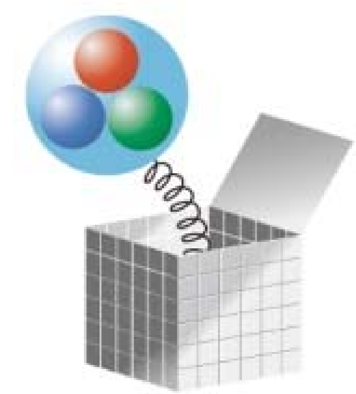
Pion form factor

Shoji Hashimoto (KEK)
@ "Hadron Physics on the Lattice,"
Milos, Greece, Sep 10, 2007.

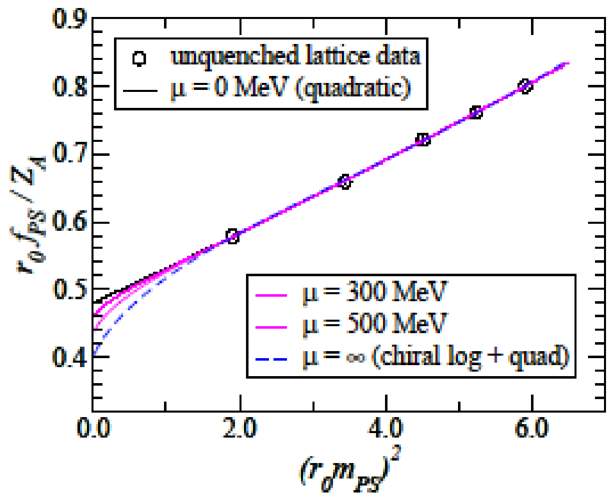


Lattice (want to) meet χPT

Unquenched lattice QCD is now entering the region of χPT.

- Was not sure, say 5 years ago.
  - Most likely due to too heavy sea quarks
  - Perhaps, also due to the broken chiral symmetry of Wilson fermion

JLQCD (2002)



$$\frac{f_\pi}{f} = 1 - \frac{N_f}{2} \frac{m_\pi^2}{(4\pi f)^2} \ln \frac{m_\pi^2}{\mu^2 + m_\pi^2} + \cdots$$

5 years back; many concerns

- Wilson fermion may contain fundamental problems near the chiral limit?
  - Exceptional configurations (trajectories)

    $\Rightarrow$ Not true in large enough volume (CERN)
  - (Unphysical) phase transition

    $\Rightarrow$ True, but only near the chiral limit for small enough a (Sharpe-Shingleton, preETMC, …)
  - Cost scales too badly ~ m_q^{-3}

    $\Rightarrow$ overcame by Hasenbusch and RHMC; lightest pion 500 MeV $\rightarrow$ 300 MeV

Now; much better shape

- Wilson fermion will be okay. Even better formulations on the market.
  - Not just Wilson

    TMQCD explicitly avoids the instability
  - Chiral

    domain-wall, overlap now feasible: No (or little) problem of explicit chiral violation
  - χPT

    now available for non-chiral lattice fermions and even for mixed actions

Time to test the consistency

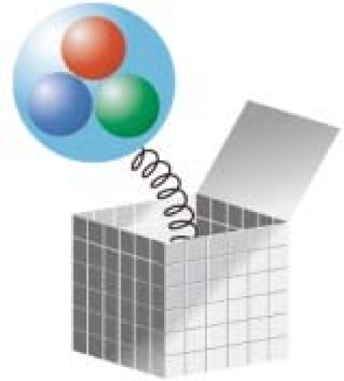
- With m_π~300 MeV, many groups are now testing the consistency with χPT.
  - Can we see the χlog?
  - Extract Low Energy Constants L_1~L_{10}

- From the easiest to more difficult
  - Pion mass & decay constant (also kaon)
  - Pion form factor (also kaon)
  - pi-pi scattering, …

Plan

1. Pion mass and decay constant
   - Taking our (JLQCD's) recent data as an example
2. Pion form factor
   - How to calculate, with better precision
   - Fit forms: VMD, χPT, …
3. Testing chiral log with pion form factor
   - Chiral extrapolation of charge radius

1. Pion mass and decay constant



With dynamical overlap

JLQCD collaboration (2006~)

- Exact chiral symmetry
  - No complication due to modified χPT (fit with additional unknown parameters)
  - Chiral limit exists: no instability, no phase transition as occurred for Wilson
  - ε-regime possible
  - At the cost of limiting physical volume…

Overlap fermion

$$D = \frac{1}{a}\left[1 + \gamma_5 \,\mathrm{sgn}(H_W)\right]$$

Implementation:

- Use with the standard Wilson kernel
- Low-lying modes of H_W projected out, treated exactly
- Rest of the eigenmodes approximated by a rational function to a $10^{-(7\text{-}8)}$ precision = "exact" chiral symmetry

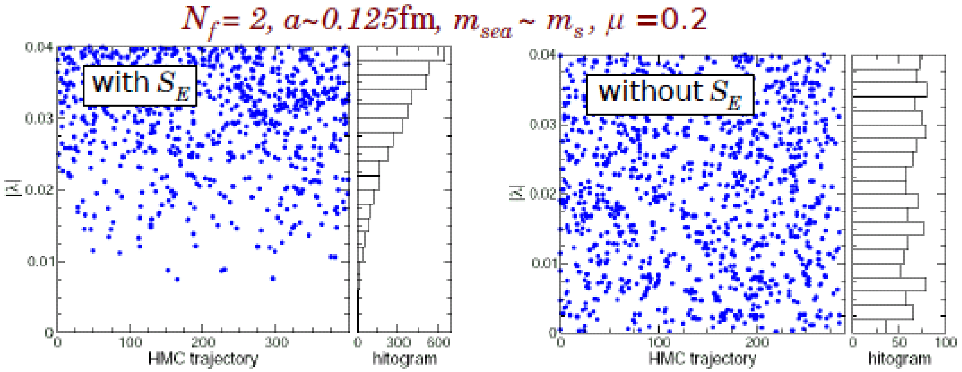
Near-zero mode suppression

= Key for the feasibility

- By adding unphysical (heavy) fermions and bosons

$$\det\left(\frac{H_W^2}{H_W^2+\mu^2}\right)=\int d\chi^\dagger d\chi \exp[-S_E]$$

  overlap operation gets much cheaper.



- No extra-cost due to eigenmodes passing zero

Topology conservation

- Topological charge Q freezes during HMC, problem? --- Yes and No.
  - θ vacuum is not sampled correctly.
  - Property of the continuum gauge action
    - If your simulation changes topology, you are still far away from continuum!
  - Fixed Q induces finite volume effect $\sim 1/(\chi_t V)$
    - You don't have to know the topological charge of the whole universe!
    - Topological fluctuation is on-going locally.

