Form factors for the nucleon-Δ system

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Workshop: 'Hadron physics on the lattice'

Outline

Motivation

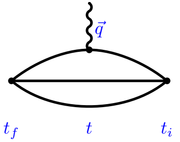
- Form factors provide crucial information about hadrons
  - size
  - magnetization
  - deformation
- Many form factors accessible experimentally
- Phenomenological models

Lattice QCD provides a tool to calculate them from first principles

Matrix elements

We are interested in QCD matrix elements

$$\langle h'(p',s')|X|h(p,s)\rangle$$



where

- $h'(p',s')$, $h(p,s)$ are hadron states with initial (final) momentum $p(p')$ and spin $s(s')$
  - nucleon
  - Δ-baryon
- X is a current or density
  - Electromagnetic current $V_\mu(x) = \frac{2}{3}\bar{u}(x)\gamma_\mu u(x) - \frac{1}{3}\bar{d}(x)\gamma_\mu d(x)$
  - Axial current $A^a_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
  - Pseudoscalar density $P^a(x) = \bar{\psi}(x)\gamma_5\frac{\tau^a}{2}\psi(x)$

Electromagnetic nucleon form factors

The electromagnetic matrix element of the nucleon can be expressed in terms of two form factors.

$$
\begin{aligned}
\langle N(p',s')|V_\mu(0)|N(p,s)\rangle &= \sqrt{\frac{m_N^2}{E_{N(\vec{p}')}E_{N(\vec{p})}}}\bar{u}(p',s')\mathcal{O}_\mu u(p,s) \\
\mathcal{O}_\mu &= \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)
\end{aligned}
$$

F_1, F_2 are the Dirac form factors.
$q = p' - p$ is the momentum transfer

$$
\begin{aligned}
G_E(q^2) &= F_1(q^2) + \frac{q^2}{(2m_N)^2}F_2(q^2) \\
G_M(q^2) &= F_1(q^2) + F_2(q^2)
\end{aligned}
$$

G_E, G_M are the electric and magnetic Sachs form factors.

Axial nucleon form factors

The axial current matrix element of the nucleon can be expressed in terms of the form factors G_A and G_p.

$$\begin{aligned}\langle N(p',s')|A_\mu^3(0)|N(p,s)\rangle &= i\sqrt{\frac{m_N^2}{E_{N(\vec{p}')}E_{N(\vec{p})}}}\bar{u}(p',s')\mathcal{O}_\mu u(p,s) \\ \mathcal{O}_\mu &= \left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu}{2m_N}G_p(q^2)\right]\frac{\tau^3}{2}\end{aligned}$$

Nucleon pseudoscalar matrix element

The pseudoscalar matrix element defines the form factor $G_{\pi NN}$

$$\langle N(p',s')|P^3(0)|N(p,s)\rangle = \frac{i}{2m_q}\sqrt{\frac{m_N^2}{E_{N(\vec{p}')}E_{N(\vec{p})}}}\bar{u}(p',s')\mathcal{O}_\mu u(p,s)$$

$$\mathcal{O}_\mu = \gamma_5 \frac{f_\pi m_\pi^2}{m_\pi^2 - q^2} G_{\pi NN}(q^2)$$

- m_q renormalized quark mass
- m_π pion mass
- f_π pion decay constant

The $N \longrightarrow \Delta$ transition

When one of the hadrons is a spin-$\frac{3}{2}$ particle

$$\langle \Delta(p',s')|X|N(p,s)\rangle = i\sqrt{\frac{m_\Delta m_N}{E_{\Delta(\vec{p}')}E_{N(\vec{p})}}}\bar{u}_\tau(p',s')\mathcal{O}^{\tau(\mu)}u(p,s)$$

$u_\tau(p,s)$ is a Schwinger-Rarita spinor

- vector-spinor
- each vector component satisfies the Dirac equation
- in addition auxiliary conditions
  - $\gamma_\mu u^\mu(p,s) = 0$
  - $p_\mu u^\mu(p,s) = 0$

Form factors in $\mathcal{O}^{\tau(\mu)}$:

- Axial current matrix element: C_3^A, C_4^A, C_5^A, C_6^A
  [L.S. Adler, Ann. Phys. 50, 189 (1968)]
  Dominant: C_5^A, C_6^A correspond to G_A, G_p
- Pseudoscalar matrix element: $G_{\pi N\Delta}$

Electromagnetic form factors of the Δ

The Electromagnetic matrix element can be decomposed in terms of *four* independent vertex-function coefficients a_1, a_2, c_1, c_2

$$\langle \Delta^+(p_f, s_f) | V^\mu | \Delta^+(p_i, s_i) \rangle = \sqrt{\frac{m_\Delta^2}{E_{\Delta(\vec{p}_f)} E_{\Delta(\vec{p}_i)}}} \, \bar{u}_\sigma(p_f, s_f) \, \mathcal{O}^{\sigma\mu\tau} \, u_\tau(p_i, s_i)$$

with (Euclidean notation)

$$\begin{aligned} \mathcal{O}^{\sigma\mu\tau} = & -\delta_{\sigma\tau} \left[a_1 \gamma^\mu - i \frac{a_2}{2m_\Delta} P^\mu \right] \\ & + \frac{q^\sigma q^\tau}{4m_\Delta^2} \left[c_1 \gamma^\mu - i \frac{c_2}{2m_\Delta} P^\mu \right] \end{aligned}$$

