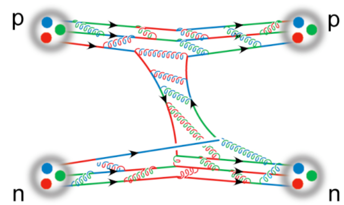
(The) Nuclear Force from lattice QCD



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in collaboration with

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★ Plan of the talk:

- PHASE 1
  - ✓ Introduction and Backgroud
  - ✓ Formalism
  - ✓ Lattice QCD results
- PHASE 2
  - ✓ Subtleties of our potential (at short distance)
  - ✓ Inverse scattering
- Summary

START

For detail, see
N.Ishii, S.Aoki, T.Hatsuda,
Phys.Rev.Lett.**99**,022001(2007).

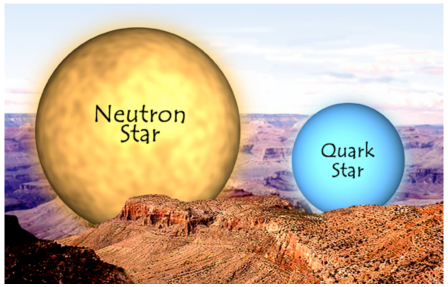
Introduction

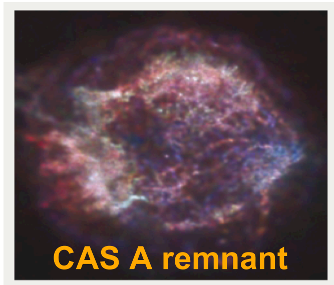
✓ The nuclear force is one of the most important building blocks in nuclear physics

✓The attraction in the medium to long distance is responsible for bound nuclei.

✓The repulsive core at short distance plays an important role for the stability of nuclei.

✓ The repulsive core is important also for astro-physics
--- the maximum mass of the neutron star
--- the ignition of the type II supernova explosion

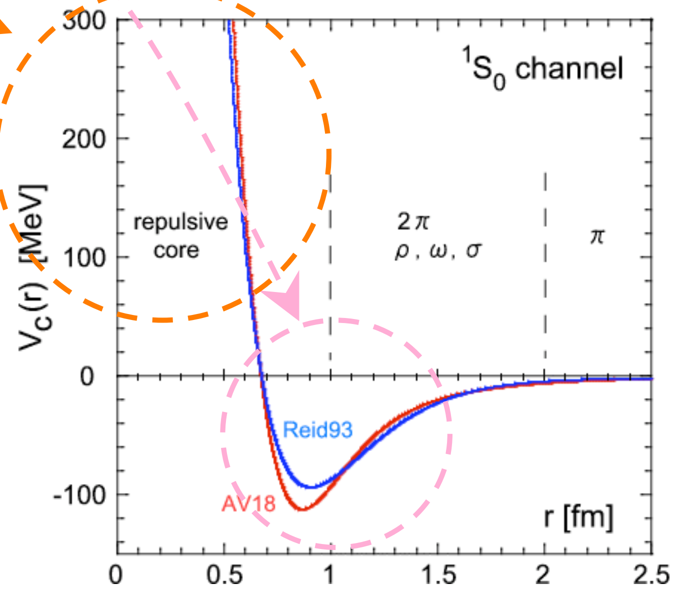




✓ Enormous efforts have been devoted to the theoretical studies of the nuclear force starting from the Yukawa's original paper 72 yeas ago:

- H. Yukawa, Proc. Math. Phys. Japan **17**, 48 (1935)
- R.Machleidt, I.Slaus, J.Phys.**G27**,R69(2001).

The nuclear force



AV16 is from
R.B.Wiringa et al., PRC**51**, 38 (1995).

Reid93 is from
V.G.J.Stoks et al., PRC**49**, 2950 (1994).

Introduction (cont'd)

Nature of nuclear force is understood in the three spatial regions.

✓ **long distance (r > 2fm)**
OPEP（one pion exchange）[H.Yukawa (1935)]

✓ **medium distance (1fm < r < 2fm)**
meson-based theories

— heavier meson exchanges such as "σ", ρ, ω,...

— multi pion exchange ∼2π

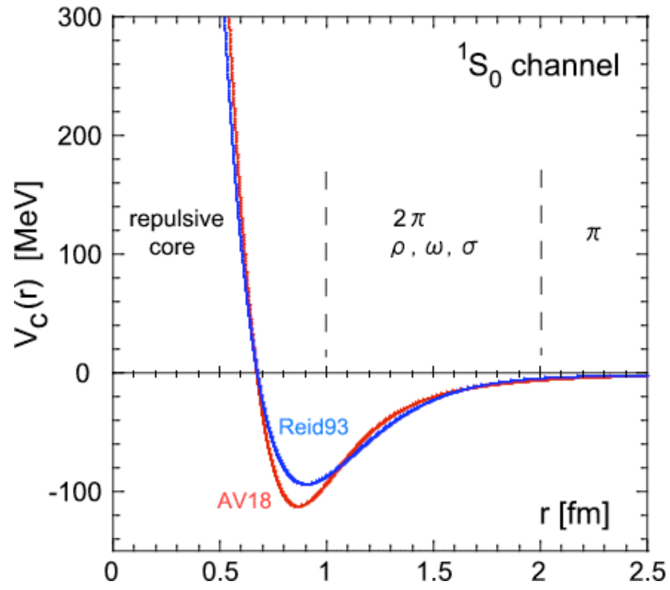
✓ **short distance (r < 1fm)**
understanding is delayed

— Physical origin of the repulsive core has no yet been settled
(1) phenomenological repulsive core model
(2) vector meson exchange model
(3) constituent quark model
Pauli forbidden states
color magnetic interaction
(4) etc.

— This region is expected to reflect
the internal quark / gluon structure of the nucleon.

— One has desired the QCD understanding of the nuclear force for a long time.

The nuclear force



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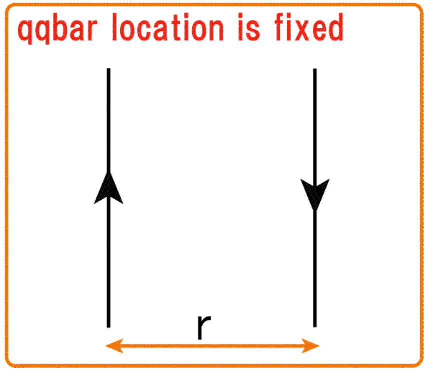
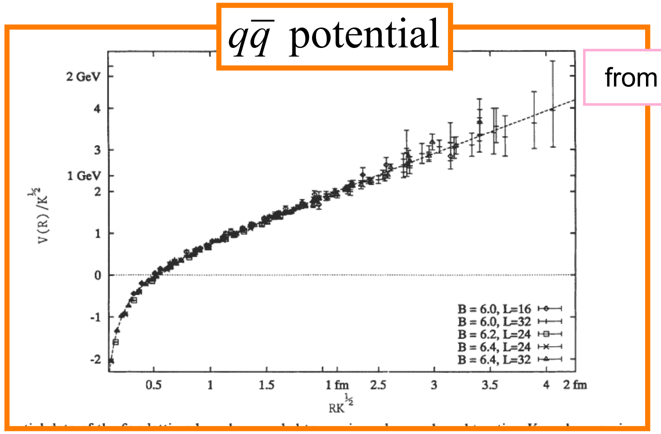
Convensional lattice QCD approach to various potentials

- static qqbar potential
  - two static quarks (Wilson lines) are introduced to fix the locations of (anti-) quark.

$$\Delta x \, \Delta v \approx \hbar / m \approx 0 \ \text{ as } m \to \infty$$

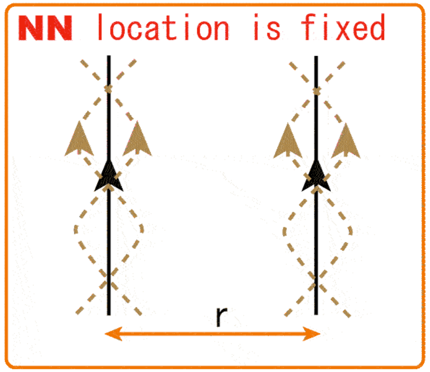


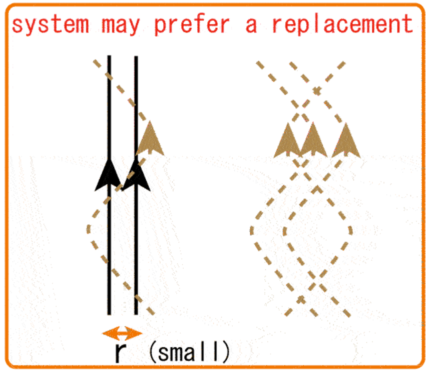


from G.S.Bali et al., PRD**46**,2636('92)

- One has attempted to extend it to the NN potential
  - A static quark in introduced in each nucleon to fix the location of the two nucleons

cf) T.T.Takahashi et al., AIP Conf.Proc.**842**,246(2006).