QCD at fixed Q

Brower et al, PLB560, 64 (2003); Aoki et al., arXiv:0707.0396 [hep-lat]

- θ=0 physics can be reconstructed
  - Fixed θ and fixed Q are related to each other by a Fourier transform
  - Q distribution primarily governed by χ_t.

$$Z(\theta) = \exp\left[-V\left(\frac{\chi_t}{2}\theta^2 + O(\theta^4) \right) \right]; \; Z_Q = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta \, Z(\theta) e^{i\theta Q}$$

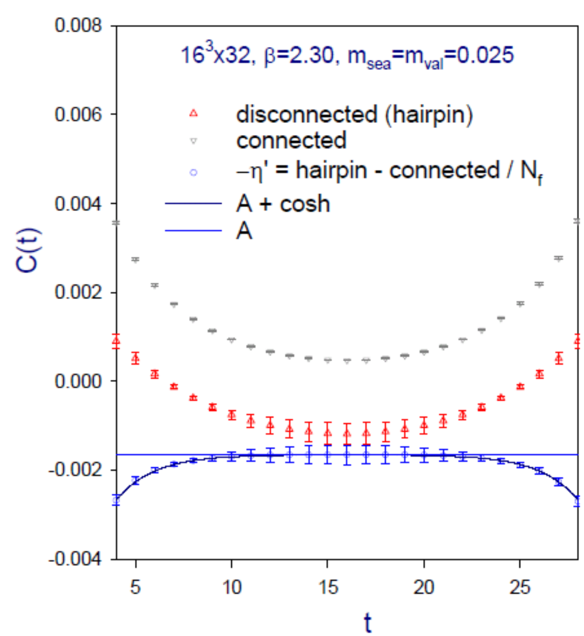
  - General n-point function related as

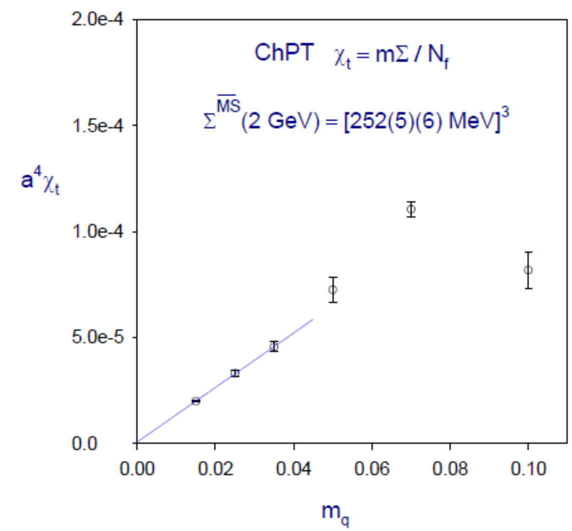
$$G_{Q\ (\mathrm{even})} = G(\theta=0) + G^{(2)} \frac{1}{2\chi_t V}\left[1 - \frac{Q^2}{\chi_t V} \right] + O(1/V^2)$$

Topological susceptibility

- Applying the formula for the flavor-singlet PS density, χ_t can be extracted.

$$\lim_{x\to\infty}\langle mP(x)mP(0)\rangle_Q = -\frac{1}{V}\left(\chi_t - \frac{Q^2}{V} + O(1/V)\right) + O(e^{-m_{\eta'}x})$$





JLQCD and TWQCD (2007)

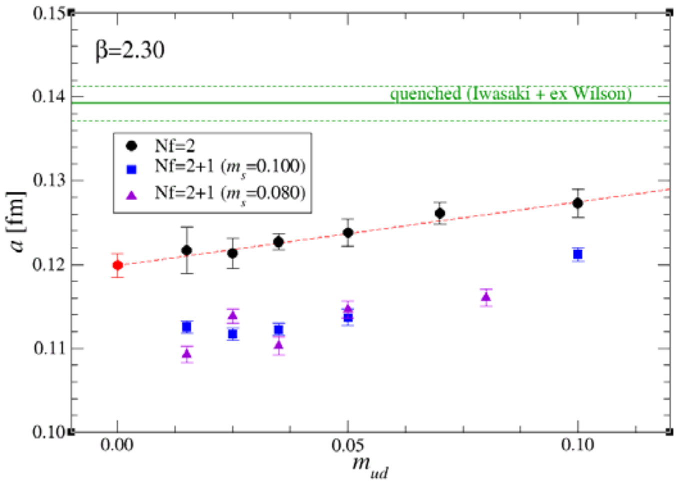
$$\chi_t = \frac{m\Sigma}{N_f}$$

Local topological fluctuation is indeed active as expected.

Simulation

Dynamical overlap runs

- N_f=2 runs completed; now 2+1 running
- a = 0.11~0.12 fm, 16^3x32 lattice
- 6 sea quark masses in $m_q=m_s/6$~m_s
- 10,000 traj for each run

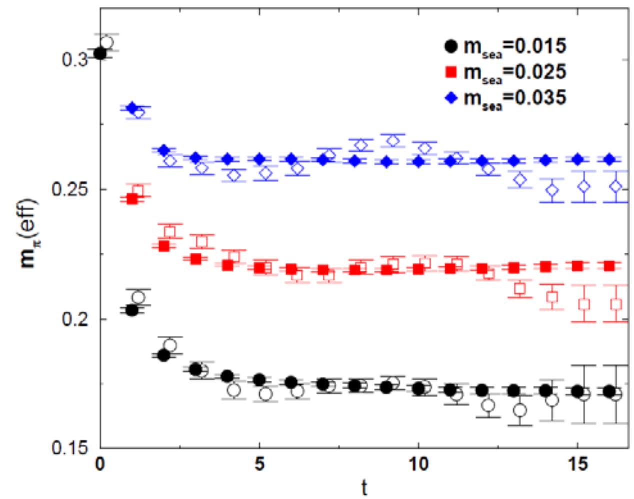


- Also done an ε-regime run at m=3 MeV.

