Two-Hadron Interactions on the Lattice: NN *and* $\pi\pi$

Hadron Physics on the Lattice
IASA: EINN 2007

André Walker-Loud

University of Maryland

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Silas R. Beane (U. New Hampshire)

Tom C. Luu (LLNL)

Kostas Orginos (William and Mary / JLAB)

Assumpta Parreno (Barcelona)

Martin J. Savage (U. Washington Seattle)

Aaron Torok (U. New Hampshire)

Jiunn-Wei Chen (National Taiwan U.)

Donal O'Connell (IAS Princeton)

Ruth S. Van de Water (FermiLAB)

Paulo F. Bedaque (U. Maryland)

Preview

- Motivation
- Two-Hadrons on the Lattice
  - 4-Point Green's Functions in Euclidean Space-Time
  - 2-Particle Interaction Energy and Lüshcer's Method
- $I = 2 \ \pi\pi$ Scattering
  - Numerical Calculation
  - Mixed Actions and Chiral Extrapolations
- $I = 1 \ KK$ Scattering $I = 3/2 \ K\pi$ Scattering f_K/f_π
- $NN \quad YN$
- Restless Pions: Orbifold boundary conditions and noise suppression in Lattice QCD
- Conclusions

Motivations

Motivations

- Lattice QCD has entered the precision era - at least for Gold-Plated quantities
- For two-pion (two-meson) interactions, lattice QCD is competitive with the best theoretical and phenomenological determinations of the non-scalar scattering channel
- One would like to understand nuclear phenomenology from first principles - a direct connection with QCD is needed

  A necessary first step is understanding the two-nucleon system

  can one find the deuteron?

  for what values of the quark masses does the deuteron remain finely- tuned

- In particular, lattice QCD has a great opportunity to make a large impact on understanding the interactions of strange hadrons

  RHIC: STAR is beginning to use kaon interferometery

  Neutron star properties: kaon-condensation? phase-diagram?

- Personally: scattering is just damn cool!

Two-Hadrons on the Lattice

Maiani-Testa No-Go-Theorem

S-Matrix elements can not be extracted away from threshold from infinite volume Euclidean correlation functions ($n \geq 3$)

C. Michael	NPB 327 (1989)
L. Maiani, M. Testa	PLB 245 (1990)

Easy to understand

- Minkowski space; S-matrix elements are complex functions above kinematic thresholds
- Euclidean space; S-matrix elements are real functions for all kinematics - lost information??

Volume Dependence

- Luscher's method for extracting infinite volume scattering parameters from the volume dependence of 2-particle states

K. Huang, C.N. Yang	Phys. Rev 105 (1957)
H.W. Hamber, E. Marinari, G. Parisi, C. Rebbi	NPB 225 (1983)
M. Lüscher	Comm. Math. Phys. 105 (1986)
M. Lüscher	NPB 354 (1991)
S.R. Beane, P.F. Bedaque A. Parreno, M.J. Savage	PLB 585 (2004)

2-hadron states in Finite Volume

Luscher's method for extracting infinite-volume scattering parameters essentially amounts to solving the eigenvalue equation,

$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n}} \frac{f(\vec{n})}{\vec{n}^2 - \left(\frac{kL}{2\pi}\right)^2}$$

For some regular function, $f(\vec{n})$. This equation can be expanded for large volume (compared to the scattering length, and $f(\vec{n}) = 1$), and solved for the energy difference from threshold,

$$\Delta E_0 \simeq -\frac{4\pi a_0}{m\mathrm{L}^3} \left[1 + c_1 \frac{a_0}{\mathrm{L}} + c_2 \left(\frac{a_0}{\mathrm{L}}\right)^2 + \mathcal{O}\left(\frac{1}{\mathrm{L}^3}\right)\right]$$

This method also works for large scattering lengths, as in the nucleon-nucleon system, where the box size is still larger than the range of the interaction

$$a \gg L \quad , \quad L > \frac{1}{m_\pi}$$

S.R. Beane, P.F. Bedaque
A. Parreno, M.J. Savage PLB 585 (2004)

$I = 2\ \pi\pi$ scattering

$I = 2\ \pi\pi$ scattering

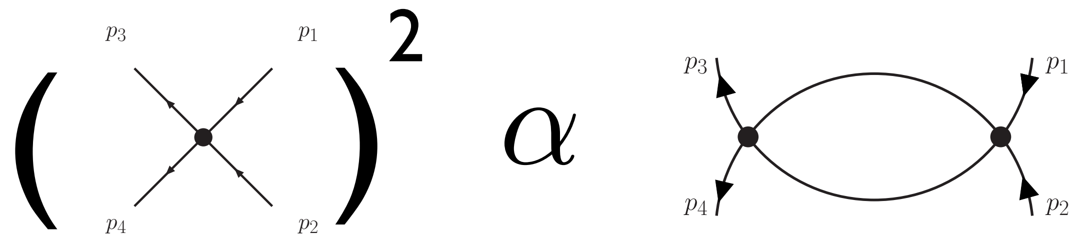
Why not $I = 0\ \pi\pi$ Scattering?

- Numerically much more expensive - requires all-to-all propagators (disconnected diagrams)

- For mixed-action schemes (or partially quenched) the unitarity violations are much more problematic, as the unphysical particles can go on shell in the s-channel diagrams, invalidating the use of Lüscher's method.

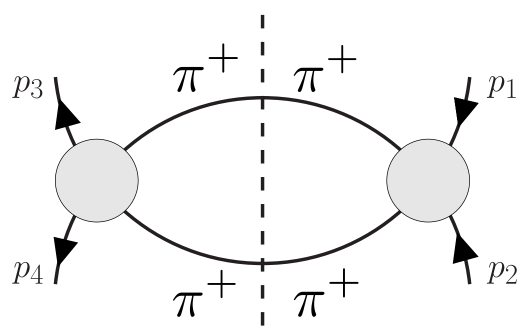
$I = 2\ \pi\pi$ scattering

Luscher's method relies upon unitarity (optical theorem)



For I=2 pion scattering - the only particles which participate in the optical theorem are the π^+ s

➡ This relation holds in a partially quenched - mixed action theories



To all orders in perturbative expansion!!

J-W. Chen, D. O'Connell, R. S. Van De Water, AW-L
PRD 73(2006)

No hairpin diagrams (unitarity violating effects) in s-channel diagram for particles below 4-pi inelastic threshold.

$I = 2\ \pi\pi$ scattering

Quenched

Sharpe, Gupta, Kilcup	NPB 383 (1992)
Gupta, Patel, Sharpe	PRD 48 (1993)
Kuramashi, Fukujita, Mino, Okawa, Ukawa	PRL 71 (1993)
	hep-lat/9301016
	PRL 73 (1994)
	hep-lat/9501024
Fiebig, Rabitsch, Markum, Mihaly	hep-lat/9911025
Liu, Zhang, Chen, Ma	hep-lat/0109010
	NPB 624 (2002)
Gattringer, Hierl, Pullirsch	hep-lat/0409064
JLQCD	PRD 66 (2002)
CP-PACS	PRD 67 (2003)
	hep-lat/0503025
	PRD 71 (2005)
CLQCD	hep-lat/0703015

Dynamical

	PRD 70 (2004)
CP-PACS 2 flavors	
NPLQCD 2+1 flavors	PRD 73 (2006)
	arXiv:0706.3026

$I = 2\ \pi\pi$ scattering Resources

Mixed Action (*hybrid*) calculation using domain-wall valence fermions and rooted staggered sea fermions with 2+1 dynamical fermion flavors

- scheme developed by LHP Collaboration

Coarse MILC ($b \sim 0.125$ fm)	Dimensions	bm_l	bm_s	bm_l^{dwf}	bm_s^{dwf}	m_π (MeV)	m_K (MeV)	$N_{cfg} \times N_{source}$
2064f21b676m007m050	$20^3 \times 64$	0.007	0.050	0.0081	0.081	290	580	468×16
2064f21b676m010m050	$20^3 \times 64$	0.010	0.050	0.0138	0.081	350	595	658×20
2064f21b679m020m050	$20^3 \times 64$	0.020	0.050	0.0313	0.081	490	640	486×24
2064f21b681m030m050	$20^3 \times 64$	0.030	0.050	0.0478	0.081	590	675	564×8
Fine MILC ($b \sim 0.09$ fm)								
2896f2b709m0062m031	$28^3 \times 96$	0.0062	0.031	0.0080	0.0423	320	538	506×1

$$L \sim 2.5 \text{ fm}$$

$I = 2$ $\pi\pi$ scattering   Resources

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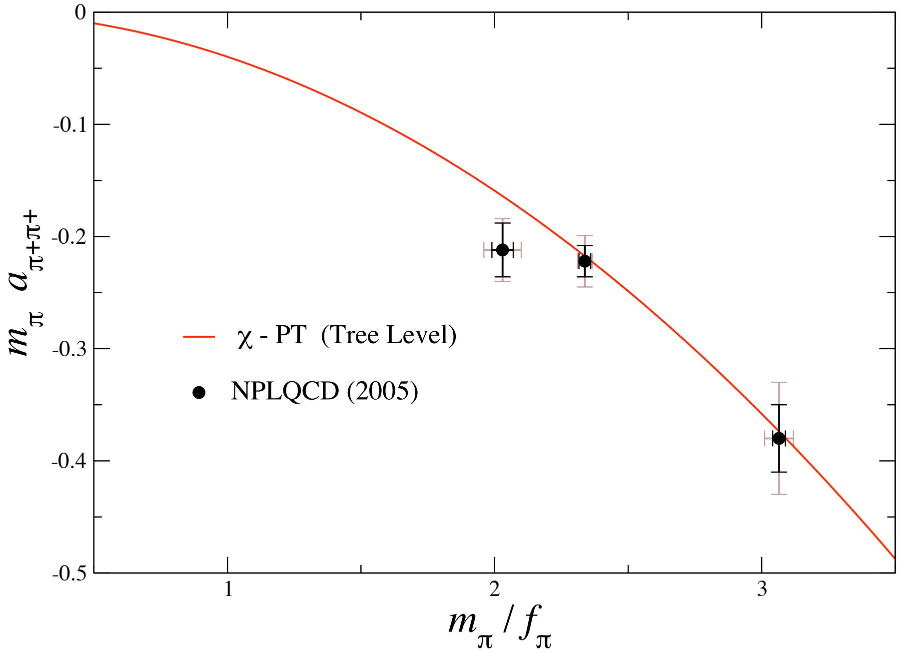
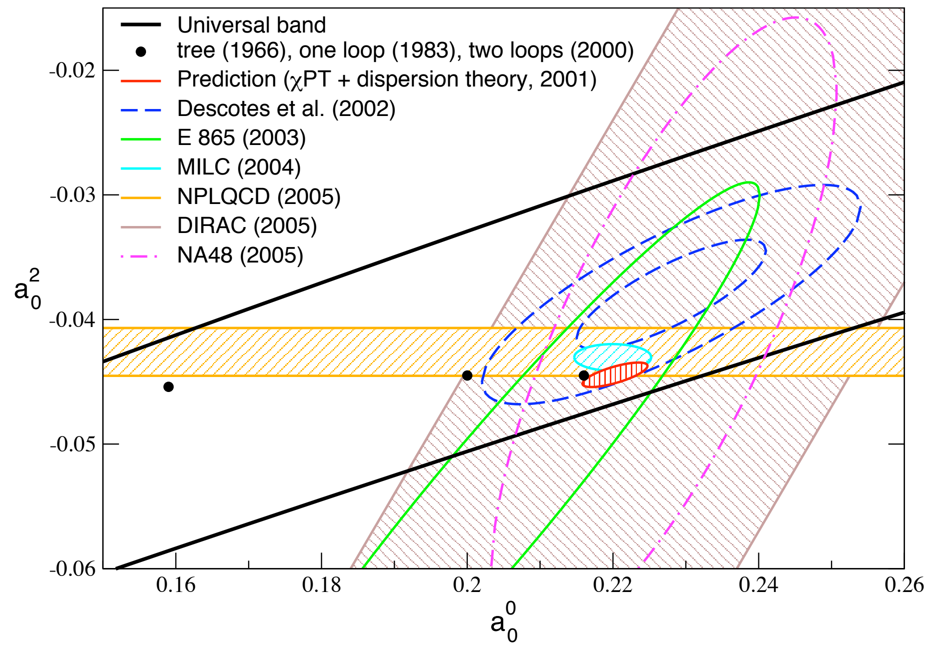
Peculiarities of staggered fermion formulation lead to 5 tastes of pions, with different masses. The mass splitting between these multiplets are lattice spacing artifacts, vanishing in the continuum limit.

One of these pions, the taste-5 pion, is the pseudo-Goldstone mode of a remnant axial symmetry, and thus is protected from additive lattice spacing dependent mass renormalizations. Consequently, this taste-5 pion is the lightest, and the pion used to tune the domain-wall valence pion mass to (within a few percent).

