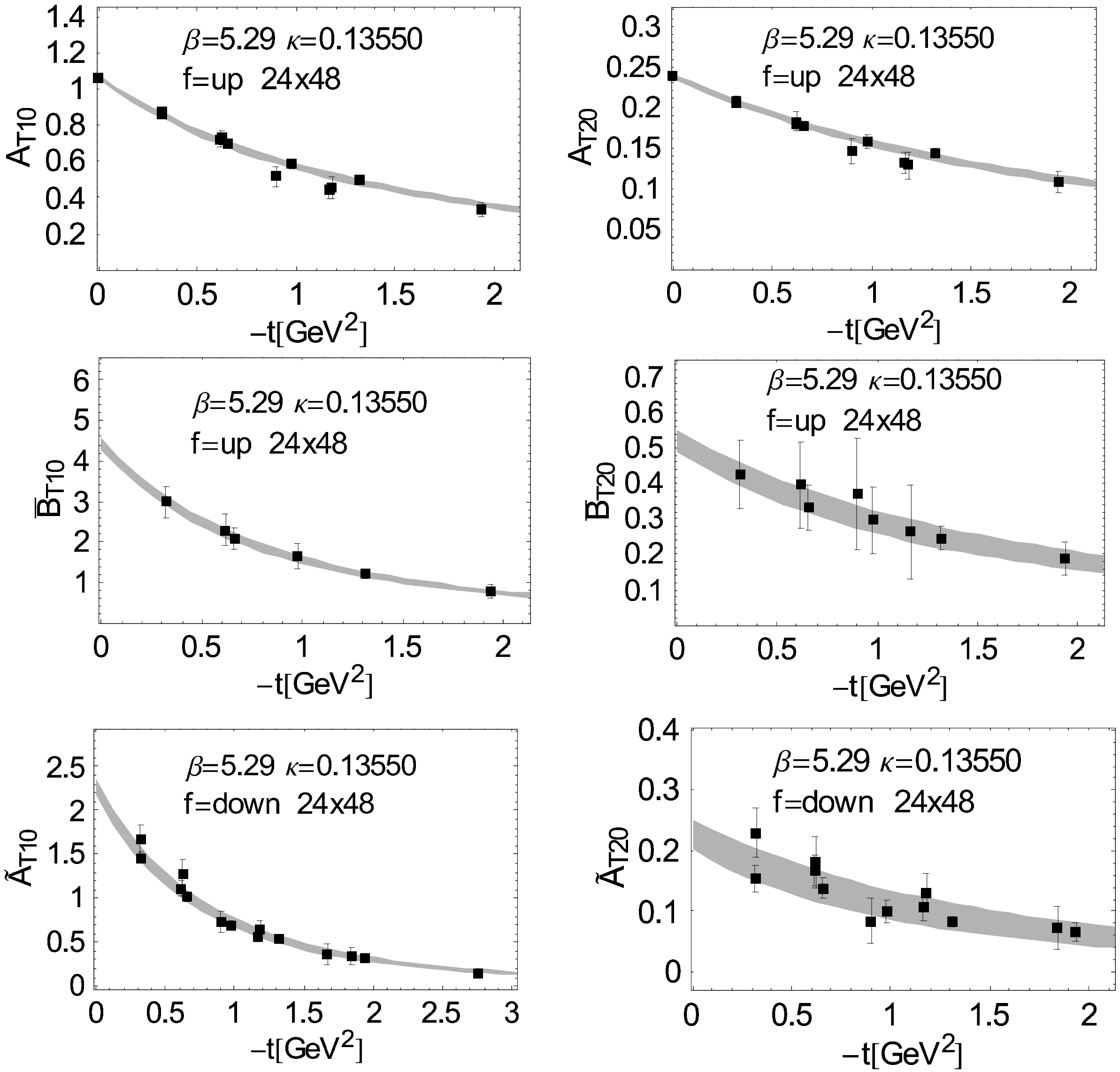
$\lambda_\perp$ quark spin
$s_\perp$ nucleon spin

$$\langle p_+, s_\perp | \bar{q}(\mathbf{b}_\perp) \left[ \gamma_+ - \lambda_{\perp i}\, \sigma_{+j}\, \gamma_5 \right] q(\mathbf{b}_\perp) | p_+, s_\perp \rangle = \Bigg\{ A_1^q(\mathbf{b}_\perp^2) + \lambda_{\perp i}\, s_{\perp i} \Big[ A_1^{Tq}(\mathbf{b}_\perp^2) - \frac{1}{4m_N^2} \Delta_{b_\perp} \tilde{A}_1^{Tq}(\mathbf{b}_\perp^2) \Big] - \frac{1}{m_N} \epsilon_{ij} b_{\perp j} \Big[ s_{\perp i} B_1^q(\mathbf{b}_\perp^2)' + \lambda_{\perp i}\, \bar{B}_1^{Tq}(\mathbf{b}_\perp^2)' \Big] + \frac{1}{m_N^2} \lambda_{\perp i} \left( 2 b_{\perp i}\, b_{\perp j} - \mathbf{b}_\perp^2 \delta_{ij} \right) s_{\perp j}\, \tilde{A}_1^{Tq}(\mathbf{b}_\perp^2)'' \Bigg\}$$