Benefit from low modes

Measurements at every 20 traj ⇒ 500 conf / m_{sea}

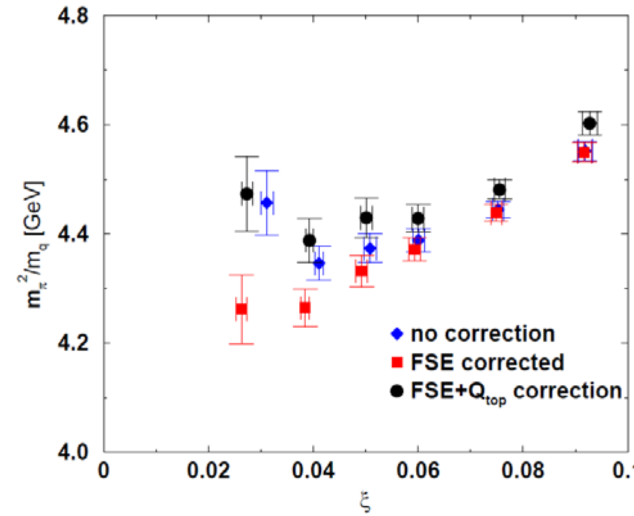
- Improved measurements
  - 50 pairs of low modes calculated and stored.
  - Used for low mode preconditioning (deflation) ⇒ (multi-mass) solver is then x8 faster
  - Low mode averaging (and all-to-all)

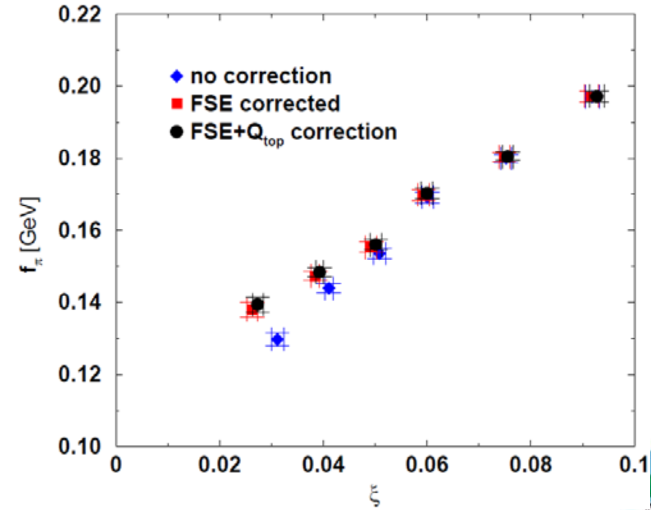


Finite volume corrections

At L~1.9 fm (smallest $M_\pi L$~3), FSE is not negligible.

- χPT at NNLO
  - Colangelo-Durr-Haefeli, NPB721, 136 (2005)
- Fixed topology
  - Aoki et al., arXiv:0707.0396 with NLO χPT and measured χ_t.





NNLO analysis

NNLO χPT predicts the mass dependence as

$$\frac{m_\pi^2}{m_q} = 2B_0\left[1+\xi\ln\xi+\frac{7}{2}(\xi\ln\xi)^2+\left(\frac{2L_4}{f}-\frac{4}{3}(\tilde{L}+16)\right)\xi^2\ln\xi\right] + L_3(\xi-9\xi^2\ln\xi)+K_1\xi^2$$

$$f_\pi = f\left[1-2\xi\ln\xi+5(\xi\ln\xi)^2-\frac{3}{2}\left(\tilde{L}+\frac{53}{2}\right)\xi^2\ln\xi\right] + L_4(\xi-10\xi^2\ln\xi)+K_2\xi^2$$

simultaneous fit

input: $\tilde{L} = 7\ln\left(\frac{\Lambda_1}{4\pi f}\right)^2+8\ln\left(\frac{\Lambda_2}{4\pi f}\right)^2$ from phenomenology

$$\xi = \left(\frac{m_\pi}{4\pi f_\pi}\right)^2$$

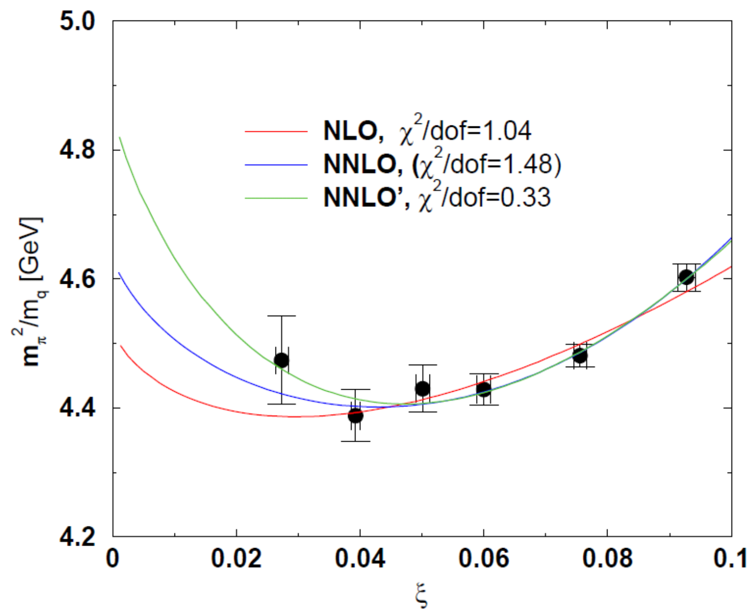
NNLO analysis

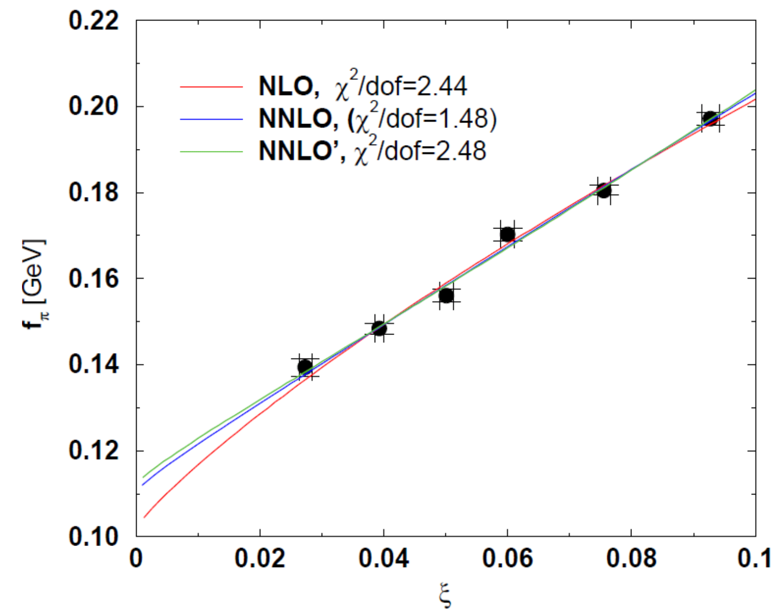
Also, NNLO'

$$\frac{m_\pi^2}{m_q} = 2B_0\left(1+\xi\ln\xi+\frac{7}{2}(\xi\ln\xi)^2\right)+L_3\xi+K_1{}'\,\xi^2$$

$$f_\pi = f\left(1-2\xi\ln\xi+5(\xi\ln\xi)^2\right)+L_4\xi+K_2{}'\,\xi^2$$

Noaki at Lattice 2007



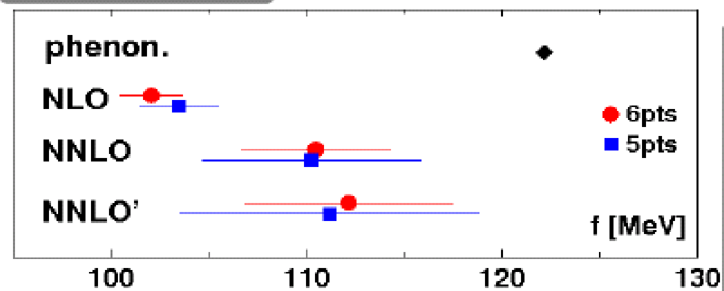


Data slightly favor NNLO, though statistically not significant.