$I = 2\ \pi\pi$ scattering 2005 - first dynamical 2+1 flavor





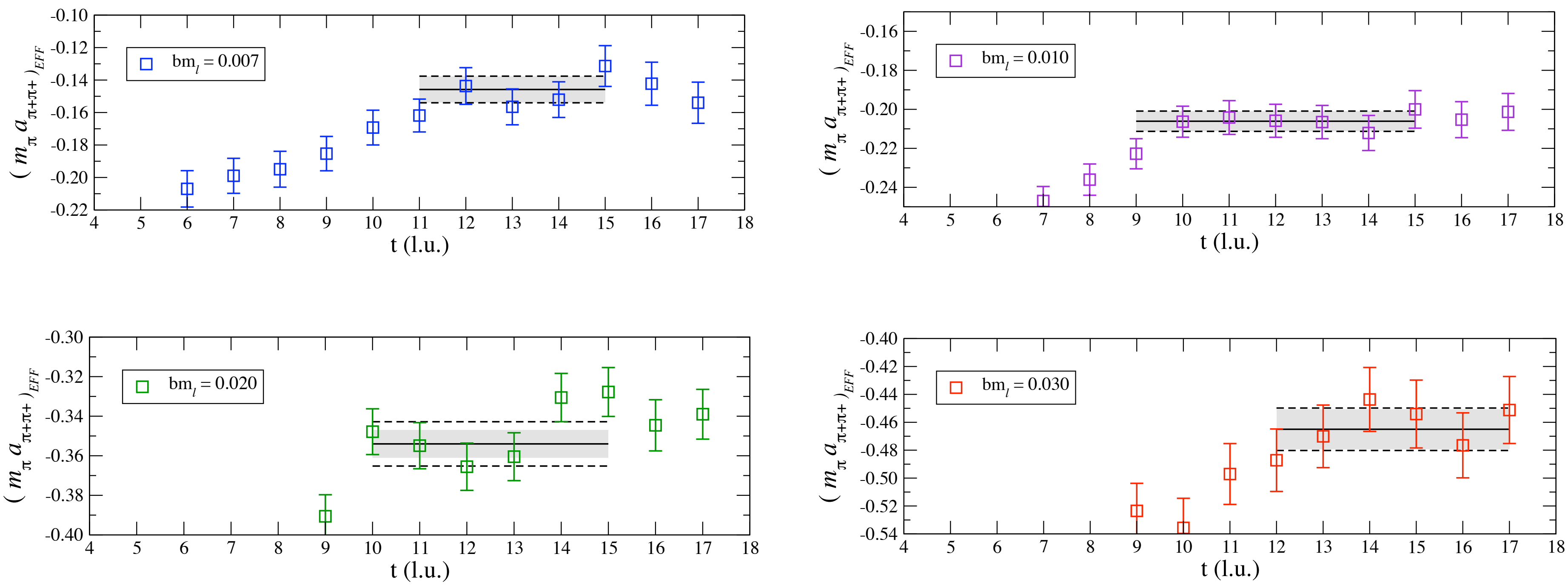
$$m_\pi a_{\pi\pi}^{I=2} = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\left[3\ln\left(\frac{m_\pi^2}{\mu^2}\right) - 1 - l_{\pi\pi}^{I=2}(\mu)\right]\right\}$$

$$m_\pi a_{\pi\pi}^{I=2} = -0.0426 \pm 0.006 \pm 0.0003 \pm 0.0018$$

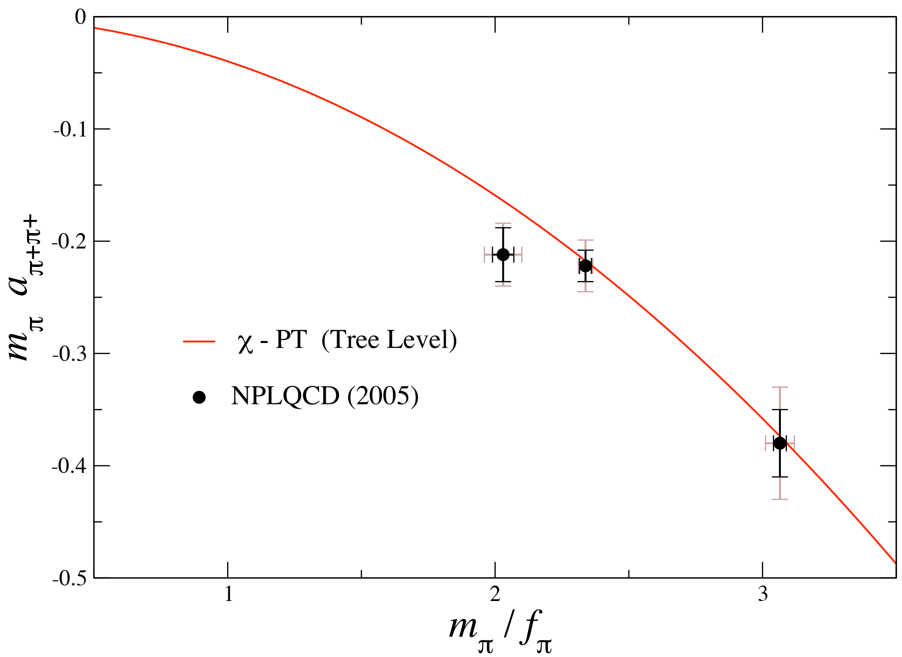
NPLQCD
PRD 73 (2006)

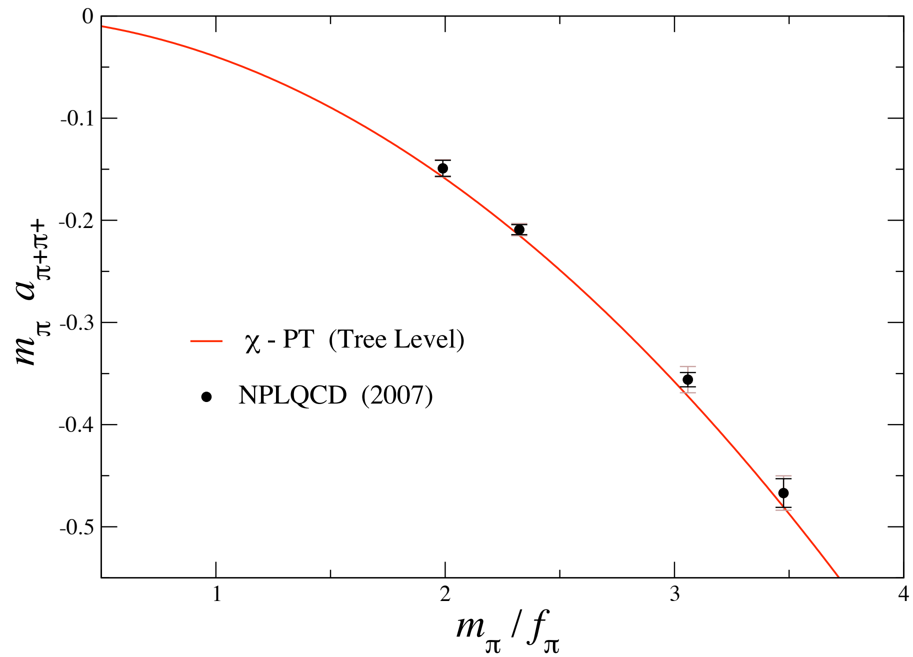
- Used continuum formula for extrapolation - now MA formula exists
- Significant increase in statistics

$I = 2\ \pi\pi$ scattering 2007 - precision results arXiv:0706.3026



$I = 2\ \pi\pi$ scattering 2007 - precision results arXiv:0706.3026

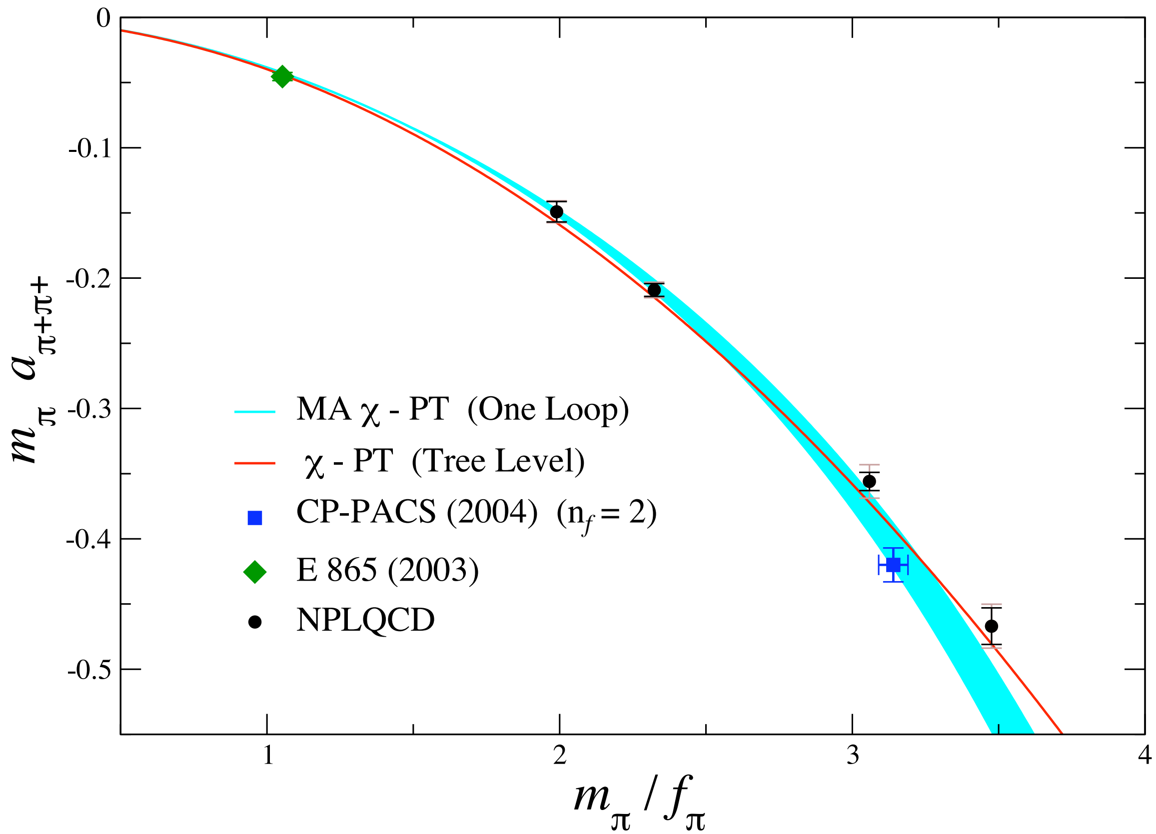




Can address all sources of systematic error (except for staggered action)

- Mixed Action Extrapolation formula (including estimates of NNLO)
- Exponential Corrections to Lüscher's formula
- Residual chiral symmetry breaking from the domain-wall action
- Effective Range corrections

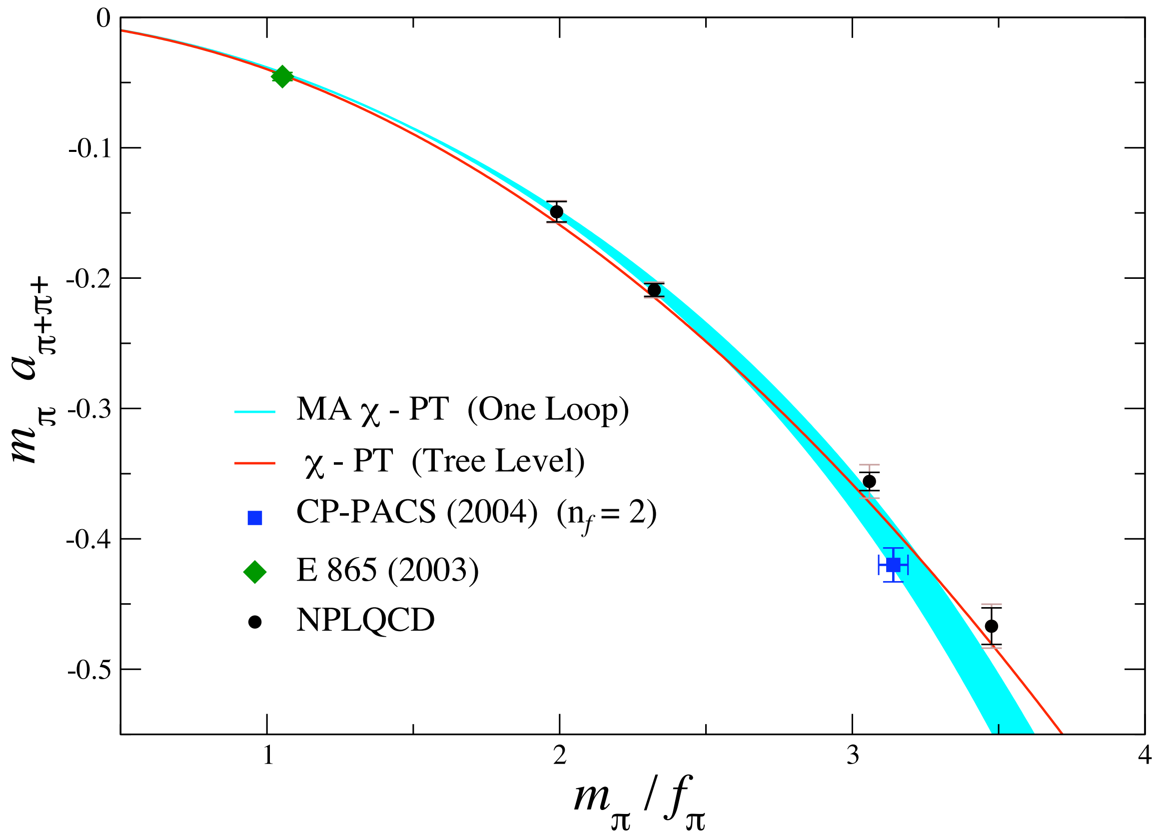
$I = 2$ $\pi\pi$ scattering 2007 - precision results



$$m_\pi a_{\pi\pi}^{I=2} = -0.04330 \pm 0.00042$$

NPLQCD (AW-L)
arXiv:0706.3026

$I = 2\ \pi\pi$ scattering 2007 - precision results



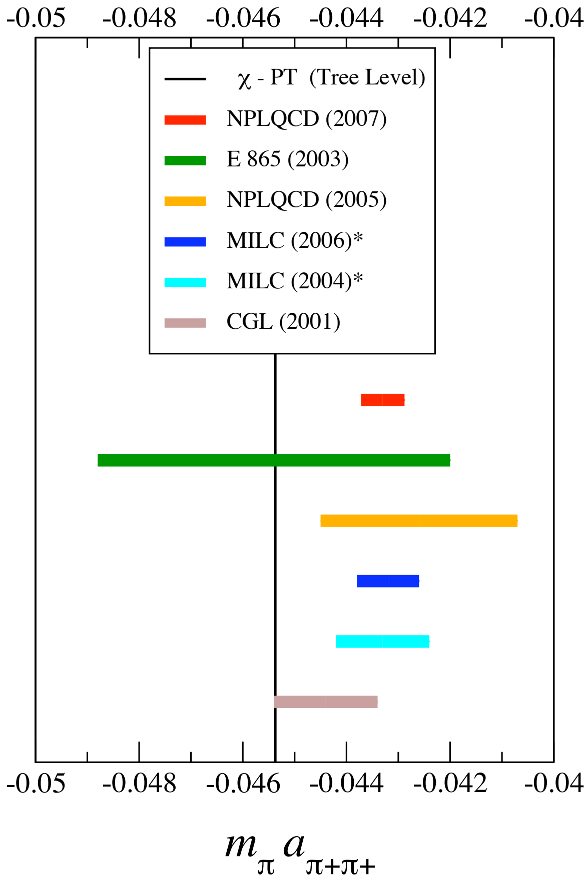
$$m_\pi a_{\pi\pi}^{I=2} = -0.04330 \pm 0.00042$$