$\Uparrow$

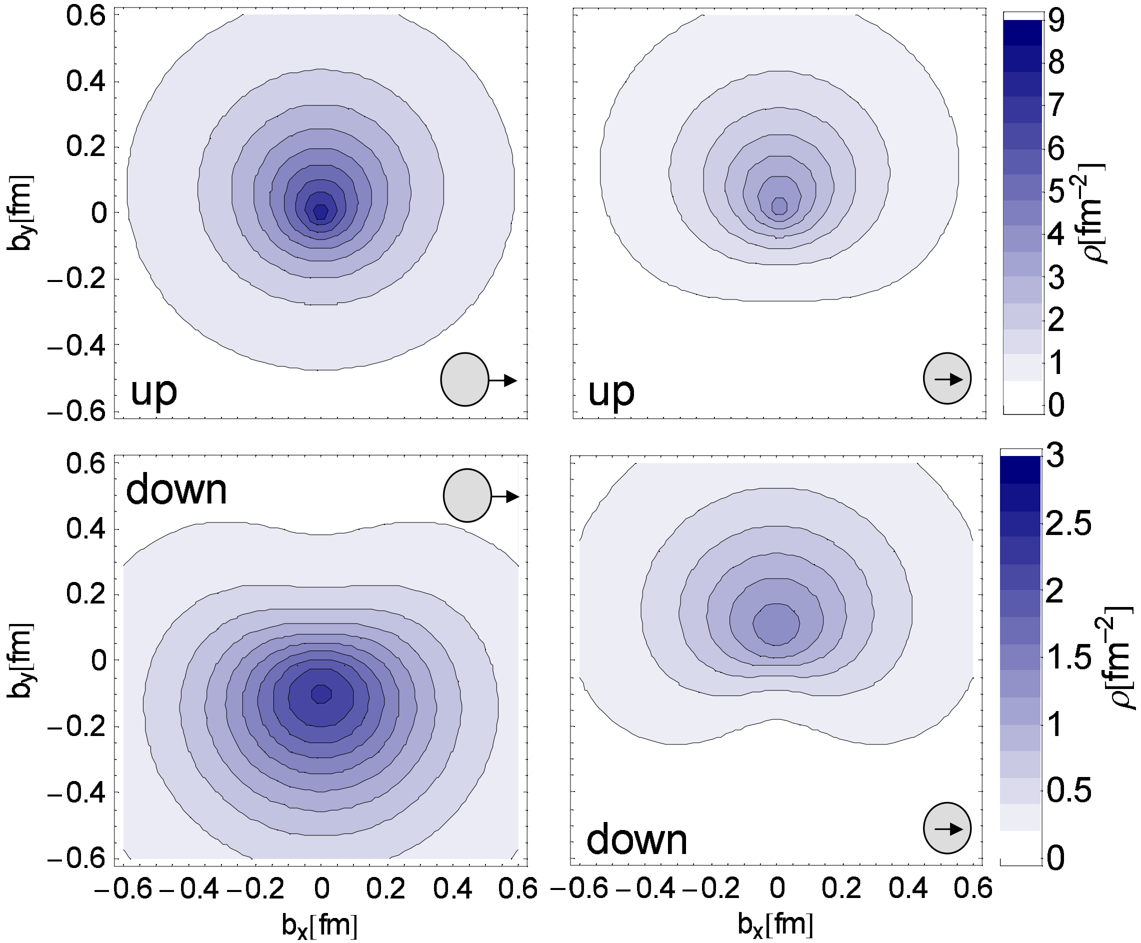
Quadrupole

Diehl & Hägler



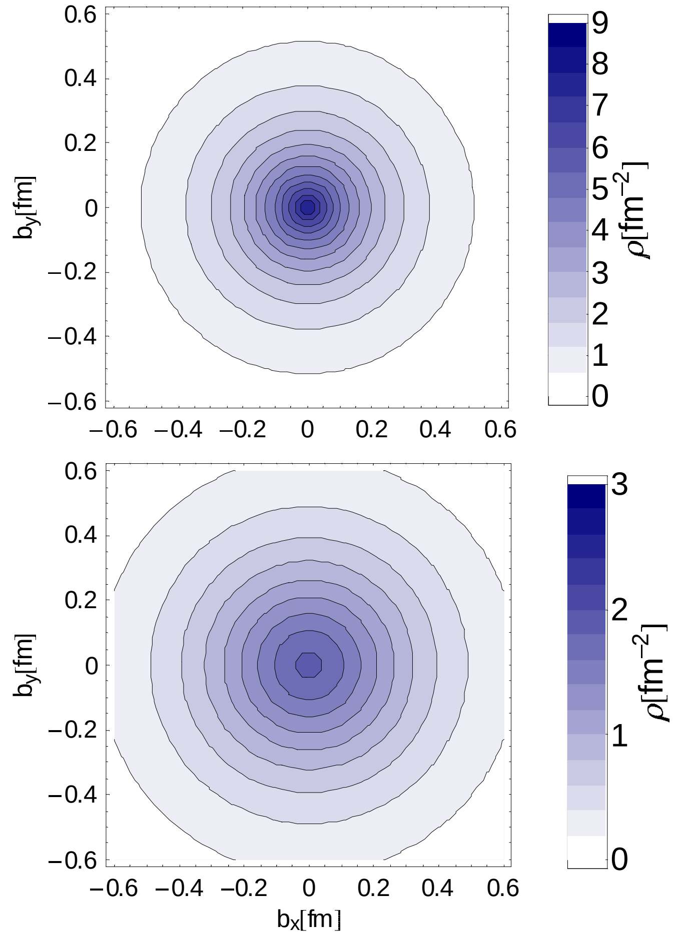

Dipole fit $\qquad$ $1^{st}$ moment

To be extrapolated to chiral limit




