In medium hadronic properties:

QCD in extreme conditions

Chris Allton

Swansea University

Overview

	$T = \mu = 0$
quarks are	confined
accuracy of predictions:	< 5%
has similarities with:	atomic physics (bound states)
fundamental properties:	*masses & transition mx els*

Overview

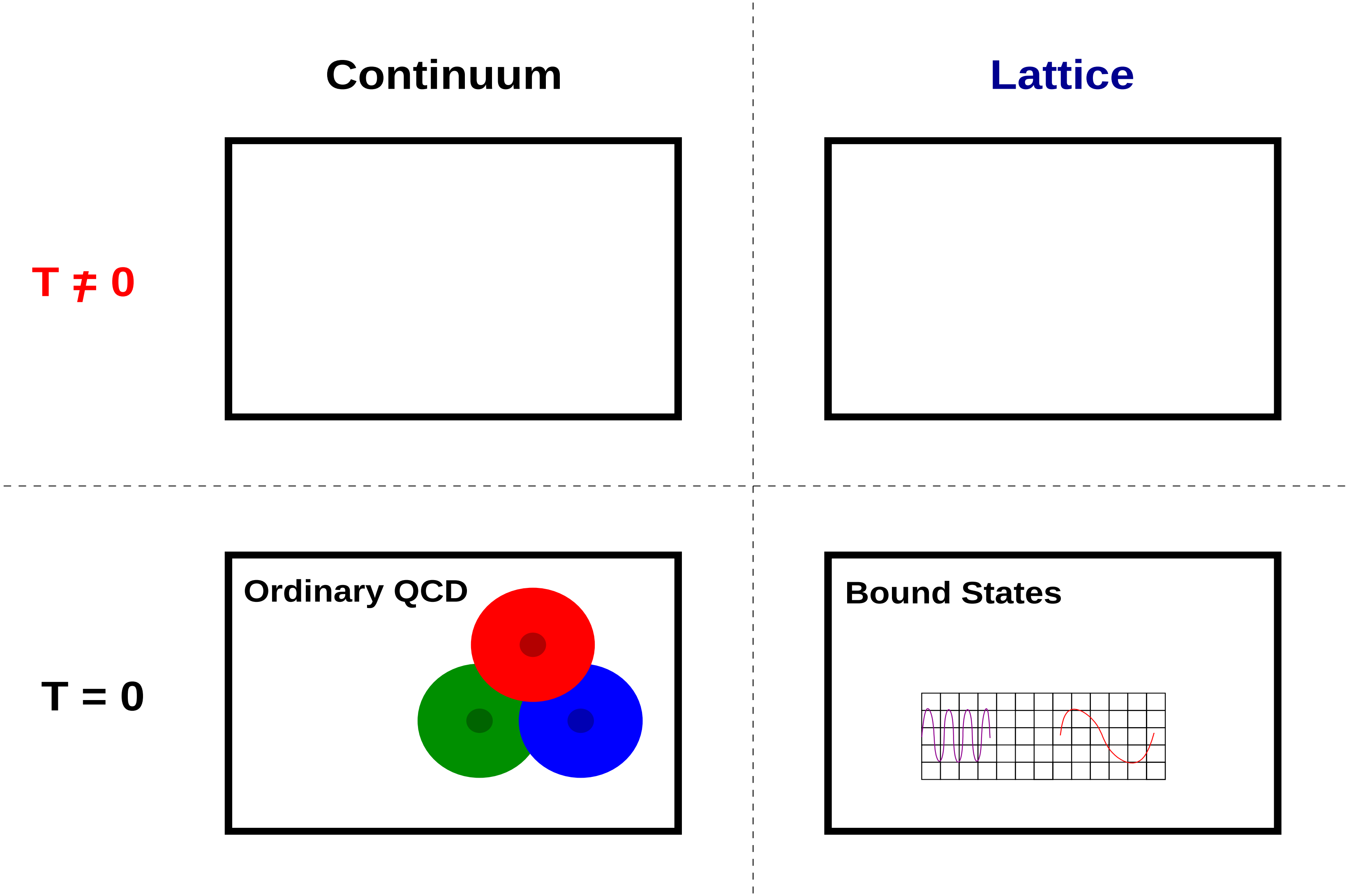
	$T = \mu = 0$	T or $\mu \neq 0$
quarks are	confined	*de-confined*
accuracy of predictions:	< 5%	~ 20%
has similarities with:	atomic physics (bound states)	plasma/fluid (spectral functions)
fundamental properties:	*masses & transition mx els*	*pressure, transport coefficients*

Overview

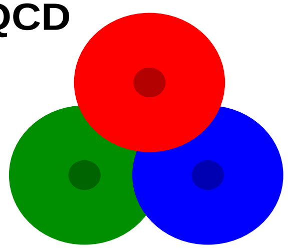
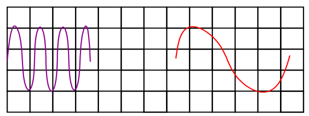
	$T = \mu = 0$	T or $\mu \neq 0$
quarks are	confined	*de-confined*
accuracy of predictions:	< 5%	~ 20%
has similarities with:	atomic physics (bound states)	plasma/fluid (spectral functions)
fundamental properties:	*masses & transition mx els*	*pressure, transport coefficients*

- Need to change our preconceptions...

$T = 0$

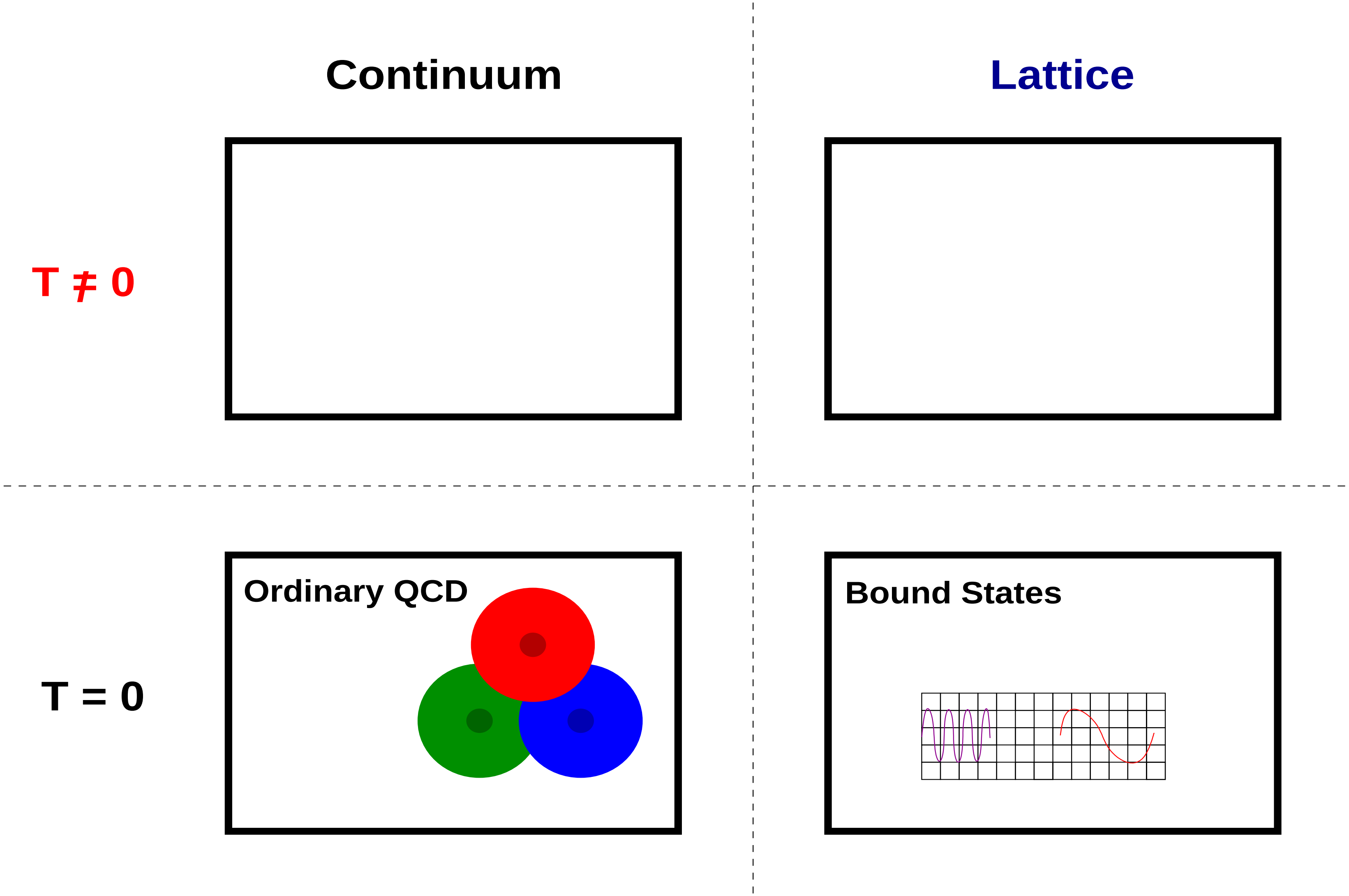


$T = 0$

	Continuum	Lattice
T ≠ 0		
T = 0	Ordinary QCD 	Bound States 

perturbation theory?

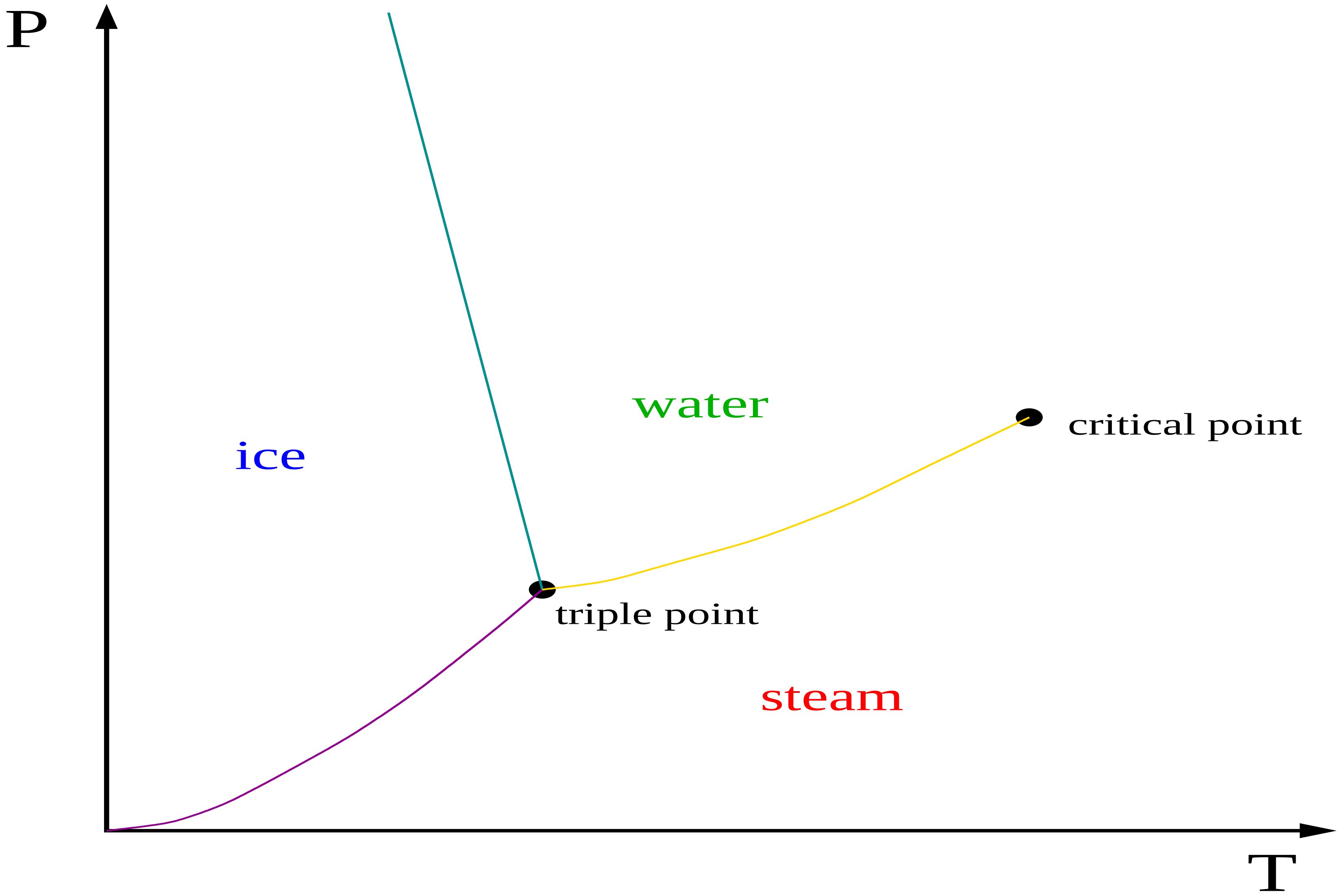
$T = 0$



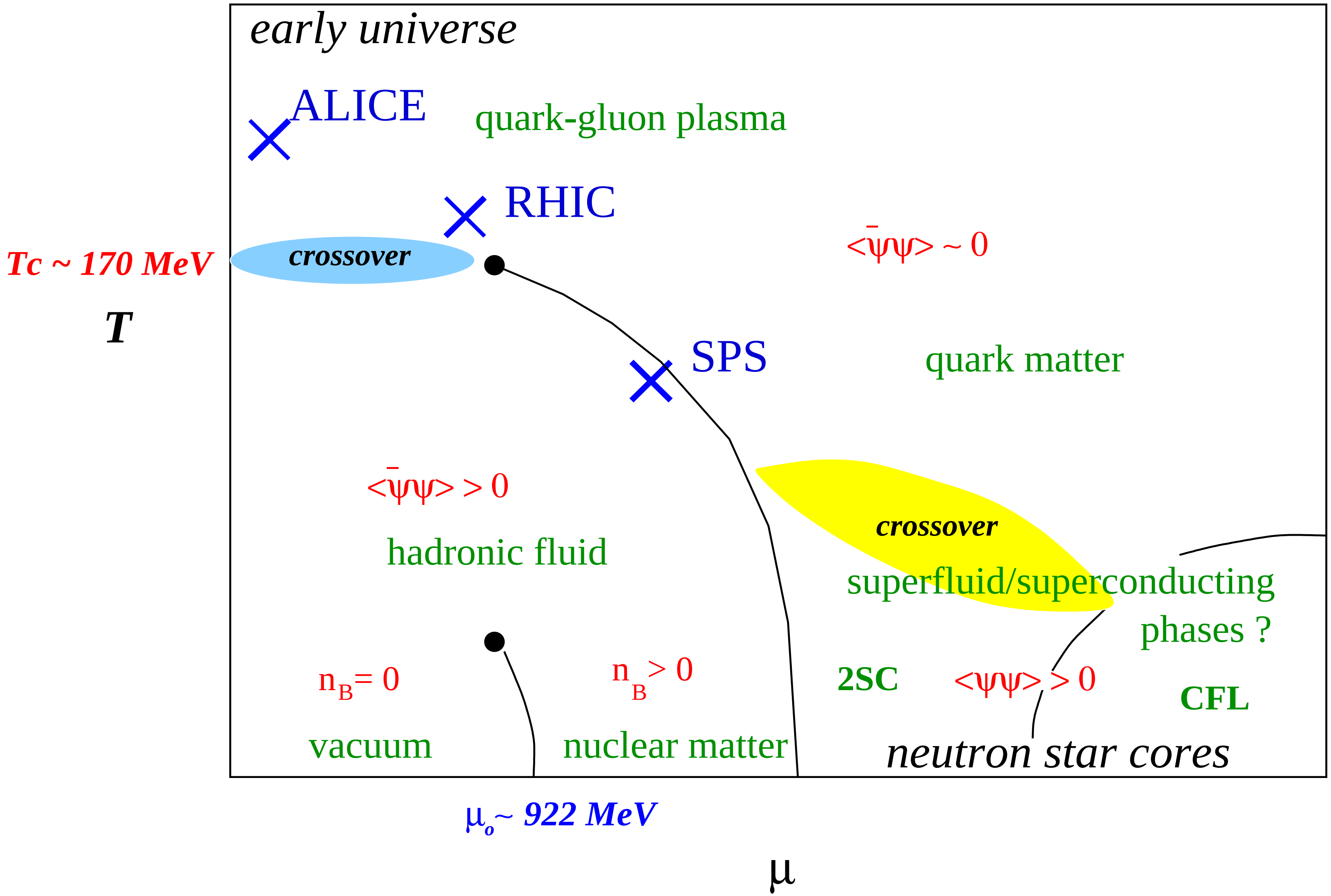
perturbation theory?