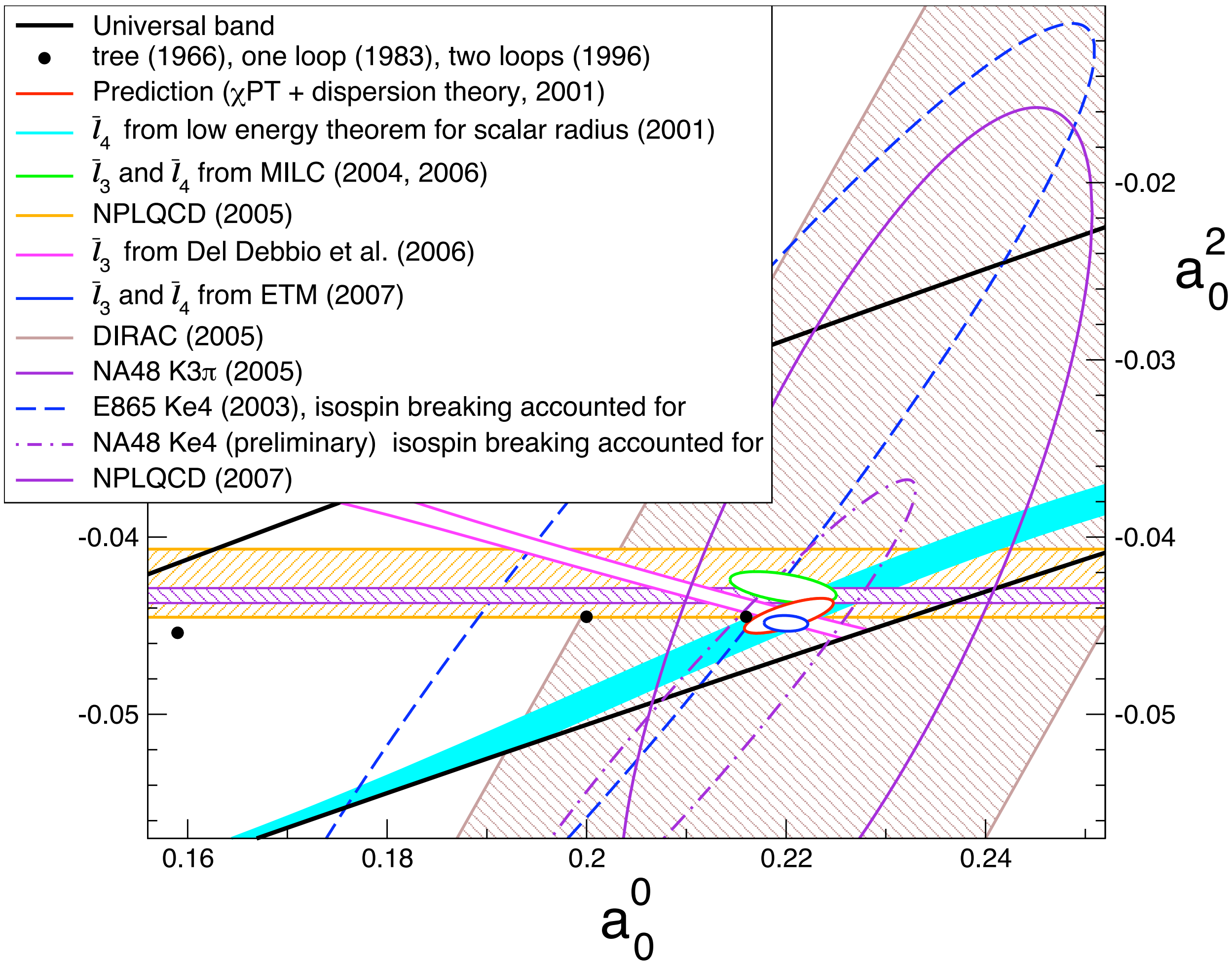
NPLQCD (AW-L)
arXiv:0706.3026

$$m_\pi a_{\pi\pi}^{I=2}(LO) = -0.04438$$

Weinberg
1966

$I = 2\ \pi\pi$ scattering 2007 - precision results arXiv:0706.3026





Mixed Action $\chi\mathrm{PT}$

- J-W. Chen, D. O'Connnell, R.S. Van de Water, A.W-L PRD 73(2006) (hep-lat/0510024)
- J-W. Chen, D. O'Connnell, A.W-L PRD 75(2007) (hep-lat/0611003)
- J-W. Chen, D. O'Connnell, A.W-L arXiv:0706.0035
- K.Orginos, A.W-L arXiv:0705.0572

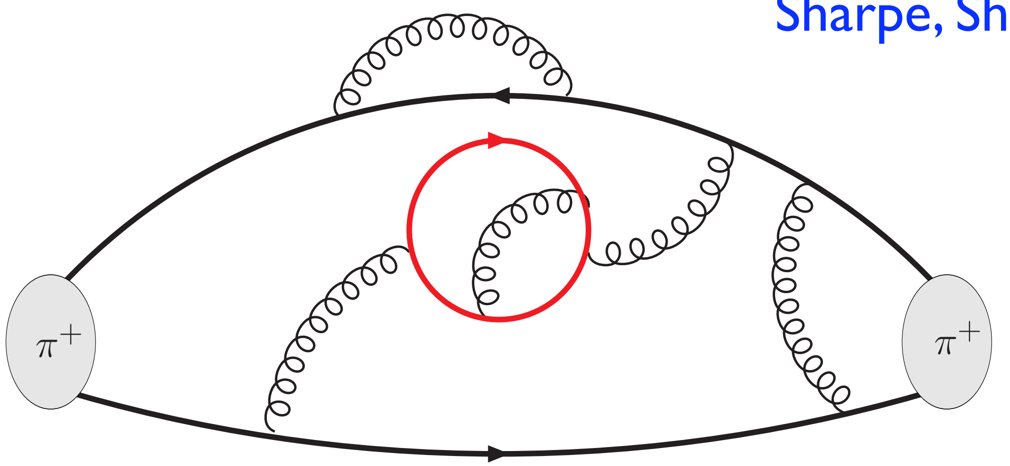
Mixed Actions (MA) and Partial Quenching (PQ)

Bar, Rupak, Shoresh
PRD 67 (2003)
PRD 70 (2004)

Bernard, Golterman PRD 46 (1992)

Sharpe PRD 56 (1997)

Sharpe, Shoresh PRD 62 (2000)
PRD 64 (2001)



$$\langle \pi^\dagger(y)\pi(x)\rangle = \frac{1}{\mathcal{Z}[0]}\int \mathcal{D}\mathcal{A}\,\mathrm{Det}\left(\not{D}_{sea} + m_{sea}\right) e^{-S[\mathcal{A}]}$$

$$\times \mathrm{Tr}\left(\gamma_5 \left(\not{D}_{val} + m_{val}\right)^{-1}_{xy} \gamma_5 \left(\not{D}_{val} + m_{val}\right)^{-1}_{yx}\right)$$

$$\not{D}_{sea} - \not{D}_{val} = \mathcal{O}(b)$$

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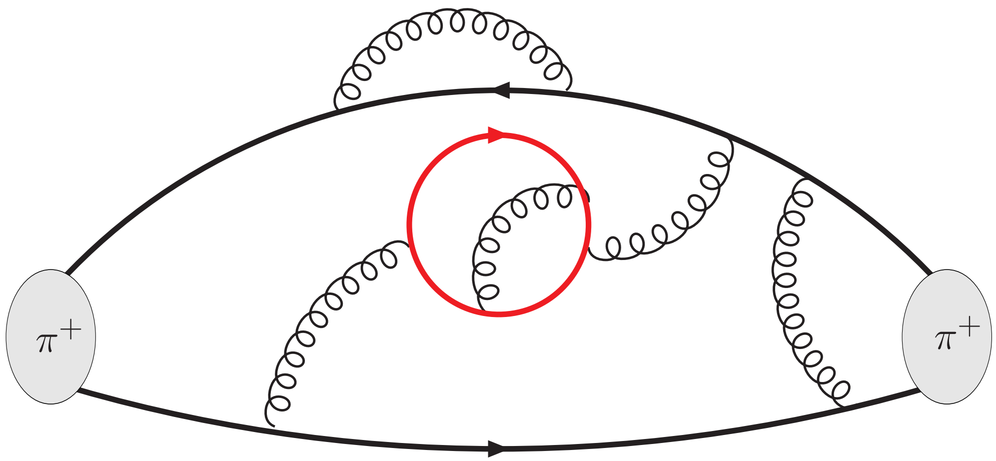

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$$\not{D}_{sea} - \not{D}_{val} = \mathcal{O}(b)$$

$m_{sea} = m_{val}$: QCD

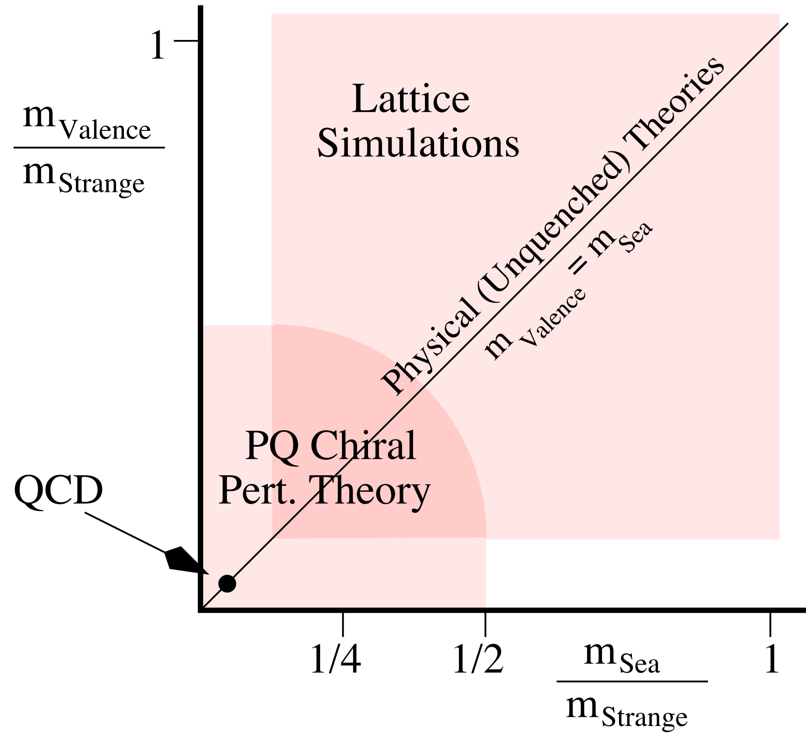
$b \to 0$

$\not{D}_{sea} = \not{D}_{val}$: Partially Quenched QCD

~~$m_{sea} = \infty$: Quenched QCD~~

Why consider PQ or MA theories?

- simulating light sea quarks numerically costly: valence quarks are cheaper



- chiral symmetry of Ginsparg-Wilson quarks ideal: currently prohibitavely costly
- larger parameter space to match effective theory to: QCD limit of theory
- provide means to test effective field theories (EFT):
  do PQ and MA EFTs completely encode all the unitarity violation which is manifest in the low energy dynamics?

Mixed Actions (MA) and Partial Quenching (PQ)

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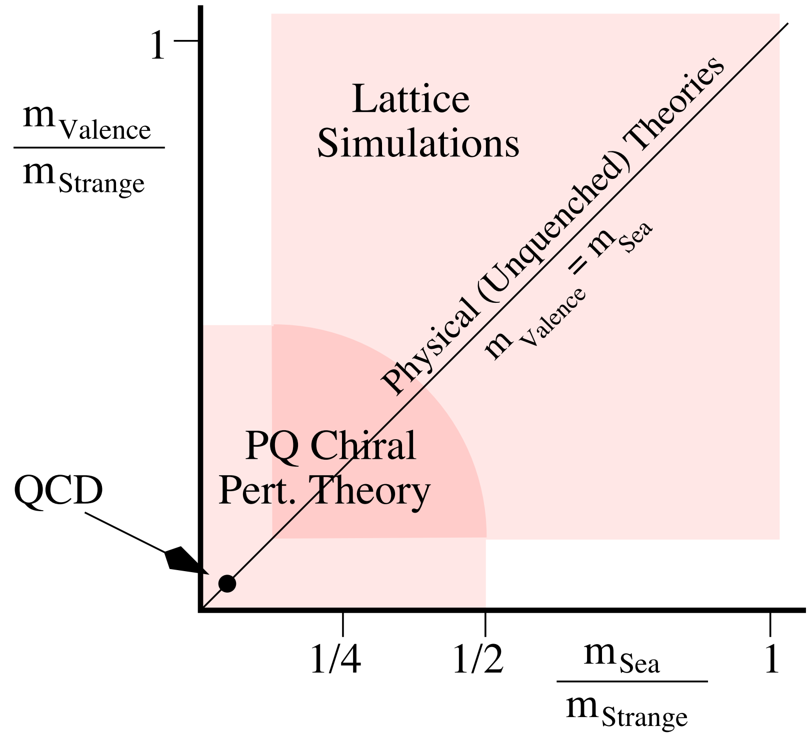

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Mixed Action theories with Ginsparg-Wilson valence quarks

most lattice discretization schemes violate chiral symmetry

Wilson	$\mathcal{O}(b)$
clover Wilson	$\left.\right\}$ $\mathcal{O}(b^2)$
twisted mass (Wilson)	
staggered	

Ginsparg-Wilson fermions have a lattice-chiral symmetry

domain-wall	D.B. Kaplan	Phys.Lett.B (1992)
	Y. Shamir	Nucl.Phys.B (1993)
	V. Furman Y. Shamir	Nucl.Phys.B (1995)
overlap	R. Narayanan H. Neuberger	
	PRL (1993) Nucl.Phys.B (1994,1995)	

Ginsparg-Wilson fermions numerically expensive

MA EFT at Leading Order (LO)

Bar, Rupak, Shoresh
PRD 67 (2003)
PRD 70 (2004)