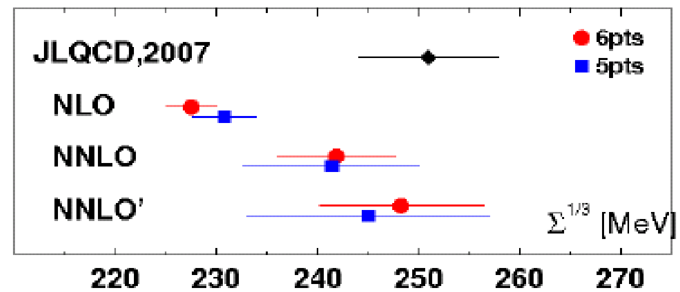
LECs

Noaki at Lattice 2007

$f = 122.2\,\text{MeV}$ Gasser & Leutwyler,1984

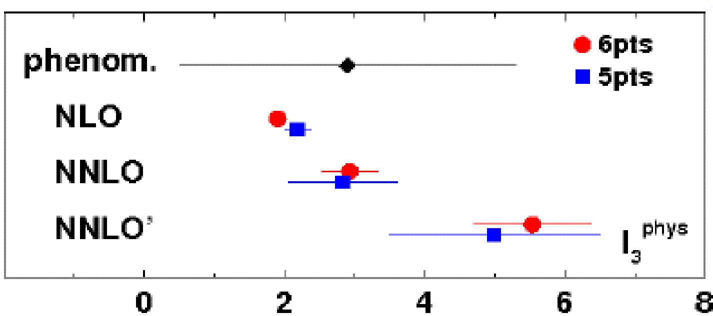


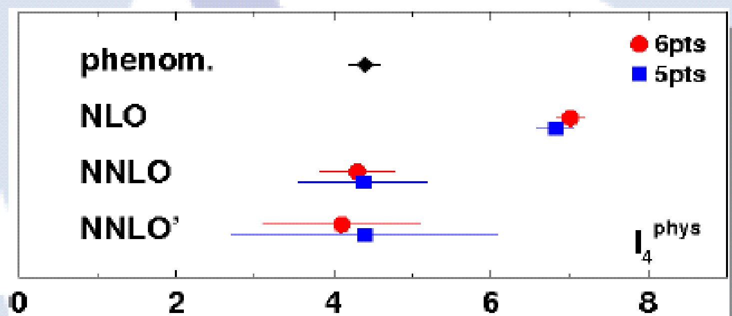
$\Sigma = B_0 \cdot f^2 / 2$



$$l_3^{\text{phys}} = \ln\left(\frac{\Lambda_3}{m_\pi^{\text{phys}}}\right)^2 = \ln\left(\frac{4\pi f}{m_\pi^{\text{phys}}}\right)^2 - \frac{L_3}{2B_0}$$

$$l_4^{\text{phys}} = \ln\left(\frac{\Lambda_4}{m_\pi^{\text{phys}}}\right)^2 = \ln\left(\frac{4\pi f}{m_\pi^{\text{phys}}}\right)^2 + \frac{L_4}{2f}$$



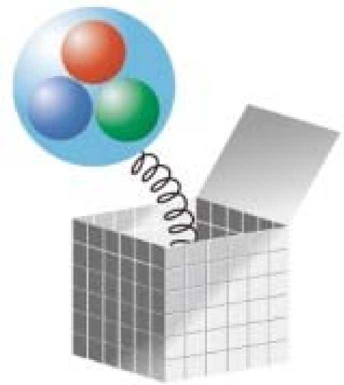


Lessons, if not conclusions

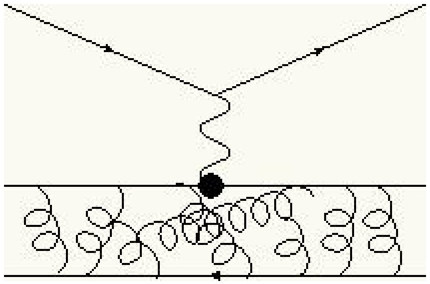
With exact chiral symmetry, test of χPT is possible.

- Need very good precision for the test, otherwise χlog is not significant.
- With our mass range, NLO and NNLO lead to different chiral limit = NNLO is necessary.
- Finite size effect is important for L<2 fm.

2. Pion form factor



Pion form factor



- The simplest form factor

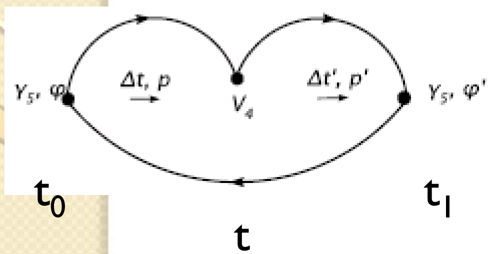
$$\langle \pi(p')|V_\mu|\pi(p)\rangle = i(p_\mu + p_\mu')F_V(q^2), \quad q_\mu \equiv p_\mu' - p_\mu$$

  - Momentum transfer q_μ by a virtual photon. Space-like ($q^2<0$) in the $\pi e \to \pi e$ process.
  - Vector form factor $F_V(q^2)$ normalized as $F_V(0)=1$, because of the vector current conservation.

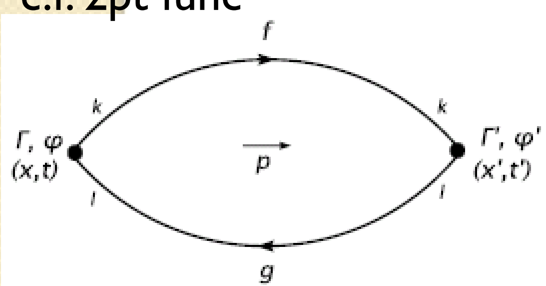
$$F_V(q^2) = 1 + \frac{1}{6}\langle r^2\rangle_V^\pi q^2 + O(q^4),$$

  - Vector (or EM) charge radius $\langle r^2\rangle_V^\pi$ is defined through the slope at $q^2=0$.