2-point correlators

H_2O phase diagram

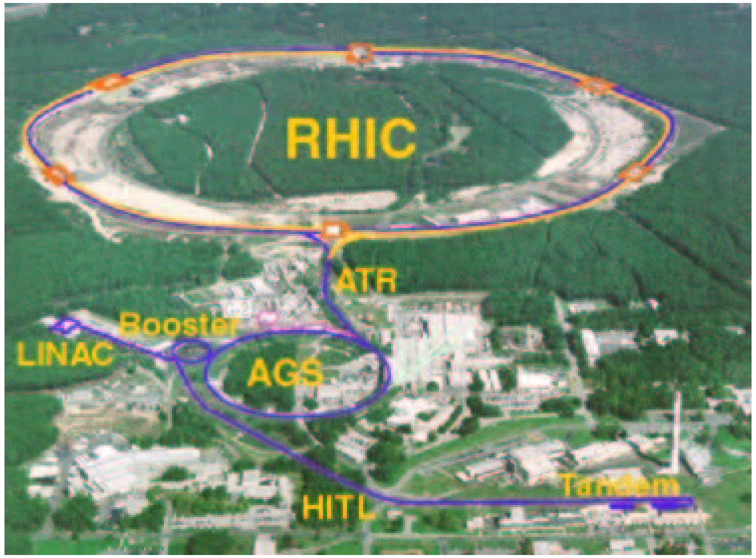


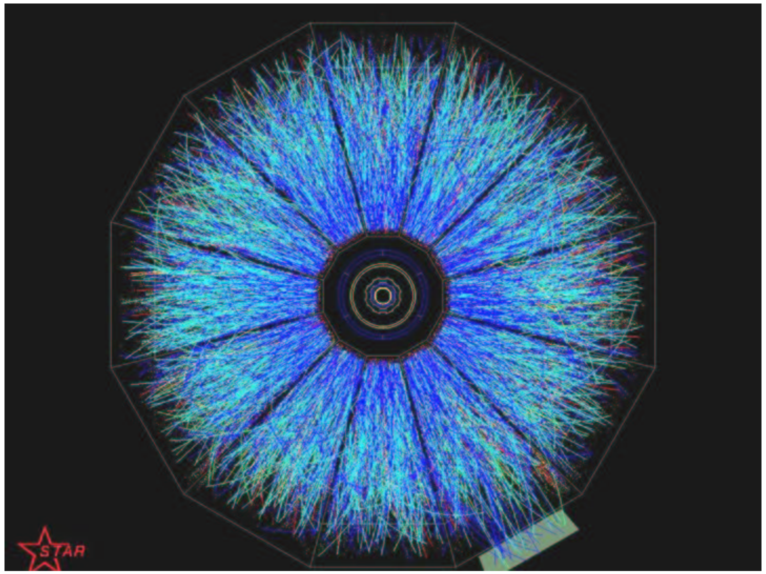
QCD phase diagram



Experiments of QCD at $T \neq 0$

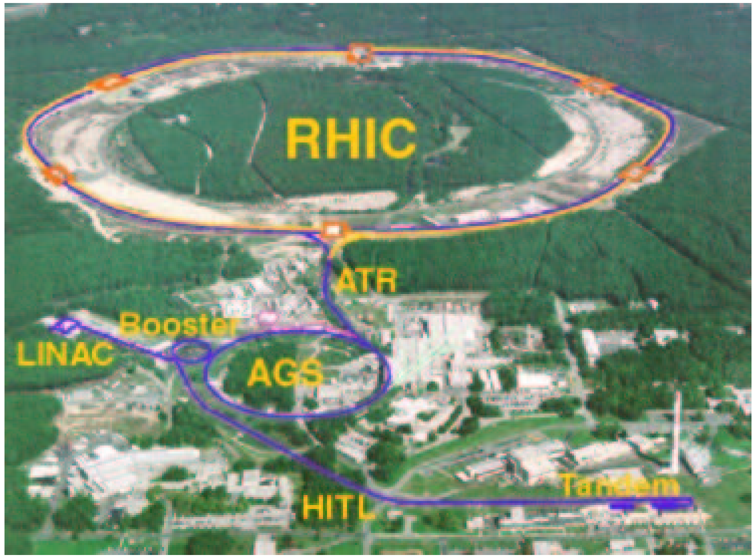
RHIC Experiment @ BNL

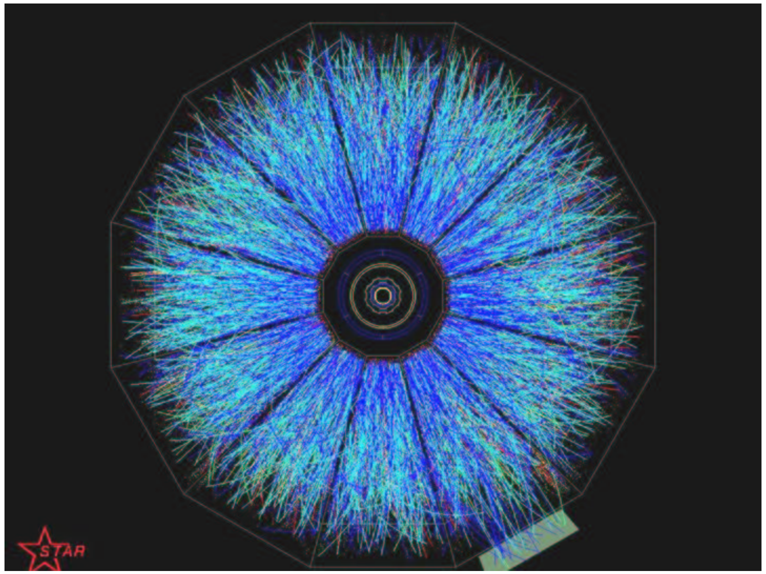




Experiments of QCD at $T \neq 0$

RHIC Experiment @ BNL





- First thought that quarks and gluons virtually free
- Now known that they are strongly interacting
  - *Strongly Coupled Quark-Gluon Plasma*

Hadronic states/resonances

- $T = 0 \quad \rightarrow$ Bound states (hadrons)
- $T \neq 0 \quad \rightarrow$ resonances/melted states?

J/ψ suppression [Matsui and Satz 1986]

i.e. $c - \overline{c}$ pairs created early in collision can move apart before being able to form J/ψ

Motivates the study of hadronic states/resonances

Qualitative features of QCD at $T \neq 0$

- *NOT* weakly coupled...

- Very low viscosity
  - symptom of finite coupling

Qualitative features of QCD at $T \neq 0$

- *NOT* weakly coupled...

- Very low viscosity
  - symptom of finite coupling

Qualitative features of QCD at $T \neq 0$

- *NOT* weakly coupled...

- Very low viscosity
  - symptom of finite coupling

Qualitative features of QCD at $T \neq 0$

- *NOT* weakly coupled...
- Very low viscosity
  - symptom of finite coupling

RHIC serves the Perfect Fluid

ig-Noble aside

2005 Ig-Nobel Prize for Physics awarded to the “Pitch Drop” experiment by:

Profs. Mainstone and Parnell from the University of Queensland, Brisbane, Australia.

Pitch has viscosity 10^{11} times water’s...

ig-Noble aside

2005 Ig-Nobel Prize for Physics awarded to the "Pitch Drop" experiment by:

Profs. Mainstone and Parnell from the University of Queensland, Brisbane, Australia.

Pitch has viscosity 10^{11} times water's...

Quantitative features of QCD at $T \neq 0$

- Weak coupling [Arnold, Moore and Yaffe]:

$$\eta/s \sim 1/g^4$$

i.e. predicts large η *(shear viscosity)* (s =*entropy density*)

- $\mathcal{N} = 4$ SYM $\Leftrightarrow$ AdS$_5 \times$S^5 [Son, Starinets, Policastro, Kovtun]

$$\eta/s \geq \frac{1}{4\pi}, \qquad N_c, \;\; g^2 N_c \to \infty$$

i.e. predicts small η

(Conjectured lower bound for all matter)

Quantitative features of QCD at $T \neq 0$

- Weak coupling [Arnold, Moore and Yaffe]:

$$\eta/s \sim 1/g^4$$

i.e. predicts large η *(shear viscosity)* (s =*entropy density*)

- $\mathcal{N} = 4$ SYM $\Leftrightarrow$ AdS$_5 \times$S^5 [Son, Starinets, Policastro, Kovtun]

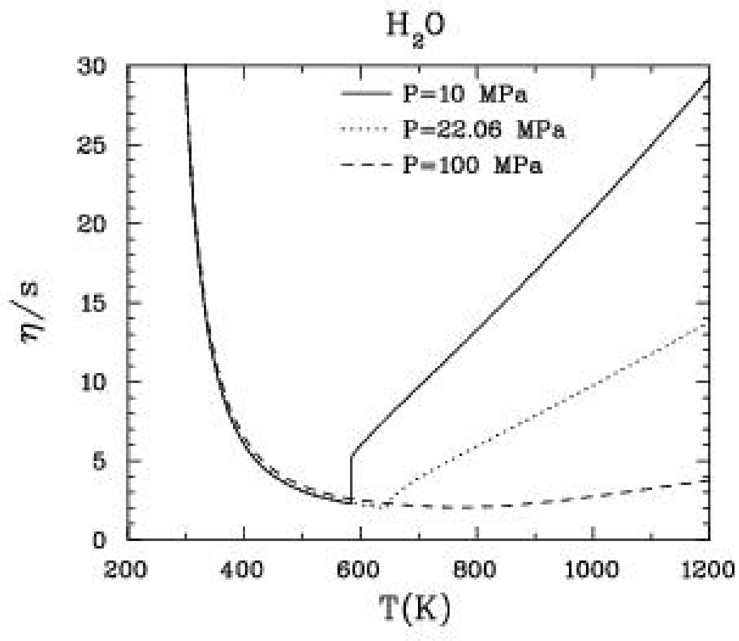
$$\eta/s \geq \frac{1}{4\pi}, \qquad N_c, \;\; g^2 N_c \to \infty$$

i.e. predicts small η

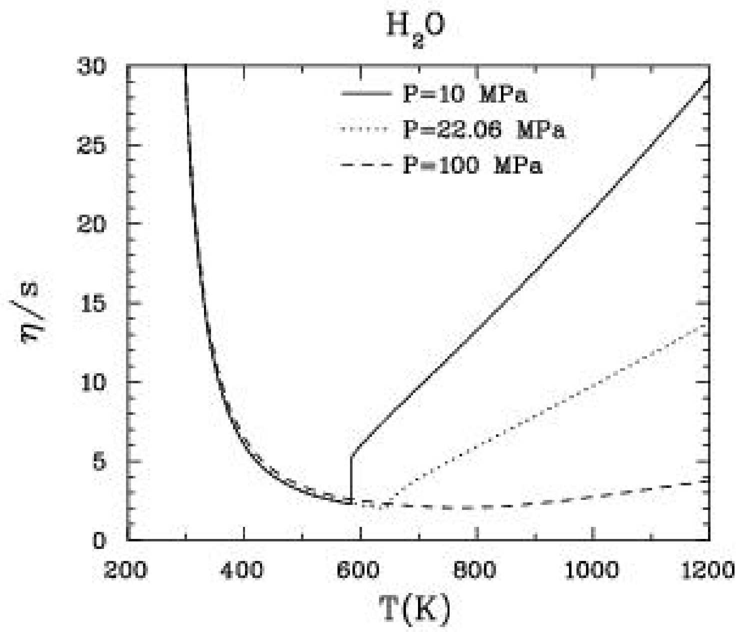
(Conjectured lower bound for all matter)

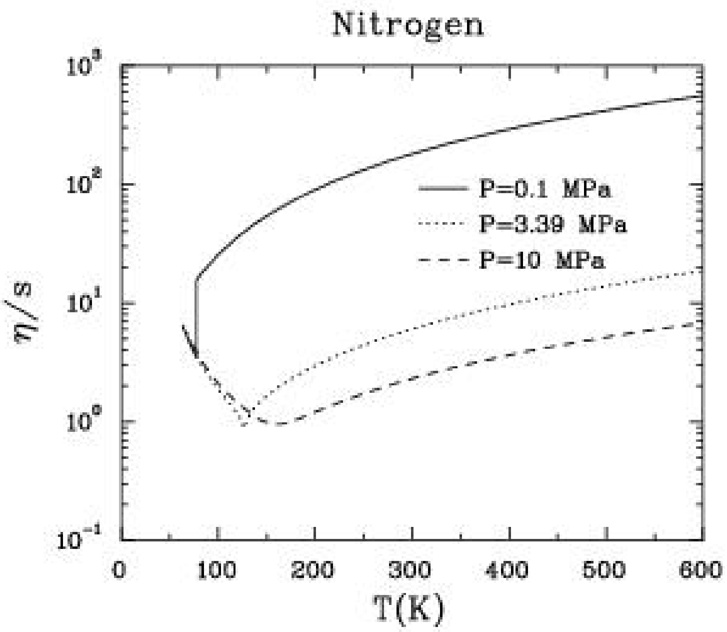
Finally string theory makes contact with nature...

Physical values for η: [Csernai, Kapusta, McLerran, 2006]

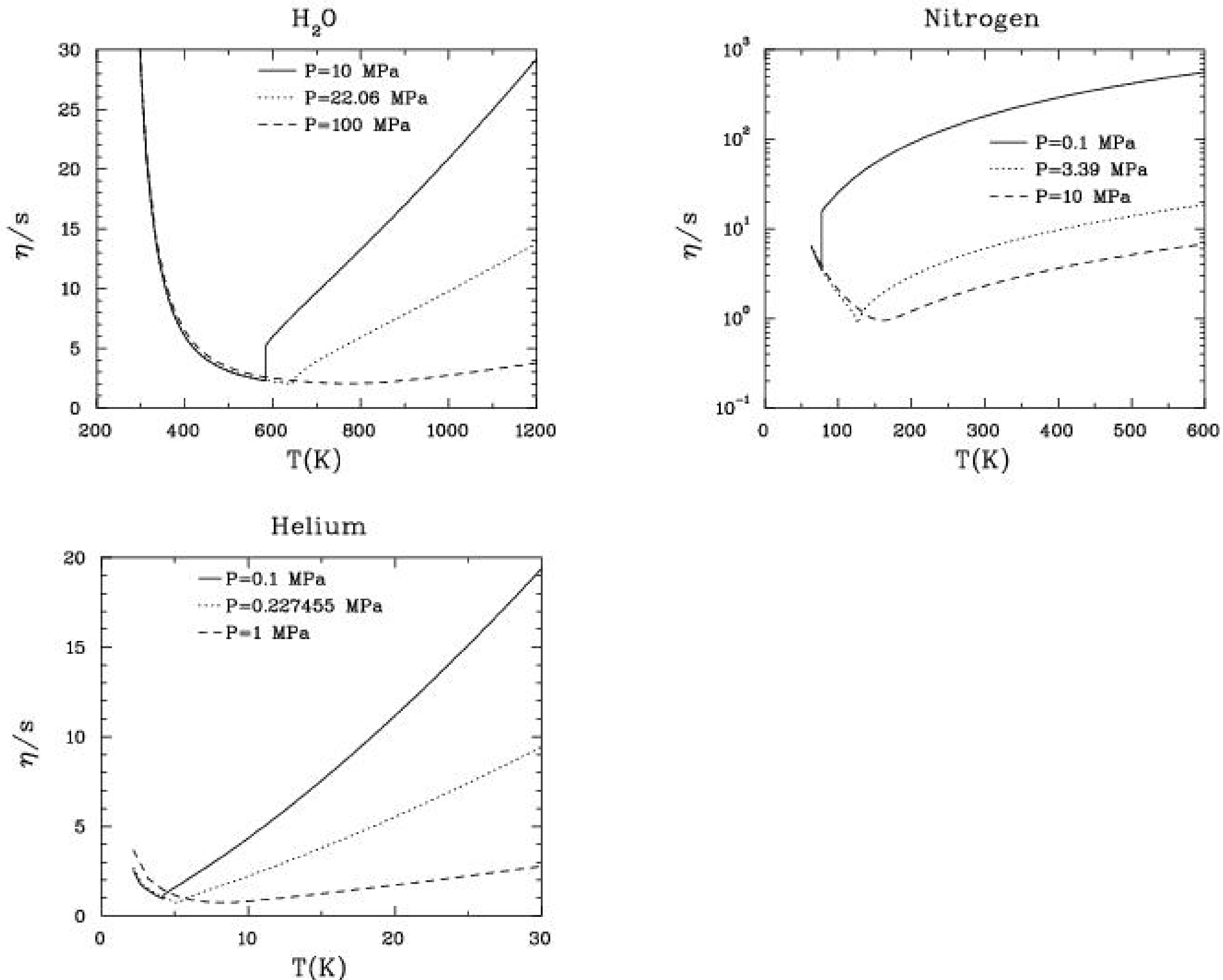


Physical values for η: [Csernai, Kapusta, McLerran, 2006]

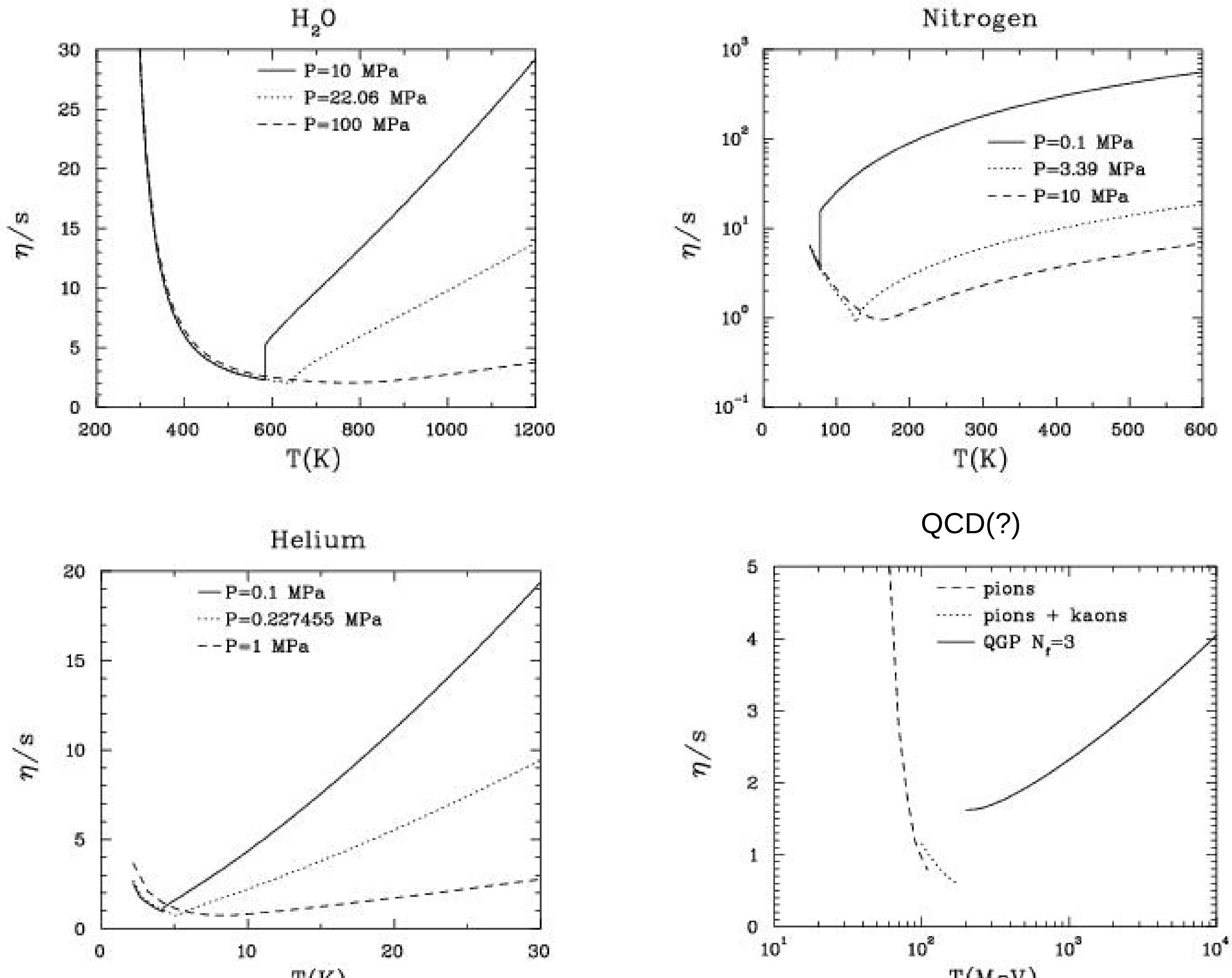




Physical values for η: [Csernai, Kapusta, McLerran, 2006]



Physical values for η: [Csernai, Kapusta, McLerran, 2006]



Other transport coefficients

transport coefficients from behaviour of *spectral functions* at $\omega \to 0$.