Ginsparg-Wilson valence
Wilson sea

Bar, Bernard, Rupak, Shoresh
PRD 72 (2005)

Ginsparg-Wilson valence
Staggered sea

today

Ginsparg-Wilson valence
anything sea

→ QCD

Mixed Action Effective Field Theory

Symmetries of mixed action

Valence Fermions: chiral symmetry, CPT, O(4)

Sea Fermions: chiral ~~symmetry~~, CPT, O(4)

Ghost Fermions: chiral symmetry, CPT, O(4)

introduced to remove valence contributions from dynamical quark-antiquark loops: mathematical "trick"

MA EFT at LO: Meson Masses

LO Lagrangian

$$\mathcal{L} = \frac{f^2}{8}\mathrm{str}\left(\partial_\mu \Sigma\, \partial^\mu \Sigma^\dagger\right) + \frac{f^2 B}{4}\mathrm{str}\left(\Sigma m_Q^\dagger + m_Q \Sigma^\dagger\right)$$

$$\mathcal{L}_{MA} = b^2\ \left(\mathcal{U}_{VS} - \mathcal{U}_{sea}\right)$$

form of mixed valence-sea potential is universal

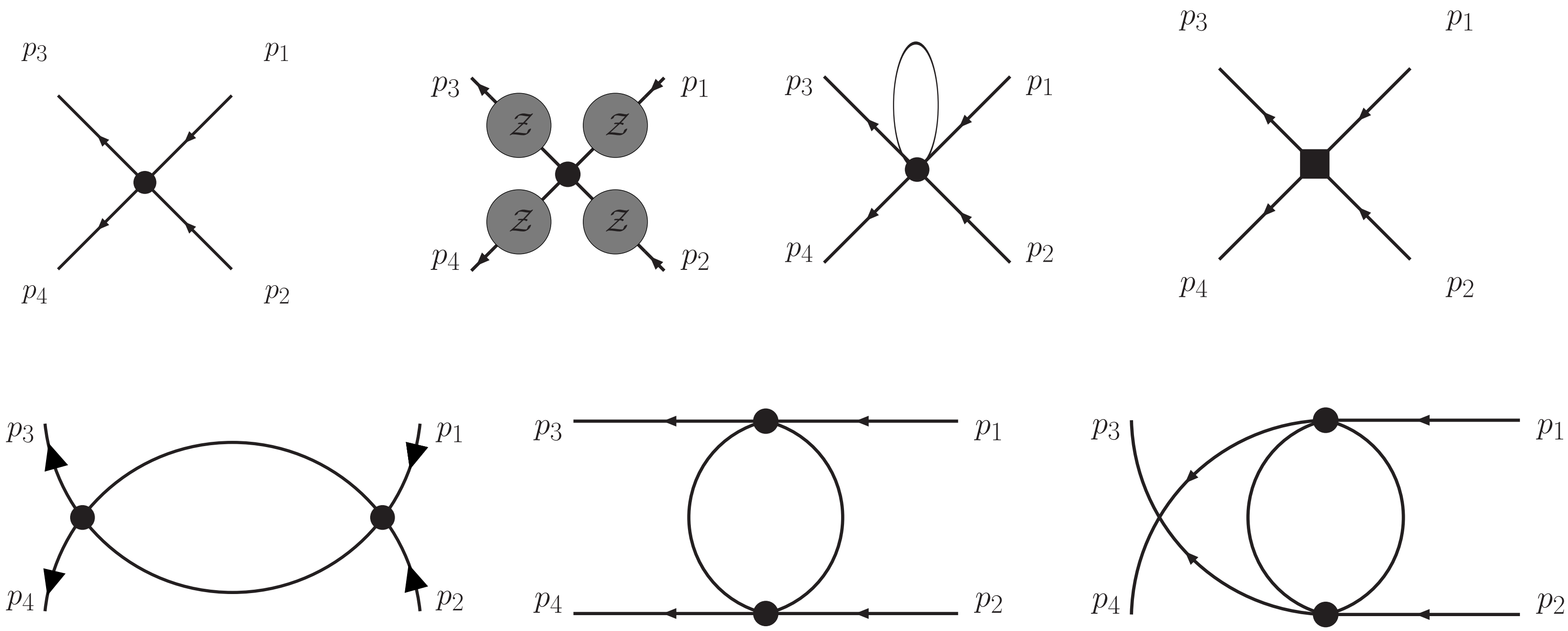
$$\mathcal{U}_{VS} = C_{Mix}\,\mathrm{str}\left(T_3 \Sigma T_3 \Sigma^\dagger\right) \qquad T_3 = \mathcal{P}_S - \mathcal{P}_V$$

valence-valence $\quad m_{vv}^2 = 2B_0 m_v$

valence-sea $\quad m_{vs}^2 = B_0(m_v + m_s) + b^2 \Delta_{Mix} \qquad \Delta_{Mix} = \frac{16 C_{Mix}}{f^2}$

sea-sea $\quad m_{ss}^2 = 2B_0 m_s + b^2 \Delta_{sea}$

$I = 2$ $\pi\pi$ scattering



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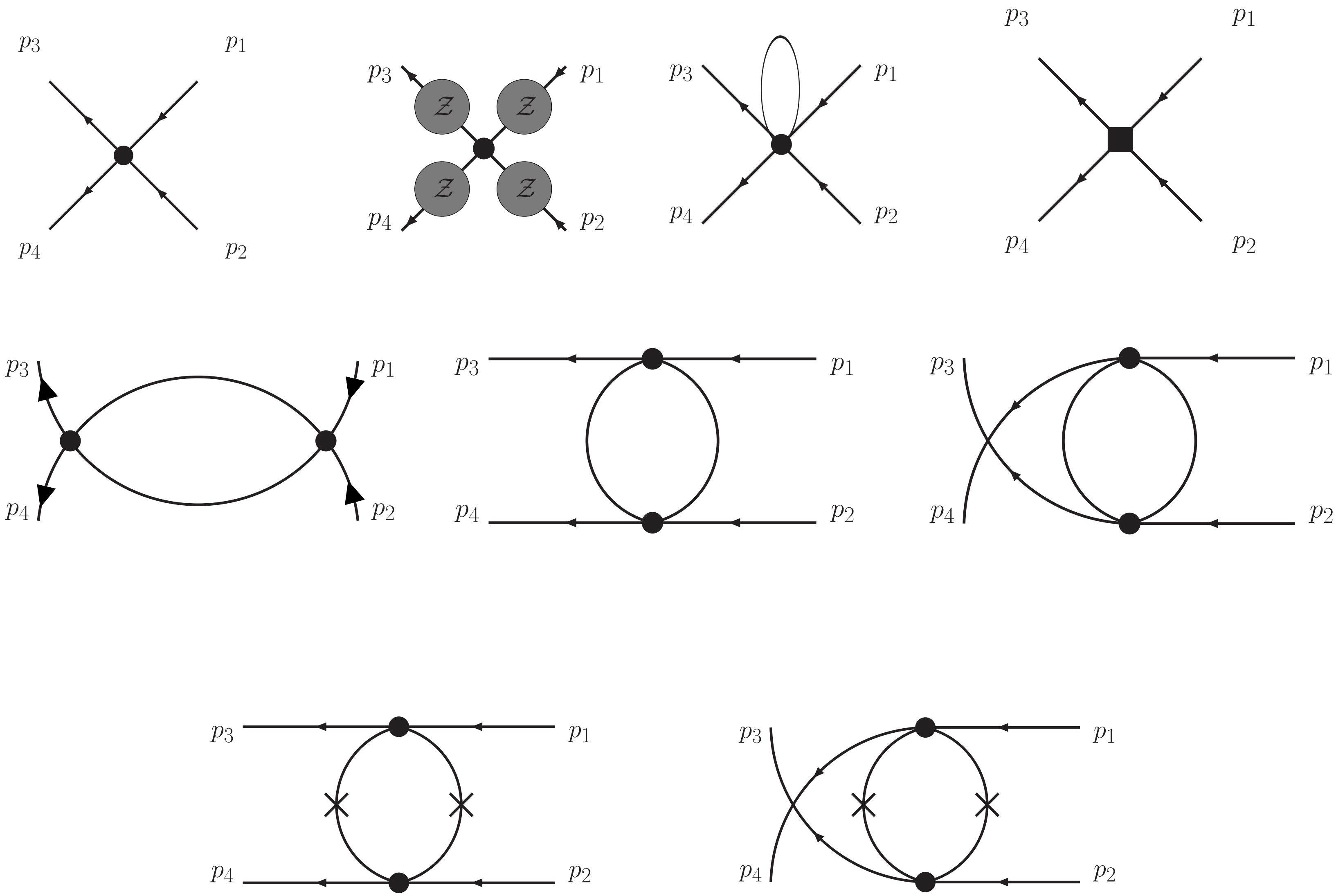


$I = 2\ \pi\pi$ scattering

Adding mixed action and partial quenching effects

$$m_\pi a_2 = -\frac{m_{uu}^2}{8\pi f^2}\left\{1 + \frac{m_{uu}^2}{(4\pi f)^2}\left[4\ln\left(\frac{m_{uu}^2}{\mu^2}\right) + 4\frac{\tilde{m}_{ju}^2}{m_{uu}^2}\ln\left(\frac{\tilde{m}_{ju}^2}{\mu^2}\right) + l'_{\pi\pi}(\mu)\right.\right.$$

$$\left. - \frac{\tilde{\Delta}_{PQ}^2}{m_{uu}^2}\left[\ln\left(\frac{m_{uu}^2}{\mu^2}\right)\right] - \frac{\tilde{\Delta}_{PQ}^4}{6m_{uu}^4}\right]$$

$$\left. + \frac{\tilde{\Delta}_{PQ}^2}{(4\pi f)^2} l'_{PQ}(\mu) + \frac{b^2}{(4\pi f)^2} l'_{b^2}(\mu)\right\}$$

$$\tilde{\Delta}_{PQ}^2 = m_{jj}^2 + \Delta_{sea}(b) - m_{uu}^2$$

$$\tilde{\Delta}_{PQ}^2 = m_{jj}^2 + b^2\Delta_I - m_{uu}^2$$ staggered sea

$$\tilde{\Delta}_{PQ}^2 = m_{jj}^2 + bW_0 - m_{uu}^2$$ Wilson sea

$$\tilde{m}_{ju}^2 = B_0(m_u + m_j) + b^2\Delta_{Mix}$$

Every sickness expected is apparent:

partial quenching $(\tilde{\Delta}_{PQ})$ lattice discretization effects (b)

$I = 2$ $\pi\pi$ scattering

lattice-physical parameters (mass and decay constant measured directly from correlators) the scattering length is given by

$$m_\pi a_{\pi\pi}^{I=2} = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\left[3\ln\left(\frac{m_\pi^2}{\mu^2}\right) - 1 - l_{\pi\pi}^{I=2}(\mu)\right]\right\}$$

$I = 2\ \pi\pi$ scattering

Adding mixed action and partial quenching effects,

$$m_\pi a_{\pi\pi}^{I=2} = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\left[3\ln\left(\frac{m_\pi^2}{\mu^2}\right) - 1 - l_{\pi\pi}^{I=2}(\mu)\right] - \frac{m_\pi^2}{(4\pi f_\pi)^2}\frac{\tilde{\Delta}_{PQ}^4}{6m_\pi^4}\right\}$$

The explicit dependence on the lattice spacing has exactly cancelled - up to a calculable effect from the hairpin interactions!!!

This is independent of the type of sea-quarks

MA EFT at next-to-leading order (NLO)

J-W. Chen, D. O'Connell, AW-L   PRD 75(2007)

MA EFT at NLO:

$I = 2\ \pi\pi$ scattering length in SU(3) and SU(2):

SU(3): chiral symmetry dictates that any strange-quark mass dependence at NLO must be of the form $m_\pi^2 m_K^2$

SU(2):
$$m_\pi a_2^{QCD} = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\left[3\ln\left(\frac{m_\pi^2}{\mu^2}\right) - 1 + l_{\pi\pi}(\mu)\right]\right\}$$

⟹ there can not be any strange-quark mass dependence in the on-shell renormalized scattering length in SU(3)

observed by M. Knecht, B. Moussallam, J. Stern, N.H. Fuchs Nucl.Phys.B (1995)

MA EFT at NLO

- Through the order we are working, $m_\pi^4,\ b^2 m_\pi^2,\ b^4$
all problematic lattice-spacing artifacts can be absorbed as multiplicative renormalizations of the continuum low-energy constants, the chiral condensate, the pion decay constant and the Gasser-Leutwyler constants

$$\delta\mathcal{L}_{GL} = 4B_0\, L_4 \,\mathrm{str}\left(\partial_\mu\Sigma P_V\, \partial^\mu\Sigma^\dagger P_V\right) \mathrm{str}(m_q) + 16 B_0^2\, L_6 \,\mathrm{str}\left(m_q\Sigma^\dagger P_V + P_V\Sigma m_q^\dagger\right) \mathrm{str}(m_q).$$

$$\delta\mathcal{L}_{MA} = b^2 L_{b^2}^{\partial} \,\mathrm{str}\left(\partial_\mu\Sigma P_V\, \partial^\mu\Sigma^\dagger P_V\right)\, \mathrm{str}\left(P_S f(\Sigma)\, P_S f'(\Sigma^\dagger)\right)$$

$$+ b^2 L_{b^2}^{m_q} \,\mathrm{str}\left(m_q\Sigma^\dagger P_V + P_V\Sigma m_q^\dagger\right)\, \mathrm{str}\left(P_S\, g(\Sigma)\, P_S\, g'(\Sigma^\dagger)\right)$$

⇒ Use of a lattice-physical (on-shell) renormalization scheme absorbs all sea-quark effects into the LO parameters, $f,\ B_0$ and thus removes any explicit sea-quark dependence from meson scattering processes

- This holds for all mesonic quantities!!!