★ If we include meson-meson potential and color SU(2) case, there are quite many articles.
(published ones only)

- ✓ D.G.Richards et al., PRD42, 3191 (1990).
- ✓ A.Mihaly et la., PRD55, 3077 (1997).
- ✓ C.Stewart et al., PRD57, 5581 (1998).
- ✓ C.Michael et al., PRD60, 054012 (1999).
- ✓ P.Pennanen et al, NPPS83, 200 (2000).
- ✓ A.M.Green et al., PRD61, 014014 (2000).
- ✓ H.R Fiebig, NPPS106, 344 (2002); 109A, 207 (2002).
- ✓ T.Doi et al., AIP Conf. Proc. 842, 246 (2006).

- This is an elaborate methods. However, it has not yet successfully reproduced NN potential so far.
- This method does not provide a realistic potential for light flavor hadrons, which is faithful to the scattering data ---scattering length and phase shifts.

We use a totally different method.

We will extend the method recently proposed by CP-PACS collaboration,

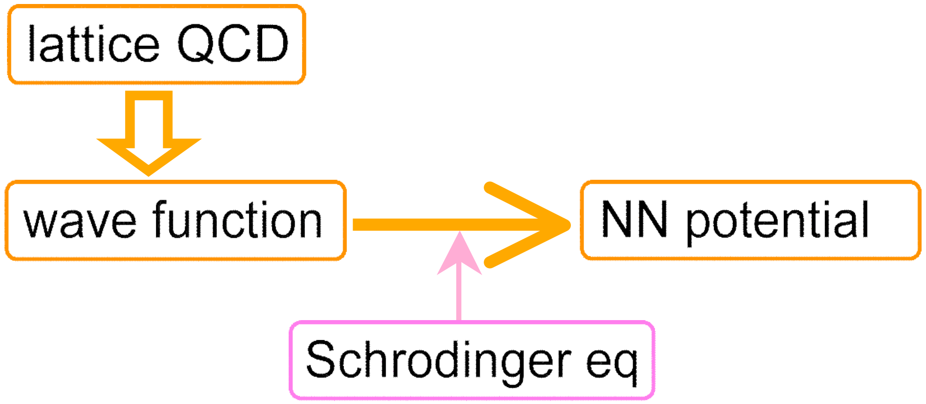
CP-PACS collab., S. Aoki et al., PRD**71**,094504(2005)

in studying pion-pion scattering length.

Sketch of our method (PHASE 1):

(1) NN wave function is constructed by using lattice QCD.

(2) The NN potential is reconstructed from the wave function
by demanding that the wave function should satisfy the Schrodinger equation.



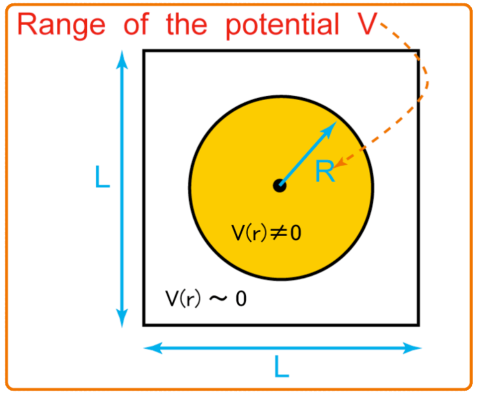
Symbolic expression

$$V(r) = E + \frac{1}{2\mu} \frac{\vec{\nabla}^2 \psi(\vec{x})}{\psi(\vec{x})}$$

GOOD FEATURE

Since our potential is constructed from the wave function,
it is expected to provided an NN potential, which is faithful to the NN scattering data.

The Formalism (PHASE 1)



☆ **Schroedinger-like eq.** for NN system.

$$\left(\vec{\nabla}^2 + k^2\right)\phi(\vec{r}) = m_N \int d^3r' \, U(\vec{r},\vec{r}')\phi(\vec{r}')$$

☆ For derivation, see C.-J.D.Lin et al., NPB619,467 (2001).
S.Aoki et al., CP-PACS Collab., PRD71,094504(2005).
S.Aoki, T.Hatsuda, N.Ishii in preparation.

☆ The interaction kernel U(r,r') is non-local.

☆ Various symmetries restrict possible forms of U(r,r').
Derivative expansion at low energy leads us to

$$V_{NN}(\vec{r},\vec{\nabla})\delta(\vec{r}-\vec{r}') \equiv U(\vec{r},\vec{r}').$$

$$V_{NN} = V_C(r) + V_T(r)\, S_{12} + V_{LS}(r)\, \vec{L}\cdot\vec{S} + O(\vec{\nabla}^2).$$

$$S_{12} \equiv 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

Explicit expression for φ is

$$\phi(\vec{r}) \Leftrightarrow \phi_{i_1,\alpha_1;i_2,\alpha_2}(\vec{x}_1,\vec{x}_2)$$

$$\vec{r} \equiv \vec{x}_2 - \vec{x}_1$$

These three interactions play the most important role in the conventional nuclear physics

$V_C(r)$ **central "force"**
$V_T(r)$ **tensor "force"**
$V_{LS}(r)$ **LS "force"**

⇒ NN Schrodinger equation

$$-\frac{\vec{\nabla}^2}{2\mu}\phi(\vec{r}) + V_{NN}\,\phi(\vec{r}) = E\phi(\vec{r})$$

$E \equiv \frac{k^2}{2\mu}, \mu = \frac{m_N}{2}$ reduced mass

☆ If we have wave function φ(r), the potential may be schematically expressed as

$$V_{NN} = E + \frac{1}{2\mu}\frac{\vec{\nabla}^2\phi(\vec{r})}{\phi(\vec{r})}$$

←only schematical sense
∵ V_{NN} involves derivative and matrix structure

General form of NN potential

★ By imposing following constraints:

- Probability (Hermiticity):
- Energy-momentum conservation:
- Galilei invariance:
- Spatial rotation:
- Spatial reflection:
- Time reversal:
- Quantum statistics:
- Isospin invariance:

The most general (off-shell) form of NN potential is given as follows:
[see S.Okubo, R.E.Marshak,Ann.Phys.4,166(1958)]

$$V = V^0 + V^\tau \cdot (\vec{\tau}_1 \cdot \vec{\tau}_2)$$

$$V^i = V_0^i + V_\sigma^i \cdot (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_{LS}^i \cdot (\vec{L} \cdot \vec{S}) + \{V_T^i, S_{12}\} + \frac{1}{2}\{V_{\sigma p}^i, (\vec{\sigma}_1 \cdot \vec{p})(\vec{\sigma}_2 \cdot \vec{p})\} + \frac{1}{2}\{V_Q^i, Q_{12}\}$$

$$Q_{12} \equiv \frac{1}{2}\left[(\vec{\sigma}_1 \cdot \vec{L})(\vec{\sigma}_2 \cdot \vec{L}) + (\vec{\sigma}_2 \cdot \vec{L})(\vec{\sigma}_1 \cdot \vec{L})\right]$$

where $V_j^i = V_j^i(\vec{r}^{\,2}, \vec{p}^{\,2}, \vec{L}^2), \quad \vec{p} \equiv i\vec{\nabla}$

★ If we keep the terms up to O(p), we are left with the convensional form of the potential in nuclear physics:

$$V = V_0(r) + V_\sigma(r)(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_{LS}(r)\vec{L} \cdot \vec{S} + V_T(r) S_{12} + O(\vec{\nabla}^2).$$

$V_0(r) + V_\sigma(r)(\vec{\sigma}_1 \cdot \vec{\sigma}_2) \leftrightarrow V_C(r)$

$^1S_0$ channel (The schematical expression becomes mathematically sound)

cf) Deuteron ⇔ $^3S_1$

★ L=0, S=0 ⇒ only $V_C(r)$ survives

$$V_{NN} = V_C(r) + V_T(r)S_{12} + V_{LS}(r)\vec{L}\cdot\vec{S} + O(\vec{\nabla}^2) \\ \cong V_C(r)$$

★ We are left with a conventional Schroedinger equation.

$$-\frac{\vec{\nabla}^2}{2\mu}\phi(\vec{r}) + V_C(r)\phi(\vec{r}) = E\,\phi(\vec{r})$$

★ $V_C(r)$ is an ordinary function.