Matrix element can also be expressed in terms of multipole form factors G_{e0}, G_{e2}, G_{m1}, G_{m3}. The linear relation between the two formulations is known [Leinweber, Nozawa Phys. Rev. D42, 3567 (1990)]

Lattice techniques: Interpolating fields

We need to excite states $\chi|\Omega\rangle$ that have an overlap with the desired baryon ground states

$$\begin{aligned} \langle\Omega|\chi^N(0)|N(p,s)\rangle &= Z^N u(p,s) \\ \langle\Omega|\chi^\Delta_\sigma(0)|\Delta(p,s)\rangle &= Z^\Delta u_\sigma(p,s) \end{aligned}$$

proton:

$$\chi^P_\alpha(x) = \epsilon^{abc}(\mathbf{u}^{a\top}(x)\mathcal{C}\gamma_5\mathbf{d}^b(x))\mathbf{u}^c_\alpha(x)$$

Δ^+ baryon:

$$\begin{aligned} \chi^{\Delta^+}_{\sigma\alpha}(x) &= \frac{1}{\sqrt{3}}\epsilon^{abc}\Big[2(\mathbf{u}^{a\top}(x)\mathcal{C}\gamma_\sigma\mathbf{d}^b(x))\mathbf{u}^c_\alpha(x) \\ &\qquad +(\mathbf{u}^{a\top}(x)\mathcal{C}\gamma_\sigma\mathbf{u}^b(x))\mathbf{d}^c_\alpha(x)\Big] \end{aligned}$$

Lattice techniques: Smearing

Gauge invariant Gaussian smearing:

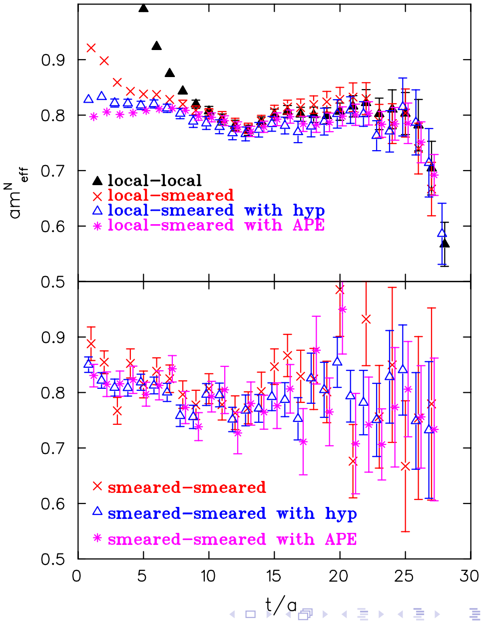
$$\mathbf{d}_\beta(t,\vec{x}) = \sum_{\vec{x}\vec{y}} [\mathbb{1} + \alpha H(\vec{x},\vec{y})]^n \, d_\beta(t,\vec{y})$$

$$H(\vec{x},\vec{y};U(t)) = \sum_{\mu=1}^{3} \left(U_\mu(\vec{x},t)\delta_{\vec{x},\vec{y}-\hat{\mu}} + U_\mu^\dagger(\vec{x}-\hat{\mu},t)\delta_{\vec{x},\vec{y}+\hat{\mu}} \right)$$

- Better overlap with the baryon ground-state
  $\longrightarrow$ Ground state dominance after only 2-3 time slices
- But: increased statistical noise
  $\longrightarrow$ apply HYP or APE smearing to the gauge-field entering $H(\vec{x},\vec{y})$

Effect of smearing

- Nucleon effective mass
- Smearing is crucial for the calculation of form factors



Two-point and three-point functions

Measure two-point and three-point functions (here: EM $\Delta \to \Delta$)

$$\Gamma_{\sigma\tau}(T^\nu, \vec{p}, t_f - t_i) = \int d^3x_f\, e^{i\vec{x}_f\cdot\vec{p}} T^\nu_{\alpha'\alpha} \langle \chi_{\sigma\alpha}(t_f, \vec{x}_f) \bar{\chi}_{\tau\alpha'}(t_i, \vec{x}_i) \rangle$$

$$\Gamma_{\sigma\ \tau}^{\ \mu}(T^\nu, \vec{q}, t) = \int d^3x \int d^3x_f\, e^{i\vec{x}_f\cdot\vec{p}_f - i\vec{x}\cdot\vec{q}} T^\nu_{\alpha'\alpha} \langle \chi_{\sigma\alpha}(t_f, \vec{x}_f) V^\mu(t, \vec{x}) \bar{\chi}_{\tau\alpha'}(t_i, \vec{x}_i) \rangle$$

we use $T^k = \frac{1}{2}\begin{pmatrix} \sigma^{(k)} & 0 \\ 0 & 0 \end{pmatrix}$ and $T^4 = \frac{1}{2}\begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix}$

Numerical evaluation of correlation functions

- Starting point: Euclidean pathintegral formulation of QCD

$$\frac{1}{Z}\int DU\, D\psi\, D\bar{\psi}\; \mathcal{O}[U,\bar{\psi},\psi]\; e^{-\bar{\psi}D\psi}\; e^{-S_G}$$

- Integrate out the fermionic fields

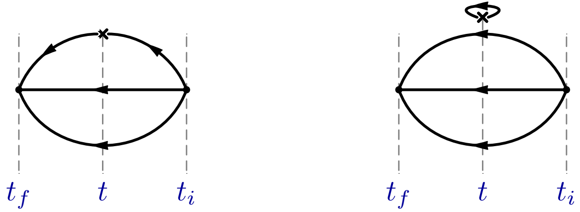
$$\frac{1}{Z}\int DU\; \mathcal{O}[U,D^{-1}]\, \det[D]\;\; e^{-S_G}$$