Sivers effect

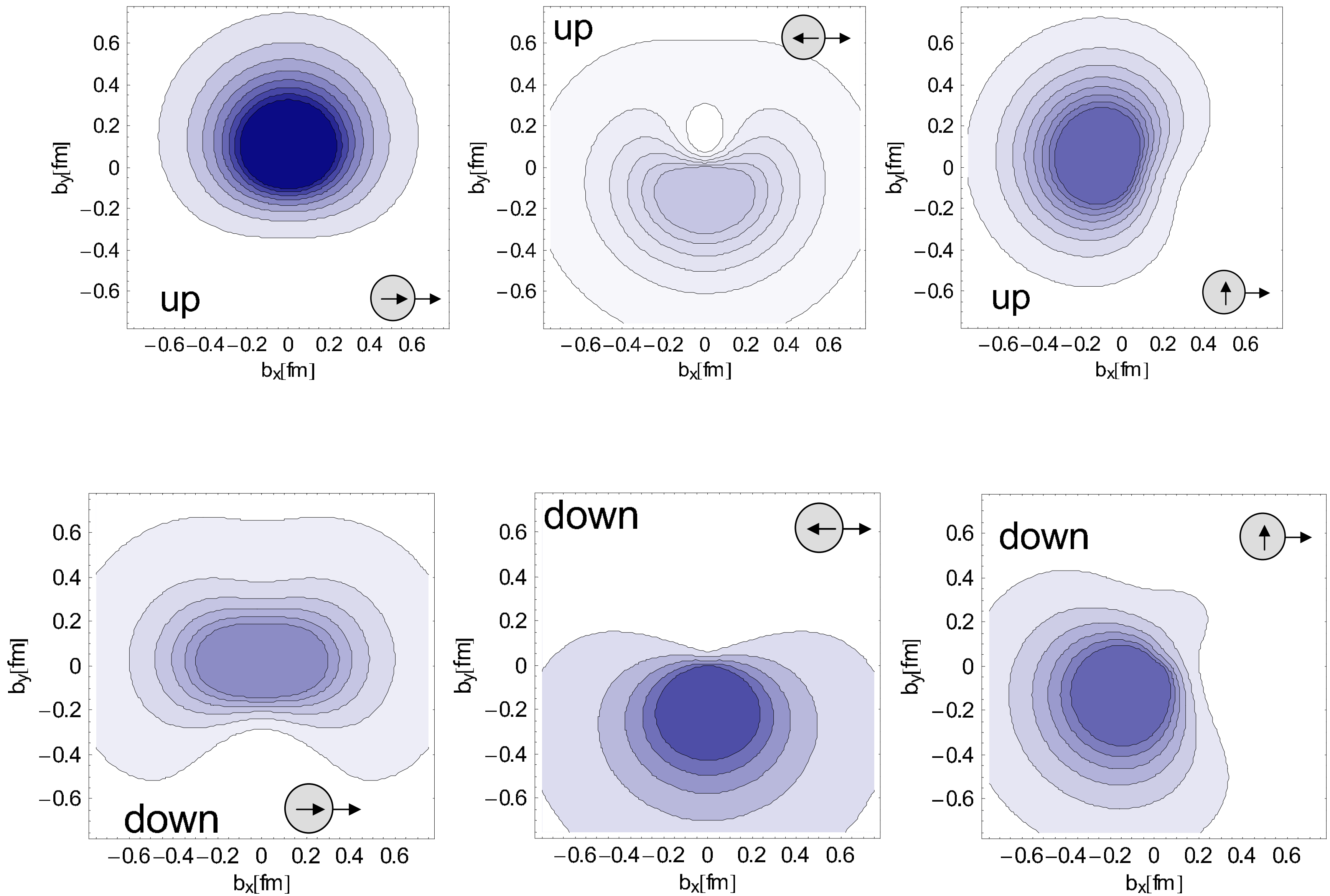
Boer-Mulders effect




Unpolarized

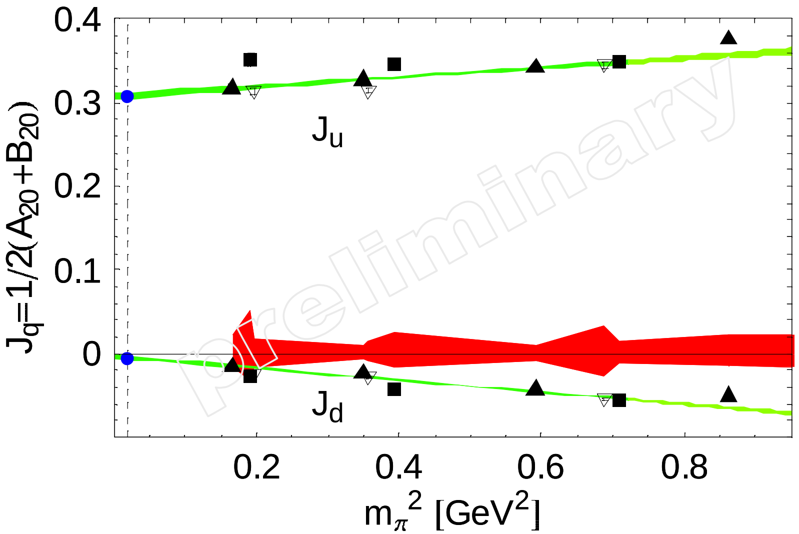
$$r_T^d > r_T^u$$