⇒ **$V_C(r)$ can be expressed as**

$$V_C(r) = E + \frac{1}{2\mu}\frac{\vec{\nabla}^2\phi(\vec{r})}{\phi(\vec{r})}$$

Mathematically sound expression
We will use this to calculate NN potential.

NN wave function

- In QCD, the non-rela. NN wave function is only an approximate concept.
- The closest concept is provided by

  equal-time **Bethe-Salpeter(BS) wave function**

$$\phi_{\alpha\beta}(\vec{x}-\vec{y}) \equiv \left\langle 0 \left| \mathrm{T}\left[p_\alpha(\vec{x},t)\, n_\beta(\vec{y},0)\right] \right| pn \right\rangle \Big|_{t\to+0}$$

$$p_\alpha(x) \equiv \varepsilon_{abc}(u_a C\gamma_5 d_b) u_{c,\alpha}$$
$$n_\alpha(y) \equiv \varepsilon_{abc}(u_a C\gamma_5 d_b) d_{c,\alpha}$$

- It is a probability amplitude to find three quarks at x and another three quarks at y.
- Asympotoic behavior at large r ≡ |x−y|,

(s-wave)

$$\phi(r) \approx e^{i\delta_0(k)} \frac{\sin\left(kr+\delta_0(k)\right)}{kr} + \cdots$$

- BS wave function for NN is obtained from the nucleon four point function.

$$F_{NN}(\vec{x},\vec{y},t;\,t_0)$$
$$\equiv \left\langle 0 \left| p(\vec{x},t)\, n(\vec{y},t)\ \overline{p}(\vec{0},t_0)\, \overline{n}(\vec{0},t_0) \right| 0 \right\rangle$$

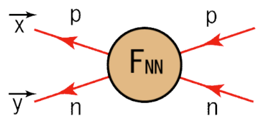
$$\mathbf{1} = \sum_m |m\rangle\langle m|$$

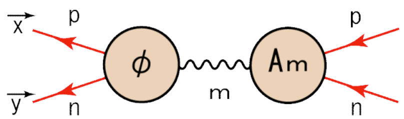
$$= \sum_m \left\langle 0 \left| p(\vec{x})\, n(\vec{y}) \right| m \right\rangle e^{-E_m(t-t_0)} \left\langle m \left| \overline{p}(\vec{0})\, \overline{n}(\vec{0}) \right| 0 \right\rangle$$

$$\phi(\vec{x}-\vec{y};m) \qquad A_m$$

$$= \sum_{\vec{k}} A_{pn(k)}\, e^{-E_{pn}(\vec{k}^2)(t-t_0)}\, \phi(\vec{x}-\vec{y};\ pn(k)) + \cdots$$

Contribution from non-NN intermediate states





projected wave function

total spin=0

★ $J^P=0^+$ (S=0, L=0)

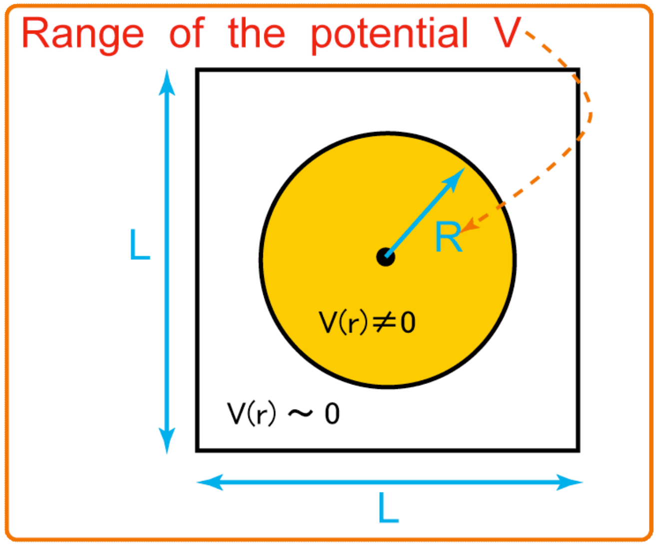
$$\phi(\vec{x};k) \equiv \frac{1}{24}\sum_{R\in O}\frac{1}{L^3}\sum_{\vec{X}}(\sigma_2)_{\alpha\beta}\left\langle 0\left|p_\alpha(R\cdot\vec{x}+\vec{X})\,n_\beta(\vec{X})\right|pn;k\right\rangle$$

Sum over spatial sites（total momentum=0）

Relative distance between pn

Sum over cubic rotation group

”asympototic momentum” between pn.



Lattice QCD parameters

1. **Quenched QCD is used.**

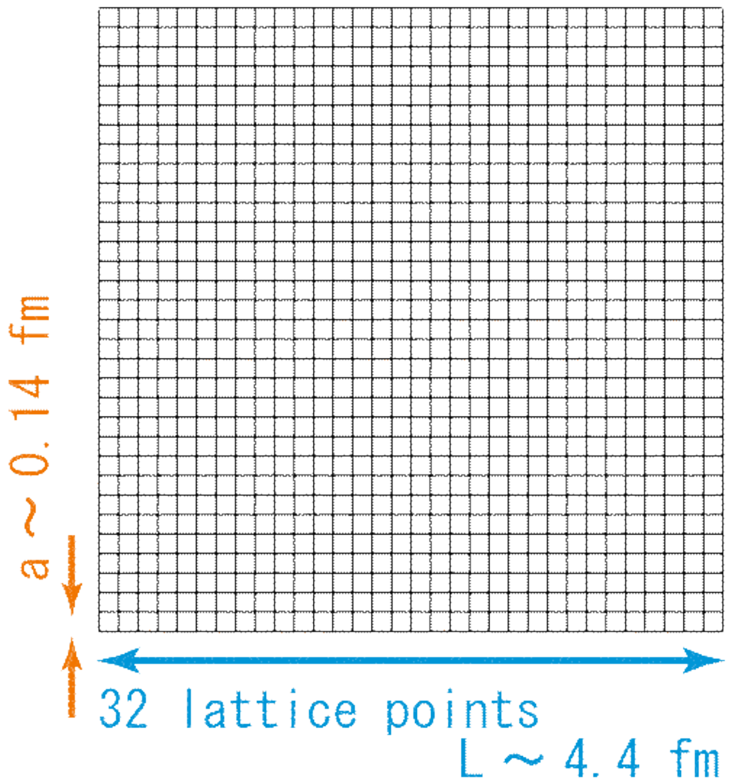
2. **Standard Wilson gauge action.**

   ✓ **$\beta = 6/g^2 = 5.7$**

   ✓ **$a \sim 0.14$ fm**
   (from ρmass in the chiral limit)

   ✓ 32^4 lattice (**$L \sim 4.4$ fm**)

   ✓ 1000-2000 gauge configs
   (3000 sweeps for thermalization.
   The gauge config is separated by 200 sweeps)



Standard Wilson quark action

✓ **κ=0.1665**

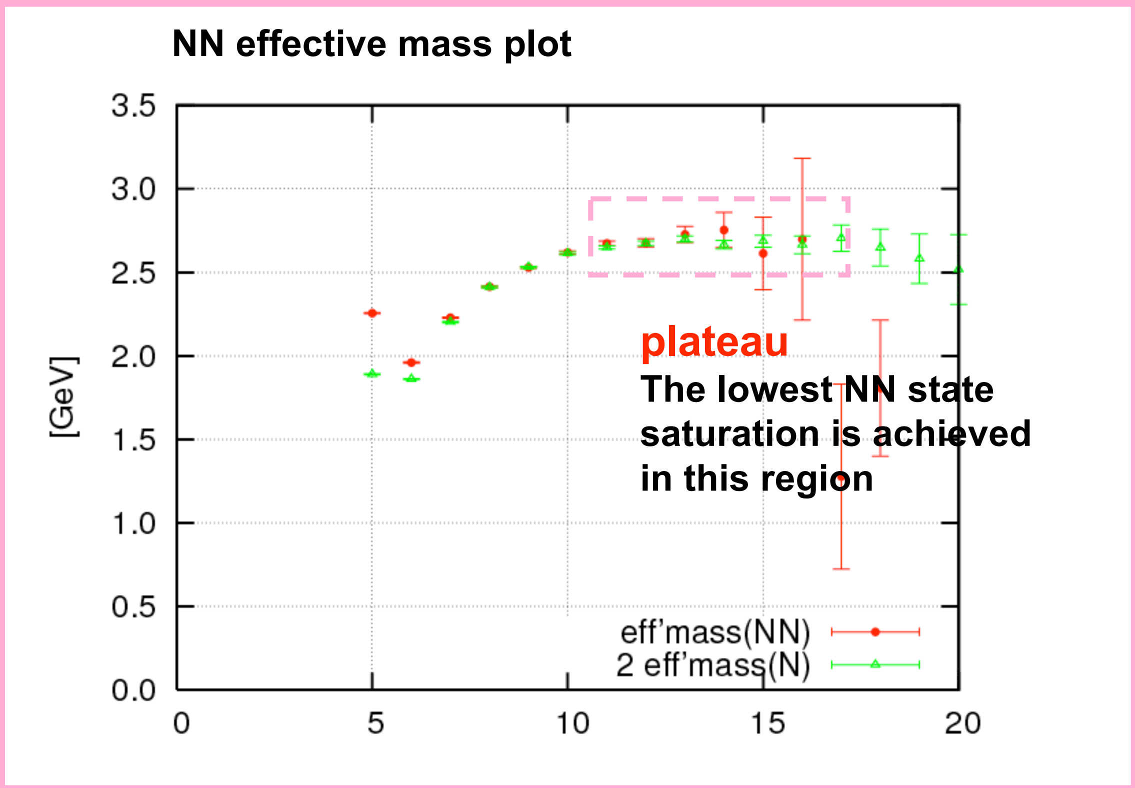
✓ **$m_\pi \sim 0.53$ GeV**, $m_\rho \sim 0.88$ GeV, $m_N \sim 1.34$ GeV