Lattice calculation of 3pt function



c.f. 2pt func

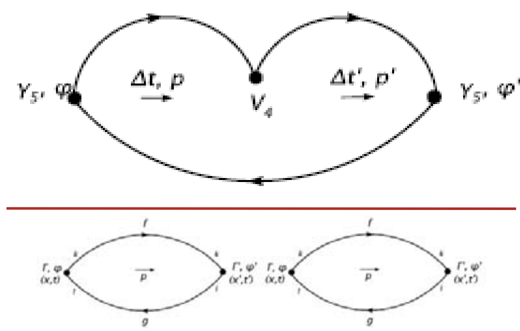


- $\pi(p) \to \pi(p')$
  - An interpolating operator for the initial state $\pi(p)$ at $t=t_0$
  - Another interpolating operator for the final state $\pi(p')$ at $t=t_1$
  - Current insertion V_μ in the middle t.
  - Spatial momentum inserted at two operators.
- (sequential) source method
  - Calculate a quark propagator starting from a previous quark propagator at t.

$$(\not{D}+m)S_2(x) = e^{i\mathbf{q}.\mathbf{x}}\Gamma S_1(x)\delta(x_0 - t)$$

Lattice calculation of 3pt function

- At large enough time separations $\Delta t = t - t_0$, $\Delta t' = t_1 - t$, the ground state pions dominate. Extra factors can be taken off with 2pt functions.



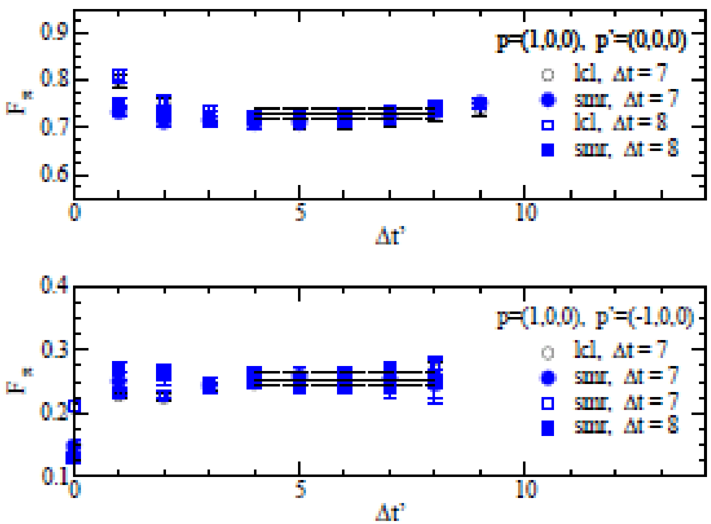
$$R(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') = \frac{C^{\pi\pi}_{\mu,\mathrm{smr},\mathrm{smr}}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{C^{\pi}_{\mathrm{smr},\mathrm{lcl}}(\Delta t; \mathbf{p})\, C^{\pi}_{\mathrm{lcl},\mathrm{smr}}(\Delta t'; \mathbf{p}')}$$

- Then, consider a ratio

$$F_\pi(q^2) = \frac{2M_\pi}{E(p) + E(p')} \frac{R^{\pi\pi}_{\mu,\phi,\phi'}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{R^{\pi\pi}_{4,\phi,\phi'}(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0})}$$

A recent calculation

JLQCD (2007)
dynamical overlap (Nf=2)
(talk by Kaneko at lat07)



- Lattice signal
  - Look for a plateau, where the ground state pion dominates.
  - Noisier for larger pion momentum.
  - Note:
    - The actual data were obtained using the all-to-all technique, so that the data points at different t_0,t,t_1 and different momentum combinations can all be averaged.

All-to-all

To improve the signal

- Usually, the quark propagator is calculated with a fixed initial point (one-to-all)
- Average over initial point (or momentum config) will improve statistics; possible with all-to-all

$$D^{-1}(x,y) = \sum_{k=1}^{N_{ev}} \frac{1}{\lambda^{(k)}} u^{(k)}(x) u^{(k)\dagger}(y) + \sum_{d=1}^{N_d} \left[D_{high}^{-1} \eta^{(d)} \right](x) \eta^{(d)}(y)$$

Low mode contribution

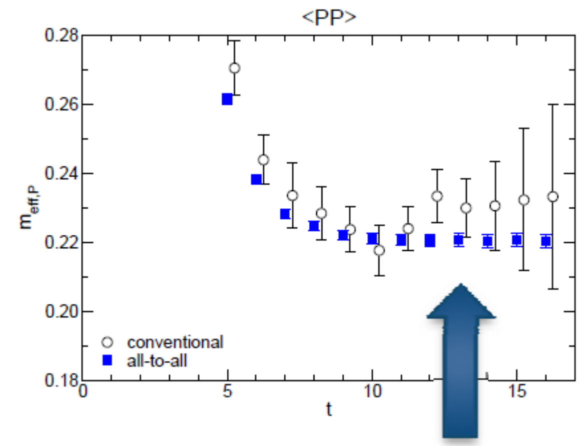
High mode propagation
From the random noice

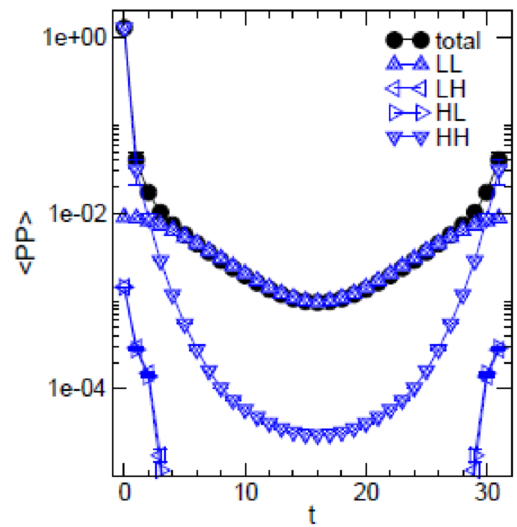
Random noise

An example: two-point func

Dramatic improvement of the signal, thanks to the averaging over source points

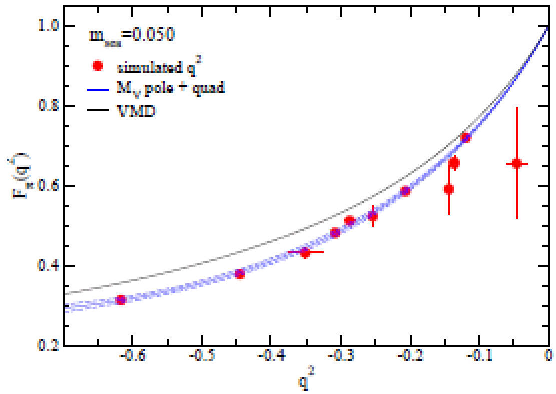
- Similar to the low mode averaging; but all-to-all can be used for any n-point func.
- PP correlator is dominated by the low-modes