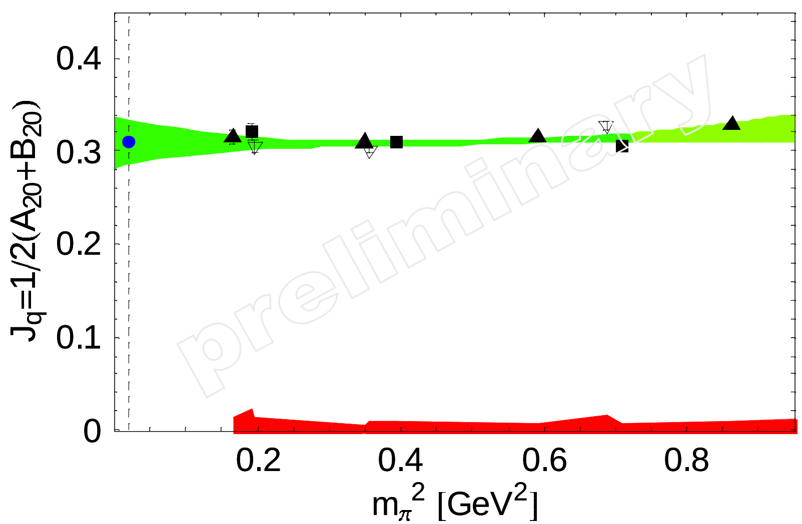
Nucleon and quarks both polarized



# (Orbital) Angular Momentum

$$J^q = \frac{1}{2}\left(A_2^q(0) + B_2^q(0)\right) \equiv \frac{1}{2}\Delta\Sigma^q + L^q$$