✓ Dirichlet (periodic) BC along temporal (spatial) direction
Wall source on the time-slice $t = t_0 = 5$
NN wave function is measured on the time-slice $t-t_0 = 6$

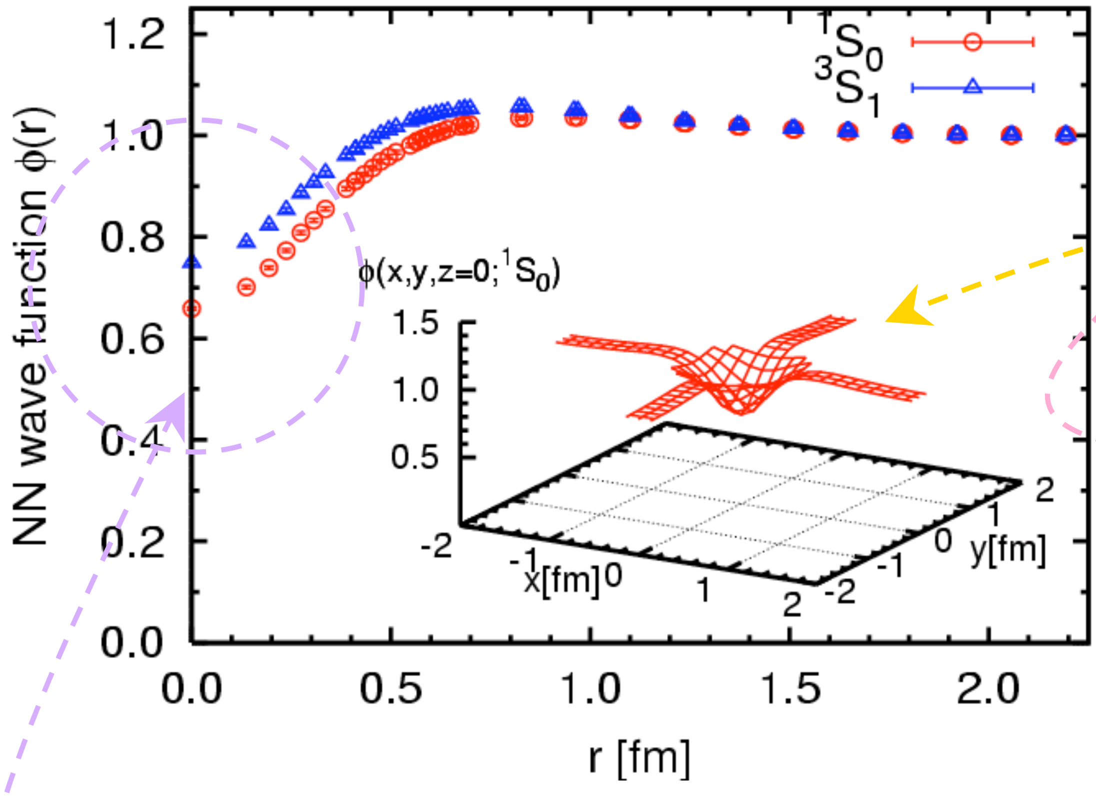
cf) M.Fukugita et al., Phys. Rev. D**52**, 3003 (1995).

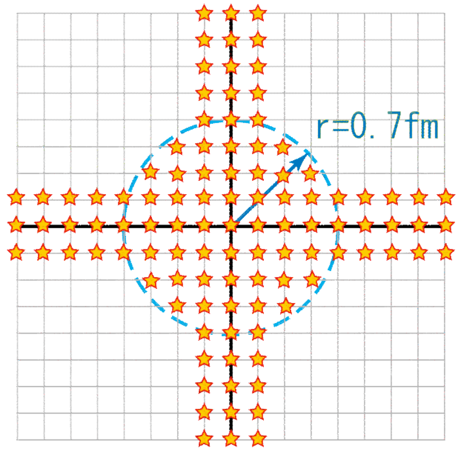
Blue Gene/L at KEK has been used for the Monte Carlo calculations.

★ effective mass plot of NN



Lattice QCD result for NN wave function





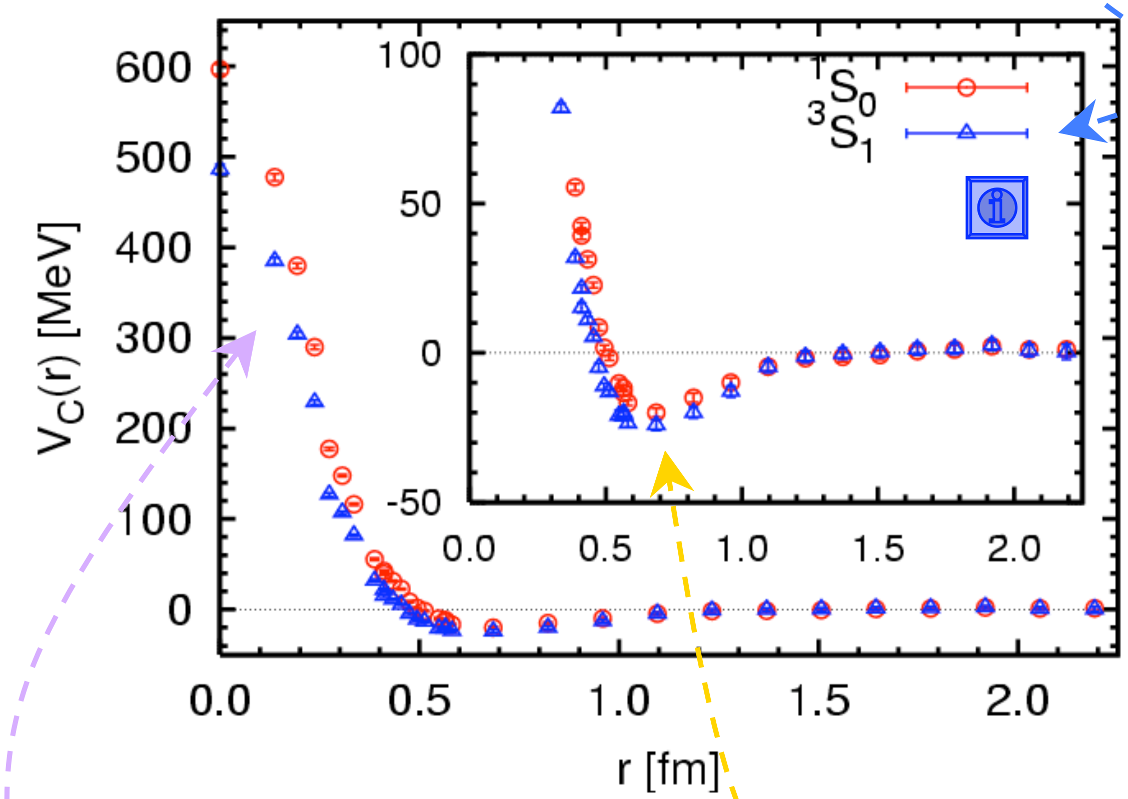
★ **3D plot of wave function**

✓ For r > 0.7 fm,
calculation is performed only in the neighborhood of the coordinate axis to save the computational time.
To obtain all the data, the calculation becomes 60 times as tough.