- Caution: This does breakdown at the next order - we understand how

MA from PQ

J-W. Chen, D. O'Connell, AW-L arXiv:0706.0035

MA EFT at NLO: Symanzik Action

Mixed Action effects break Symmetry Between Valence and Sea Fermions

$$SU(N_v + N_s|N_v)_L \otimes SU(N_v + N_s|N_v)_R \underbrace{\longrightarrow}_{b \neq 0}$$

$$SU(N_v|N_v)_L \otimes SU(N_v|N_v)_R \otimes SU(N_s)_L \otimes SU(N_s)_R$$

Symanzik Lagrangian $\mathcal{O}(b^2)$ contains terms which distinguish valence and sea fermions

$$\mathcal{L}_{Mix}^{(b^2)} = b^2 C_{Mix}^V \left(\bar{Q}\gamma_\mu \mathcal{P}_V Q\right)\left(\bar{Q}\gamma_\mu \mathcal{P}_S Q\right) + b^2 C_{Mix}^A \left(\bar{Q}\gamma_\mu\gamma_5 \mathcal{P}_V Q\right)\left(\bar{Q}\gamma_\mu\gamma_5 \mathcal{P}_S Q\right)$$

$\mathcal{P}_S^2 = \mathcal{P}_S$ Sea projector

$\mathcal{P}_V^2 = \mathcal{P}_V$ Valence projector

$$\mathcal{P}_S + \mathcal{P}_V = 1$$

MA EFT at NLO: Hadronic Lagrangian: spurion analysis

$$\mathcal{L}_{Mix}^{(b^2)} = b^2 C_{Mix}^{V} \left(\bar{Q}\gamma_\mu \mathcal{P}_V Q\right)\left(\bar{Q}\gamma_\mu \mathcal{P}_S Q\right) + b^2 C_{Mix}^{A} \left(\bar{Q}\gamma_\mu\gamma_5 \mathcal{P}_V Q\right)\left(\bar{Q}\gamma_\mu\gamma_5 \mathcal{P}_S Q\right)$$

$$\mathcal{P}^L_{V(S)} \rightarrow L\mathcal{P}^L_{V(S)}L^\dagger \qquad \downarrow \qquad \mathcal{P}^R_{V(S)} \rightarrow R\mathcal{P}^R_{V(S)}R^\dagger$$

$$\mathcal{L}_{Mix}^{(b^2)} = b^2 \left(\mathcal{U}_M + \mathcal{U}_N + \mathcal{U}_{NN}\right)$$

Hadronic Field Transformation under chiral symmetry

$$\Sigma \rightarrow L\Sigma R^\dagger \qquad \xi \rightarrow L\xi U^\dagger = U\xi R^\dagger \qquad N_V \rightarrow UN_V$$

Projector Transformation under chiral symmetry

$$\left(\xi^\dagger \mathcal{P}^L_{V(S)}\xi\right) \rightarrow U\left(\xi^\dagger \mathcal{P}^L_{V(S)}\xi\right)U^\dagger \qquad \left(\xi \mathcal{P}^R_{V(S)}\xi^\dagger\right) \rightarrow U^\dagger\left(\xi \mathcal{P}^R_{V(S)}\xi^\dagger\right)U$$

$\mathcal{U}_M = \mathrm{str}\left(T_3\Sigma T_3\Sigma^\dagger\right)$ additive mass renormalization for mixed valence-sea mesons

$\mathcal{U}_N = C^N_{Mix}\bar{N}_V N_V$ additive mass renormalization for valence nucleons (baryons)

$$\mathcal{U}_{NN} = D_{2b}^{(^1S_0)}\left(N_V^T P_i^{(^1S_0)} N_V\right)^\dagger\left(N_V^T P_i^{(^1S_0)} N_V\right) + D_{2b}^{(^3S_1)}\left(N_V^T P_i^{(^3S_1)} N_V\right)^\dagger\left(N_V^T P_i^{(^3S_1)} N_V\right)$$

Mixed Action Extrapolation Formulae from PQ ChPT

1. *mesons and quark masses:* $m_{uu} \to m_\pi$ where m_π is the pion mass measured directly from two-point correlator, the lattice-physical pion mass. Similarly, replace tree level meson (quark) masses with their corresponding lattice-physical meson masses, $2B_0 m_u \to m_\pi^2 - NLO$

2. *decay constants:* $f \to f_\pi \ (f_K)$ the lattice-physical decay constant

3. *mixed mesons:* $m_{ju}^2 \to \tilde{m}_{ju}^2 = \frac{1}{2}m_{jj}^2 + \frac{1}{2}m_\pi^2 + b^2\Delta_{Mix}$ mixed meson masses receive additive lattice spacing dependent renormalization which can be measured directly form two-point correlation functions

domain-wall / staggered $\quad b^2\Delta_{\rm Mix} = 0.0336(22) - 0.064(17)m_\pi^2 \ (l.u.)$

$(291 \pm 10 \ {\rm MeV})^2$

K.Orginos, A.W-L arXiv:0705.0572

4. *sea-sea mesons:* $m_{jr}^2 \to \tilde{m}_{jr}^2 = m_{jr}^2 + b^2\Delta_{sea}$ sea-sea mesons receive additive lattice spacing dependent mass renormalization

5. *lattice spacing dependent counterterms:* when appropriate, add lattice spacing dependent counterterms. This can largely be determined by enforcing the scale-independence of the given observable

Mixed Action Extrapolation Formulae from PQ ChPT

all meson quantities at one-loop are straightforward

most baryon observables are straightforward

twist-2 matrix elements: W. Detmold C.J.D. Lin PRD 71 (2005)

LHP Collaboration, J. Negele et. al.

PRL 96 (2006)

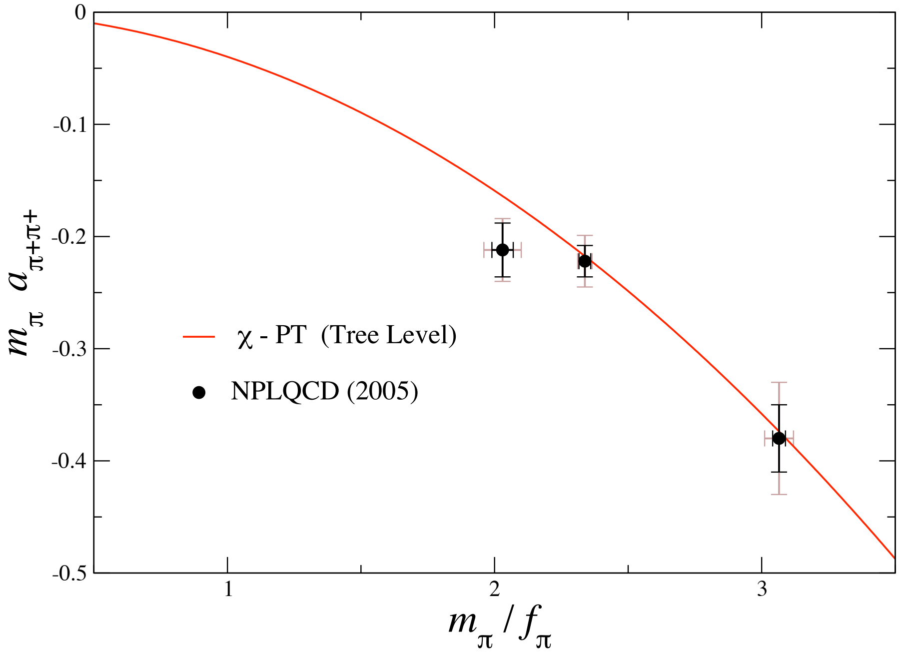
arXiv:0705.4295

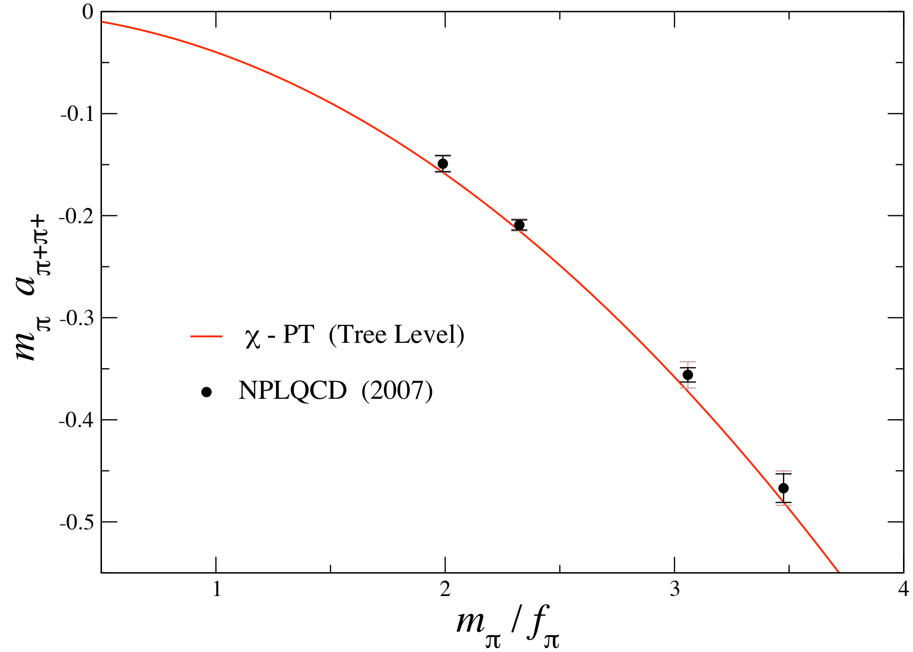
$N \to \Delta$ transitions — C. Alexandrou, Th. Leontiou, J.W. Negele A. Tsapalis PRL 98 (2007)

Nucleon, Delta,
Nucleon to Delta form factors — C. Alexandrou, Th. Korzec, Th. Leontiou, J.W. Negele, A. Tsapalis - this workshop

⋮

$I = 2\ \pi\pi$ scattering 2007 - precision results

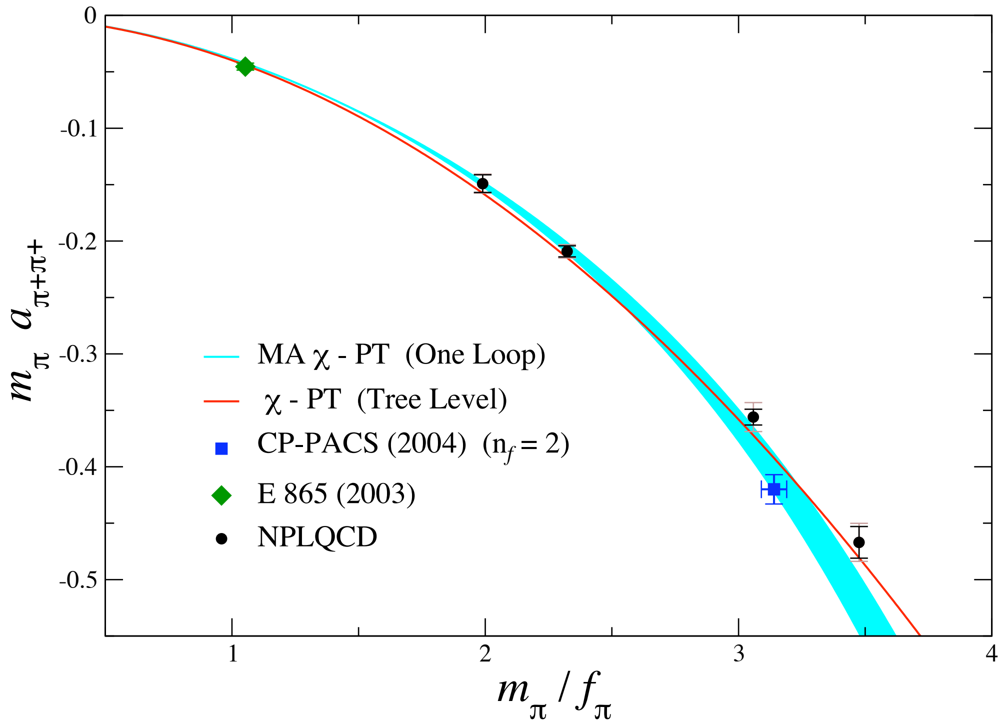




Can address all sources of systematic error (except for staggered action)

- Mixed Action Extrapolation formula (including estimates of NNLO)
- Exponential Corrections to Lüscher's formula
- Residual chiral symmetry breaking from the domain-wall action
- Effective Range corrections

$I = 2\ \pi\pi$ scattering 2007 - precision results

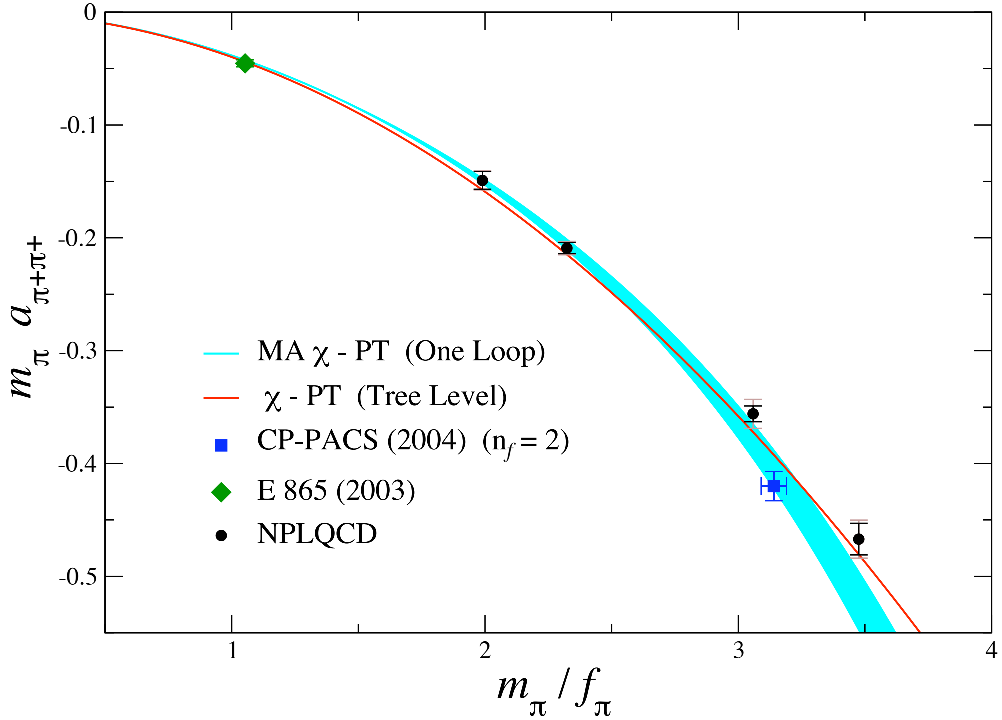


$$m_\pi a_{\pi\pi}^{I=2} = -0.04330 \pm 0.00042$$

NPLQCD (AW-L)
arXiv:0706.3026

Quantity	$m_l = 0.007$	$m_l = 0.010$	$m_l = 0.020$	$m_l = 0.030$
Fit Range	8 – 12	8 – 13	7 – 13	9 – 12
m_π (l.u.)	0.18454(58)(51)	0.22294(31)(09)	0.31132(28)(21)	0.37407(49)(12)
f_π (l.u.)	0.09273(29)(42)	0.09597(16)(10)	0.10179(12)(28)	0.10759(28)(17)
m_π / f_π	1.990(11)(14)	2.3230(57)(30)	3.0585(49)(95)	3.4758(98)(60)
Fit Range	11 – 15	9 – 15	10 – 15	12 – 17
$\Delta E_{\pi\pi}$ (l.u.)	0.00779(47)(14)	0.00745(20)(07)	0.00678(18)(20)	0.00627(23)(10)
$m_\pi a_{\pi\pi}^{I=2}$ ($b \neq 0$)	−0.1458(78)(25)(14)	−0.2061(49)(17)(20)	−0.3540(68)(89)(35)	−0.465(14)(06)(05)
$l_{\pi\pi}^{I=2}$ ($b \neq 0$)	6.1(1.9)(0.7)(0.4)	5.23(68)(24)(28)	6.53(32)(42)(16)	6.90(40)(18)(13)
δ ($b \neq 0$)(degrees)	−1.71(14)(04)	−2.181(81)(28)	−3.01(09)(12)	−3.46(17)(07)
$\lvert\mathbf{p}\rvert / m_\pi$	0.2032(60)(18)	0.1836(25)(09)	0.1480(17)(23)	0.1298(24)(10)