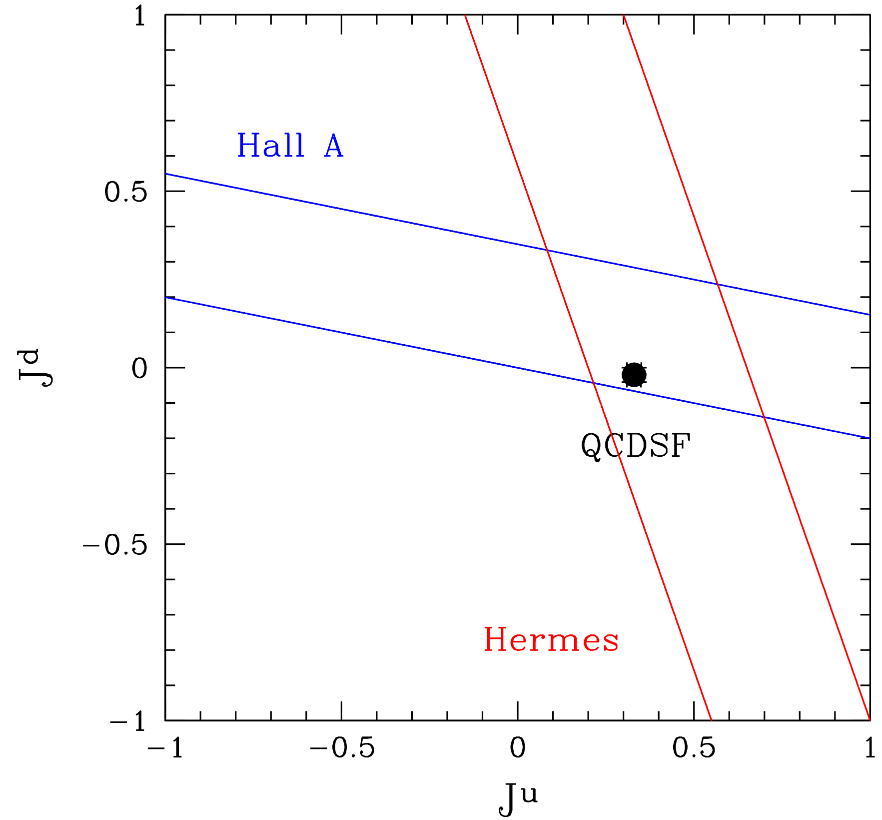



ChPT fit    Chen & Ji

$J^u = 0.33(2)$    $J^d = -0.02(2)$    $L^u = -0.13(4)$    $L^d = 0.15(4)$

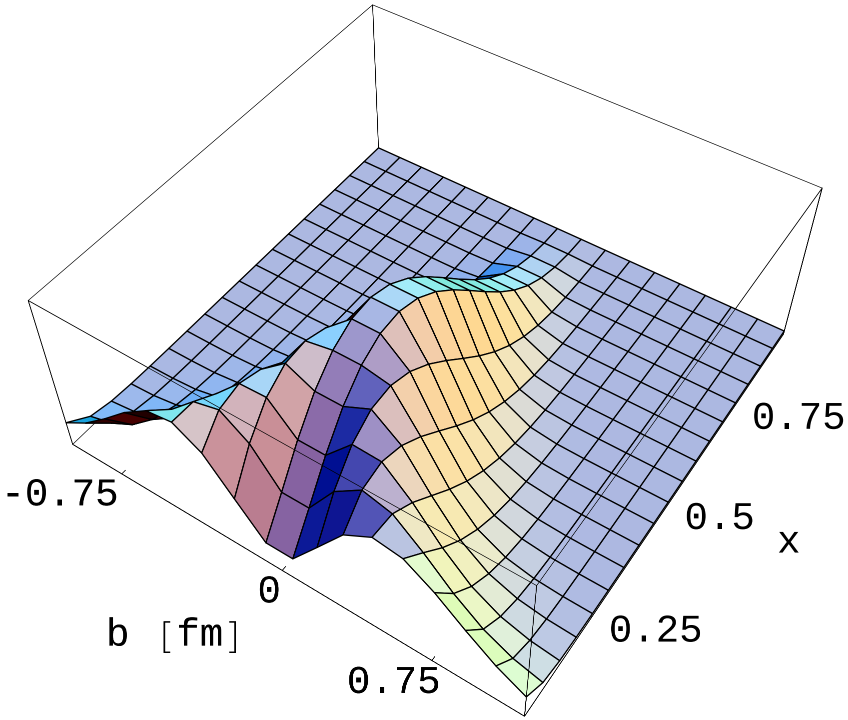
Comparison with experiment

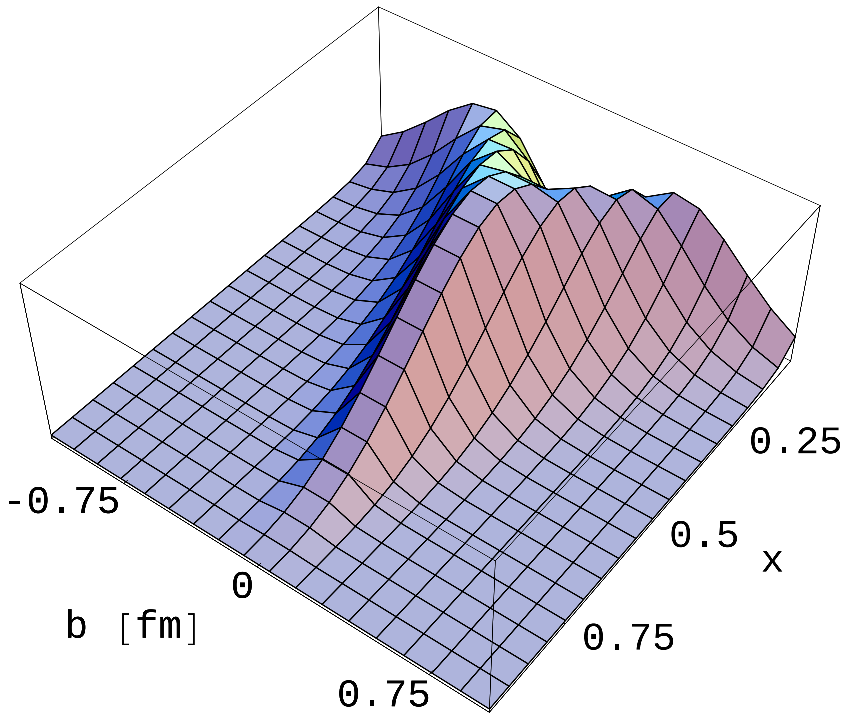


## GPDs

$H^u(x, \mathbf{b}_\perp^2)$

$Q^2 = 4$ GeV$^2$