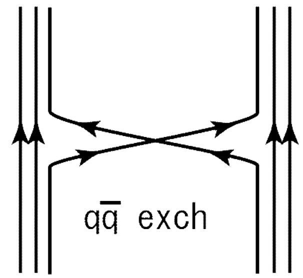
★ **shrink at short distance**
suggests an existence of repulsion.

Lattice QCD result of NN potential

The **effective central force** for $^3S_1$, which can effectively take into account the coupling of $^3D_1$ via $V_T(r)$ (and $V_{LS}(r)$) .



★ Our calculation includes the following diagram, which leads to OPEP at long distance.



★ quenched artifacts appear in the iso-scalar channel.
Cf) S.R.Beane et al., PLB535,177('02).

Repulsive core: 500 – 600MeV

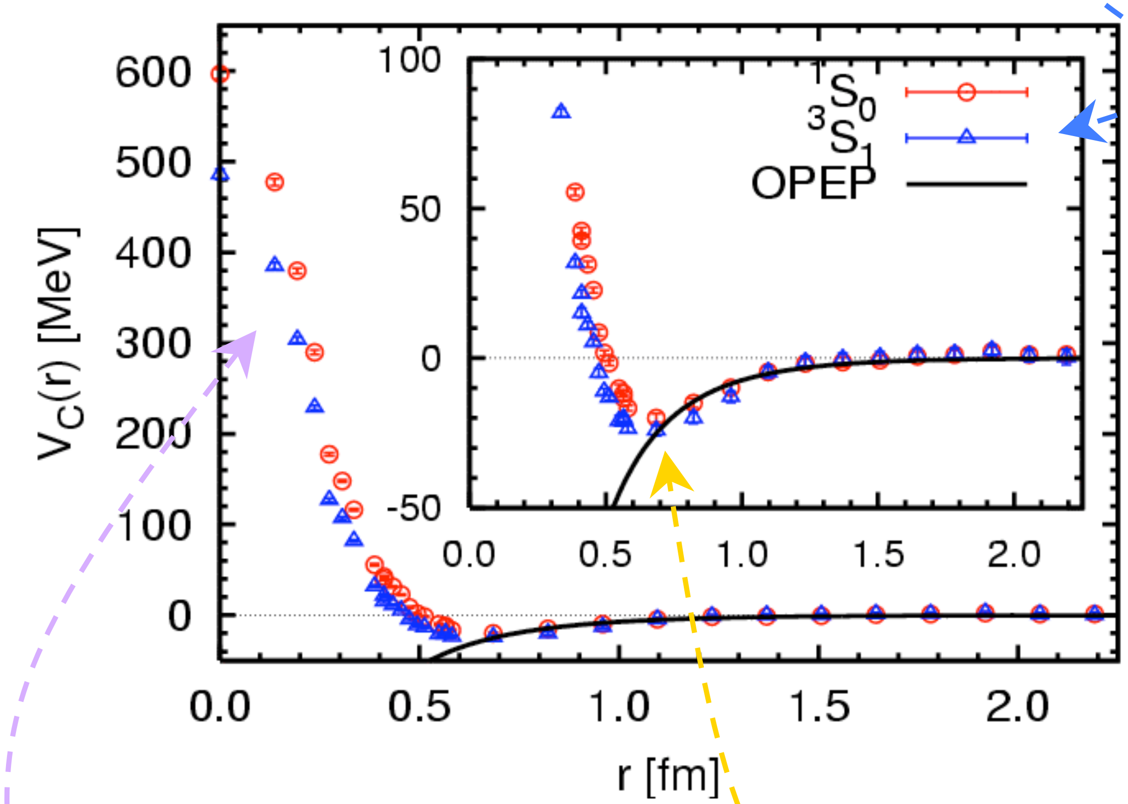
constituent quark model suggests
it is enhanced in the light quark mass region

Medium range attraction: 30 MeV

✓ heavy m_π makes it weaker..
⇒heavy virtual pion cannot propagate long distance.
For lighter pion mass, the attraction is enhanced.

✓ The effective central force in $^3S_1$ tends to be stronger than in the central force in $^1S_0$.
(This is good for deuteron)

Lattice QCD result of NN potential



The **effective central force** for $^3S_1$, which can effectively take into account the coupling of $^3D_1$ via $V_T(r)$ (and $V_{LS}(r)$) .

The following values are used for **OPEP**

m_π =528 MeV (lattice value)

m_N=1337 MeV (lattice value)

$g_{\pi N}^2/(4\pi)$=14.0 (exp value)

★ The coincidence between the data and OPEP is an accident.
(Lattice value of $g_{\pi N}$ should be used.)

Repulsive core: 500 – 600MeV

constituent quark model suggests
it is enhanced in the light quark mass region

Medium range attraction: 30 MeV

✓ heavy m_π makes it weaker..
⇒heavy virtual pion cannot propagate long distance.
For lighter pion mass, the attraction is enhanced.

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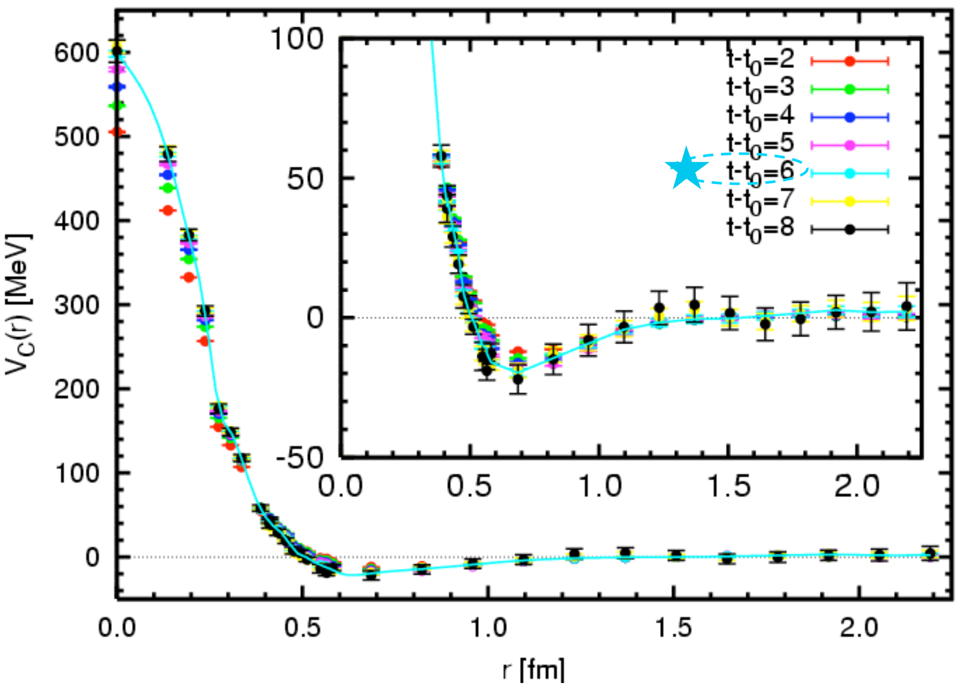
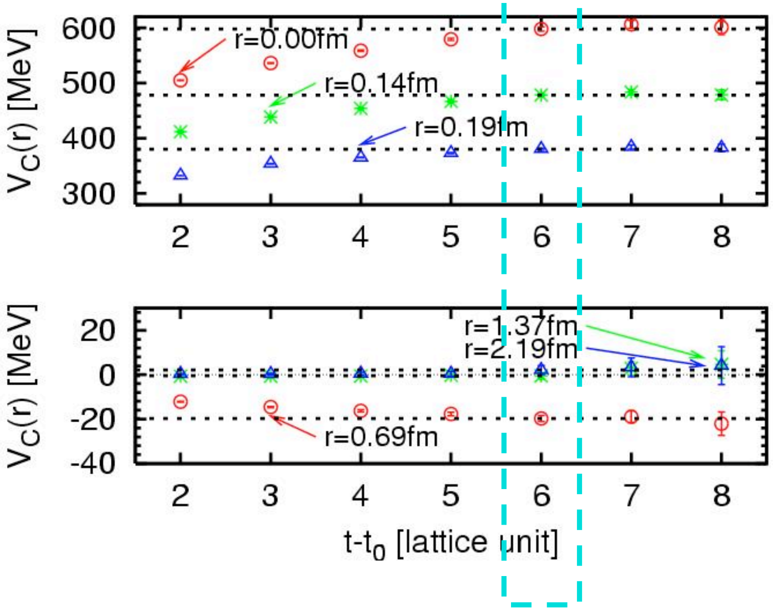
Convergence with respect to time-slice



t-dependence of each point:



✓time-slice **t−t$_0$=6** (cyan symbol) achieves the **ground state saturation**.

$$F_{NN}(\vec{x},\vec{y},t;\,t_0)$$

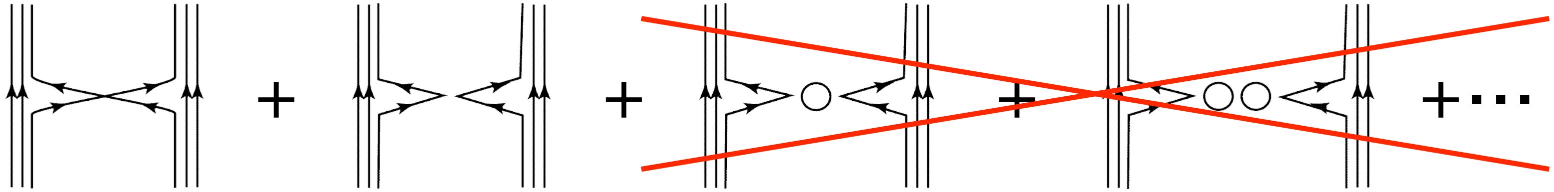
$$\equiv \left\langle 0 \middle| p(\vec{x},t)\,n(\vec{y},t)\ \ \overline{p}(\vec{0},t_0)\,\overline{n}(\vec{0},t_0) \middle| 0 \right\rangle$$

$$= \sum_m \left\langle 0 \middle| p(\vec{x})\,n(\vec{y}) \middle| m \right\rangle e^{-E_m(t-t_0)} \left\langle m \middle| \overline{p}(\vec{0})\,\overline{n}(\vec{0}) \middle| 0 \right\rangle$$

$$= \sum_{\vec{k}} A_{pn(k)}\ e^{-E_{pn}(\vec{k}^2)(t-t_0)}\ \phi(\vec{x}-\vec{y};\ pn(k)) + \cdots$$

Quenched QCD artifact

★ quenched QCD includes only a part of iso-scalar exchange diagram



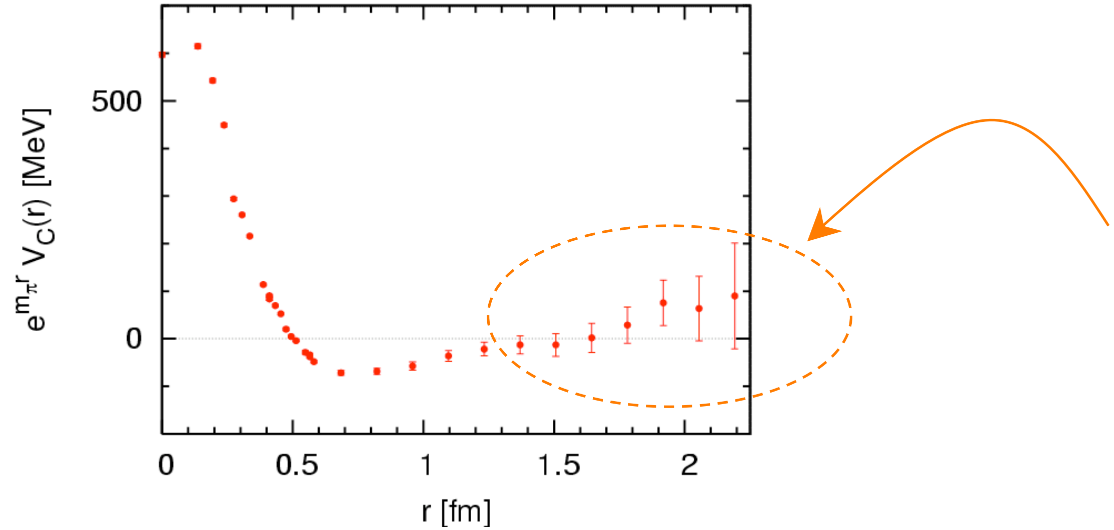
leading to the following contribution, which can spoil the OPEP behavior at long distance.

Stronger than Yukawa

$$V_C^{\eta}(r) = \frac{g_{\eta N}^2}{4\pi}\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{3}\left(\frac{m_\pi}{2m_N}\right)^2\left(\frac{1}{r}-\frac{m_0^2}{2m_\pi}\right)e^{-m_\pi r}$$

For detail, see
S.R.Beane, M.J.Savage PLB535,177(2002).

★ We have not observed this artifact yet. ($g_{\eta N}$ may be small)



Consistent with zero

$^3S_1$ channel (channel of Deuteron)

★ coupling with $^3D_1$, $V_T(r)$ and $V_{LS}(r)$ survives in this channel.

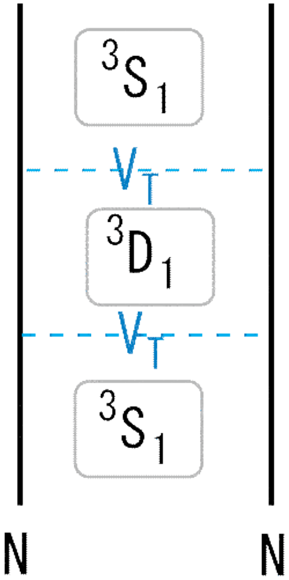
$$V_{NN} = V_C(r) + V_T(r) S_{12} + V_{LS}(r) \vec{L}\cdot\vec{S} + O(\vec{\nabla}^2)$$

★ To obtain these potentials, we have to consider coupled equations such as

$^3S_1$, $^3D_1$, $^3D_2$

⇒ $V_C(r)$, $V_T(r)$, $V_{LS}(r)$ are obtained simultaneously.

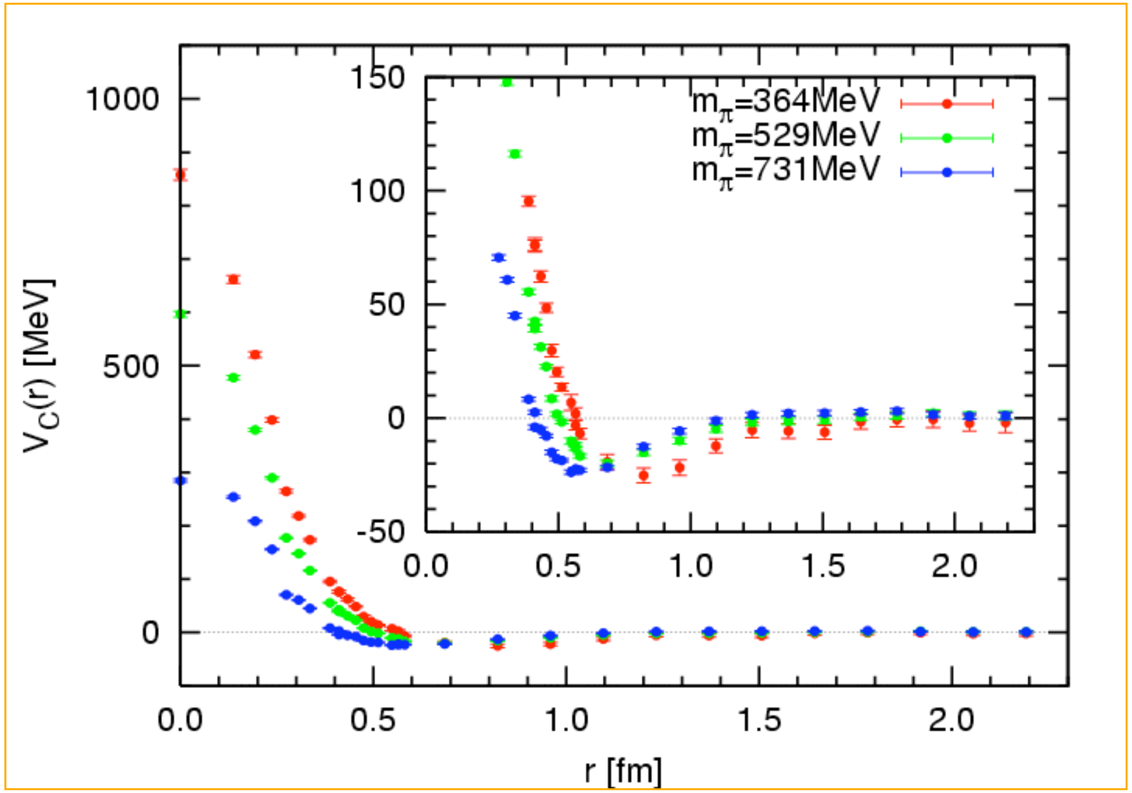
★ In this talk, for simplicity, we apply the same procedure as $^1S_0$ to $^3S_1$.