- shear viscosity η
  - *off-diagonal* energy-momentum tensor
    - (gluonic correlators)
- bulk viscosity ζ
  - *diagonal* energy-momentum tensor
    - (gluonic correlators)
- electrical conductivity σ
  - vector correlators
    - $(\bar{\psi}\gamma_i\psi)$

$\eta, \zeta, \sigma \sim$ *LOW ENERGY CONSTANTS*

Kubo Relations

electrical conductivity: $\sigma = \lim_{\omega \to 0} \frac{\rho_{x,x}(\omega)}{2\omega}$

shear viscosity: $\eta = \lim_{\omega \to 0} \frac{\rho_{xy,xy}(\omega)}{2\omega}$

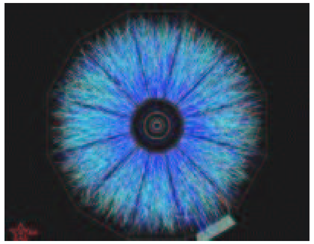
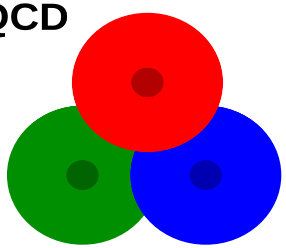
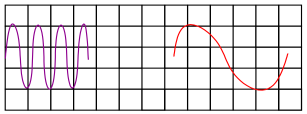
spectral densities: $\rho_{\mu\nu}(\omega) = \int d^4x \, \langle [j_\mu(x), j_\nu(0)] \rangle_{\text{eq}}$

$$\rho_{\mu\nu,\sigma\tau}(\omega) = \int d^4x \, \langle [T_{\mu\nu}(x), T_{\sigma\tau}(0)] \rangle_{\text{eq}}$$

where $j_\mu = \overline{\psi}\gamma_\mu\psi =$ EM current, $j_\mu = \overline{\psi}\gamma_\mu\psi =$ EM current

i.e. transport coefficients $\sim$ intercept of $\rho(\omega)/\omega$ at $\omega = 0$

Summary - QCD at $T \neq 0$

	Continuum	Lattice
$T \neq 0$	Extreme QCD 	
$T = 0$	Ordinary QCD 	Bound States 

- cores of neutron stars & early universe physics

Lattice input

Both:

- $\omega \to 0$ physics (transport coefficients)

and

- $\omega \neq 0$ physics (hadronic resonances)

are intrinsically non-perturbative and can be addressed by the lattice.

Lattice input

Both:

- $\omega \to 0$ physics (transport coefficients)

and

- $\omega \neq 0$ physics (hadronic resonances)

are intrinsically non-perturbative and can be addressed by the lattice.

Spectral functions can answer both:

- Do hadronic states persist in “quark-gluon” plasma phase?

- What are the transport coefficients?

Spectral Functions

Euclidean Correlator $\downarrow$ (Lattice) Kernel $\downarrow$

$$G(t,\vec{p}) \;=\; \int \rho(\omega,\vec{p})\; K(t,\omega)\; d\omega$$

Spectral Functions

Euclidean Correlator $\downarrow$

(Lattice) Kernel $\downarrow$

$$G(t,\vec{p}) = \int \rho(\omega,\vec{p}) \; K(t,\omega) \; d\omega$$

$\downarrow$ Spectral Function

Spectral Functions

Euclidean Correlator $\downarrow$ (Lattice) Kernel $\downarrow$

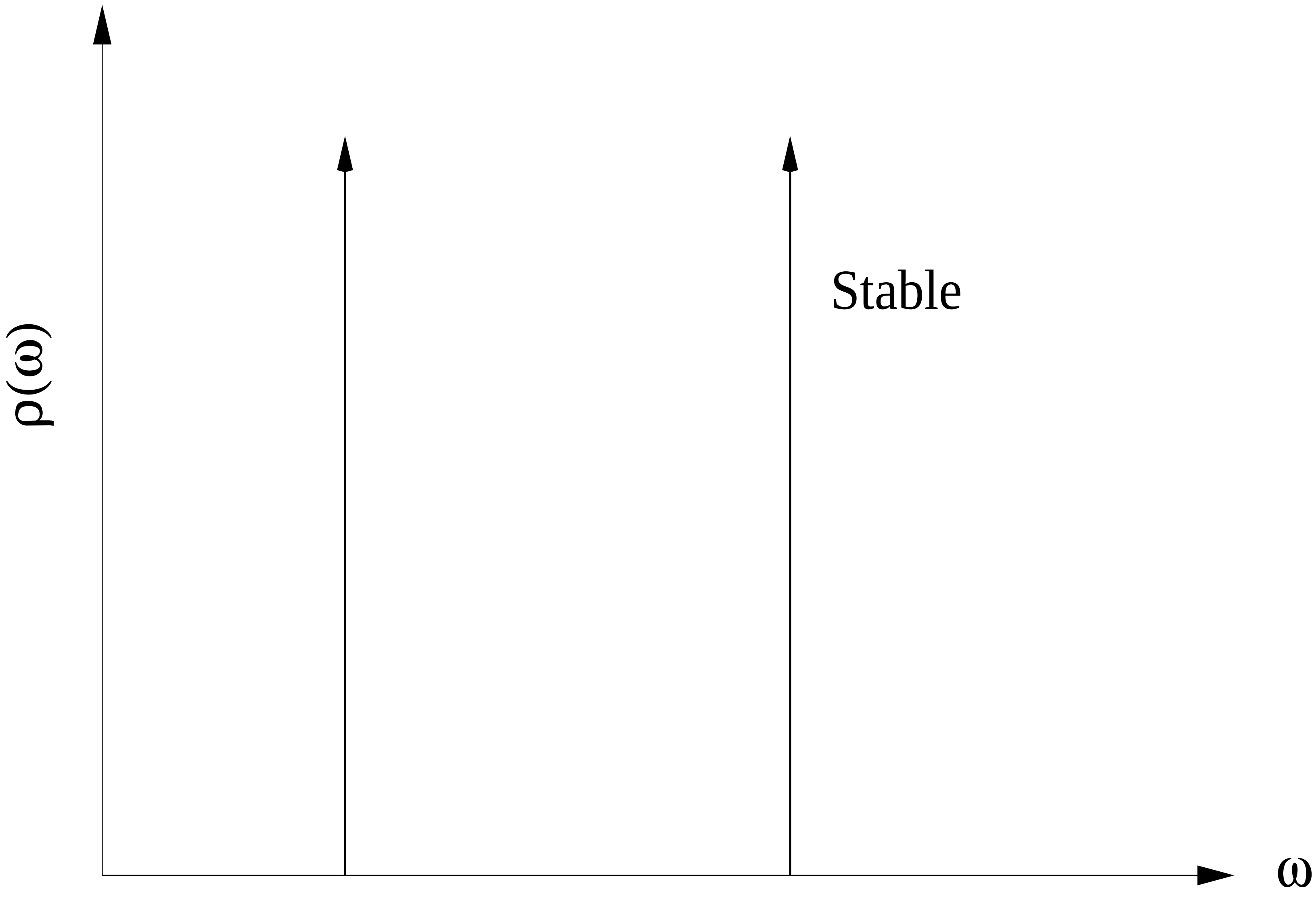
$$G(t,\vec{p}) \quad = \quad \int \rho(\omega,\vec{p})\; K(t,\omega)\; d\omega$$

$\downarrow$ Spectral Function

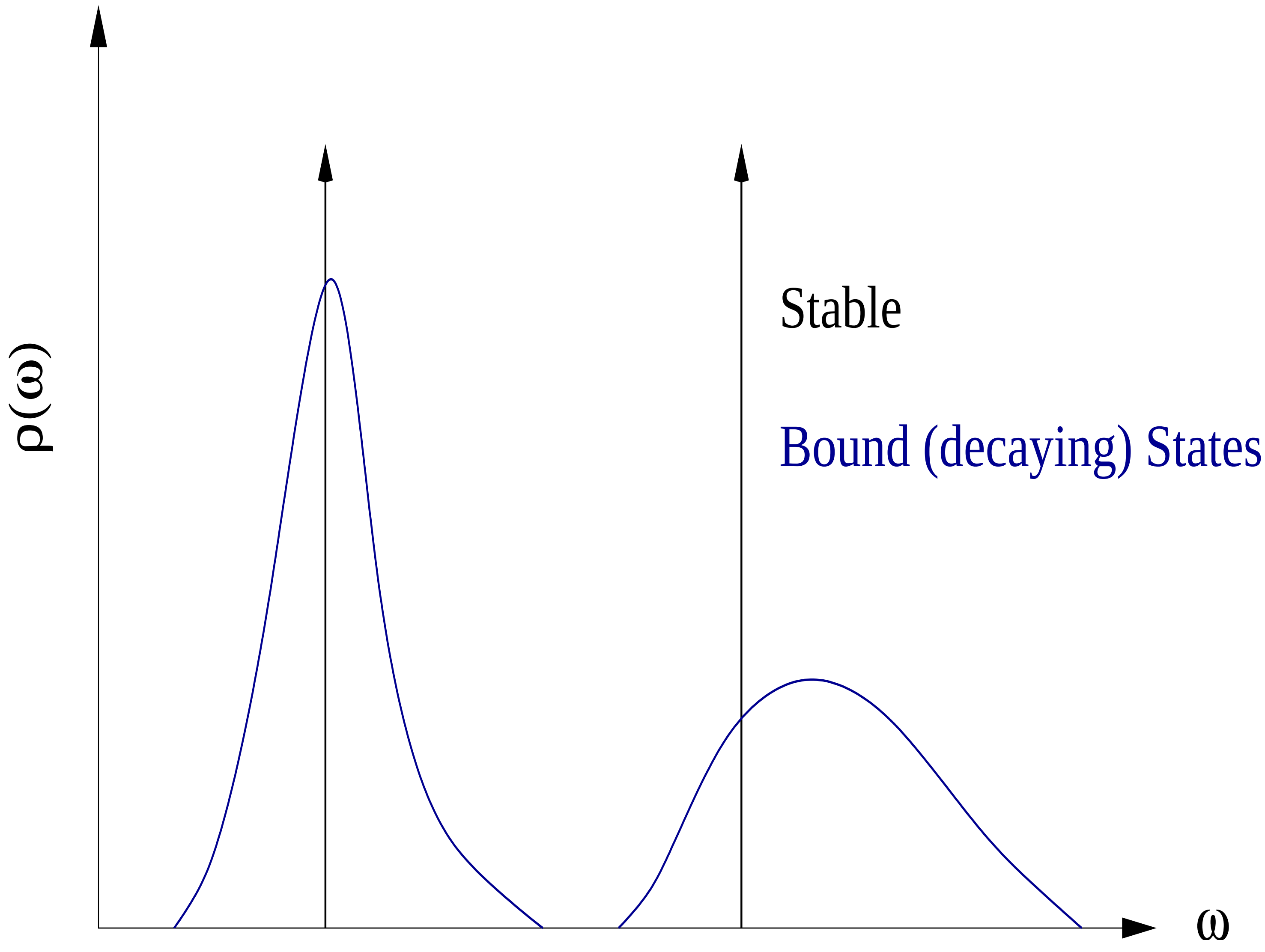
where the (lattice) Kernel is:

$$K(t,\omega) = \frac{\cosh[\omega(t-N_t/2)]}{\sinh[\omega/(2T)]} \sim \exp[-\omega t]$$

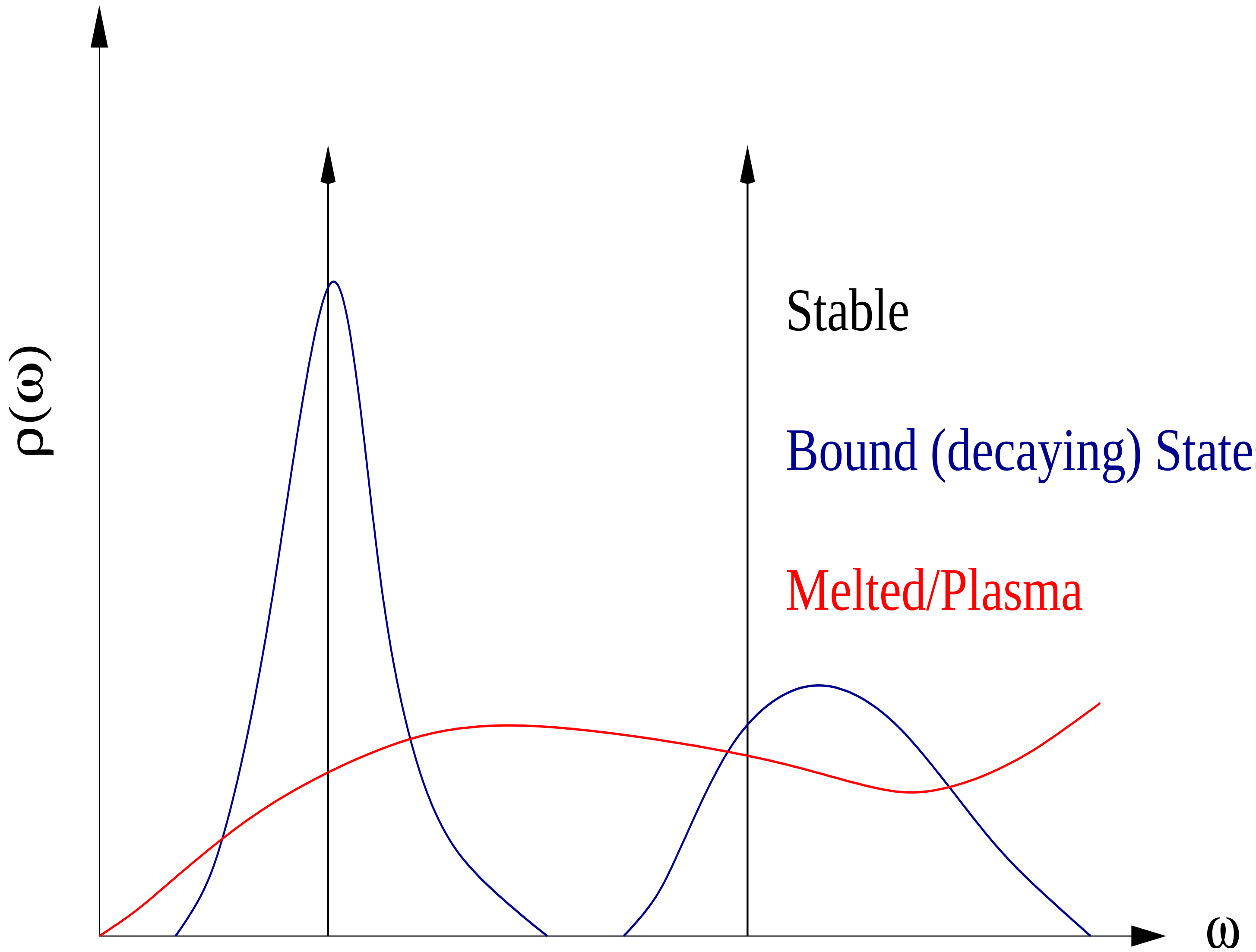
Example Spectral Functions: Stable



Example Spectral Functions: Decaying



Example Spectral Functions: Melted

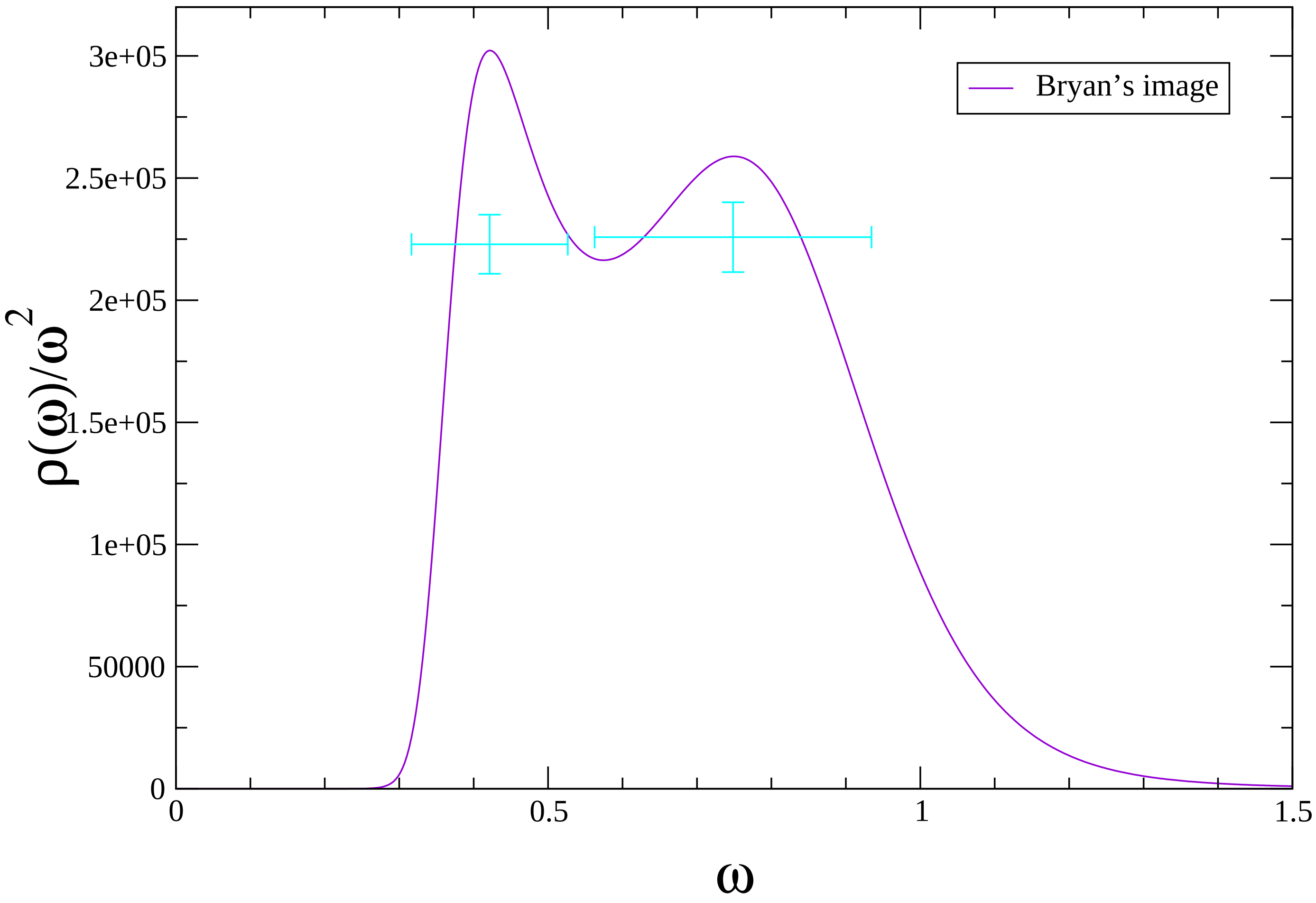


Spectral Functions via MEM

- Extraction of a spectral density from a lattice correlator is an ill-posed problem:
  - *Given* $G(t)$ *derive* $\rho(\omega)$
  - *More* ω *data points then* t *data points!*
  - *uses an SVD of the kernel to obtain working basis*
- Requires the use of Bayesian analysis - Maximum Entropy Method (MEM)
  - $P(\rho) = exp[-(\chi^2(\rho) - \alpha S(\rho))]$
- Takes prior information into account which includes some plausible "default model" for the spectral density
  - *used to normalise the entropy*
- The result of the analysis must be stable under:
  - changes in the default model
  - sensible variations in time window . . .