- Estimate integrals over gauge-field by Monte-Carlo methods
  - quenched approximation: $\det[D] \rightarrow 1$

Can't calculate the full inverse ${D^{-1}}^{ab}_{\alpha\beta}(x,y)$
With fixed y,b,β one can obtain D^{-1} for all x,a,α by solving the linear system
$D^{a'a}_{\alpha'\alpha}(x',x){D^{-1}}^{ab}_{\alpha\beta}(x,y) = \delta_{a',b}\delta_{\alpha',\beta}\delta_{x',y}$
$\Rightarrow$ 12 "inversions" for all Dirac and color components

Disconnected diagrams

Wick contractions for 3-point function: two different types of contributions



disconnected diagram contains factor $\sum_{\vec{x}} \mathrm{tr}^{\mathrm{s,f}}[D^{-1}(x,x)\Gamma]$
$\rightarrow$ vanishes for

- $\Gamma = \gamma_\mu \gamma_5 \tau^a$ axial current
- $\Gamma = \gamma_5 \tau^a$ pseudoscalar density
- $\Gamma = \gamma_\mu \tau^3$ isovector current

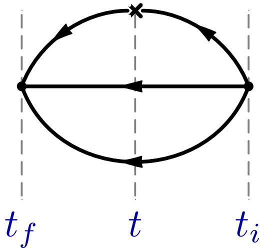
We calculate V_μ^{iv} *or* $V_\mu^{\mathrm{connected}}$

note: we use the (symmetrized) lattice conserved current $\Rightarrow Z_V = 1$.

Sequential inversion through the sink

Our setup

- Source at $t_i = 0, \vec{x} = 0$
- Sink at $t = t_f, \vec{p}_f = 0$
- Operator X at $t, \vec{q} = -\vec{p}_i$
- No new inversions for different operator $X(t, \vec{q})$
- But: new inversions necessary for different interpolating fields



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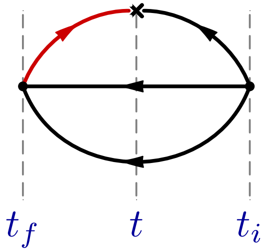
$$D^{-1\dagger} = \gamma_5 D^{-1} \gamma_5$$



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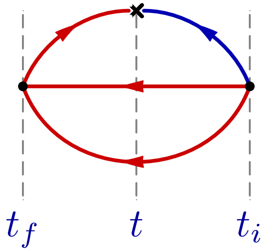


For large Euclidean time separations:

Two-point function (here $\Delta \to \Delta$ electromagnetic):

$$\begin{aligned}
\Gamma_{\sigma\tau} &\to e^{-E_{\Delta(\vec{p})}(t_f - t_i)} \, |Z|^2 \, c(\vec{p}) \, \mathrm{tr}[T \, \Lambda_{\sigma\tau}] \\
\Lambda_{\sigma\tau} &= \sum_s u_\sigma(p,s)\bar{u}_\tau(p,s) \\
&= -\frac{-i\not{p} + m_\Delta}{2m_\Delta}\left[\delta_{\sigma\tau} - \frac{\gamma_\sigma\gamma_\tau}{3} + \frac{2p_\sigma p_\tau}{3m_\Delta^2} - i\frac{p_\sigma\gamma_\tau - p_\tau\gamma_\sigma}{3m_\Delta}\right]
\end{aligned}$$

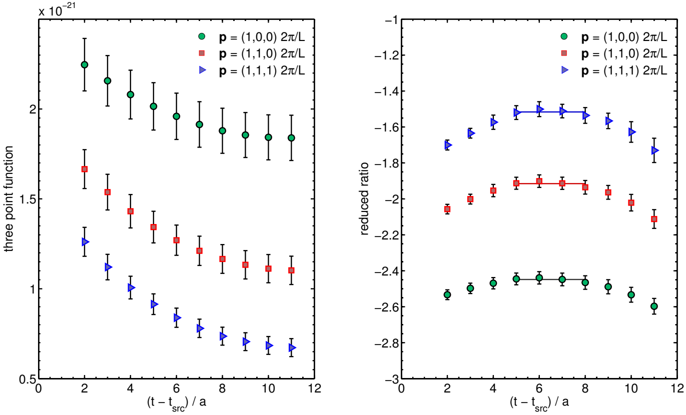
Three-point function:

$$\begin{aligned}
\Gamma_{\sigma\ \tau}^{\ \mu} &\to e^{-m_\Delta(t_f - t)} \, e^{-E_{\Delta(\vec{p}_i)}(t - t_i)} \, |Z|^2 \, c(\vec{p}_i)c(\vec{p}_f)\sqrt{\frac{m_\Delta^2}{E_{\Delta(\vec{p}_f)}E_{\Delta(\vec{p}_i)}}} G_{\sigma\ \tau}^{\ \mu} \\
G_{\sigma\ \tau}^{\ \mu} &\equiv \mathrm{tr}\left[T \, \Lambda_{\sigma\sigma'}(p_f)\mathcal{O}^{\sigma'\mu\tau'}\Lambda_{\tau'\tau}(p_i)\right]
\end{aligned}$$