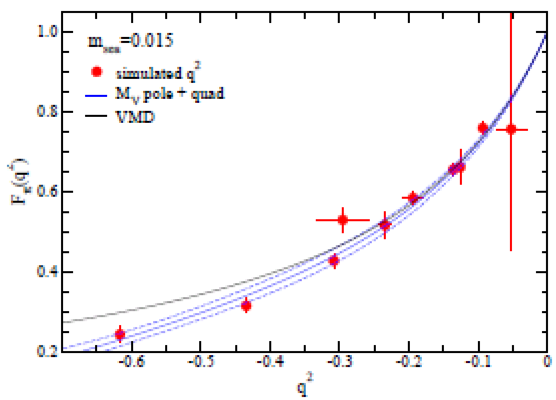


Form factor at a glance

$m_q \sim m_s/2$



$m_q \sim m_s/6$



- All-to-all ⇒ many momentum combinations
  - (1,0,0) → (0,1,0), etc. in units of $2\pi/L$.
- q^2 dependence well approximated by a vector meson pole + corrections

$$F_\pi(q^2) = \frac{1}{1 - q^2/m_V^2} + c_1 q^2 + \ldots$$

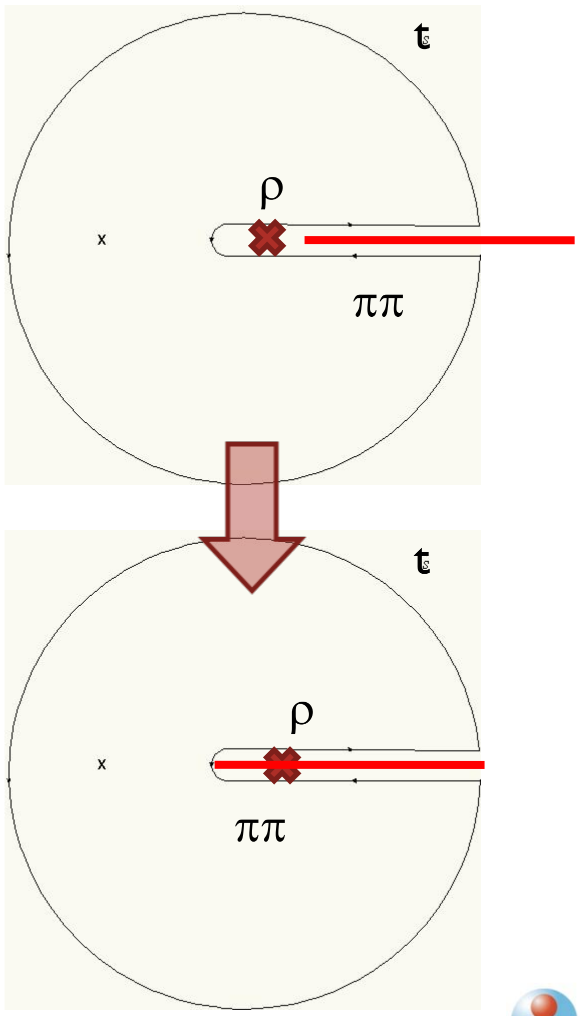
with m_V obtained at the same quark mass.

Analyticity

- Vector meson dominance: a result of the analyticity.

$$F(q^2) = \frac{1}{2\pi i}\oint dt \frac{F(t)}{t-q^2} = \frac{1}{\pi}\int_{t_0}^{\infty} dt \frac{\operatorname{Im} F(t)}{t-q^2}$$

  - In the heavier quark mass region, ρ meson is a nearest isolated pole. ππ is subleading.
  - For the physical quark mass, ππ is nearest. ρ is a part of ππ (broad resonance).



Fit forms

- Pole ansatz

$$F_\pi(q^2) = \frac{1}{1 - q^2 / m_{pole}^2}$$

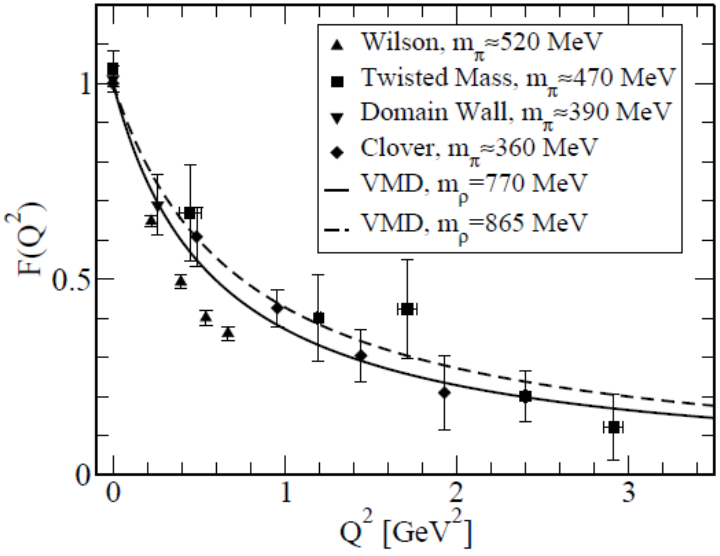
  - Fit with m_{pole} a free parameter
  - Not consistent with the analyticity.

- Fixed pole +

$$F_\pi(q^2) = \frac{1}{1 - q^2 / m_V^2} + c_1 q^2 + \ldots$$

  - Pole mass from the vector meson; model other effects by polynomials

Quenched results compiled in Abdel-Rehim, Lewis, PRD71, 014503 (2005)

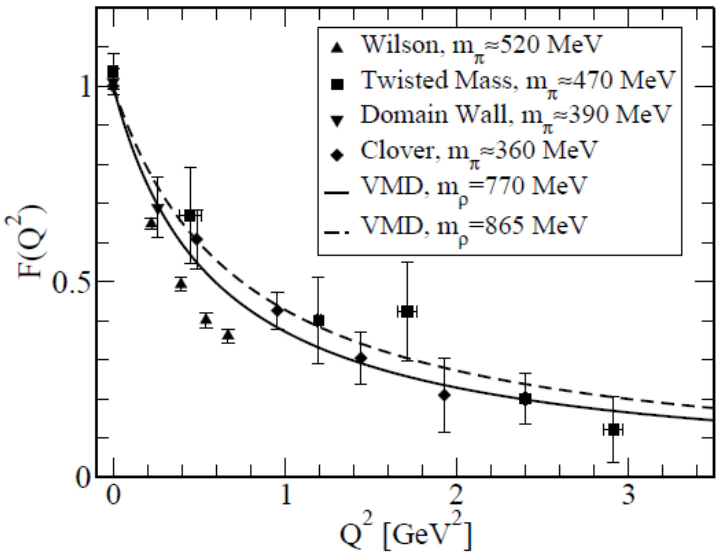


Fit form doesn't matter when the error is large.