As a result, we obtain:

what nuclear physicists call **"effective central force"**, i.e.,
It is an effective $V_C(r)$, which takes into account
the coupling with $^3D_1$ via $V_T(r)$.

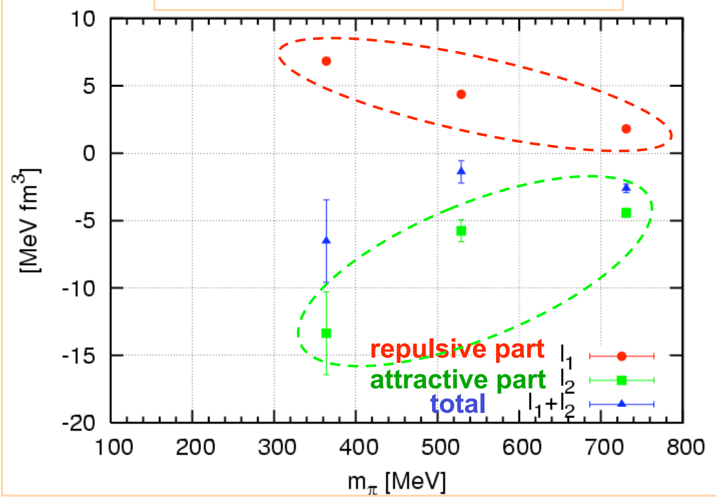
quark mass dependence



(1) m_π=364MeV: Nconf=2034
[14 exceptional configurations have been removed]
(2) m_π=529MeV: Nconf=2000
(3) m_π=731MeV: Nconf=1000

- ✓ The repulsive core at short distance grows rapidly in the light quark mass region.
- ✓ The medium range attraction is enhanced.
  - ✓ less significant in magnitude
  - ✓ range of attraction tends to become wider
- ✓ It is necessary to perform a Monte Carlo calculation in the light quark mass region.

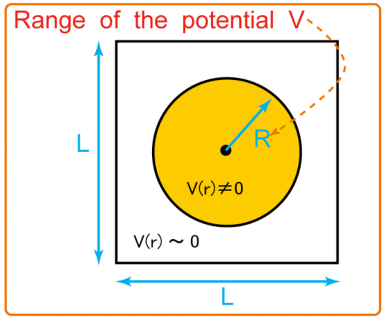
Volume integral of potential



Our potential gives “correct” scattering length by construction

The following two are formally equivalent in the limit L→∞.

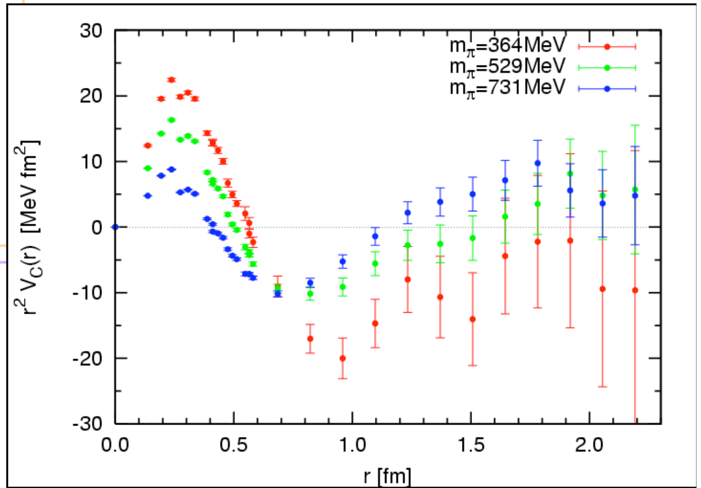
- scattering length obtained with our potential by using the standard scattering theory.
- scattering length from Luescher’s finite volume method.



Idea of proof:

Luescher’s finite volume method uses the information on the long distance part of BS wave function. Our potential is so constructed to generate exactly the same BS wave function at the energy of the input BS wave function.

Luescher’s method ⇒ a=0.123(39) fm
Our potential ⇒ a=0.066(22) fm
(The discrepancy is due to the finite size effect)



★ **comments:**

(1) **net interaction is attraction.**

Born approx. formula:

$$a_0 \cong -m_N \int V_C(r)\, r^2 dr$$

Attrractive part can hide the repulsive core inside.

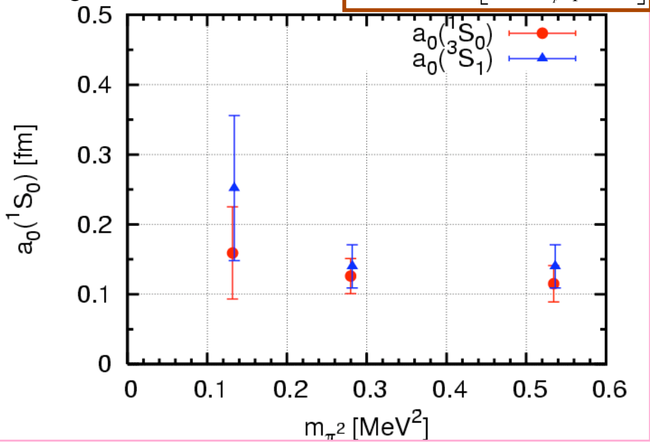
(2) **The empirical values**:

$a(^1S_0) \sim 20$fm,
$a(^3S_1) \sim -5$ fm

our value is considerably small.
This is due to heavy pion mass.

Scattering length calculated on our lattice using **Luescher’s method.**

$$a_0 = \lim_{k\to 0} \frac{1}{k} \tan^{-1}\left[\frac{4\pi}{k}\frac{1}{L^3}\sum_{\vec p}\frac{1}{\vec p^2 - k^2}\right]$$



m_q dependence of a_0 from one boson exchange potential

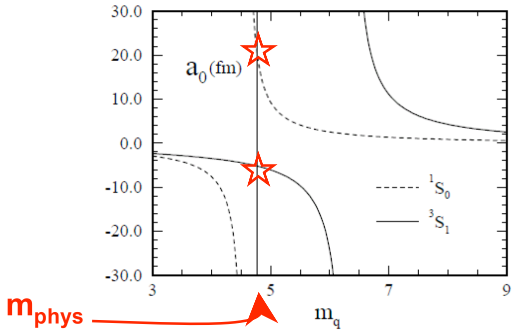


Figure 4: Quark mass m_q dependence of N-N scattering lengths based on a model of one-boson exchange potentials. Vertical line represents the physical quark mass m_q = 4.8MeV.

From Y.Kuramashi, PTPS122(1996)153.

PHASE 2 Subtleties of our potential (at short distance)

★ **Interpolating field dependence**:

Short distance part of BS wave function(W.F.) depends on a particular choice of interpolating field.

Although BS wave functions at long distance show the universal behavior

$$\phi(r) \approx e^{i\delta_0(k)} \frac{\sin\left(kr + \delta_0(k)\right)}{kr} + \cdots \quad \text{(s-wave)}$$

which leads to the operator independent way of calculating the scattering phase shift,
such universality is absent at short distance ⇒ operator dependence of BS W.F. at short distance.

★ **Energy dependent potential v.s. non-Hermitian potential**:

Most probably, the orthogonality of equal-time BS wave functions will fail, i.e.,

$$\sum_{\vec{x}} \phi^*(\vec{x}; E_1)\phi(\vec{x}; E_2) \neq 0 \text{ even for } E_1 \neq E_2$$

⇒ they are not simultaneous eigen functions of single Hermitian Hamiotonian.
⇒ Eigen functions of Energy dependent potential / Energy independent but non-Hermitian potential.

We propose to avoid these subtleties by using a knowledge of the **inverse scattering theory**, which gurantees **the existence of the unique energy-independent local potential for phase shifts $\delta_l(E)$ at all E for fixed l.**

Advantage:

- The result does not depend on a particular choice of nucleon operator.
  It is constructed from the scattering phase shift, which is free from the operator dependence problem.
- Potential is Hermitian.
- There are many ways to define non-local potentials.
  The local potential is unique. ← Inverse scattering
- Local potentials are simpler to be used in practice than non-local ones.