Reduced ratios

Form a ratio in which Z and the time dependence cancel

$$R_{\sigma\ \tau}^{\ \mu} = \frac{\Gamma_{\sigma\ \tau}^{\ \mu}(T,\vec{q},t)}{\Gamma_{kk}(T^4,0,t_f)} \sqrt{\frac{\Gamma_{kk}(T^4,\vec{p}_i,t_f-t)\Gamma_{kk}(T^4,0,t)\Gamma_{kk}(T^4,0,t_f)}{\Gamma_{kk}(T^4,0,t_f-t)\Gamma_{kk}(T^4,\vec{p}_i,t)\Gamma_{kk}(T^4,\vec{p}_i,t_f)}} \to \Pi_{\sigma\ \tau}^{\ \mu}(T,\vec{q})$$



Suitable combinations

Example: to isolate G_{m1} one can calculate

$$\Pi_{1\ 2}^{\ \mu}(T^4,\vec{q}) = f(q^2)\delta_{3,\mu}(q_2 - q_1)G_{m1}$$

$\Rightarrow$ no contributions from $\mu \neq 3$ or $\vec{q}\,||\,\hat{z}$ data
$\Rightarrow$ It's better to calculate

$$\sum_{j,k,l=1}^{3} \epsilon_{jkl}\Pi_{j\ k}^{\ \mu}(T^4,\vec{q}) = f(q^2)\left[\delta_{1,\mu}(q_3 - q_2) + \delta_{2,\mu}(q_1 - q_3) + \delta_{3,\mu}(q_2 - q_1)\right] G_{m1}$$

other "optimal" combinations

$$\sum_{k=1}^{3} \Pi_{k\ k}^{\ \mu}(T^4,\vec{q}) \quad \rightarrow \quad G_{e0},\ G_{e2}$$

$$\sum_{j,k,l=1}^{3} \epsilon_{jkl}\Pi_{j\ k}^{\ \mu}(T^j,\vec{q}) \quad \rightarrow \quad G_{e2},\ G_{m1},\ G_{m3}$$

- coefficients of G_{m1}, G_{m3} vanish for $\mu = 4 \rightarrow$ last combination isolates G_{e2}
- All coefficients satisfy $q_\mu c^\mu = 0$ ($U(1)$ vector current conservation)

Data analysis

Jackknife binning

If there are N_{q^2} different $\vec{q}$ that give the same q^2
$\Rightarrow$ up to $4 \times N_{q^2} \times \text{number of combinations}$ equations for 4 unknowns

- measure the different combinations for $\mu = 1 \ldots 4$ and the N_{q^2} directions of $\vec{q}$
- solve the linear system in the least-square sense (e.g. via SVD).
- χ^2 value of the solution should be "reasonable"

$\Rightarrow$ Jackknife errors of $G_{e0}, G_{e2}, G_{m1}, G_{m3}$ take all autocorrelation and correlation effects into account.

Calculation of electromagnetic $\Delta \rightarrow \Delta$ form factors

[Lattice 2007 proceedings: C. Alexandrou, T. K, T. Leontiou, J.W. Negele, A. Tsapalis]

Simulation parameters

- Quenched calculation
- $32^3 \times 64$ lattice points
- 200 well-separated gauge configurations
- $\beta = 6.0$ (Wilson plaquette action)
  $\Rightarrow$ lattice spacing of $a = 0.092(3)\ fm$, from nucleon mass.
- $L \approx 3\ fm$

Valence quarks: $N_f = 2$, degenerate, unimproved Wilson

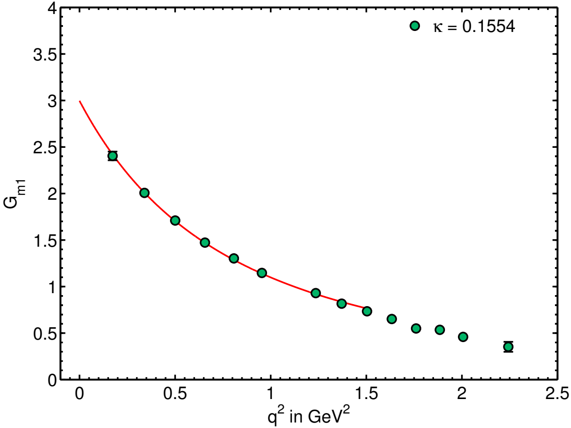
κ	m_π [MeV]	m_Δ [GeV]
0.1554	563(4)	1.470(15)
0.1558	490(4)	1.425(16)
0.1562	411(4)	1.382(15)

Results: G_{m1}

magnetic moment

$$\mu_\Delta = G_{m1}(q^2 = 0)\frac{e}{2m_\Delta}$$

m_π [MeV]	μ_Δ [μ_N]
563	2.28(5)



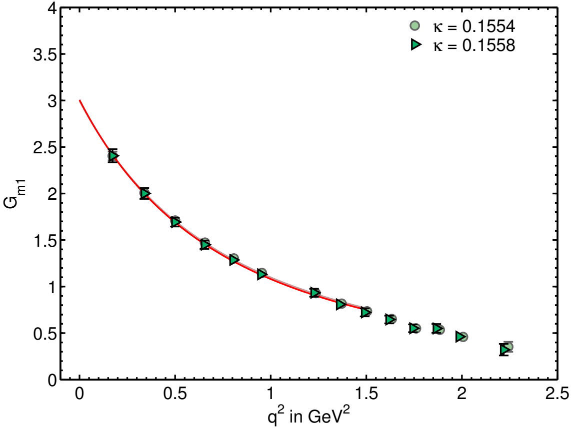
red line: fit to $\frac{G_{m1}(0)}{(1+c\,q^2)^2}$