$I = 2\ \pi\pi$ scattering 2007 - precision results



$$m_\pi a_{\pi\pi}^{I=2} = -0.04330 \pm 0.00042$$

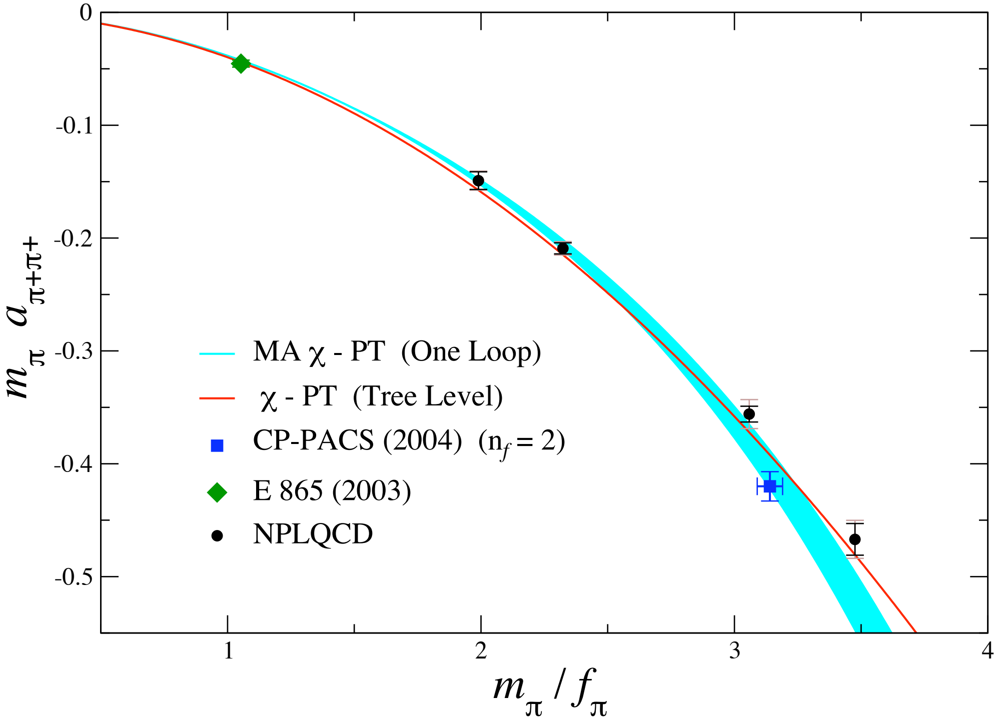
NPLQCD (AW-L)
arXiv:0706.3026

TABLE VII: Corrections and uncertainties in $m_\pi a_{\pi\pi}^{I=2}$ for $n_f = 2$.

Quantity	$m_l = 0.007$	$m_l = 0.010$	$m_l = 0.020$	$m_l = 0.030$
$\Delta_{MA}\left(m_\pi a_{\pi\pi}^{I=2}\right)$	0.0033(02)(02)	0.0030(02)(04)	0.0023(01)(10)	0.0018(01)(16)
$\Delta_{FV}\left(m_\pi a_{\pi\pi}^{I=2}\right)$	± 0.0055	± 0.0022	± 0.0003	± 0.0001
$\Delta_{m_{res}}\left(m_\pi a_{\pi\pi}^{I=2}\right)$	± 0.0032	± 0.0035	± 0.0036	± 0.0032

For pion mass and decay constant, it is found that one-loop formulae get correct order of magnitude FV corrections, but two-loop formulae are needed for accurate corrections. G. Colangelo, S. Durr, C. Haefeli NPB 721 (2005)

$I = 2\ \pi\pi$ scattering 2007 - precision results



$$m_\pi a_{\pi\pi}^{I=2} = -0.04330 \pm 0.00042$$

NPLQCD (AW-L)
arXiv:0706.3026

TABLE VII: Corrections and uncertainties in $m_\pi a_{\pi\pi}^{I=2}$ for $n_f = 2$.

Quantity	$m_l = 0.007$	$m_l = 0.010$	$m_l = 0.020$	$m_l = 0.030$
$\Delta_{MA}\left(m_\pi a_{\pi\pi}^{I=2}\right)$	0.0033(02)(02)	0.0030(02)(04)	0.0023(01)(10)	0.0018(01)(16)
$\Delta_{FV}\left(m_\pi a_{\pi\pi}^{I=2}\right)$	±0.0055	±0.0022	±0.0003	±0.0001
$\Delta_{m_{res}}\left(m_\pi a_{\pi\pi}^{I=2}\right)$	±0.0032	±0.0035	±0.0036	±0.0032

Quantity	$m_l = 0.007$	$m_l = 0.010$	$m_l = 0.020$	$m_l = 0.030$
$\Delta\left(m_\pi a_{\pi\pi}^{I=2}\right)$	0.0033(02)(02)(32)(55)	0.0030(02)(04)(35)(22)	0.0023(01)(10)(36)(03)	0.0018(01)(16)(32)(01)
$m_\pi a_{\pi\pi}^{I=2}\ (b \to 0)$	−0.1491(78)(32)	−0.2091(49)(34)	−0.356(07)(11)	−0.467(14)(09)
$l_{\pi\pi}^{I=2}\ (b \to 0)$	5.3(1.9)(1.8)	4.83(68)(73)	6.42(32)(51)	6.85(40)(27)

$I = 1$ KK Scattering

$I = 3/2$ $K\pi$ Scattering

f_K/f_π

Applications: $I=1\ KK$ Scattering

PRD 75(2007)

Mixed Action Computation

$$m_K a_{KK}^{I=1} = -\frac{m_K^2}{8\pi f_K^2}\left\{1+\frac{m_K^2}{(4\pi f_K)^2}\left[C_\pi \ln\left(\frac{m_\pi^2}{\mu^2}\right)+C_K \ln\left(\frac{m_K^2}{\mu^2}\right)\right.\right.$$
$$\left.\left.+C_X \ln\left(\frac{\tilde{m}_X^2}{\mu^2}\right)+C_{ss}\ln\left(\frac{m_{ss}^2}{\mu^2}\right)+C_0-32(4\pi)^2\, L_{KK}^{I=1}\right]\right\}$$

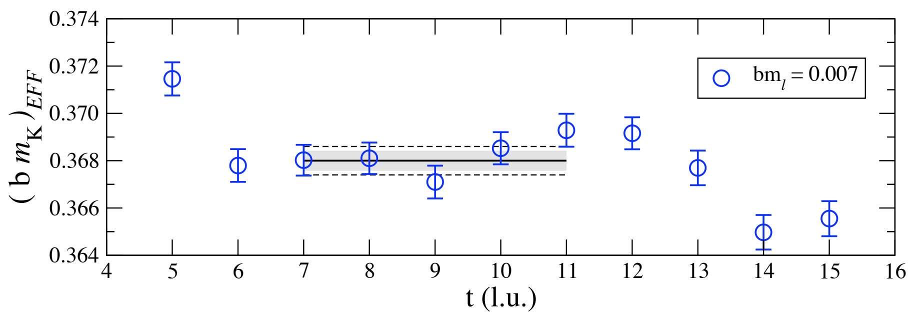
SU(3) Limit (not yet appeared in literature)

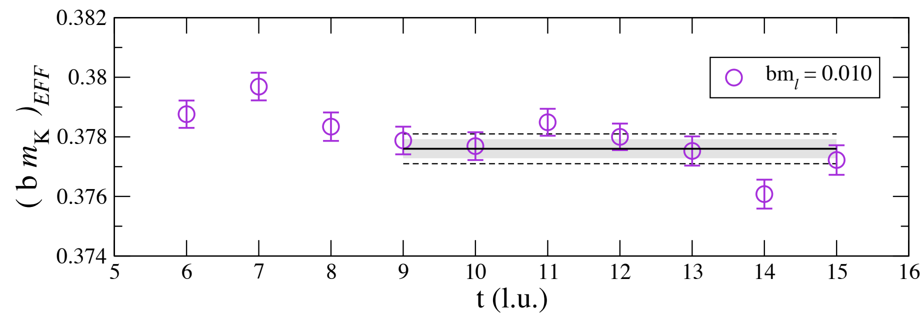
$$m_K a_{KK}^{I=1} = -\frac{m_K^2}{8\pi f_K^2}\left\{1+\frac{m_K^2}{(4\pi f_K)^2}\left[2\ln\left(\frac{m_K^2}{\mu^2}\right)-\frac{2m_\pi^2}{3(m_\eta^2-m_\pi^2)}\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right.\right.$$
$$\left.\left.+\frac{2(20m_K^2-11m_\pi^2)}{27(m_\eta^2-m_\pi^2)}\ln\left(\frac{m_\eta^2}{\mu^2}\right)-\frac{14}{9}-32(4\pi)^2\, L_{KK}^{I=1}(\mu)\right]\right\}$$