MEM Orientation

Typical MEM output:

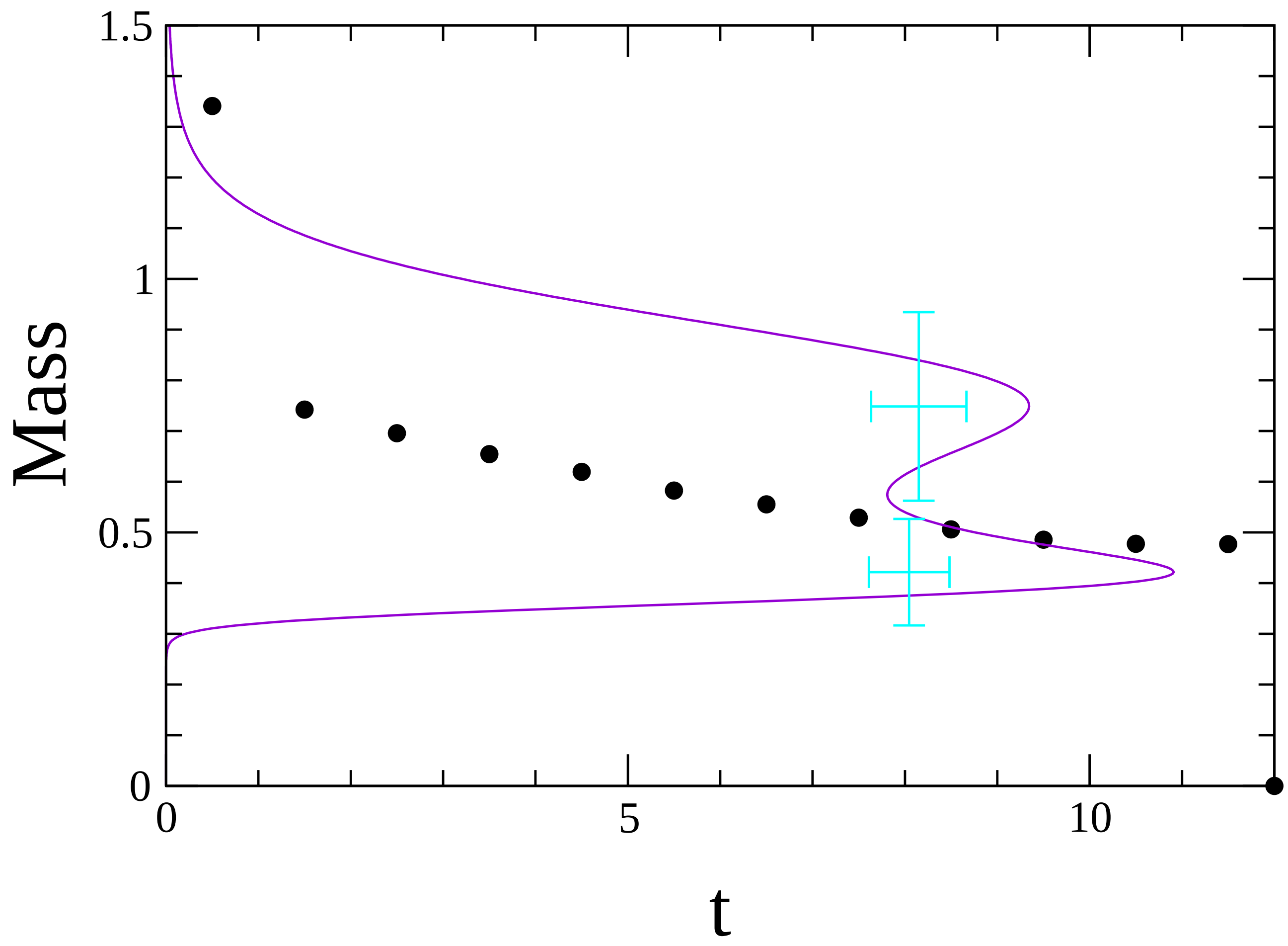


$\omega \to 0 \Longrightarrow$ *transport coefficients*

$\omega \neq 0 \Longrightarrow$ *hadron states(?)*

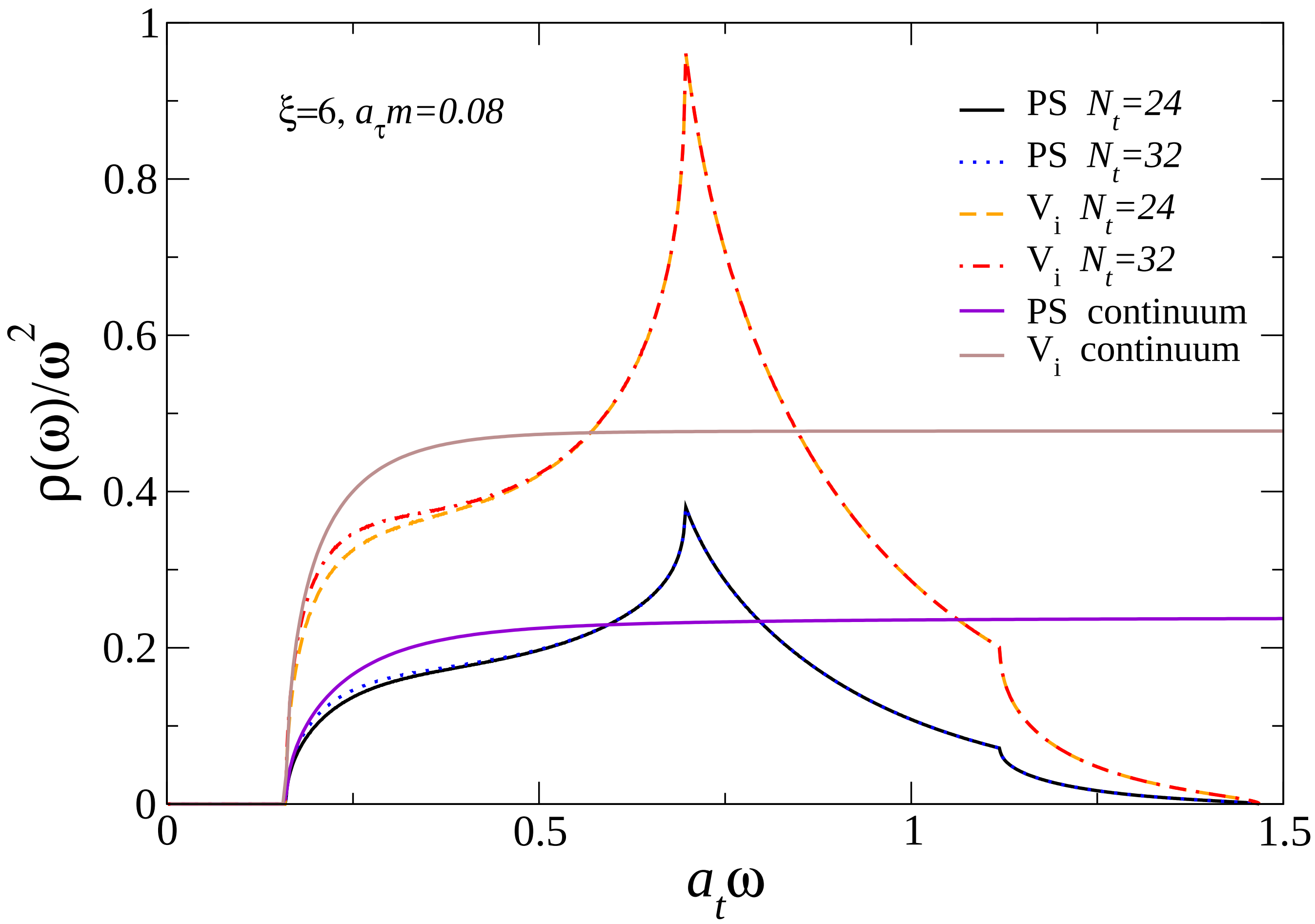
MEM Orientation

Can be superimposed on effective mass plot:



...

Free Lattice Field Theory



cusp: $a_t\omega \sim 0.7$ (lattice artefact!)

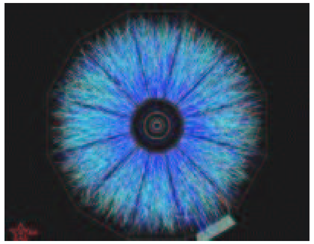
see G. Aarts and J.M. Martinez Resco, hep-lat/0507004

Summary - QCD at $T \neq 0$

Continuum | **Lattice**

T ≠ 0

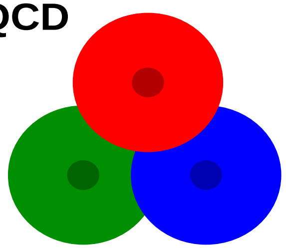
Extreme QCD



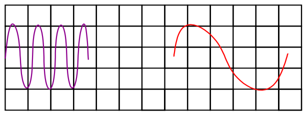
Spectral F'ns

T = 0

Ordinary QCD



Bound States



Summary of rest of talk

- $\omega \to 0$, i.e. transport coefficients
  - electrical conductivity - [Swansea-Korea]
    - modified MEM
  - shear viscosity - [Meyer]
    - two-step algorithm
- $\omega > 0$, i.e. hadron resonances
  - Quenched - [Swansea]
  - Dynamical - [Swansea-Dublin]

MEM Modification

SVD of kernel, $K(t,\omega)$:

$$G(t,\vec{p}) = \int \rho(\omega,\vec{p})\ K(t,\omega)\ d\omega$$

where $$K(t,\omega) = \frac{\cosh[\omega(t - N_t/2)]}{\sinh[\omega/(2T)]} \sim \exp[-\omega t]$$

But $K(t,\omega) \sim 1/\omega$ divergent as $\omega \to 0$.

Define modified kernel & spectral function:

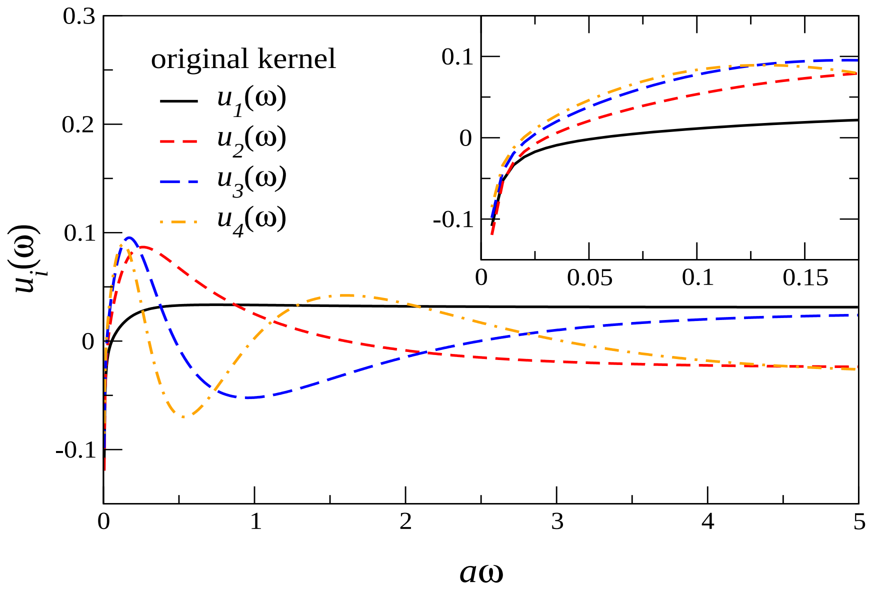
$$\overline{K}(t,\omega) = \frac{\omega}{2T} K(t,\omega), \qquad \overline{\rho}(\omega) = \frac{2T}{\omega}\rho(\omega)$$

Note $$\int \rho(\omega,\vec{p})\ K(t,\omega)\ d\omega \equiv \int \overline{\rho}(\omega,\vec{p})\ \overline{K}(t,\omega)\ d\omega$$

and $\overline{K}(t,\omega) \sim 1$ as $\omega \to 0$

SVD basis functions

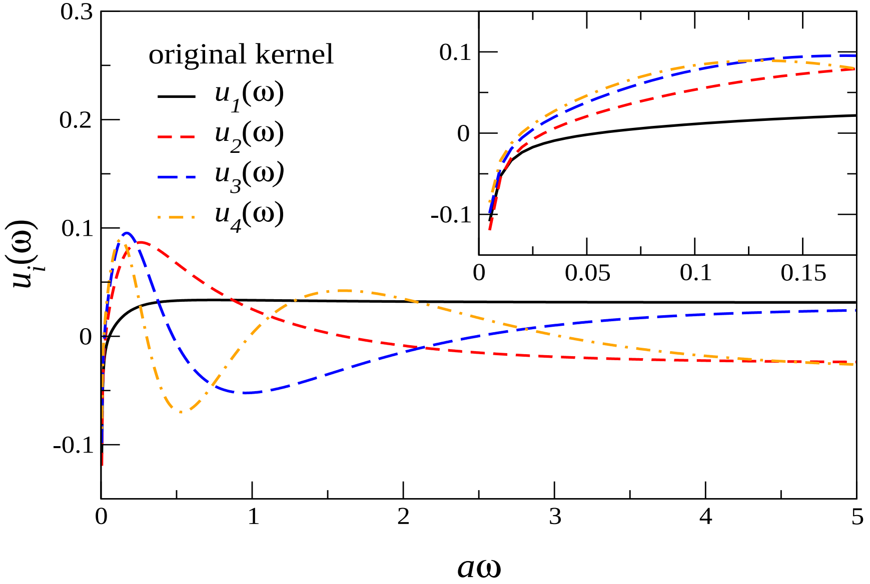
Traditional:



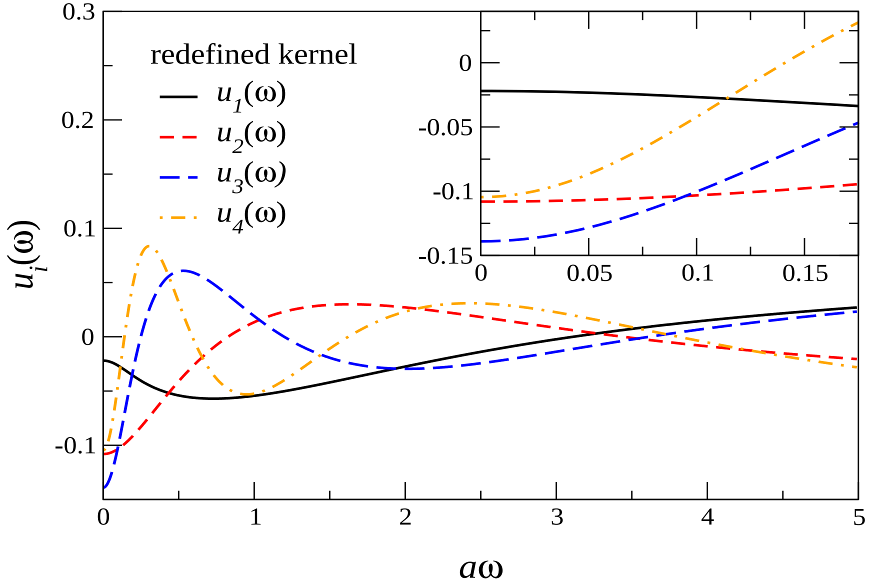
divergent as $\omega \to 0$

SVD basis functions

Traditional:



Modified:



divergent as $\omega \to 0$

finite as $\omega \to 0$

Lattice results

- Quenched, staggered QCD
- Used *modified MEM* on vector channel $\rightarrow$

σ = conductivity

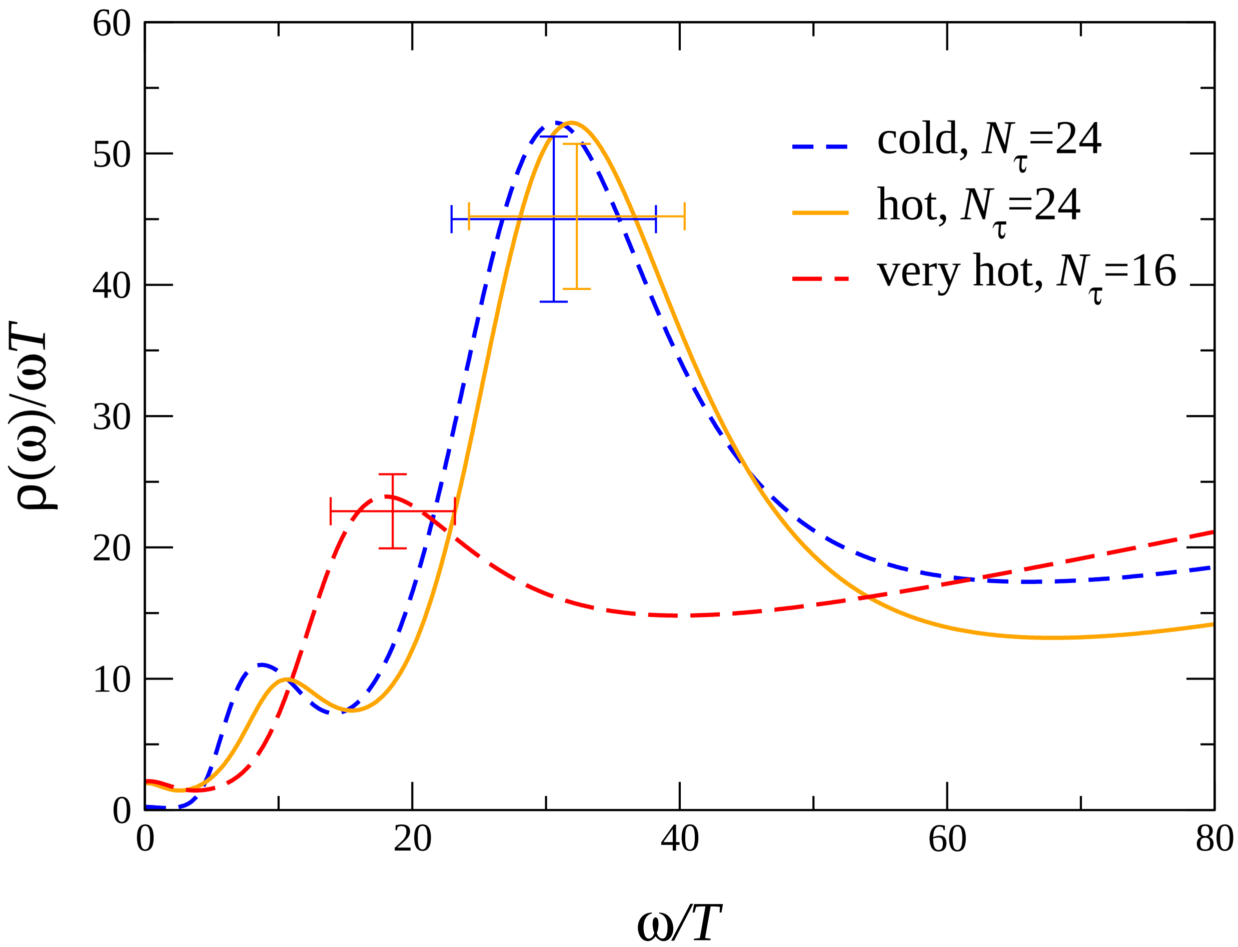
	β	a^{-1} (GeV)	$N_\sigma^3 \times N_\tau$	T/T_c	# conf
cold	6.5	4.04	$48^3 \times 24$	0.62	100
hot	7.192	9.72	$64^3 \times 24$	1.5	100
very hot	7.192	9.72	$64^3 \times 16$	2.25	50

Collaborators:

Gert Aarts, Justin Foley, Simon Hands, Seyong Kim

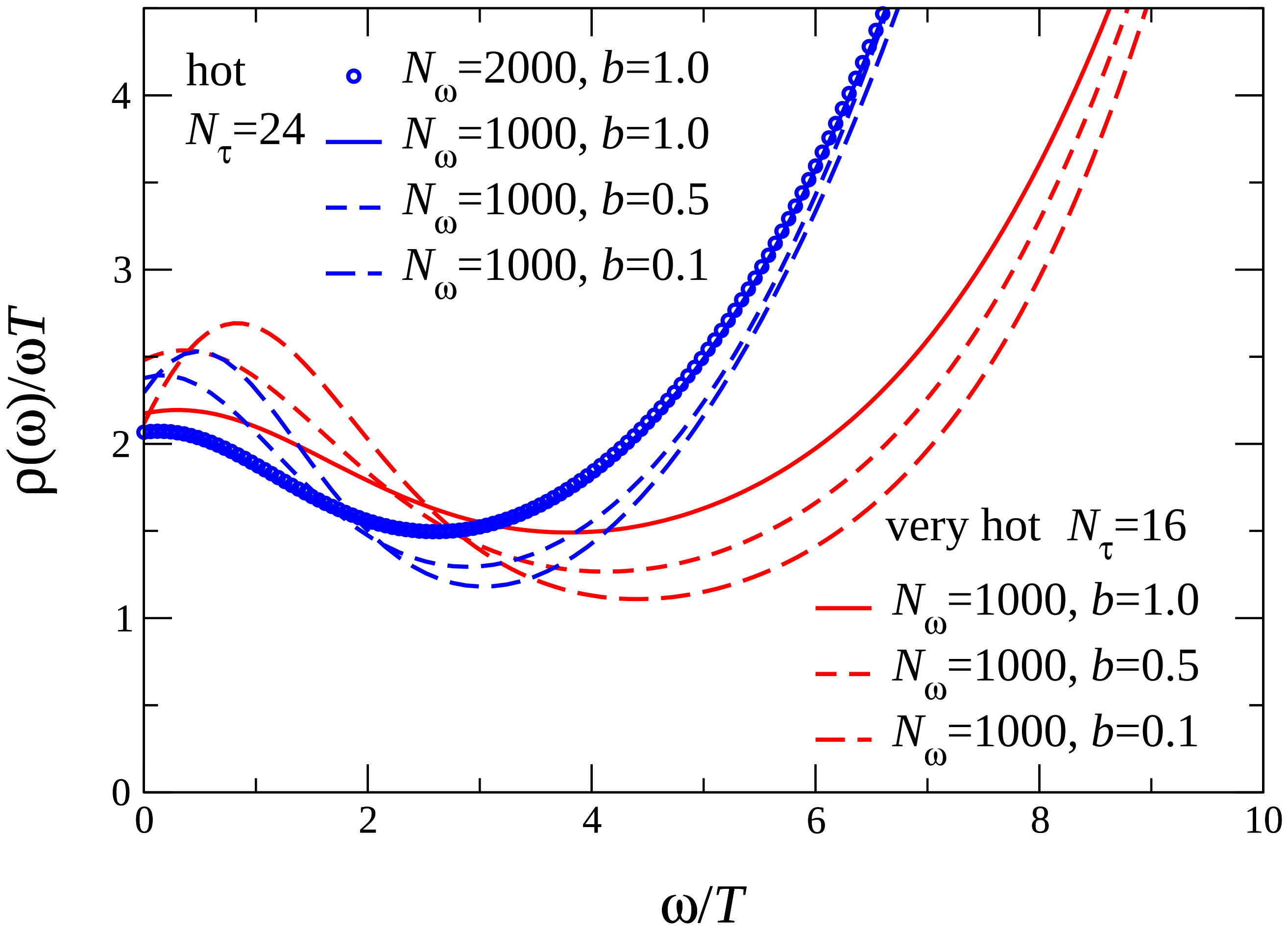
using cluster made up of undergraduate lab's PC's.

Spectral Function



- melting of ρ
- lattice artefact at $\omega/T \sim 30$

Conductivity



default model

$= m(\omega)$

$\sim \omega(b + \omega)$

- intercept robust to default model changes

intercept $\longrightarrow$ $\frac{\sigma}{T} = 0.4 \pm 0.1$

Shear Viscosity [Meyer]

- Recall η from $T_{\mu\nu}$, i.e. gluonic correlation functions
  - very noisy

Using:

- “two-level”algorithm
- model functions for ρ

Meyer found:

- “robust upper bound: $\eta/s < 1.0$
- quantitative values at T/T_c = 1.24 & 1.65

Summary of rest of talk

- $\omega \to 0$, i.e. transport coefficients
  - electrical conductivity - [Swansea-Korea]
    - modified MEM
  - shear viscosity - [Meyer]
    - two-step algorithm

- $\omega > 0$, i.e. hadron resonances
  - Quenched - [Swansea]
  - Dynamical - [Swansea-Dublin]

Lattice Choices

	Quenched	Dynamical
Isotropic	Swansea/Seoul	
Anisotropic		Dublin/Swansea

Quenched, Isotropic Study

	Quenched	Dynamical
Isotropic	*	
Anisotropic		

Collaborators

Gert Aarts
Justin Foley
Simon Hands

Seyong Kim

Run parameters

Quenched, Isotropic, Staggered (and Clover)

COLD

Lattice spacing	$a \sim 0.05$ fm	$\longrightarrow$	$a^{-1} \sim 4$ GeV
Volume	$N_s^3 \times N_t = 48^3 \times 24$	$\longrightarrow$	$T \sim \frac{1}{2} T_c$

HOT

Lattice spacing	$a \sim 0.02$ fm	$\longrightarrow$	$a^{-1} \sim 10$ GeV
Volume	$N_s^3 \times N_t = 64^3 \times 24$	$\longrightarrow$	$T \sim 1.4 T_c$

$N_{cfg} = 100$, $am_q = 0.01, 0.05, 0.125$

Run parameters

Quenched, Isotropic, Staggered (and Clover)

COLD

Lattice spacing	$a \sim 0.05$ fm	$\longrightarrow$	$a^{-1} \sim 4$ GeV
Volume	$N_s^3 \times N_t = 48^3 \times 24$	$\longrightarrow$	$T \sim \frac{1}{2} T_c$

HOT

Lattice spacing	$a \sim 0.02$ fm	$\longrightarrow$	$a^{-1} \sim 10$ GeV
Volume	$N_s^3 \times N_t = 64^3 \times 24$	$\longrightarrow$	$T \sim 1.4 T_c$

$N_{cfg} = 100$, $am_q = 0.01, 0.05, 0.125$

SELLING POINT: Many Different Momenta ...

Twisted Boundary Conditions

- Twisted B.C.'s used to access many different momenta
  - Flynn et al., hep-lat/0506016, etc.

$$\psi\left(x_i + L\right) = e^{i\theta_i}\psi\left(x_i\right)$$

- Using fermion propagators for different twist angles θ and ϕ to construct a meson correlator with momentum $\mathbf{p} = \frac{2\pi}{L_s}\mathbf{n} - \frac{\theta-\phi}{L_s}$

- Each twist requires an additional inversion of the fermion matrix per gauge configuration
  - but 4 different twist angles and fourier transforms $\longrightarrow$ 19 independent momenta

Annoying Fact of Life: Staggered Fermions

- Correlator has a contribution from unwanted "staggered" partner

- In terms of the spectral functions the correlator is given by

$$G\left(t,\mathbf{p}\right)=\int_0^\infty\frac{d\omega}{2\pi}K\left(t,\omega\right)\left[\rho(\omega,\mathbf{p})-\left(-1\right)^{t/a}\tilde{\rho}(\omega,\mathbf{p})\right]$$

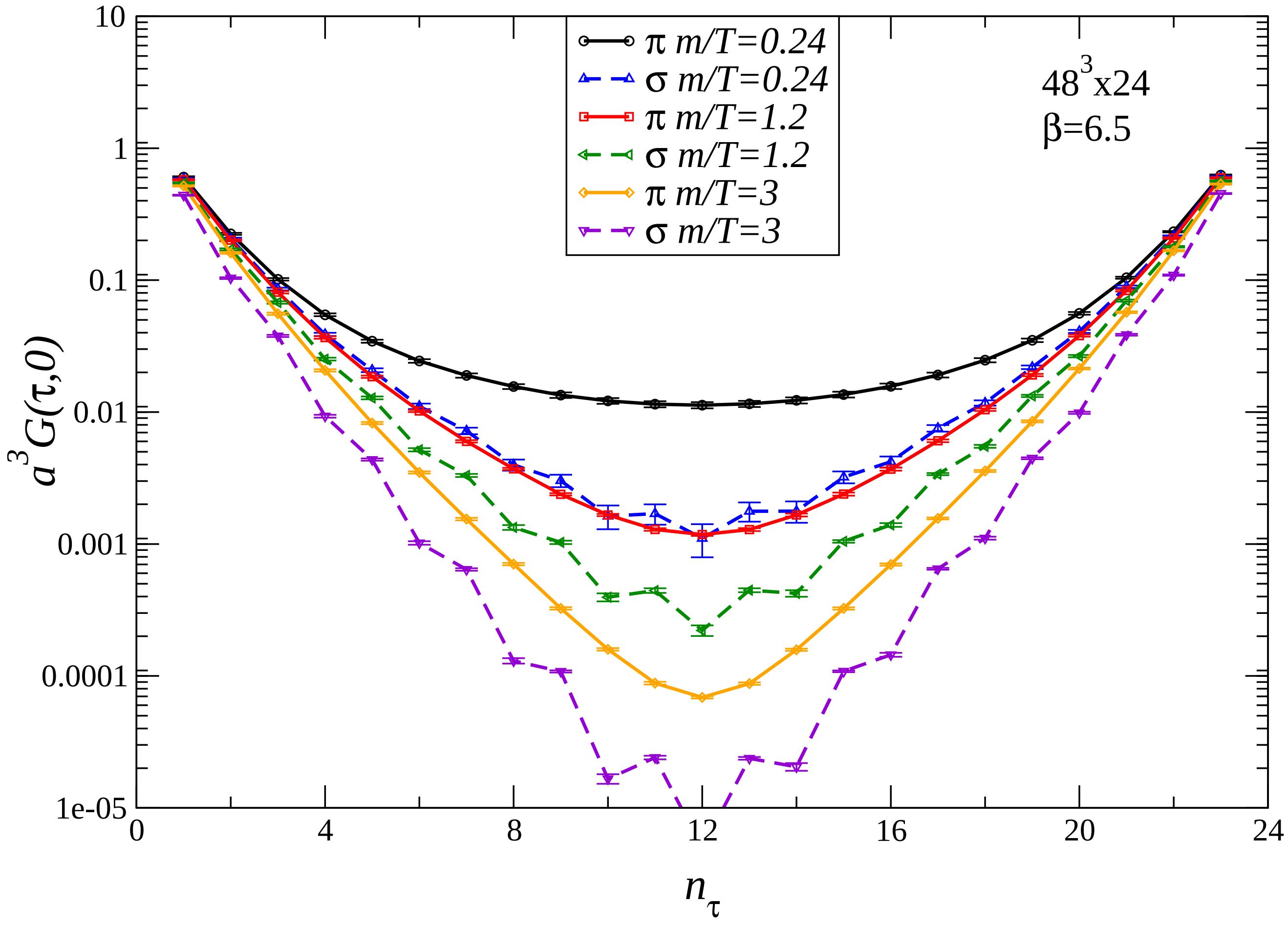
- Perform independent MEM analysis on odd and even timeslices

- Add results to obtain the desired spectral function

Disadvantage - only half the available timeslices are used in each MEM analysis

Correlator, $G(t)$, below T_c

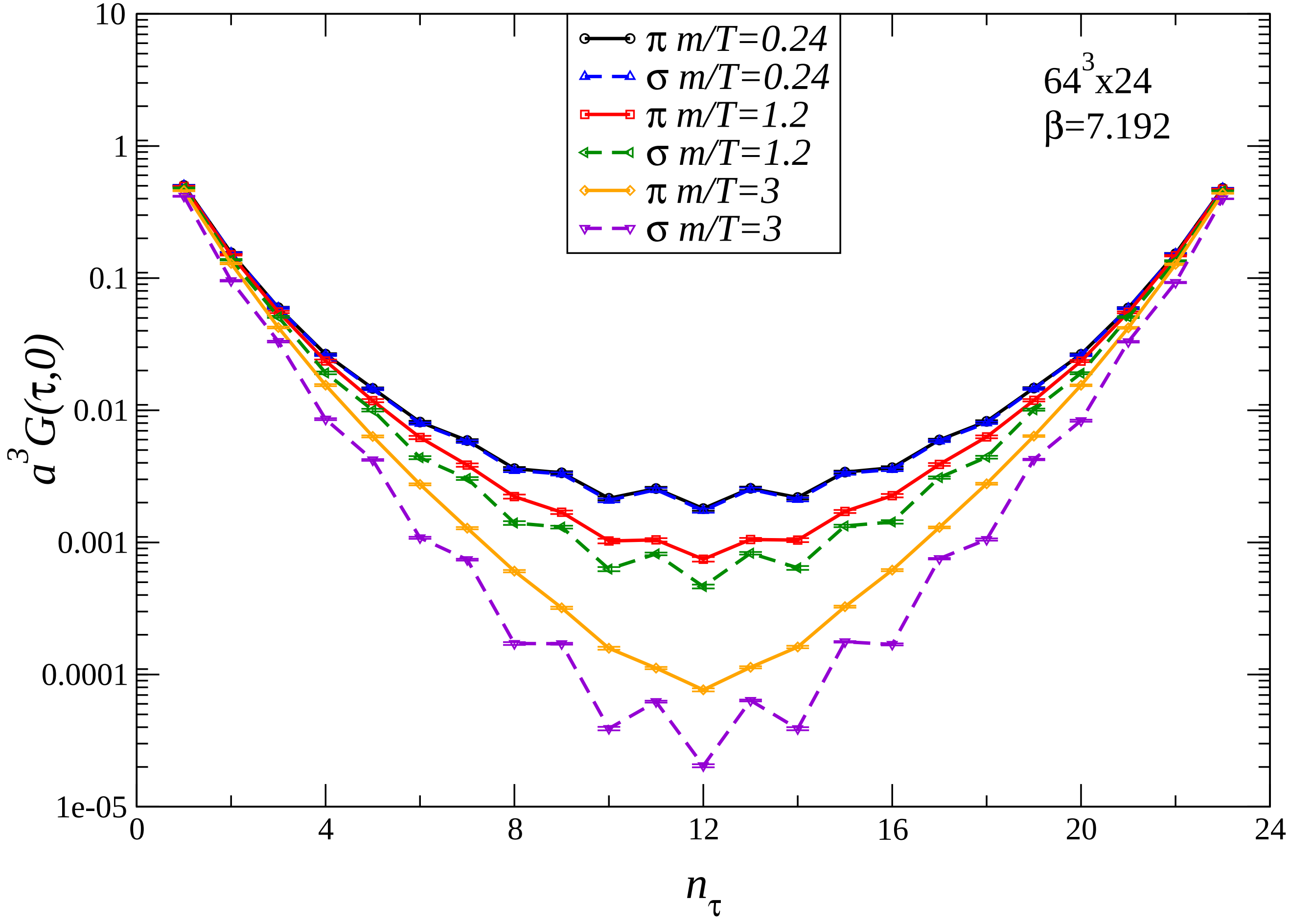
Both PS and Scalar @ 3 quark masses
(Zero Momentum)