$$\langle b^2 \rangle = \frac{7}{2} \alpha^{\vee 2} (1-x)^2 + \mathcal{O}\left((1-x)^3\right)$$

$$\langle r^2 \rangle = \frac{7}{2} \alpha^{\vee 2} + \mathcal{O}(1-x)$$

# Pion Structure

$N_f = 2$ 'Valence' quark distributions

# Form Factor

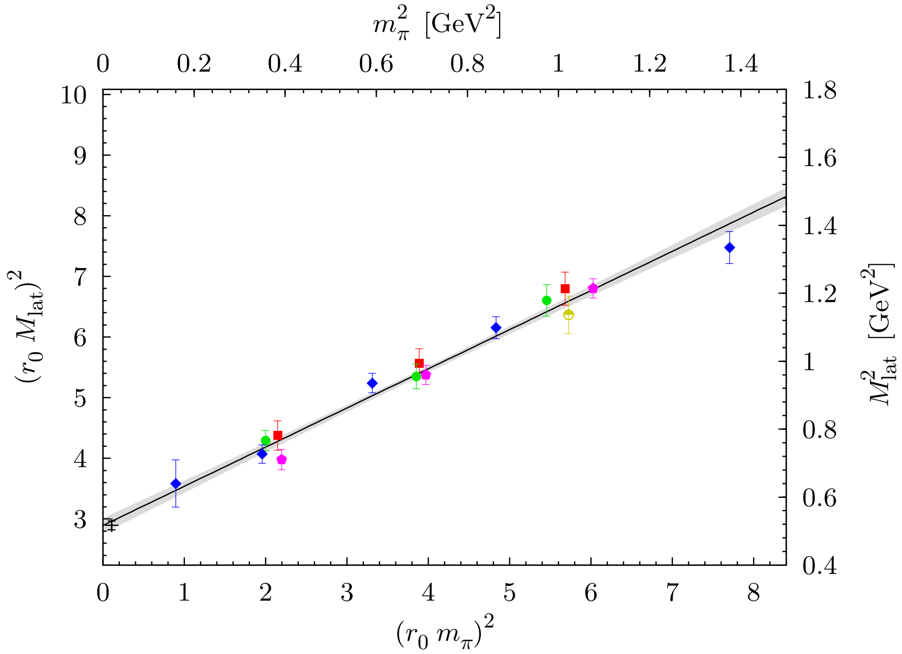
Expect (dimensional counting)