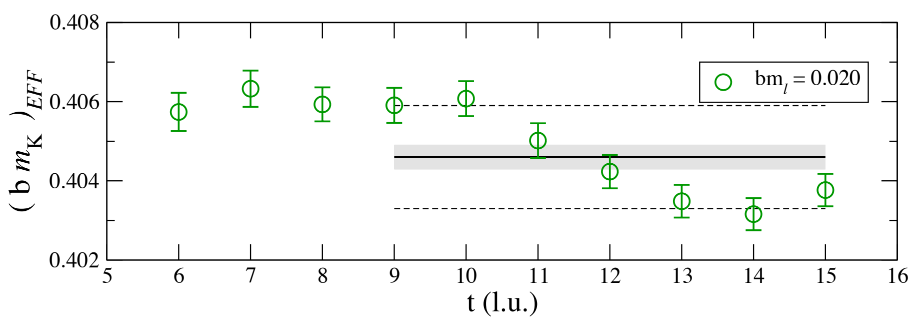
Applications: $I = 1\ KK$ Scattering

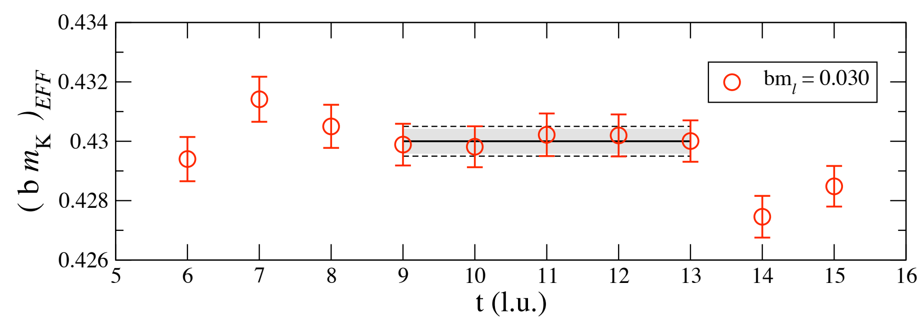
arXiv:0709.1169

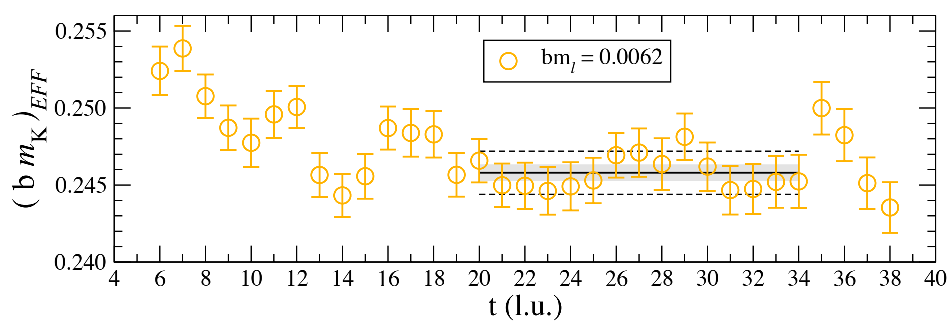
Kaon Effective Mass Plots







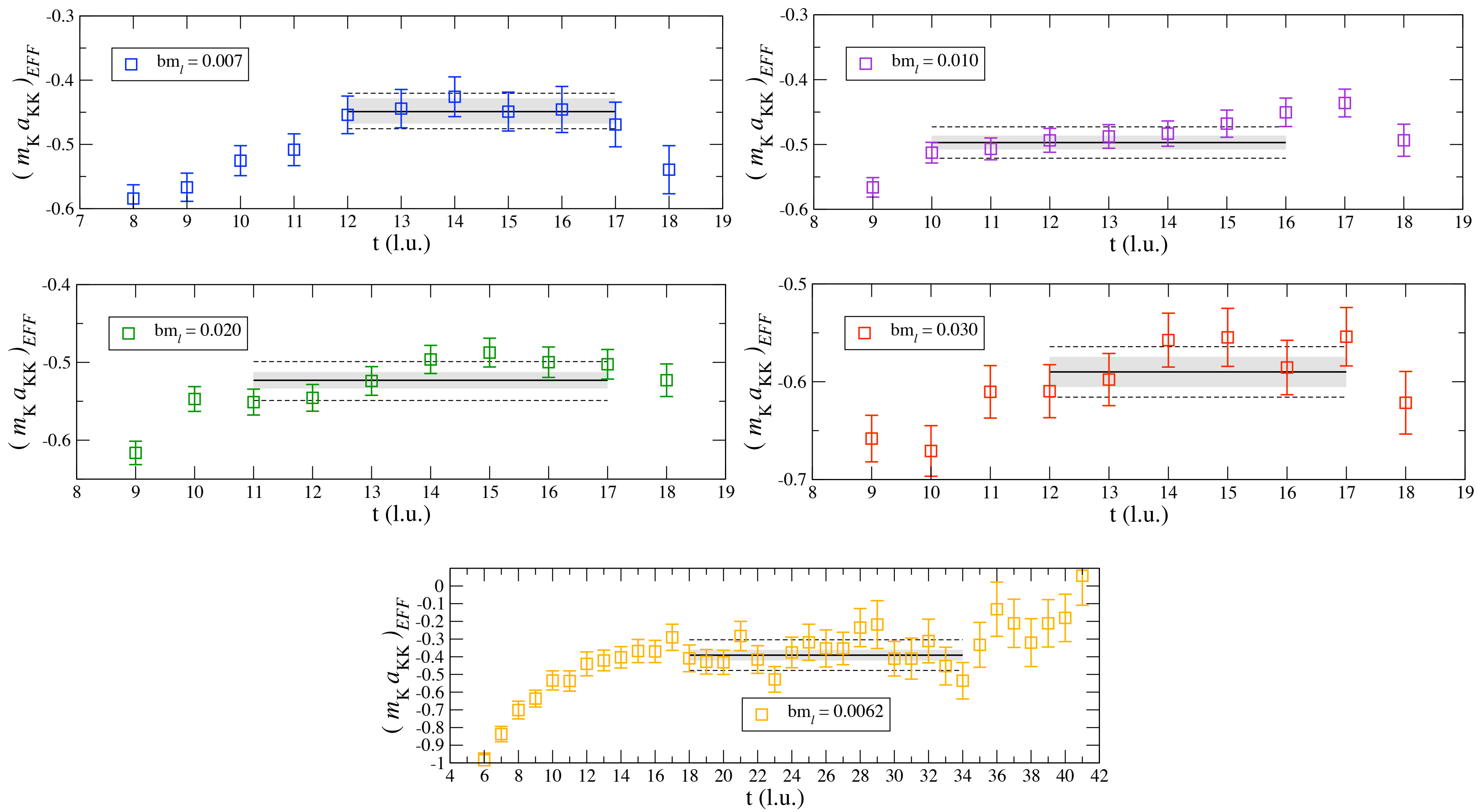




Applications: $I = 1\ KK$ Scattering

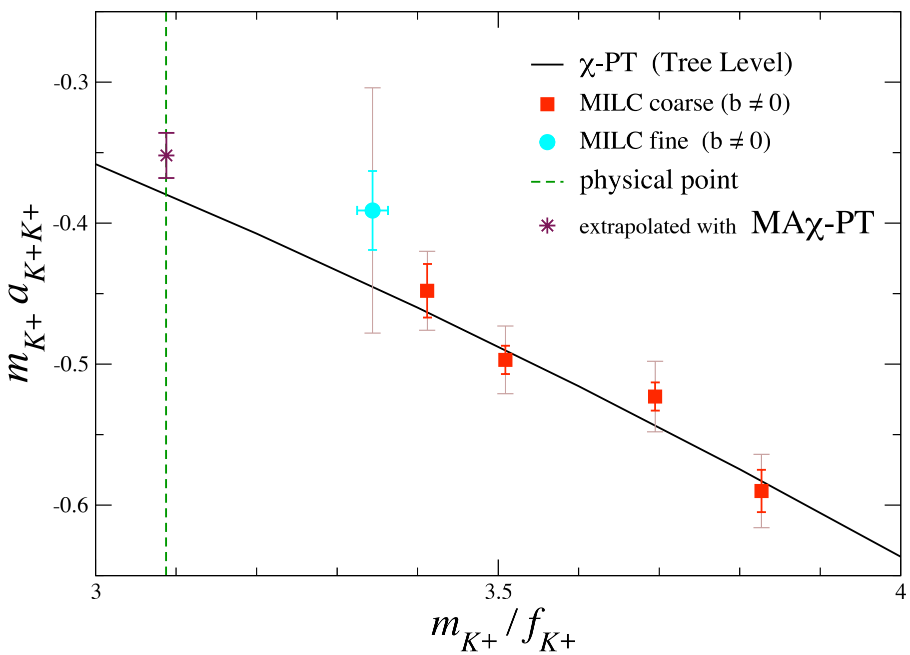
arXiv:0709.1169

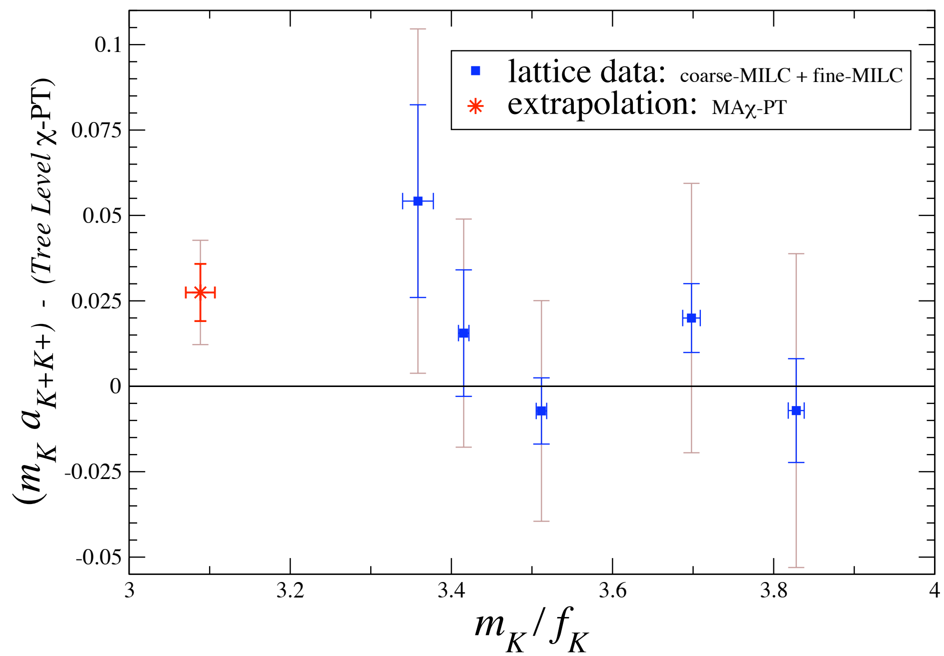
Kaon Effective Scattering Length Plots



Applications: $I = 1\ KK$ Scattering

arXiv:0709.1169

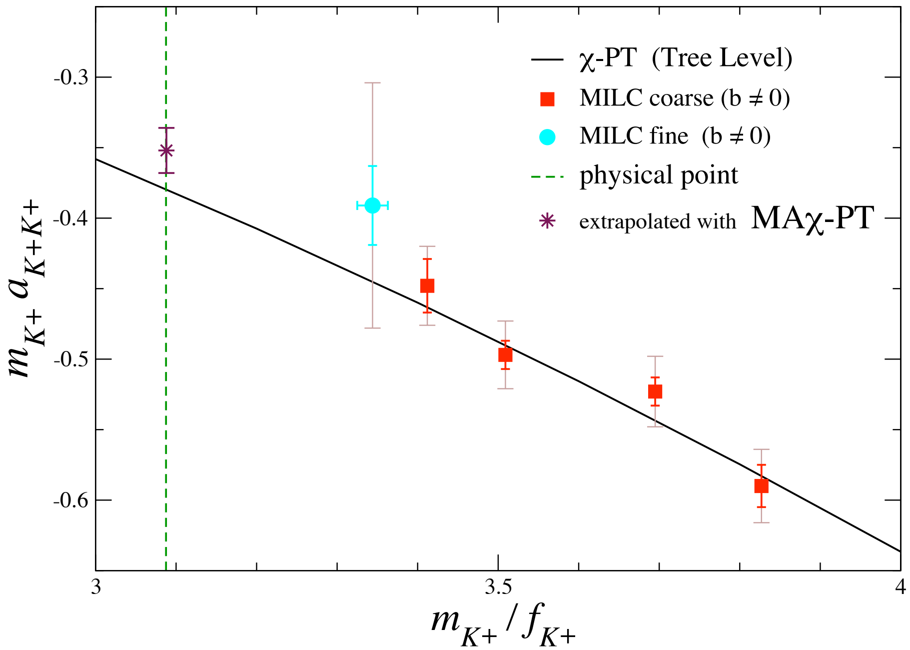


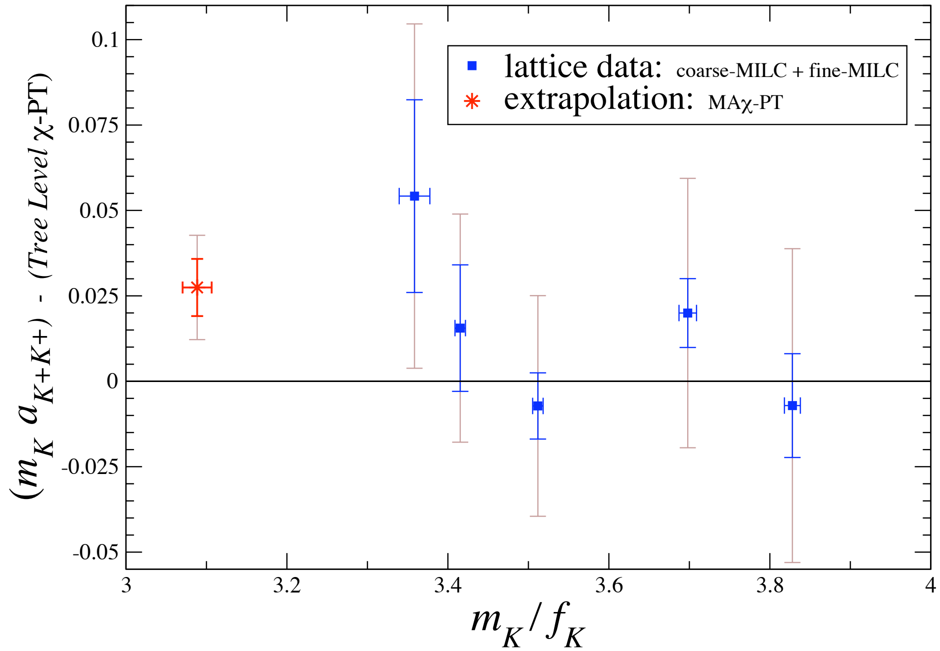


Quantity	$m_l = 0.007$	$m_l = 0.010$	$m_l = 0.020$	$m_l = 0.030$	$m_l = 0.0062$
$b\ m_\pi$	0.1846(4)(2)	0.2226(4)(3)	0.3104(3)(15)	0.3747(4)(8)	0.1453(5)(13)
Fit Range	8–14	9–13	9–15	6–13	17–39
$b\ m_K$	0.3680(4)(4)	0.3776(3)(4)	0.4046(3)(13)	0.4300(4)(3)	0.2458(5)(13)
Fit Range	7–11	9–15	9–15	9–13	20–34
m_π/f_K	1.712(4)(3)	2.069(3)(5)	2.835(3)(11)	3.335(4)(9)	1.978(15)(12)
m_K/f_K	3.412(5)(4)	3.509(3)(6)	3.695(3)(10)	3.827(4)(9)	3.344(19)(21)
ΔE_{KK}(l.u.)	0.00619(30)(32)	0.00663(15)(35)	0.00606(14)(22)	0.00613(19)(10)	0.00437(36)(105)
Fit Range	12–17	10–16	11–17	12–17	18–34
$m_{K^+}a_{K^+K^+}$ $(b \neq 0)$	-0.448(19)(20)	-0.497(10)(22)	-0.523(10)(23)	-0.590(15)(21)	-0.391(28)(82)

Applications: $I=1$ KK Scattering

arXiv:0709.1169





Quantity	$m_l = 0.007$	$m_l = 0.010$	$m_l = 0.020$	$m_l = 0.030$
$\Delta_{MA}\,(m_K a_{KK}^{I=1})$	-0.0067(14)	-0.0062(16)	-0.0052(19)	-0.0048(21)
$\Delta_{NNLO}\,(m_K a_{KK}^{I=1})$	±0.016	±0.019	±0.028	±0.037
$\Delta_{FV}\,(m_K a_{KK}^{I=1})$	±0.001	±0.001	±0.000	±0.000
$\Delta_{m_{res}}\,(m_K a_{KK}^{I=1})$	±0.007	±0.006	±0.005	±0.004
$\Delta_{range}\,(m_K a_{KK}^{I=1})$	± 0.008	±0.008	±0.008	±0.007
$m_{K^+} a_{K^+K^+}$ $(b \to 0)$	-0.441(19)(20)(19)	-0.491(10)(22)(22)	-0.518(10)(23)(30)	-0.585(15)(21)(38)
$32(4\pi)^2 L_{KK}^{I=1}(f_K)$	7.3(5)(8)	6.8(3)(8)	7.7(2)(8)	7.4(3)(8)

Quantity	$m_l = 0.0062$
$\Delta_{MA}\,(m_K a_{KK}^{I=1})$	-0.0048(15)
$\Delta_{NNLO}\,(m_K a_{KK}^{I=1})$	±0.013
$\Delta_{FV}\,(m_K a_{KK}^{I=1})$	±0.001
$\Delta_{m_{res}}\,(m_K a_{KK}^{I=1})$	±0.004
$\Delta_{range}\,(m_K a_{KK}^{I=1})$	±0.004
$m_{K^+} a_{K^+K^+}$ $(b \to 0)$	-0.387(28)(82)(14)
$32(4\pi)^2 L_{KK}^{I=1}(f_K)$	8.4(9)(2.6)