Results: G_{m1}

magnetic moment

$$\mu_\Delta = G_{m1}(q^2 = 0)\frac{e}{2m_\Delta}$$

m_π [MeV]	μ_Δ [μ_N]
563	2.28(5)
490	2.29(7)



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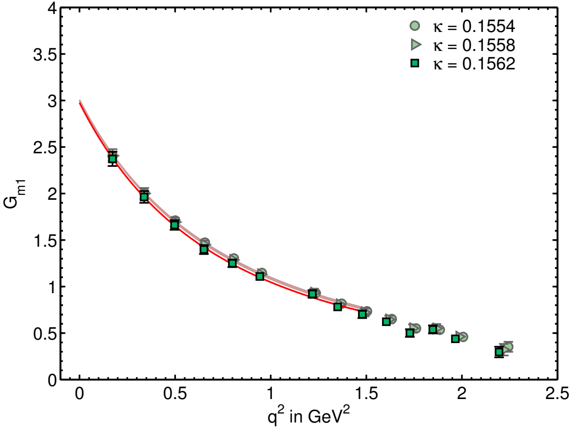

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m_π [MeV]	μ_Δ [μ_N]
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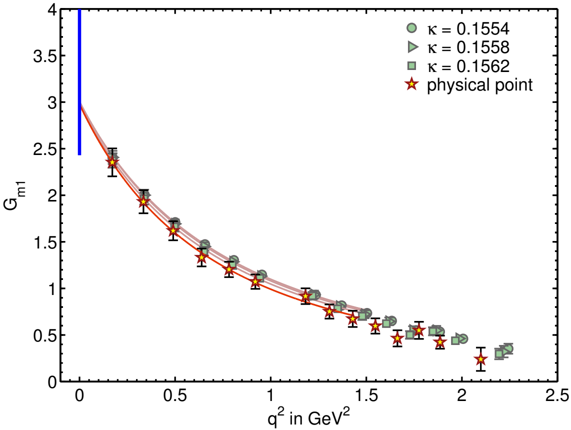
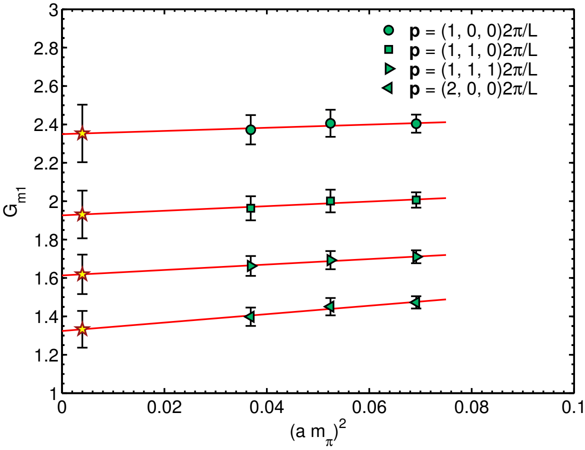
490	2.29(7)
411	2.27(7)
135	2.27(15)



red line: fit to $\frac{G_{m1}(0)}{(1+c\,q^2)^2}$

Chiral extrapolation

- No chiral PT available
- Even if: are pion masses small enough?
  $\Rightarrow$ Extrapolate linearly in m_π^2

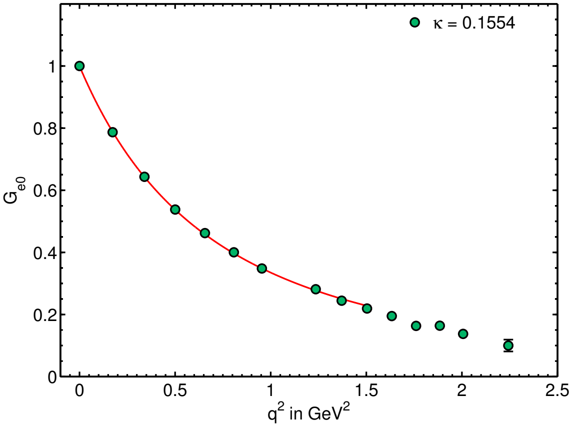


Results: G_{e0}

charge radius

$$\langle r^2 \rangle = -6 \frac{\partial G_{e0}}{\partial q^2}\Big|_{q^2=0}$$

m_π [MeV]	$\langle r^2 \rangle^{\frac{1}{2}}$ [fm]
563	0.583(2)



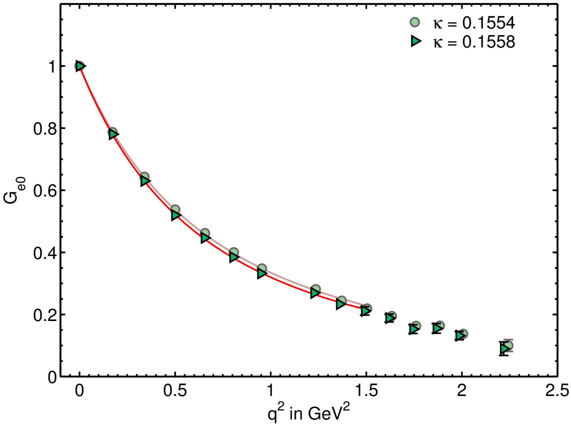
red line: fit to $\frac{1}{(1+c\,q^2)^2}$