PS and Scalar non-degenerate

Correlator, $G(t)$, above T_c

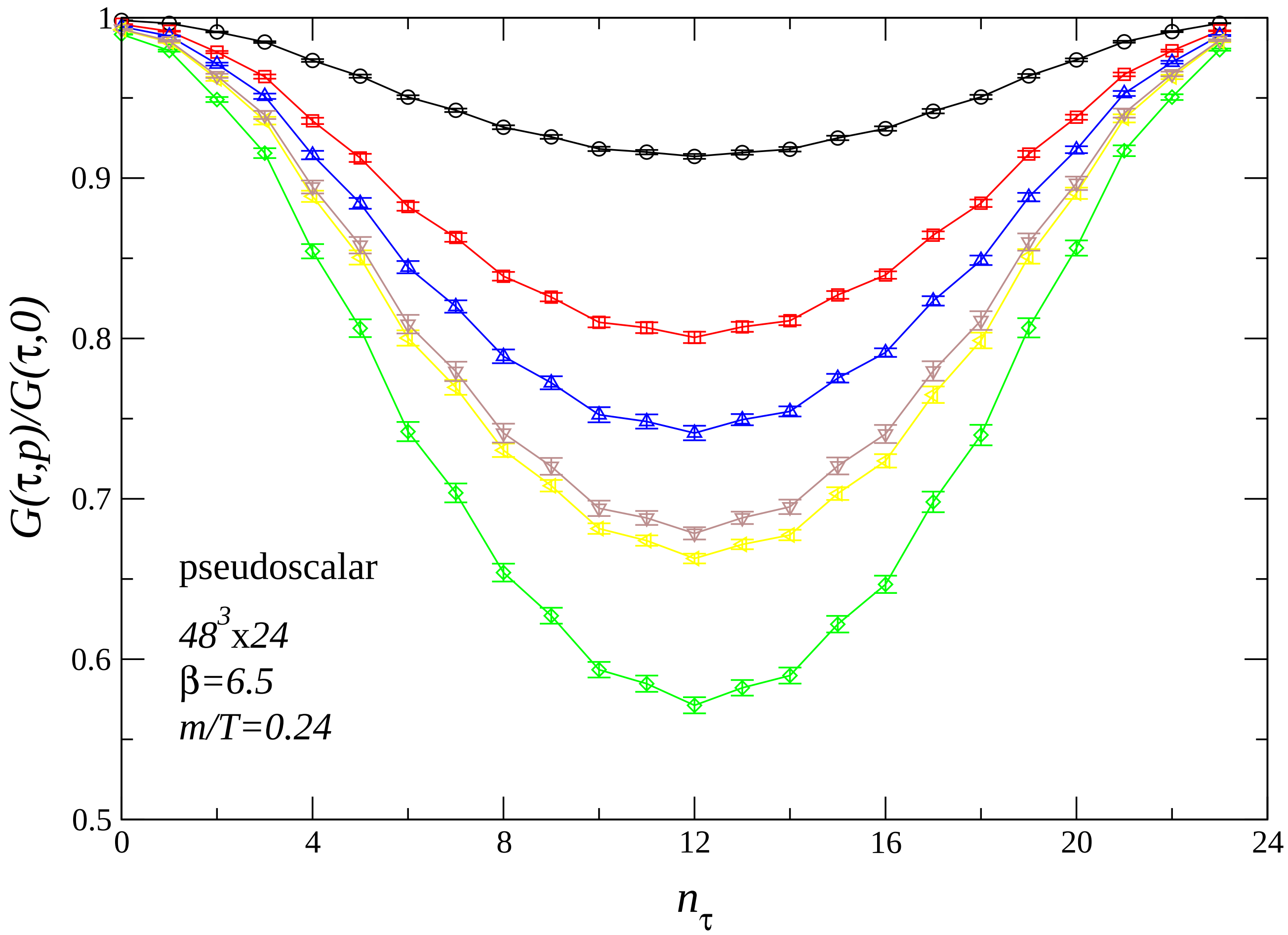
In the deconfined phase, for the lightest quark mass, pseudoscalar and scalar correlators are degenerate



Restoration of Chiral Symmetry

Correlator momentum-dependence below T_c

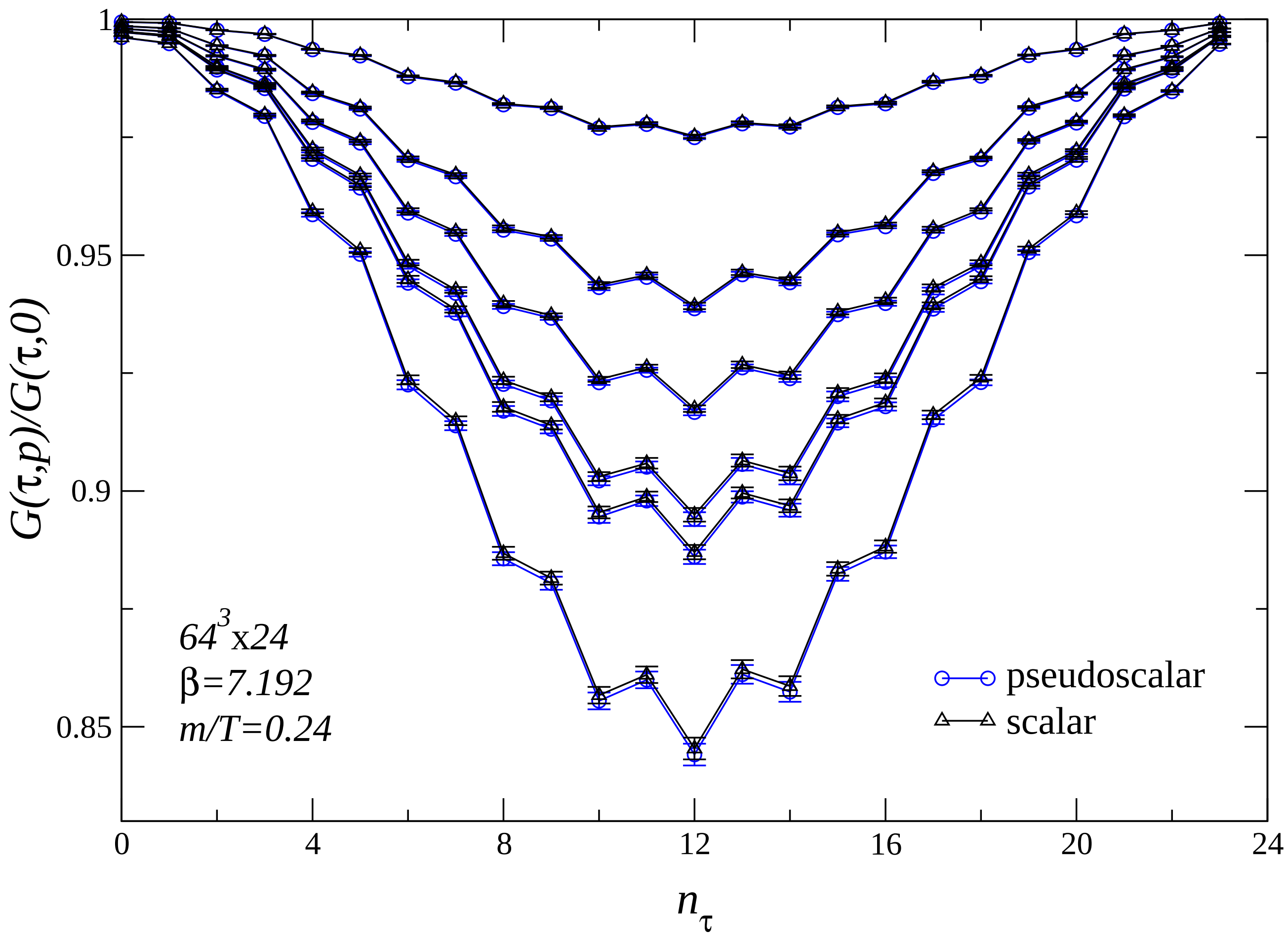
$G(t, \mathbf{p})/G(t, \mathbf{0})$ for PS meson below $\mathrm{T_c}$



$$\mathbf{p}L = 2.0, 3.14, 3.72, 4.25, 4.36, 5.2$$

Correlator momentum-dependence above T_c

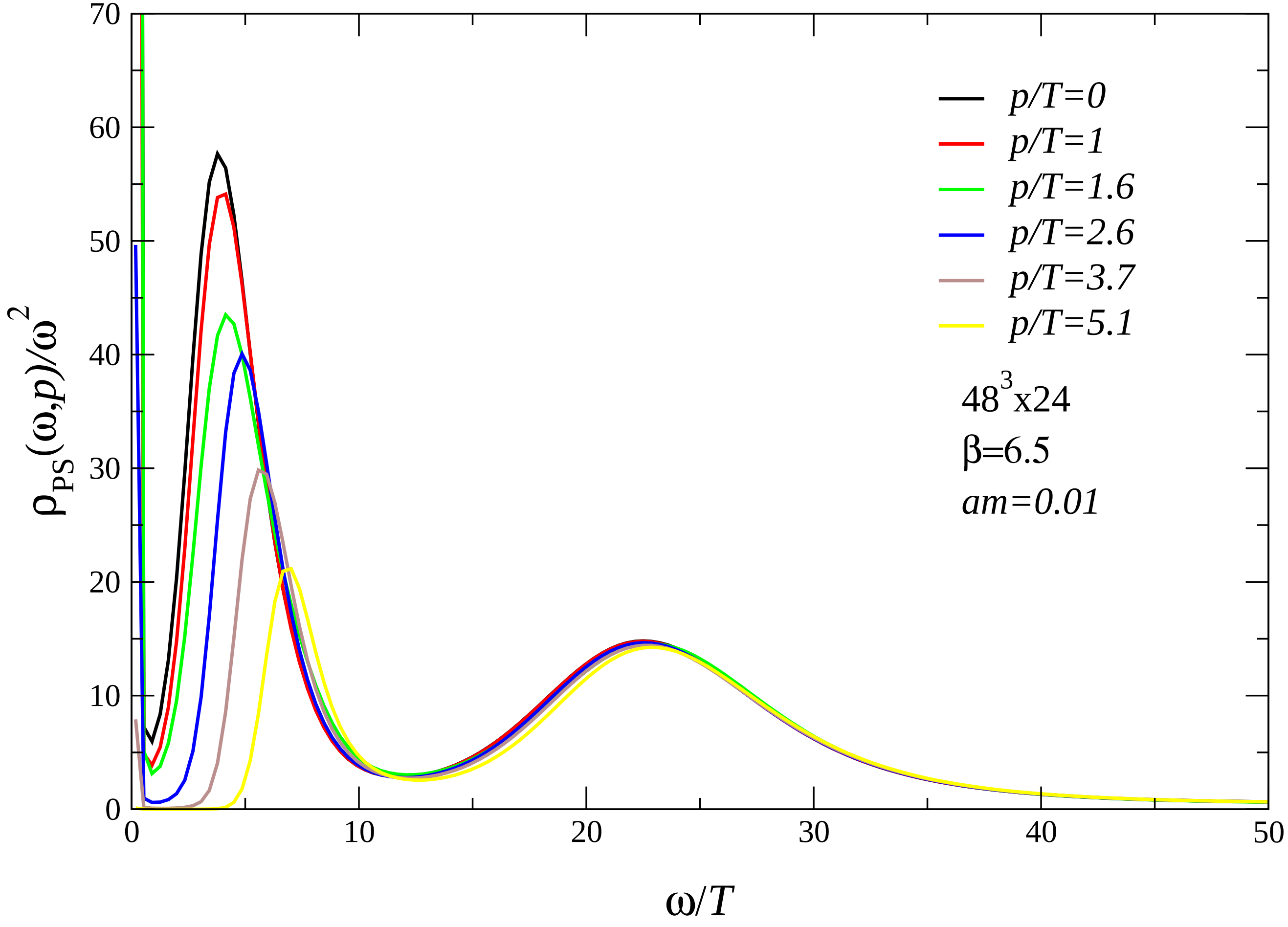
Corresponding plot above $\mathrm{T_c}$



Varying the momentum has a much smaller effect

MEM results below T_c

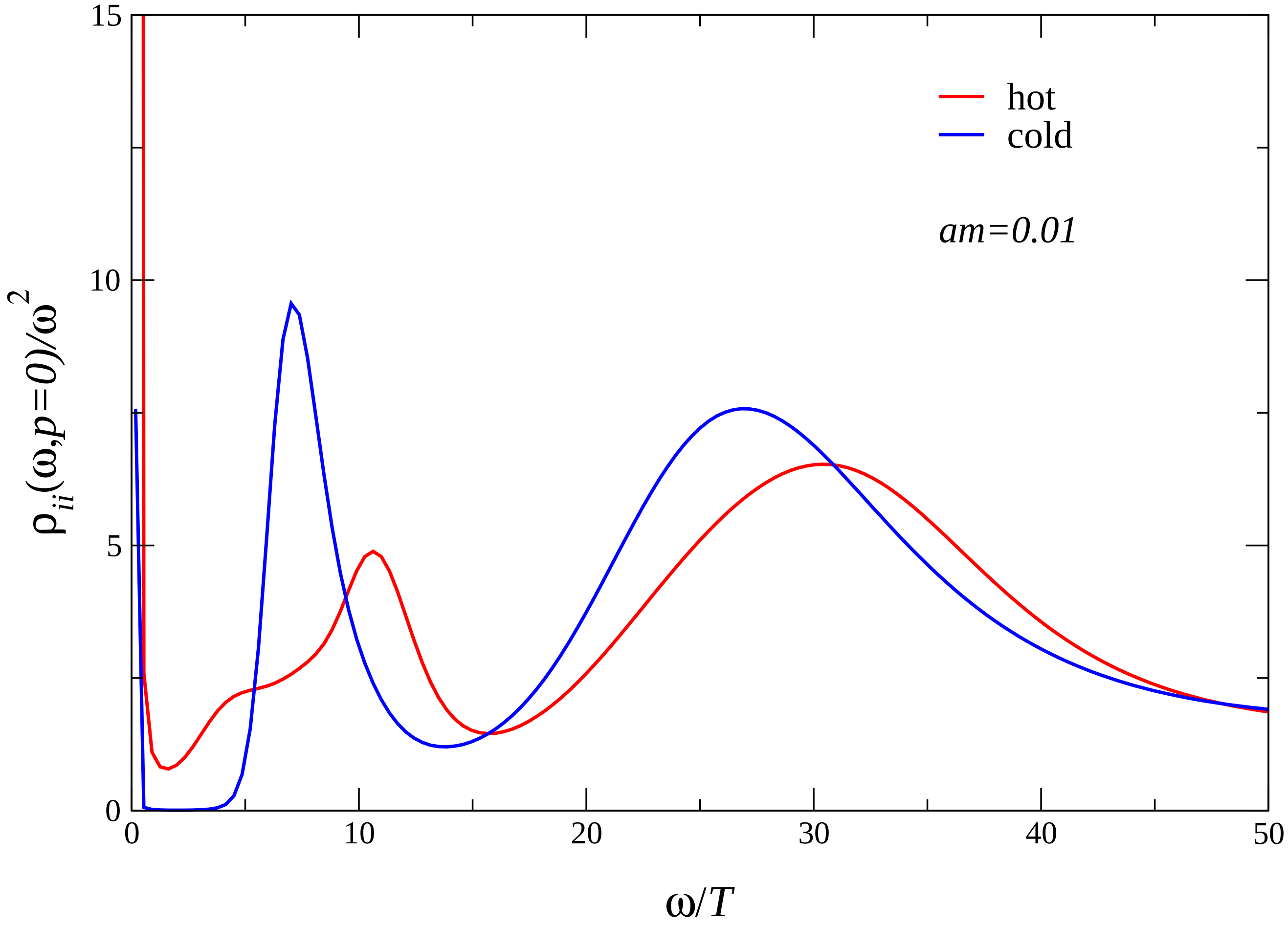
Momentum dependence of the pseudoscalar spectral function below T_c



Interpretation

- Moving bound state
- Dispersion relation yields a speed of light consistent with unity
- Temporal extent only $\sim 1.2\mathrm{fm}$
- $\Rightarrow$ Determination of the dispersion relation is not possible using conventional (maximum likelyhood) fits to exponentials

Vector Spectral Functions



Comparison of vector spectral functions at zero momentum above and below T_c

Dynamical, Anisotropic Study

	Quenched	Dynamical
Isotropic		
Anisotropic		*

Collaborators

Gert Aarts
Bugras Oktay
Mike Peardon
Jon-Ivar Skullerud

Tuning anisotropic *dynamical* lattices

Have to fine-tune couplings so that both quarks and gluons feel the *same* anisotropy in both spatial and temporal directions...