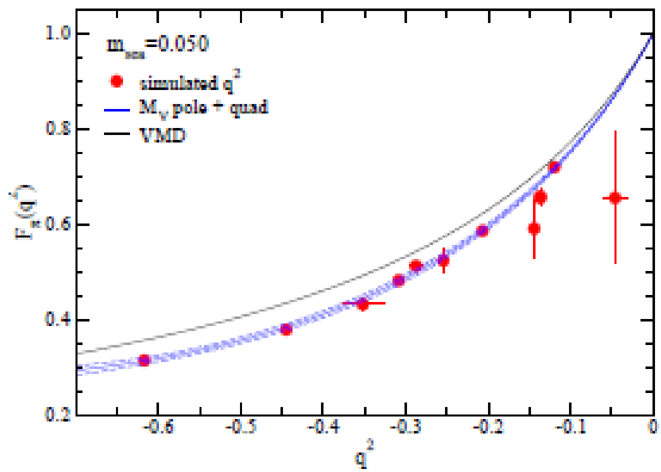
Quenched references

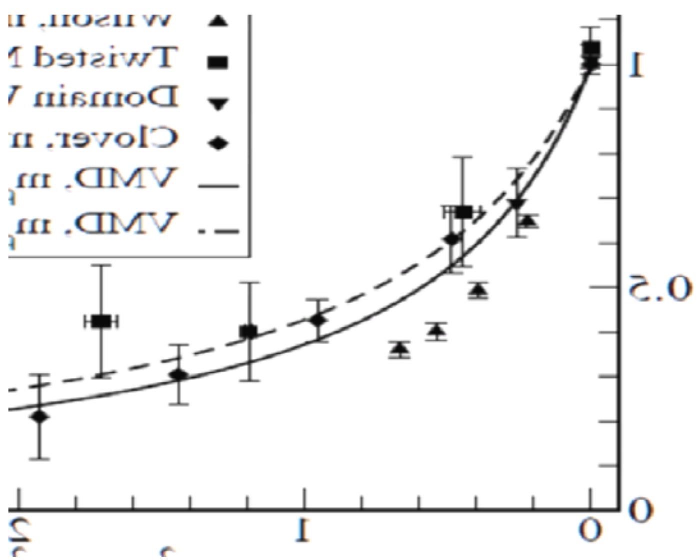
- Bonnet et al. [LHP collaboration], PRD72, 054506 (2005): Wilson.
- Van der Heide et al., PLB556, 131 (2003): clover.
- Nemoto [RBC collaboration], Lattice 2003: domain-wall.
- Abdel-Rehim, Lewis, PRD71, 014503 (2005): twisted mass.

Quenched results compiled in Abdel-Rehim, Lewis, PRD71, 014503 (2005)



Precise unquenched data





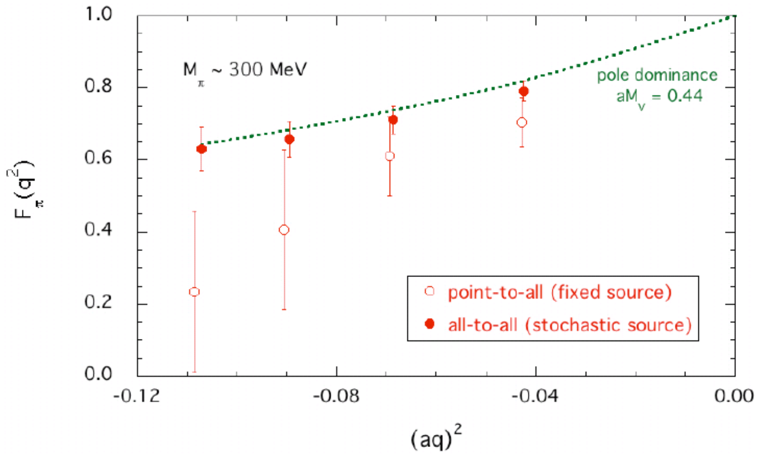
New JLQCD data

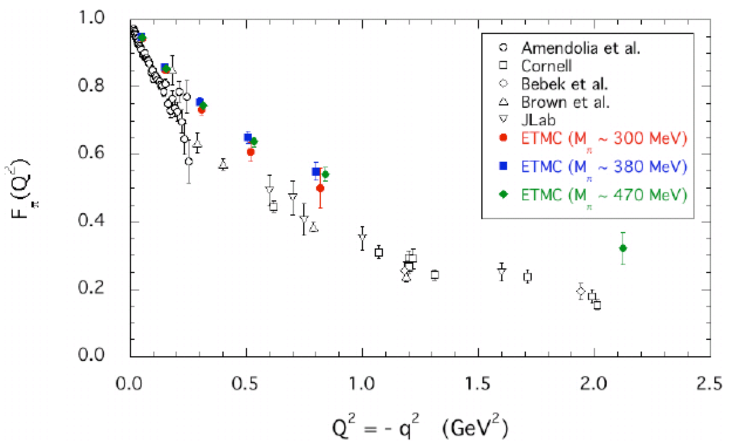
- All-to-all
  - improves statistics
  - increases data points without extra cost
- Pole fit tested
  - Correction to the single pole is visible.

ETMC

Similar improvement observed by ETMC (Simula at Lattice 2007)

- Nf=2 twisted mass fermion
- All-to-all
  - But without the low-mode projection
- twisted boundary condition
  - Momentum smaller than $2\pi/L$ accessible