Results: G_{e0}

charge radius

$$\langle r^2 \rangle = -6 \frac{\partial G_{e0}}{\partial q^2}\Big|_{q^2=0}$$

m_π [MeV]	$\langle r^2 \rangle^{\frac{1}{2}}$ [fm]
563	0.583(2)
490	0.599(2)



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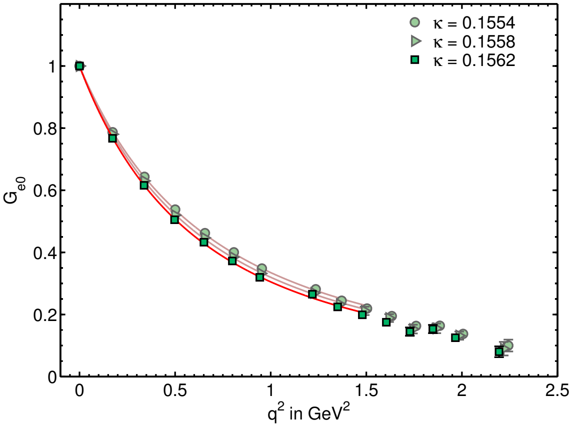
m_π [MeV]	$\langle r^2 \rangle^{\frac{1}{2}}$ [fm]
563	0.583(2)
490	0.599(2)
411	0.615(2)



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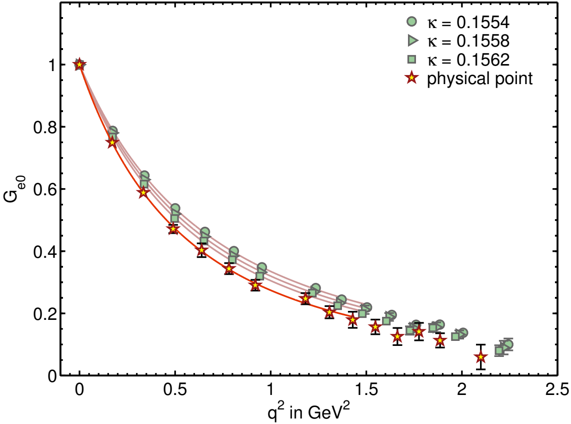
563	0.583(2)
490	0.599(2)
411	0.615(2)
135	0.652(6)



red line: fit to $\frac{1}{(1+c\,q^2)^2}$

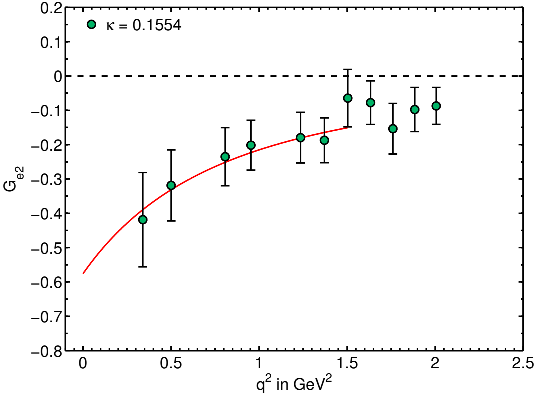
Results: G_{e2}

deformation

If spin is quantized along z-direction:

$$G_{e2}(q^2=0) \sim$$
$$m_\Delta^2 \int d^3r\, \bar{\psi}(r)\, [3z^2 - r^2]\, \psi(r)$$

$\Rightarrow$ negative $G_{e2} \leftrightarrow$ oplate Δ



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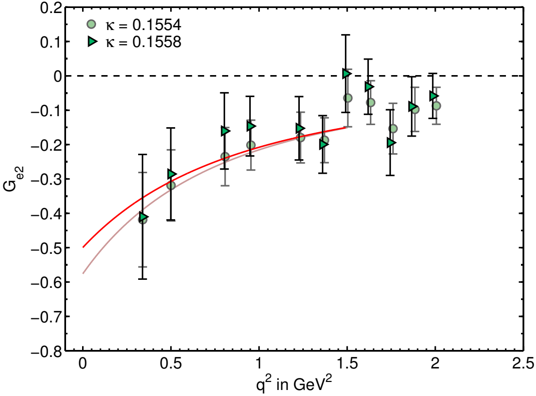