For *Quenched*: $\xi_g = f(\beta_s, \beta_t) \neq f(\xi_f^0)$

For *Dynamical*: $\xi_g = f(\beta_s, \beta_t, m_s, m_t) = f(\xi_f^0)$

See TrinLat, hep-lat/0604021

Lattice Action and Parameters

- Gluon Action: Improved anisotropic
- Fermion Action: Wilson+Hamber-Wu + stout links

Lattice Action and Parameters

- Gluon Action: Improved anisotropic
- Fermion Action: Wilson+Hamber-Wu + stout links

Light quarks	M_π/M_ρ	~ 0.5	
Anisotropy	ξ	6	
Lattice spacings	a_t	~ 0.025 fm	
	a_s	~ 0.15 fm	
Spatial Volume	N_s^3	$8^3 \;\; (\&12^3)$	
Temporal Extent	N_t	16	$\rightarrow T \sim 2T_c$
		24	$\rightarrow T \sim 1.3T_c$
		32	$\rightarrow T \sim T_c$
Statistics	N_{cfg}	~ 500	

Lattice Action and Parameters

- Gluon Action: Improved anisotropic
- Fermion Action: Wilson+Hamber-Wu + stout links

Light quarks	M_π/M_ρ	~ 0.5	
Anisotropy	ξ	6	
Lattice spacings	a_t	~ 0.025 fm	
	a_s	~ 0.15 fm	
Spatial Volume	N_s^3	$8^3 \;\; (\&12^3)$	
Temporal Extent	N_t	16	$\rightarrow T \sim 2T_c$
		24	$\rightarrow T \sim 1.3T_c$
		32	$\rightarrow T \sim T_c$
Statistics	N_{cfg}	~ 500	

SELLING POINT: DYNAMICAL +
. Systematic Effects Studied

Quantities Studied

- We have 4 channels (PS, Vector, Axial, Scalar)

- We can vary:
  - Energy resolution for MEM
  - Start time for MEM)
  - $m_c = 0.080$ or 0.092
  - Spatial Volume

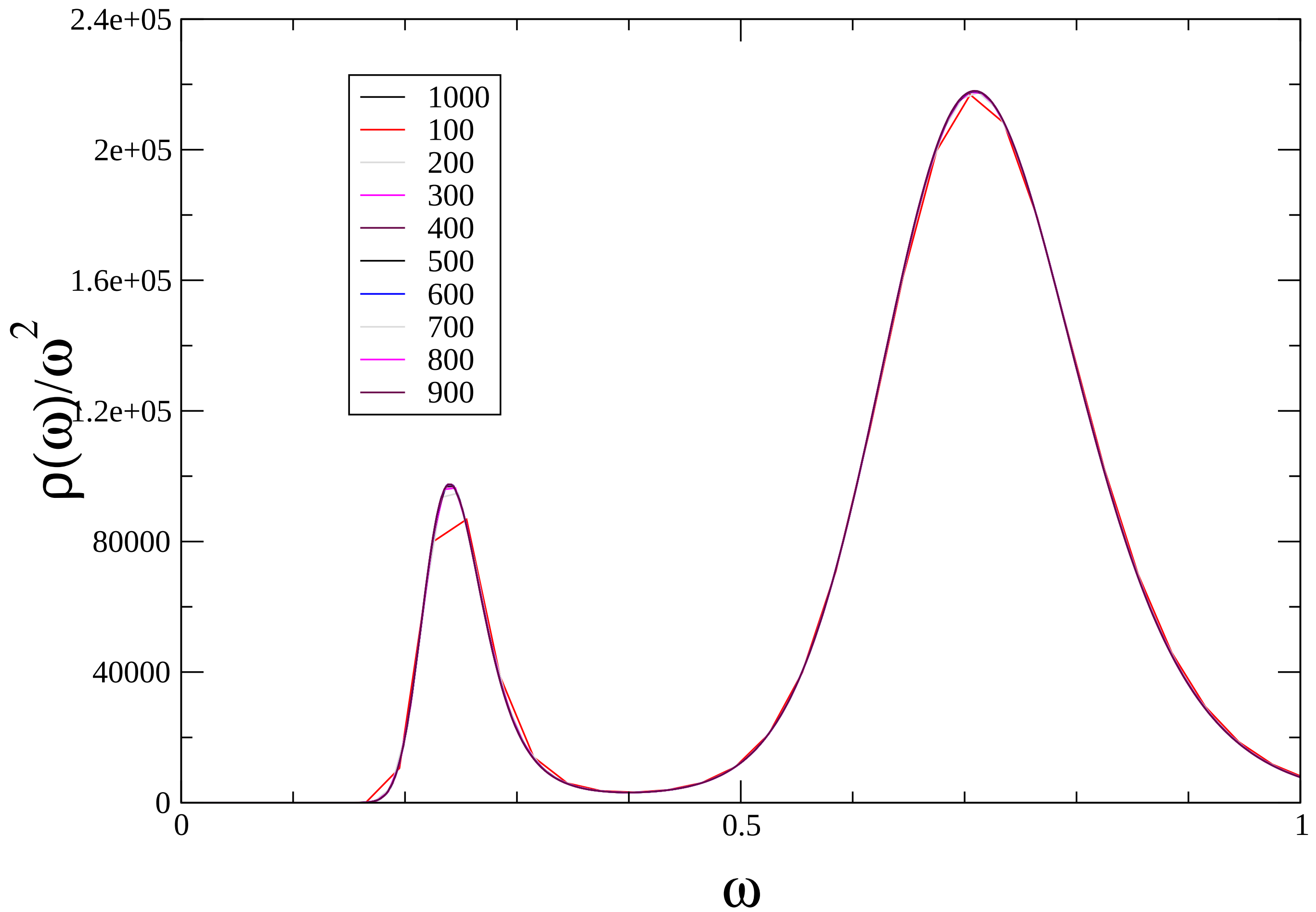
Quantities Studied

- We have 4 channels (PS, Vector, Axial, Scalar)

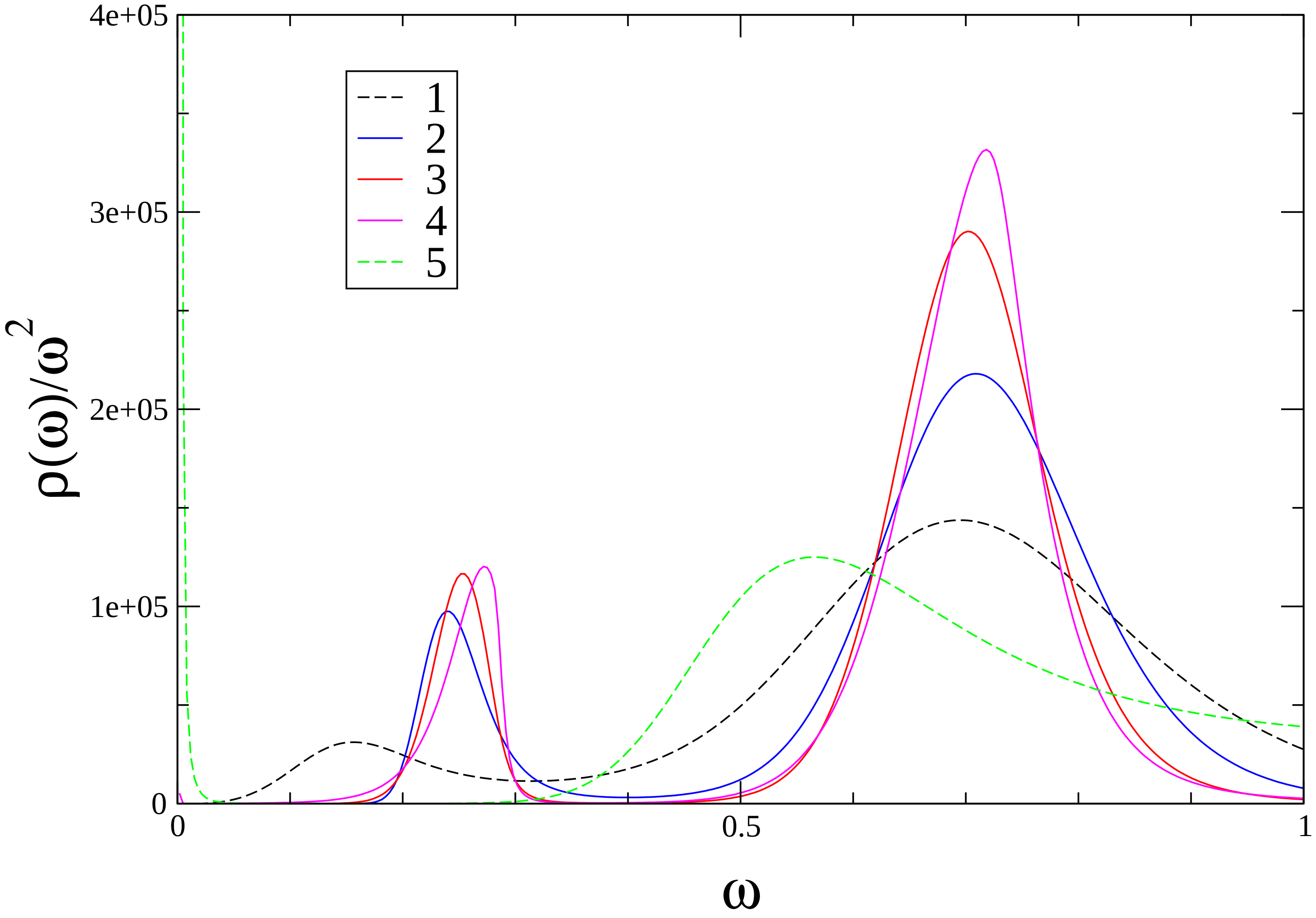
- We can vary:
  - Energy resolution for MEM
  - Start time for MEM)
  - $m_c = 0.080$ or 0.092
  - Spatial Volume

- and:
  - Temperature (i.e. N_t)