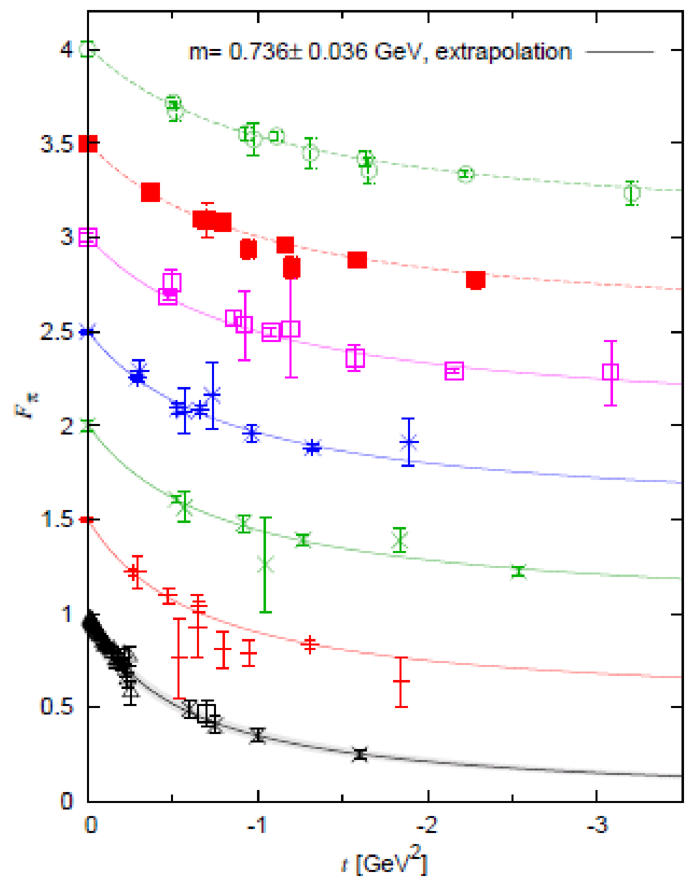


QCDSF

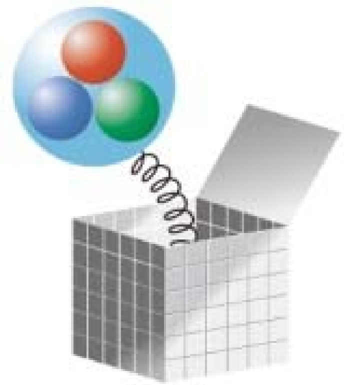
Extensive data with Nf=2 O(a)-improved Wilson fermion

(Brommel et al, Lattice 2005; Brommel et al, EPJC51, 335 (2007).)

- Data at several lattice spacings.
- Lightest sea quark corresponds to M_{π}=400 MeV.
- Data fitted to the pole ansatz.



3. Testing chiral log with pion form factor

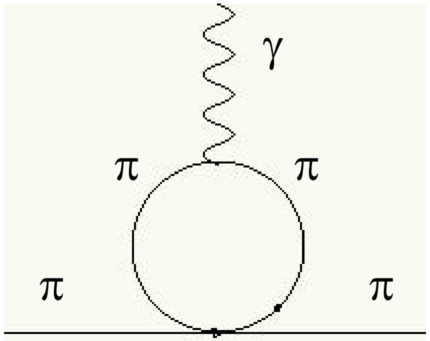


Chiral extrapolation

- χPT predicts χlog

$$\langle r^2 \rangle_V^\pi = -\frac{1}{(4\pi f_\pi)^2}\left[\ln\frac{m_\pi^2}{\mu^2} + 12(4\pi)^2 L_9 + O(m_\pi^2)\right]$$

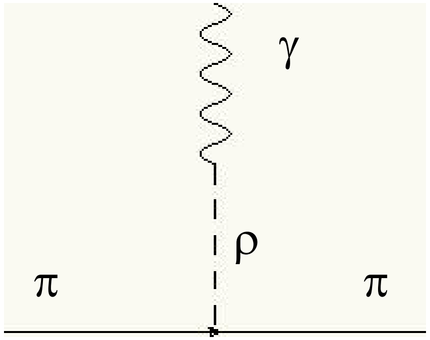
  - Must diverge in the chiral limit: pion cloud gets larger.
  - Valid only in the region where $2m_\pi < m_\rho$.



- VMD

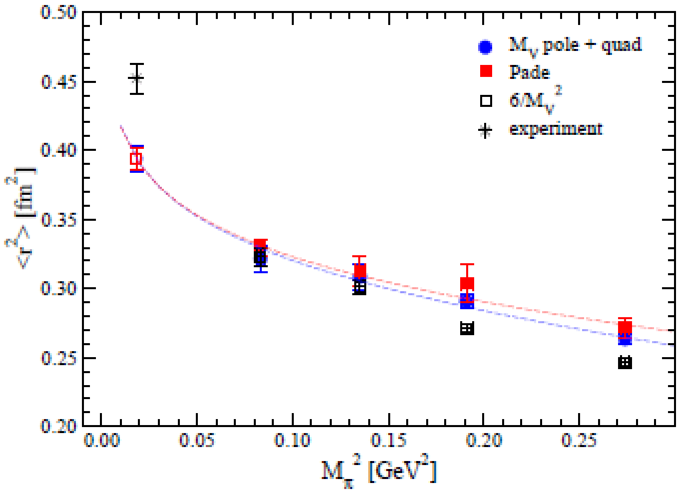
$$\langle r^2 \rangle_V^\pi = 6/m_V^2$$

  - Mass dependence not well understood.



Chiral extrapolation

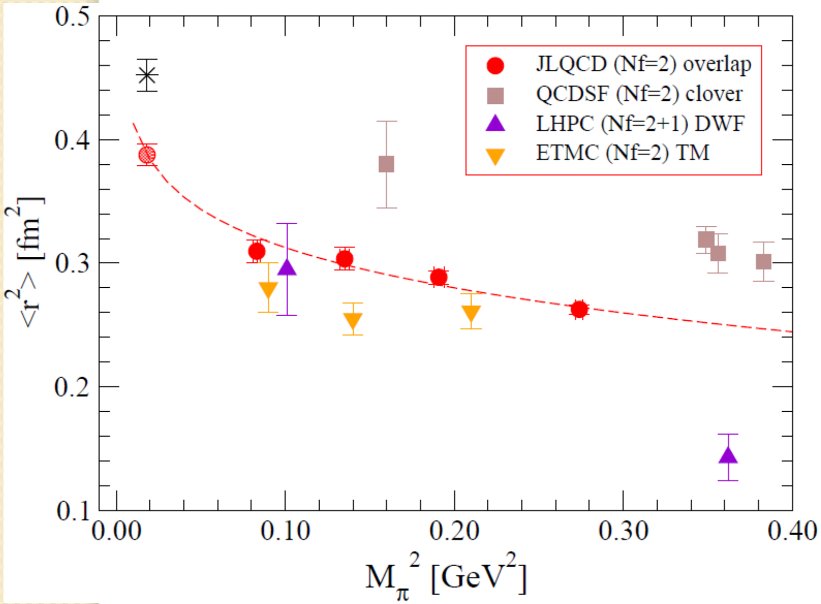
- Lattice data
  - Mass dependence very similar to VMD, but the difference is visible.
  - χlog may become significant beyond the region of lattice data.
  - Assuming the NLO χlog and NNLO analytic term.



$$\left\langle r^2 \right\rangle_V^\pi = 0.388(9)(12)\ \mathrm{fm}^2$$

Lower than the exp number, even after the chiral enhancement.

Comparison (unquenched)



ETMC points read off from the slide by Simula: my fault if it's not precise.

- Recent unquenched calculations
  - Poor agreement: due to different fit ansatz?
  - QCDSF seems consistent with the exp number.
  - χlog not clear: how to distinguish χlog from the mass dependence of $6/m_V^2$?

Discussions

- Systematic errors to be considered.
  - Finite volume effect known only at NLO.

- To ensure theoretical consistency, need a framework including the vector resonance.
  - Resonance χPT
  - Hidden local symmetry

- Scalar form factor would be less problematic (scalar pole is far), but contains disconnected loop. Calculation is on-going (JLQCD).