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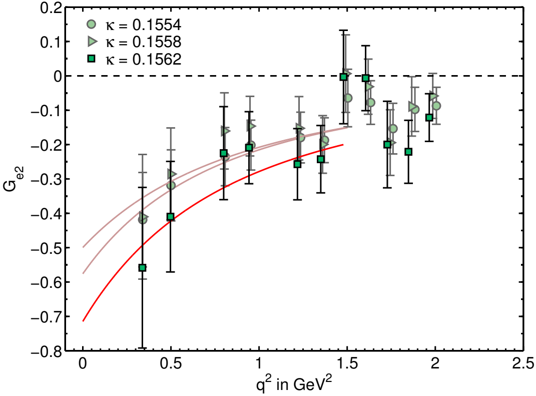
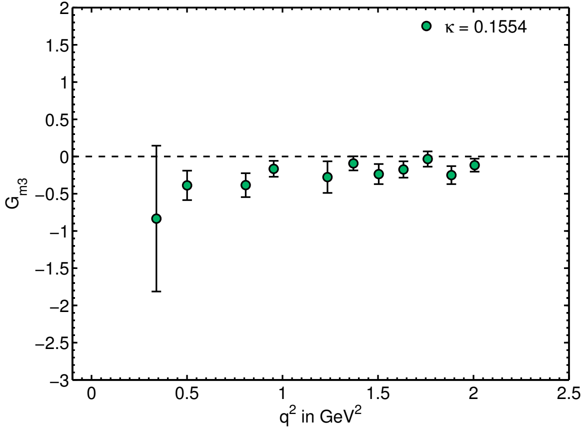
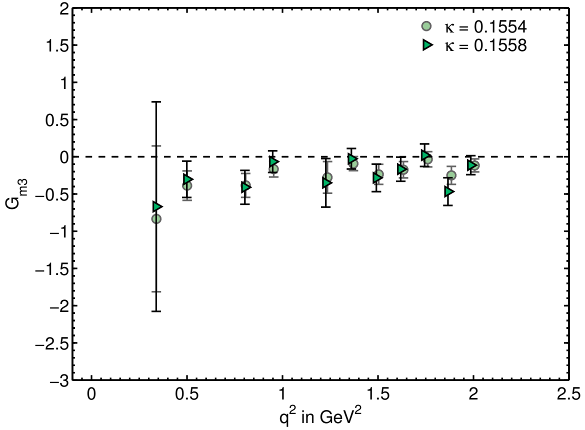


red line: fit to $\frac{G_{e2}(0)}{(1+c\,q^2)^2}$

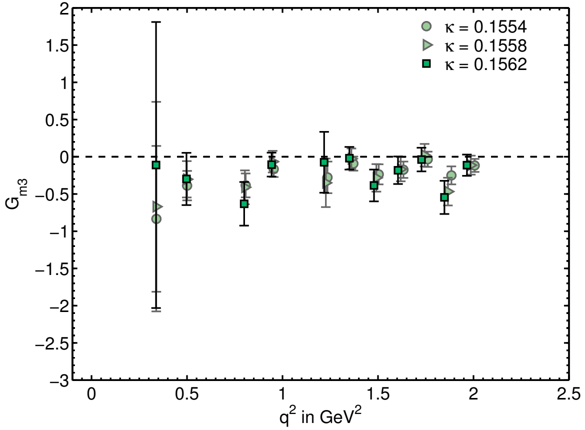
Results: G_{m3}



Results: G_{m3}



Results: G_{m3}


κ = 0.1554
κ = 0.1558
κ = 0.1562
G_{m3}
q^2 in GeV^2
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
−3
0
0.5
1
1.5
2
2.5

Calculation of axial nucleon and nucleon to Δ form factors

[arXiv:0706.3011 C. Alexandrou, G. Koutsou, T. Leontiou, J.W. Negele, A. Tsapalis]

Simulation parameters

Same quenched lattices as before, in addition:

Dynamical Wilson quarks, $N_f = 2$, $a = 0.08\,fm$, $L \approx 2\,fm$

κ	m_π $[MeV]$	m_N $[GeV]$	m_Δ $[GeV]$
0.1575	691(8)	1.485(18)	1.687(15)
0.1580	509(8)	1.280(26)	1.559(19)
0.15825	384(8)	1.083(18)	1.395(18)

Configurations created by [TχL collaboration, B. Orth et al. Phys. Rev. D72(2005)014503]

and [DESY-Zeuthen group, C. Urbach et al. Comput. Phys. Commun. 174(2006)87]

Hybrid action: asqtad / domain wall, $a = 0.125\,fm$, $L \approx 2.5\,fm$

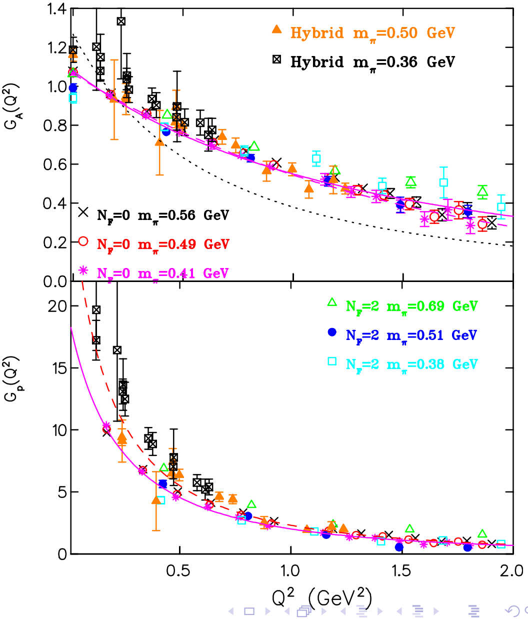
m_π $[MeV]$	m_N $[GeV]$	m_Δ $[GeV]$
594(1)	1.416(20)	1.683(22)
498(3)	1.261(17)	1.589(35)
357(2)	1.210(15)	1.514(41)

Configurations created by [MILC collab., C. Aubin et al. Phys. Rev. D70(2004) 094505]