Outline (PHASE 2)

Instead of using the inverse scattering directly on the lattice, we go the following way:

1. Construction of E−indep. non−local potential $U_{NL}(x,x')$:

$$(E_i - H_0)\psi_{E_i}(\vec{x}) \equiv K_{E_i}(\vec{x}) = \sum_{\vec{x}'} U_{NL}(\vec{x},\vec{x}')\psi_{E_i}(\vec{x}')$$

We may consider $\psi_{E_i}(\vec{x}')$ as a matrix with respect to index (x', E_i).

$$U_{NL}(\vec{x},\vec{x}') \equiv \sum_i K_{E_i}(\vec{x})\psi_{E_i}^{-1}(\vec{x}')$$

Non-local and non-hermitian, but energy independent and correct phase shift

2. Deform the short distance part of BS wave function without affecting the long distance part

$$\phi_{E_i}(\vec{x}) \equiv \sum_x \Lambda(\vec{x},\vec{x}')\psi_{E_i}(\vec{x}')$$

3. Seek for such Λ(x,x') that can lead to a local $V_L(x)$ through the relation

$$[H_0,\Lambda] + \Lambda U_{NL} = V_L \Lambda$$

$$(H_0 + U_{NL})\psi_{E_i} = E_i\psi_{E_i}$$

$$\Downarrow \; \Lambda(\vec{x},\vec{x}')$$

$$(H_0 + V_L)\phi_{E_i} = E_i\phi_{E_i}$$

Once such $V_L(x)$ is found, it is the unique solution to the inverse scattering problem, i.e., $V_L(x)$ is the unique local potential, which reproduces the same phase shift as $U_{NL}(x,x')$.

★ For practical lattice calculation, it is difficult to obtain all BS wave function.
⇒ derivative expansion to take into account the non-locality of $U_{NL}(x,x')$ term by term.

$$U_{NL}(\vec{x},\vec{x}') = \left(U_0(\vec{x}) + U_1(\vec{x})\nabla + \cdots\right)\delta(\vec{x}-\vec{x}')$$

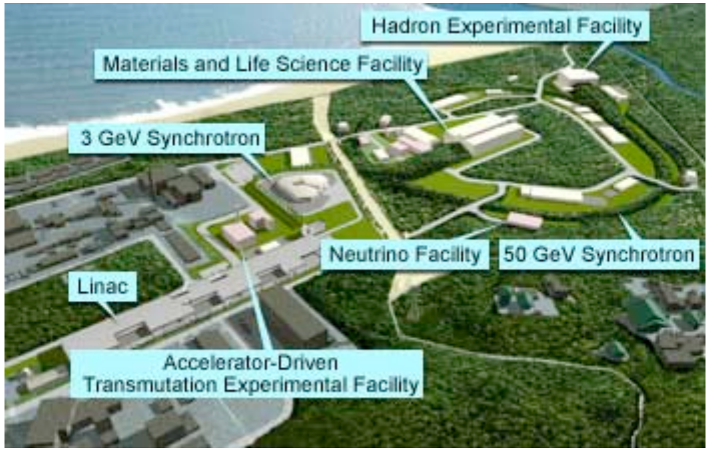
In this context, our potential at **PHASE 1** serves as 0-th step of this procedure.

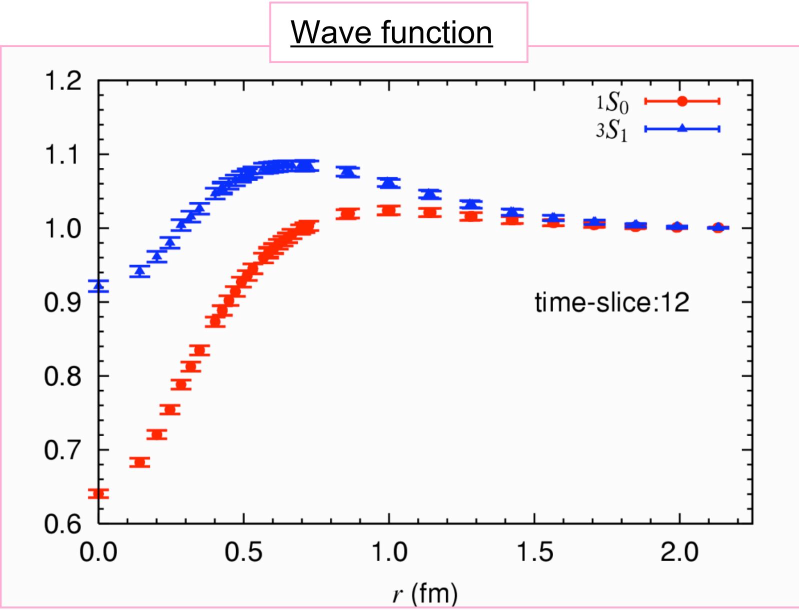
$$\Lambda(\vec{x},\vec{x}') = \delta(\vec{x}-\vec{x}'), \;\; \phi_{E_i}(\vec{x}) = \psi_{E_i}(\vec{x})$$

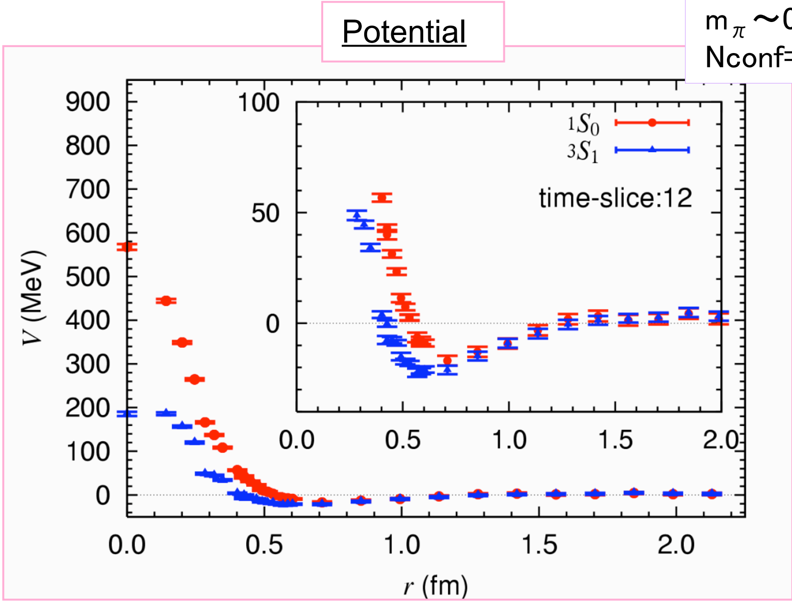
Hyperon potentials (PHASE 1)

- Our method can be applied to hyperon potentials YN & YY
- NΞ potential as a first step
  - ✓ Main target of the J-PARC DAY-1 experiment
  - ✓ Few experimental information so far
- I=1 channels ($^1S_0$, $^3S_1$)
  --- I=0 channels are not the lowest state

J-PARC







$m_\pi \sim 0.53$ GeV
Nconf=1000

※ zero-adjustment due to E is not performed.

Basic data:
m_π=509.8(5) MeV, m_K=603.7(5) MeV, m_ρ=859(2) MeV
m_N=1297(4) MeV, m_Ξ=1415(4) MeV
a∼0.142 fm(1/a∼1.39 GeV)

H.Nemura, N.Ishii, S.Aoki, T.Hatsuda in preparation

Summary

1. **PHASE 1: NN potential is calcuclated with lattice QCD.**

   We extended the method, which was recently proposed by CP-PACS collaboration in studying pion-pion scattering length.

   1. All the qualitative properties of nuclear force have been reproduced
      - ✓ **Repulsive core (∼600 MeV) at short distance (r < 0.5fm)**.
      - ✓ **Attraction (∼30 MeV) at medium distance (0.5 fm < r < 1.2 fm)**.
        (The attraction is weak due to the heavy pion (m_π∼530 MeV).)
   - Results of quark mass dependence suggest that, in the light quark mass region,
      - ✓ Repulsive core at short distance is enhanced.
      - ✓ Medium range attraction is enhanced.
   - We applied our method to hyperon potential (NΞ(I=1)).

2. **PHASE 2**: To avoid the subtleties of operator dependence at short distance, we proposed to resort to the inverse scattering theory, which provides us with the operator independent definition of the local energy-independent potential.

3. Future plans:
   - ✓ Physical origin of the repulsive core
     (dependences on the quark mass, the flavor structure, ...)
   - ✓ Hyperon potential (Sigma N, Lambda N, Lambda Lambda, Xi N, etc.) is going on.
   - ✓ LS force and tensor force, etc.
   - ✓ Performing the plan in **PHASE 2**.
   - ✓ **physical quark mass**, **unquenched QCD**, **large spatial volume,**
     finer discretization, chiral quark actions, ...