$$F^{\pi}(\Delta^2) = A_1(\Delta^2)$$

$$F^{\pi}(Q^2) \propto \frac{1}{Q^2} \qquad Q^2 = -\Delta^2$$

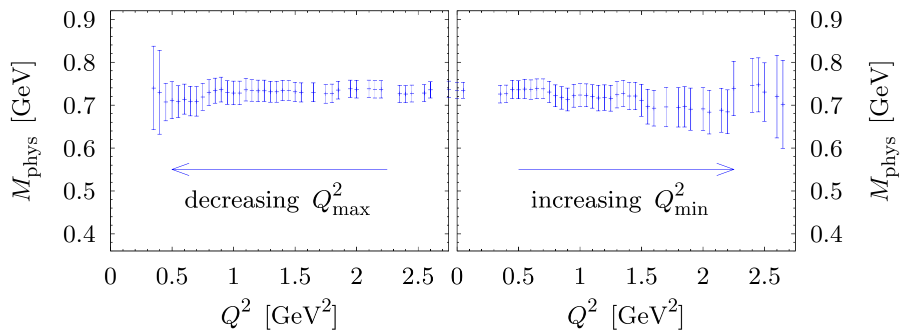
Ansatz

$$\boxed{F^{\pi}(Q^2) = 1/(1 + Q^2/m^2)^{-1}}$$




$$M_{lat} = m$$

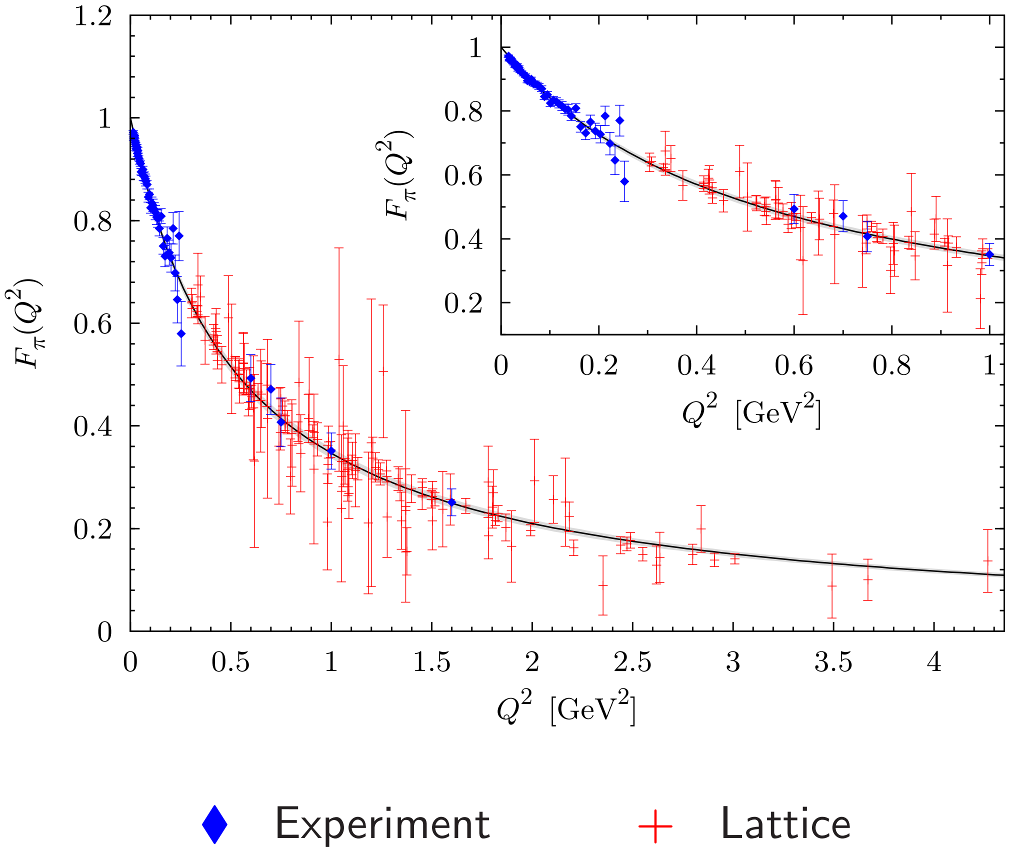
$$M_{phys} = M_{lat}\,(m_{\pi,phys})$$




$\Leftarrow$ Stability of monopole fit

$M_{phys} = 773(17)$ MeV $\quad Q^2 < 1$ GeV$^2$ $\qquad M_{phys} = 729(16)$ MeV all $Q^2$