Results: G_A and G_P

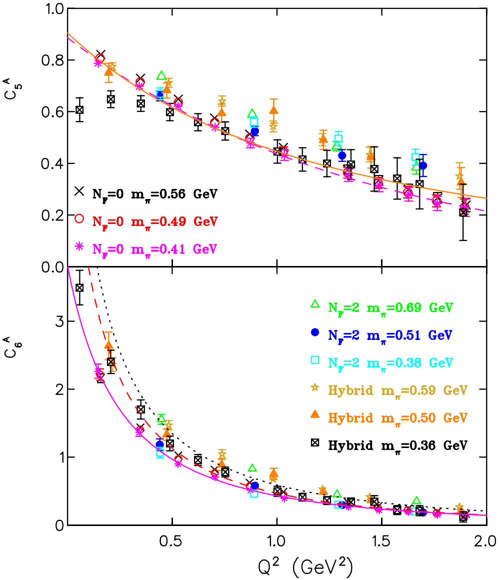
- needs Z_A
- solid magenta lines: fits of the $m_\pi = 410$ MeV data to $\frac{g_0}{(Q^2/m_A^2+1)^2}$
- Black dotted line: fit to experimental data
- dashed red line: G_p calculated from G_A via simplified GTR
- Hybrid-approach results by LHPC collaboration

  [P. Hägler et al. hep-lat/0705.4295]



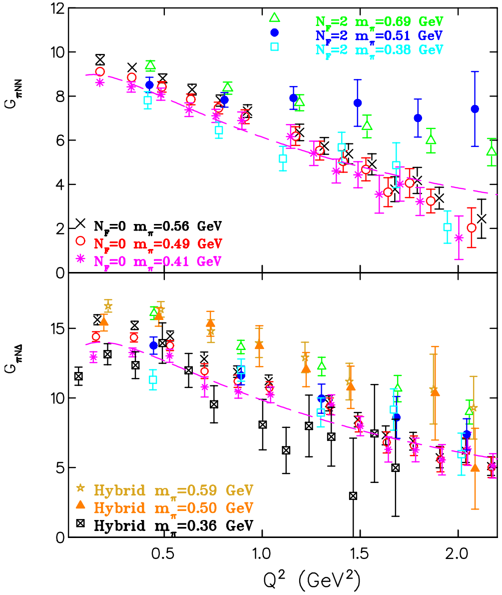
Results: C_5^A and C_6^A

- needs Z_A
- Discrepancy for low q^2
  $\Rightarrow$ lattice artifacts?



Results: $G_{\pi NN}$ and $G_{\pi N\Delta}$

- needs calculation of m_q and f_π
- dashed lines: from G_A and C_5^A via GTR
- values at $Q^2 = 0$ lower than experiment,e.g. $G_{\pi NN}(0) = 13.2(1)$



Goldberger Treiman relations

diagonal Goldberger-Treiman relation:

$$G_A(q^2) + \frac{q^2}{m_N^2} G_p(q^2) = \frac{1}{2m_N} \frac{2G_{\pi NN}(q^2) f_\pi m_\pi^2}{m_\pi^2 - q^2}$$

non-diagonal Goldberger-Treiman relation

$$C_5^A(q^2) + \frac{q^2}{m_N^2} C_6^A(q^2) = \frac{1}{2m_N} \frac{2G_{\pi N\Delta}(q^2) f_\pi m_\pi^2}{m_\pi^2 - q^2}$$

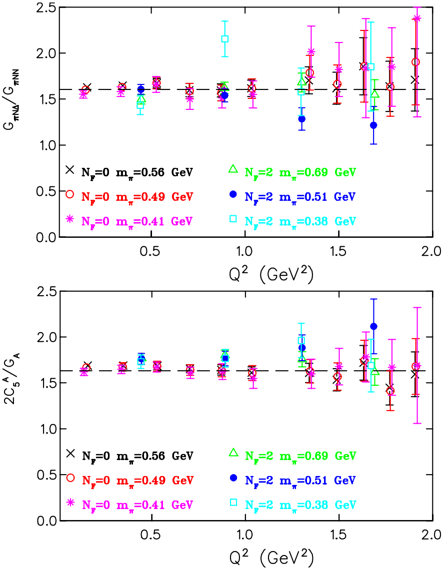
under assumption of pion pole dominance: simplification to

$$\begin{aligned} G_{\pi NN}(q^2) f_\pi &= m_N G_A(q^2) \\ G_{\pi N\Delta}(q^2) f_\pi &= 2 m_N C_5^A(q^2) \end{aligned}$$

$\Rightarrow \frac{G_{\pi NN}(q^2)}{G_{\pi N\Delta}(q^2)} = \frac{G_A(q^2)}{2C_5^A(q^2)}$

Test of simplified GTRs

- $G_{\pi N\Delta}$ and $G_{\pi NN}$ have same q^2 dependence ratio: 1.60(2) consistent with experiment
- In accordance with GTR: $2C_5^A/G_A = 1.63(1)$ also q^2 independent.



Discussion of the results

Results, electromagnetic $\Delta \to \Delta$ form factors

- look reasonable, consistent with experiment and
  [Leinweber, Draper, Woloshyn Phys. Rev. D46, 3067 (1992)]
- improvement with respect to existing calculations
  - q-dependence
  - higher precision (important for G_{e2})
  - lower pion masses

Results, axial nucleon and nucleon to Δ form factors

- $G_{\pi NN}$ and $G_{\pi N\Delta}$ have the same q^2 dependence
- Goldberger Treiman relations are satisfied

Outlook

Further reduce possible error sources

- Statistical errors: under control
- Systematical errors:
  - $\Delta \to \Delta$: quenched calculation, work with dynamical fermions in progress
  - contribution of disconnected diagrams
  - chiral extrapolations
    $\Rightarrow$ need even smaller pion masses
  - finite volume: corrections are expected to be small
  - finite resolution: effects probably significant for increasing q^2