Varying MEM's energy resolution

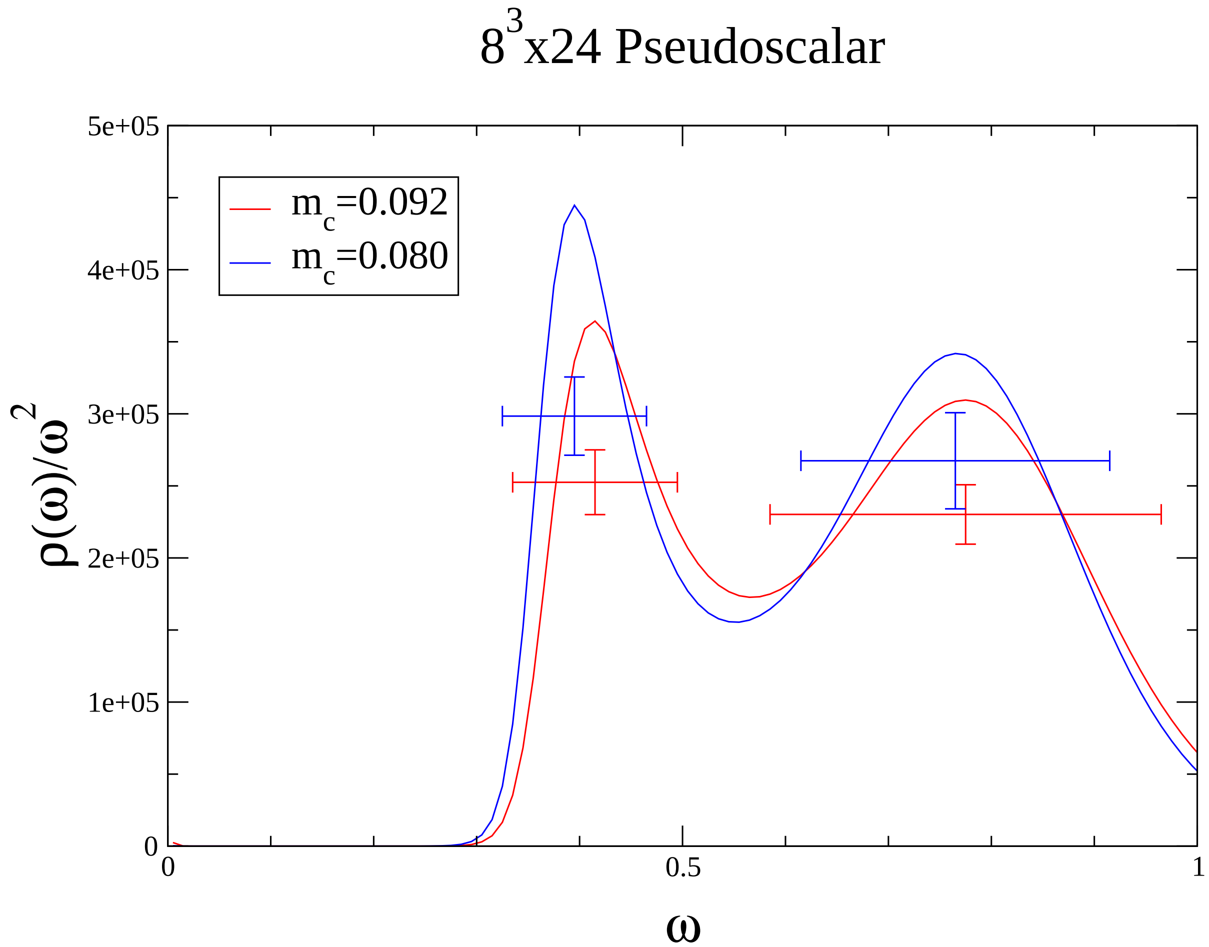


Varying MEM's start time



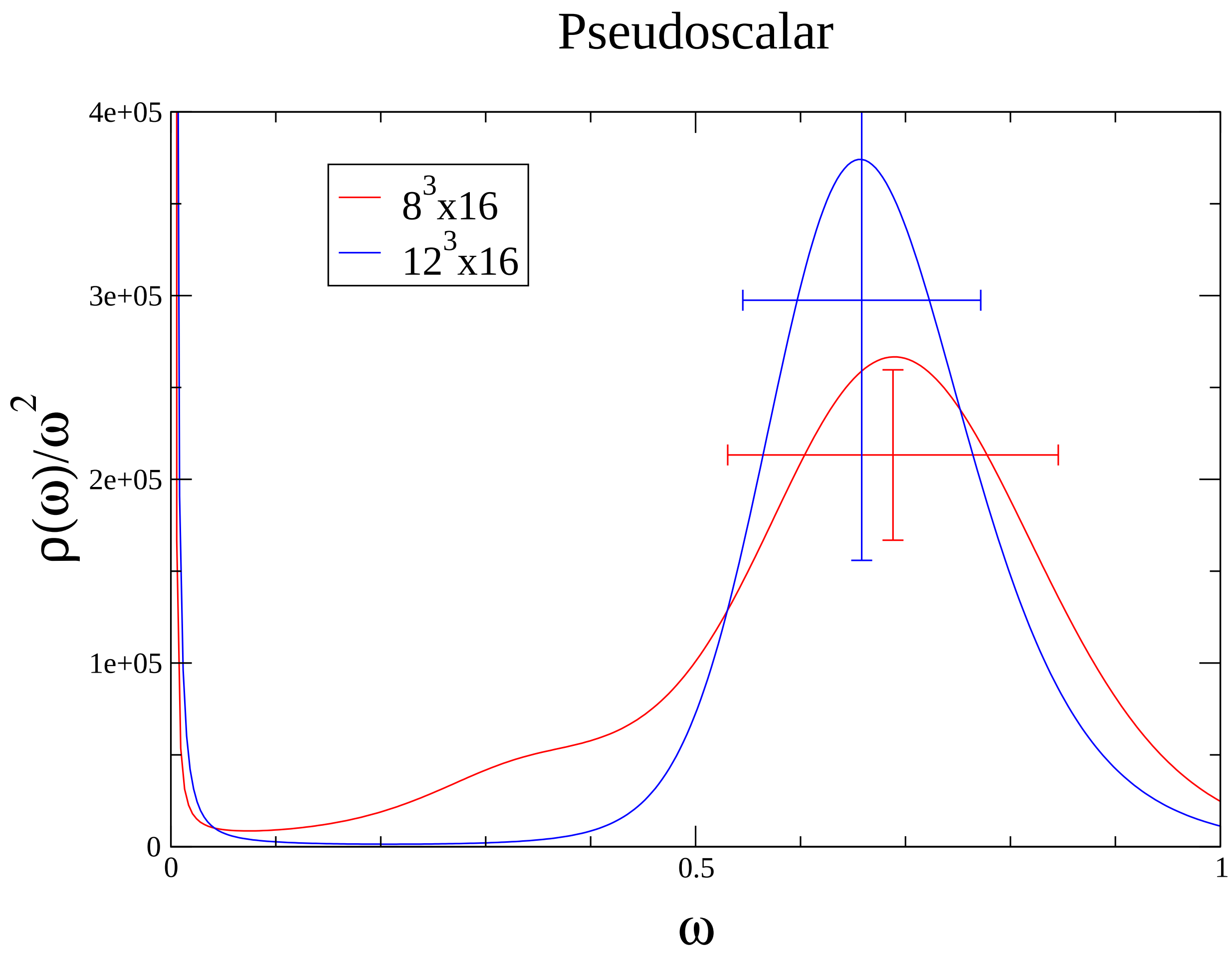
Varying m_c

$1.3\ T_c$ i.e. $N_t = 24$



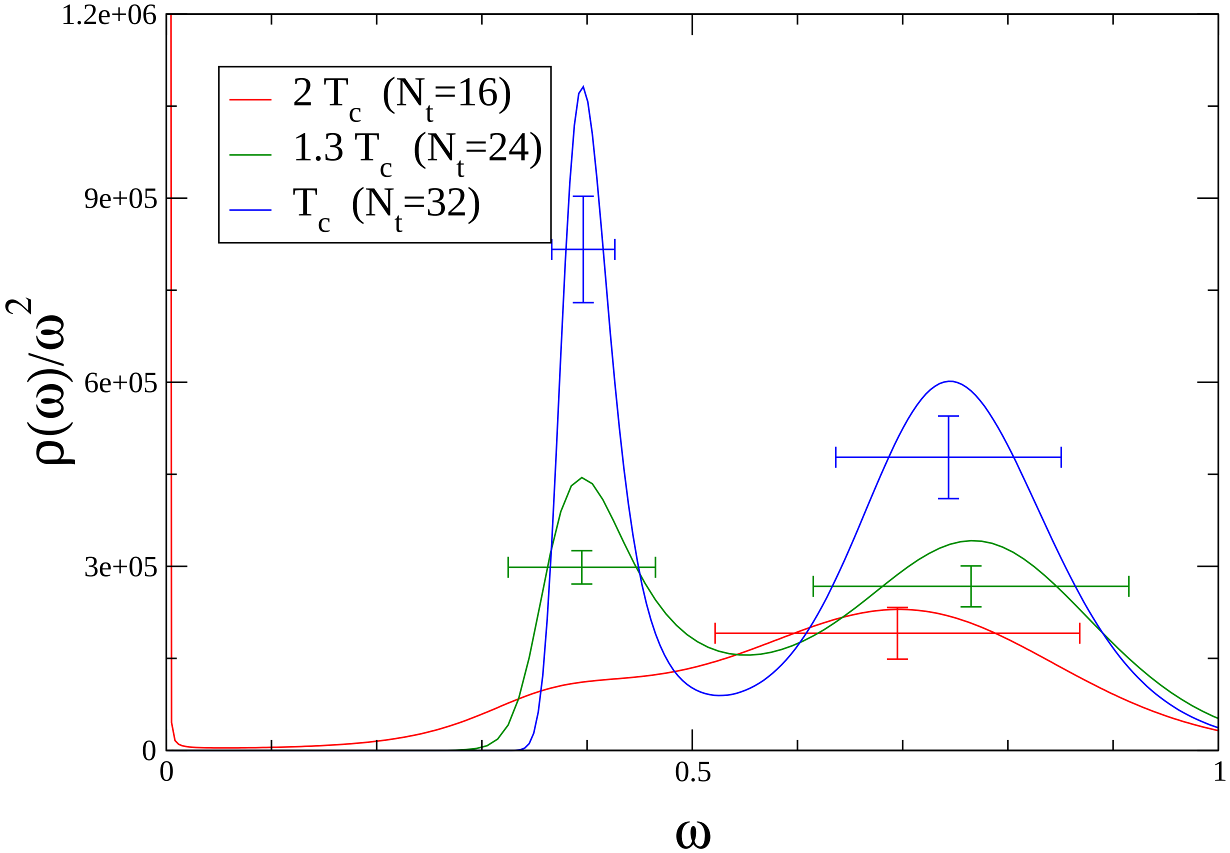
Varying Spatial Volume

$N_s = 8$ and 12



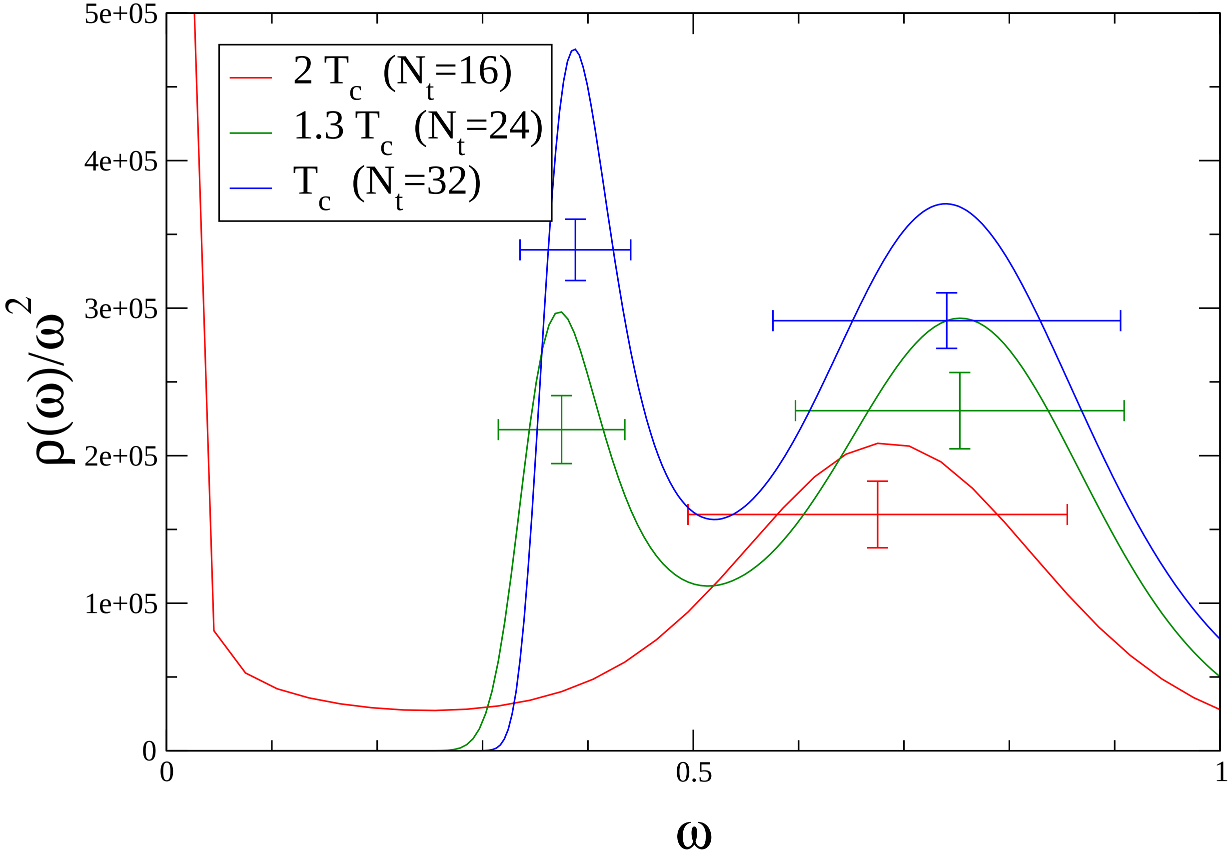
Varying Temperature!

Pseudoscalar ($am_c = 0.080$, $N_s = 8$)



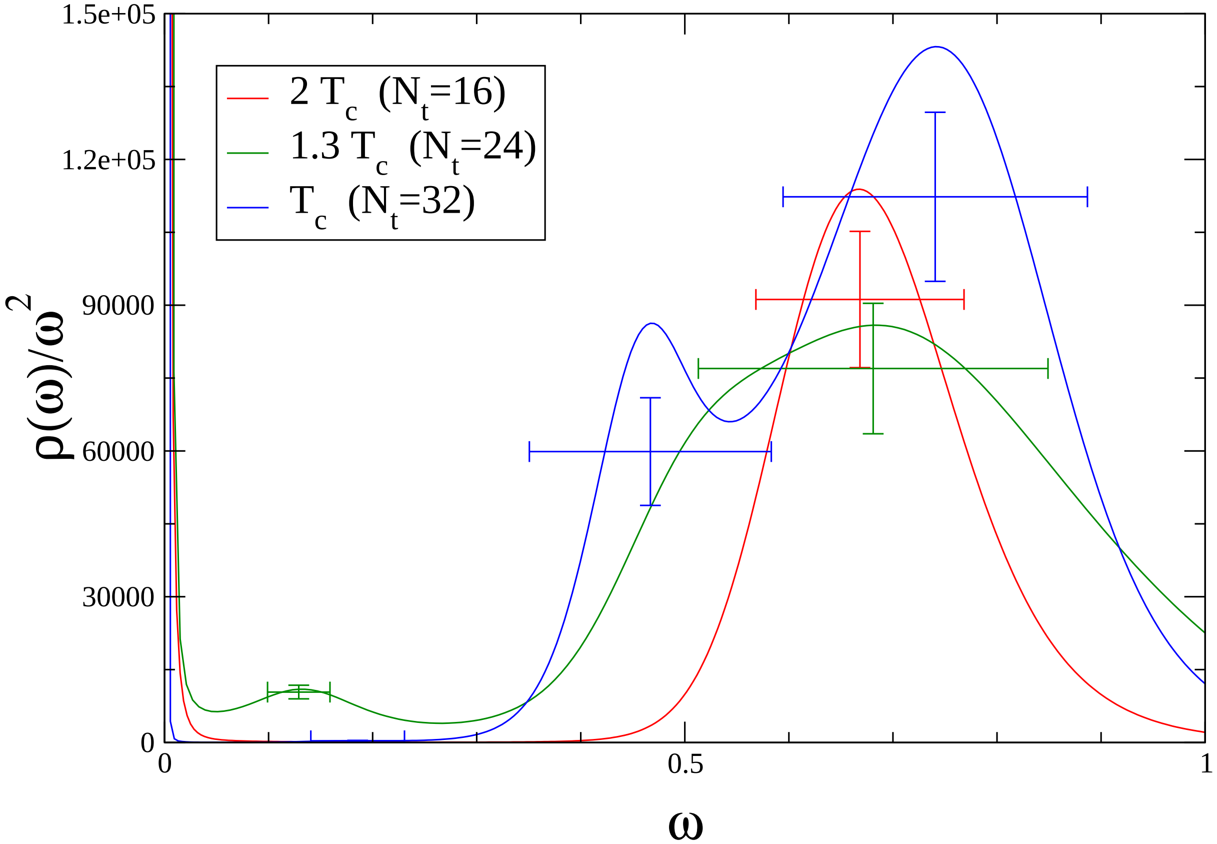
Varying **Temperature!**

Vector ($am_c = 0.080$, $N_s = 8$)



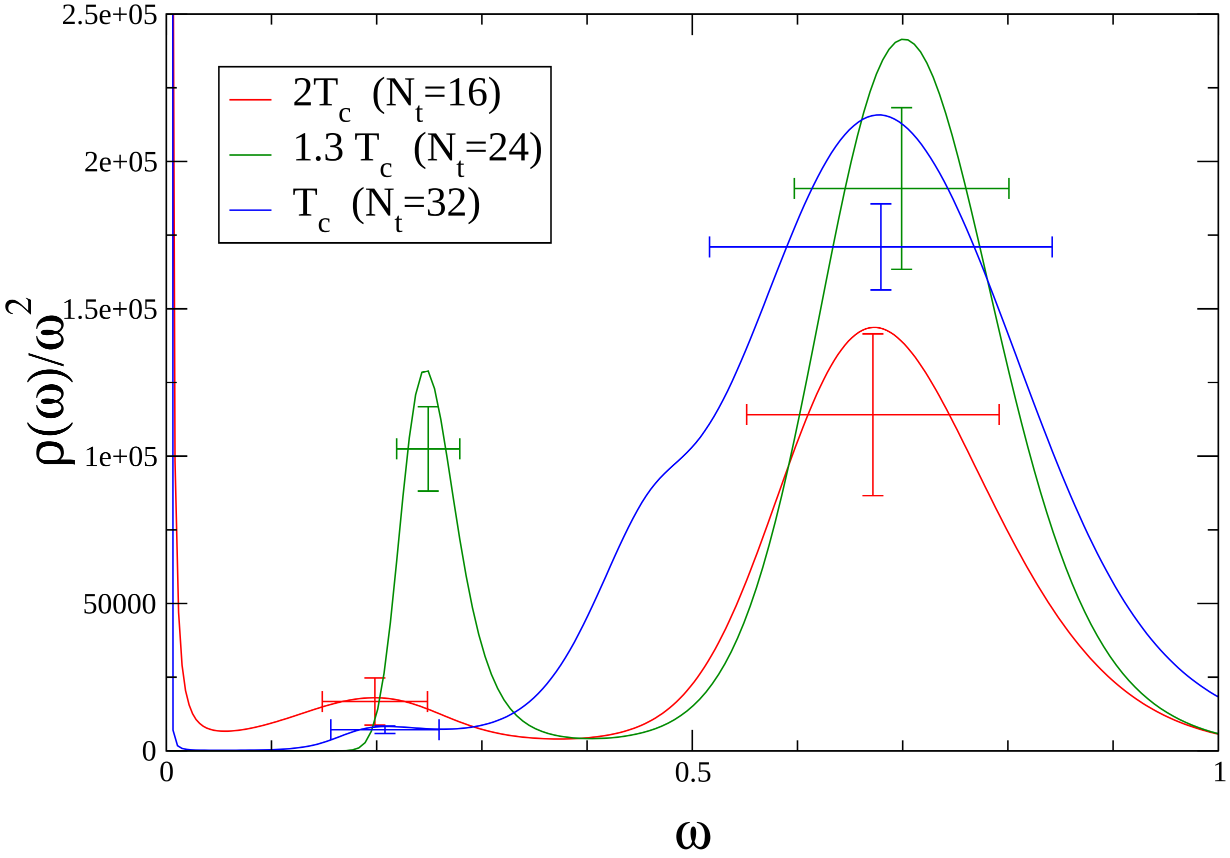
Varying Temperature!

(Spatial) Axial ($am_c = 0.080$, $N_s = 8$)

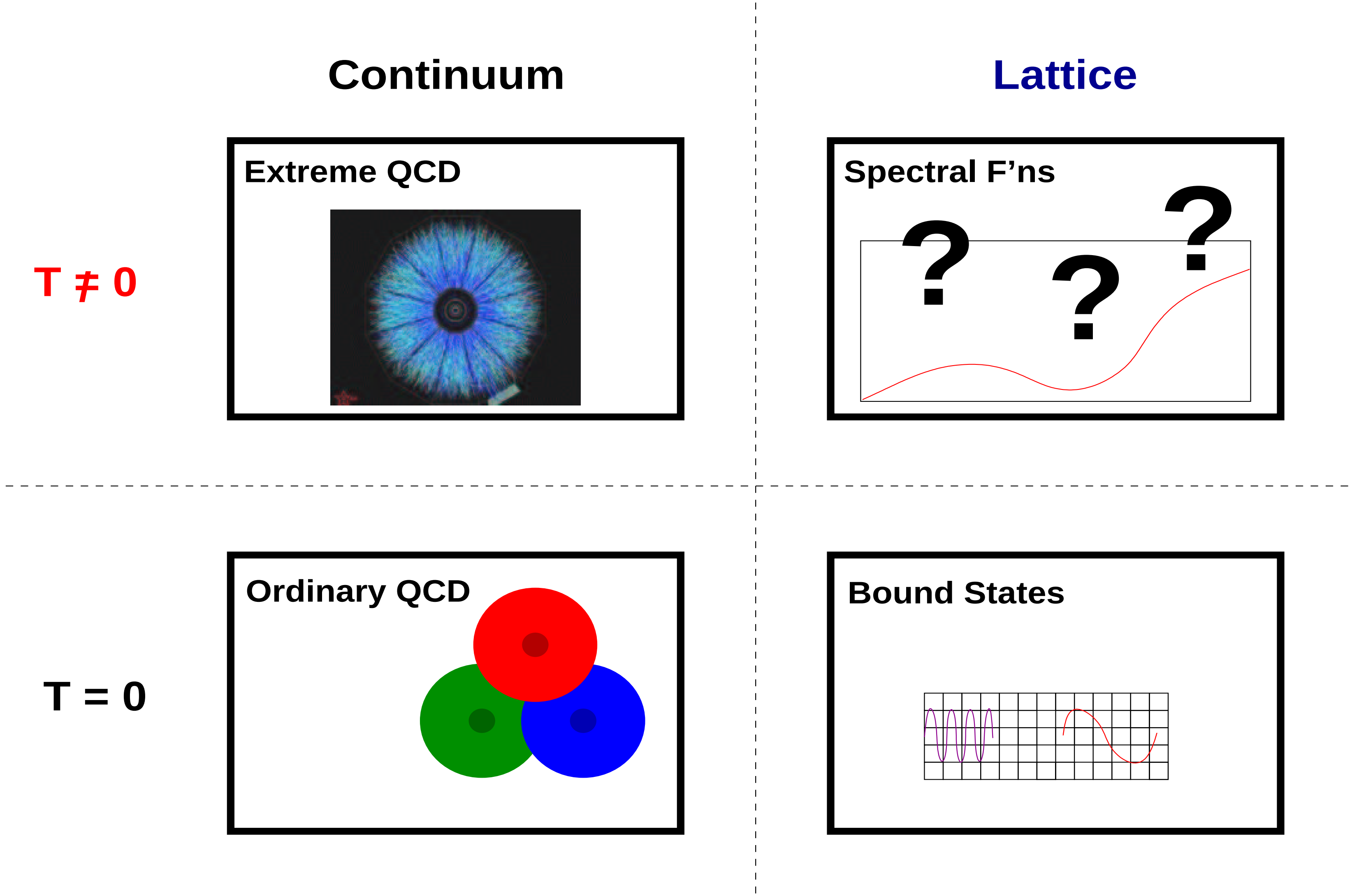


Varying **Temperature!**

Scalar ($am_c = 0.080$, $N_s = 8$)



Summary - Lattice QCD at $T \neq 0$

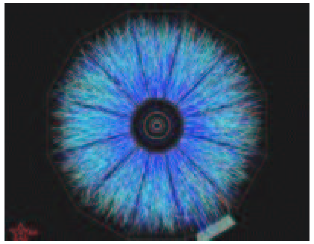


Summary - Lattice QCD at $T \neq 0$

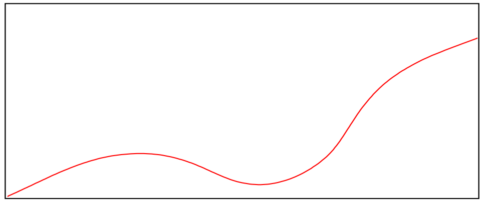
Continuum | **Lattice**

$T \neq 0$

Extreme QCD

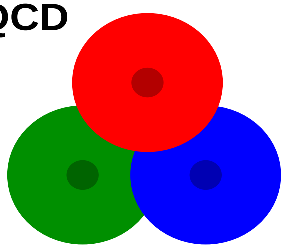


Spectral F'ns

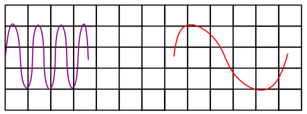


$T = 0$

Ordinary QCD



Bound States



Summary/Conclusions

- Improved MEM able to go to $\omega \to 0$
  - estimates of conductivity, σ
- Good resolution of spectral functions as T varied
  - Quenched - momentum variation also studied
  - Dynamical - anistropic lattice success!
- MEM results stable for sensible variations in MEM parameters
  - unphysical peak at the origin (?)
- Preliminary Results for Melting Temperature:
  - Pseudoscalar, Vector states melt between $1.3\,T_c$ and $2\,T_c$
- Still is work in progress ...

Thanks to:

Organisers!

The Dutch Passport Office...

Thanks to:

Organisers!

The Dutch Passport Office...