VDM




⇑

Shifted by $(1 + Q^2/M_{\rm phys}^2)^{-1} - (1 + Q^2/M_{\rm lat}^2)^{-1}$

## Spin Asymmetries

Transverse spin density

$\Downarrow$

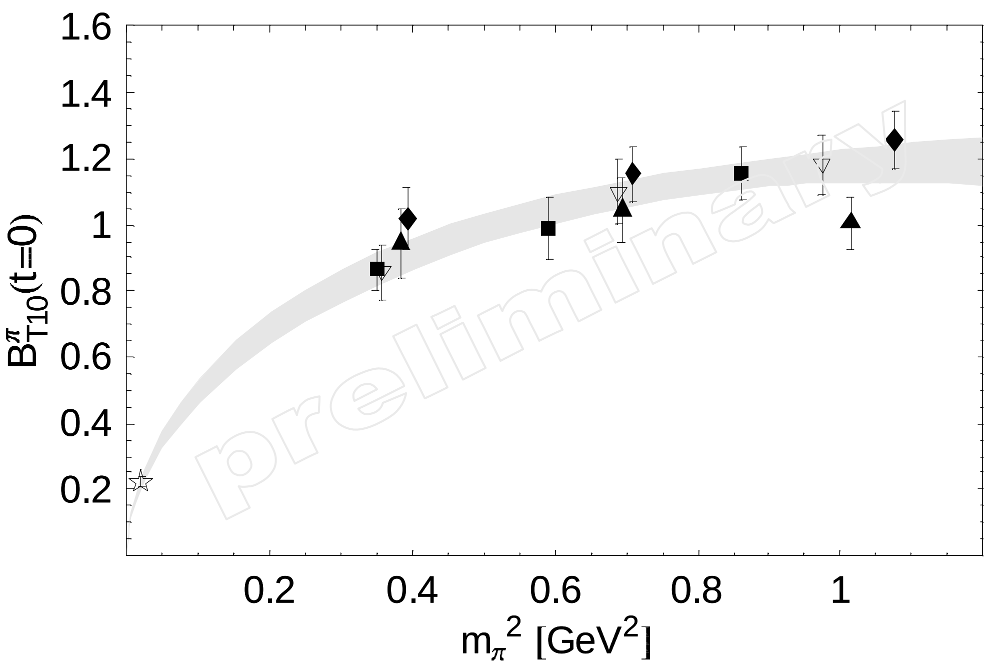
$\lambda_\perp$ quark spin

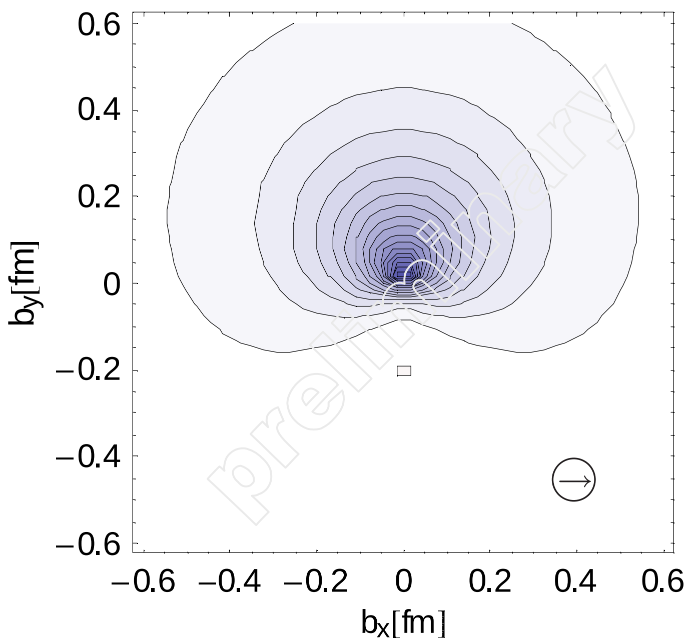
$$\langle p_+|\bar{q}(\mathbf{b}_\perp)\left[\gamma_+ - \boldsymbol{\lambda}_{\perp i}\,\boldsymbol{\sigma}_{+j}\,\gamma_5\right]q(\mathbf{b}_\perp)|p_+\rangle = \left\{A_1^q(\mathbf{b}_\perp^2) - \frac{1}{m_\pi}\epsilon_{ij}b_{\perp j}\boldsymbol{\lambda}_{\perp i}\,\bar{B}_1^{Tq}(\mathbf{b}_\perp^2)'\right\}$$

$\Uparrow$ Monopole

$\Uparrow$ Dipole

Hägler et al.




Boer-Mulders effect

# Conclusions & Outlook

- Simulations at small pion masses $m_\pi$ with Wilson-type fermions feasible now

- Improvement of algorithms
- Increase of computing power

- Extrapolation to chiral limit and infinite volume greatly improved

FS corrections surprisingly well described by ChPT

- Current simulations done at $m_\pi = O(300)$ MeV

- 2007/8: $m_\pi \longrightarrow 250$ MeV
- 2008/9: $m_\pi \longrightarrow 200$ MeV

On spatial volumes $\gtrsim (3\,\text{fm})^3$

$\Downarrow$

Resolution of pion cloud

- Challenge: Evaluation of disconnected diagrams