Applications: Kπ Scattering

PRD 75(2007)

Kaon-pion system has new effect not seen in KK or $\pi\pi$ system - at one-loop the presence of valence-sea mesons.

$$\mu_{K\pi} a_{K\pi}^{I=3/2} = -\frac{\mu_{K\pi}^2}{4\pi f_K f_\pi}\left[1 - \frac{32 m_K m_\pi}{f_K f_\pi} L_{\pi\pi}^{I=2}(\mu) + \frac{8(m_K - m_\pi)^2}{f_K f_\pi} L_5(\mu)\right]$$
$$+\mu_{K\pi}\left[a_{vv}^{K\pi,3/2}(\mu) + a_{vs}^{K\pi,3/2}(\mu)\right]$$

QCD limit, reduces to
B. Kubis U. Meissner Phys.Lett.B (2002)

$b^2 \ln(\mu^2)$ still cancels - Ginsparg-Wilson chiral valence symmetry protects amplitude from these corrections

- counter term structure of scattering length is identical to that in QCD. Mixed mesons introduce an additional unknown Δ_{Mix}
- Measured NPLQCD PRD 74 (2006) K.Orginos, A.W-L
- Mixed Action Corrections smaller than $I = 2$ $\pi\pi$ (in percentage diff)

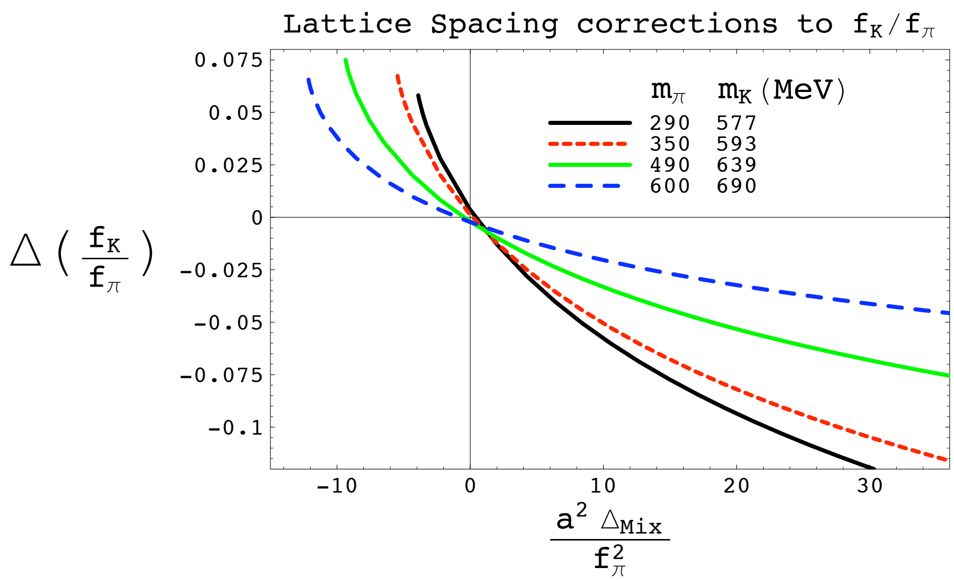
Applications: f_K/f_π

PRD 75(2007)

$$\frac{f_K}{f_\pi} = 1 + \frac{5m_\pi^2}{4(4\pi f)^2}\ln\left(\frac{m_\pi^2}{\mu^2}\right) - \frac{m_K^2}{2(4\pi f)^2}\ln\left(\frac{m_K^2}{\mu^2}\right) - \frac{3m_\eta^2}{4(4\pi f)^2}\ln\left(\frac{m_\eta^2}{\mu^2}\right) + \frac{8(m_K^2 - m_\pi^2)}{f^2} L_5(\mu)$$

- Measured NPLQCD   PRD 75(2007) - NPLQCD (hep-lat/0606023)

$$\Delta\left(\frac{f_K}{f_\pi}\right) = \frac{\left.\frac{f_K}{f_\pi}\right|_{MA} - \left.\frac{f_K}{f_\pi}\right|_{QCD}}{\left.\frac{f_K}{f_\pi}\right|_{QCD}}$$



$$\left.\frac{f_K}{f_\pi}\right|_{MA} \propto \frac{8(m_K^2 - m_\pi^2)}{f_K f_\pi} L_5$$

$$-(600\ \mathrm{MeV})^2 \lesssim b^2 \Delta_{Mix} \lesssim (800\ \mathrm{MeV})^2$$

$$f_K/f_\pi = 1.218 \pm 0.002\ {}^{+0.011}_{-0.024}$$

Applications: f_K/f_π

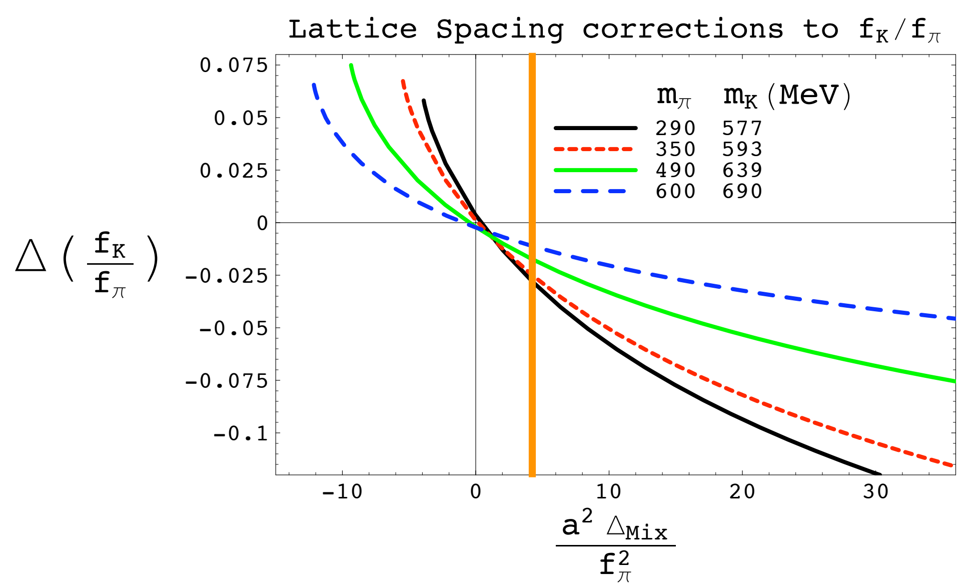
PRD 75(2007) hep-lat/0611003

$$\frac{f_K}{f_\pi} = 1 + \frac{5m_\pi^2}{4(4\pi f)^2}\ln\left(\frac{m_\pi^2}{\mu^2}\right) - \frac{m_K^2}{2(4\pi f)^2}\ln\left(\frac{m_K^2}{\mu^2}\right) - \frac{3m_\eta^2}{4(4\pi f)^2}\ln\left(\frac{m_\eta^2}{\mu^2}\right) + \frac{8(m_K^2 - m_\pi^2)}{f^2} L_5(\mu)$$

- Measured NPLQCD PRD 75(2007) - NPLQCD (hep-lat/0606023)

$$\Delta\left(\frac{f_K}{f_\pi}\right) = \frac{\left.\frac{f_K}{f_\pi}\right|_{MA} - \left.\frac{f_K}{f_\pi}\right|_{QCD}}{\left.\frac{f_K}{f_\pi}\right|_{QCD}}$$

$$\left.\frac{f_K}{f_\pi}\right|_{MA} \propto \frac{8(m_K^2 - m_\pi^2)}{f_K f_\pi} L_5$$



$$-(600\ \mathrm{MeV})^2 \lesssim b^2 \Delta_{Mix} \lesssim (800\ \mathrm{MeV})^2$$

$$f_K/f_\pi = 1.218 \pm 0.002\ {}^{+0.011}_{-0.024}$$

This deviation is within the error band of PRD 75(2007) - NPLQCD

Applications: Counter Terms

$$m_\pi \, a_{\pi\pi}^{I=2} \propto \frac{4 m_\pi^4}{\pi \, f_\pi^4} \, L_{\pi\pi}^{I=2} \qquad\qquad m_K \, a_{KK}^{I=1} \propto \frac{4 m_K^4}{\pi \, f_K^4} \, L_{KK}^{I=1}$$

$$\mu_{K\pi} a_{K\pi}^{I=3/2} \propto \frac{\mu_{K\pi}^2}{4\pi f_K f_\pi} \left[\frac{32 m_K m_\pi}{f_K f_\pi} \, L_{\pi\pi}^{I=2}(\mu) - \frac{8(m_K - m_\pi)^2}{f_K f_\pi} \, L_5(\mu) \right]$$

$$\frac{f_K}{f_\pi} \propto \frac{8(m_K^2 - m_\pi^2)}{f_\pi f_K} \, L_5 \qquad\qquad \mu_{\pi K} = \frac{m_\pi m_K}{m_\pi + m_K}$$

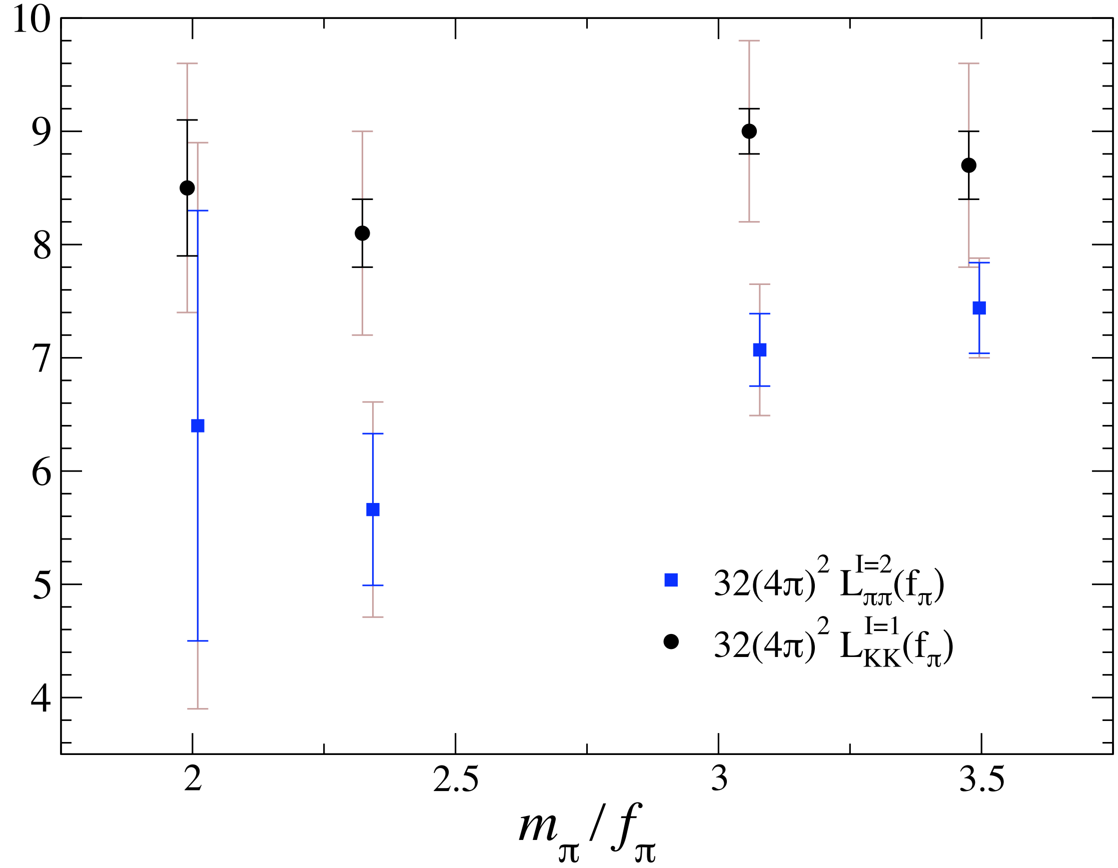
$$L_{\pi\pi}^{I=2} = 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8$$

$$L_{KK}^{I=1} = L_{\pi\pi}^{I=2}$$

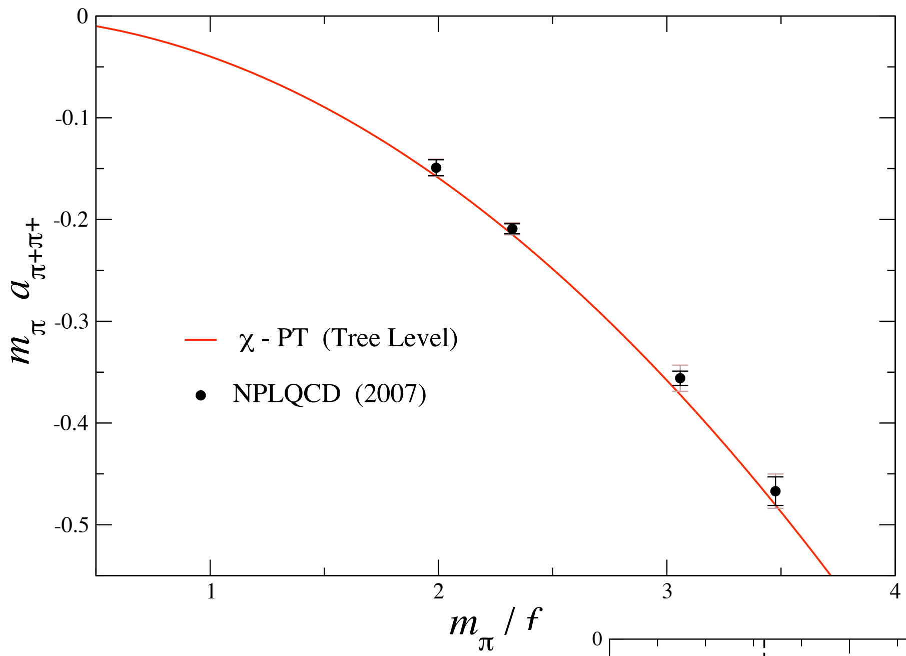
NPLQCD:

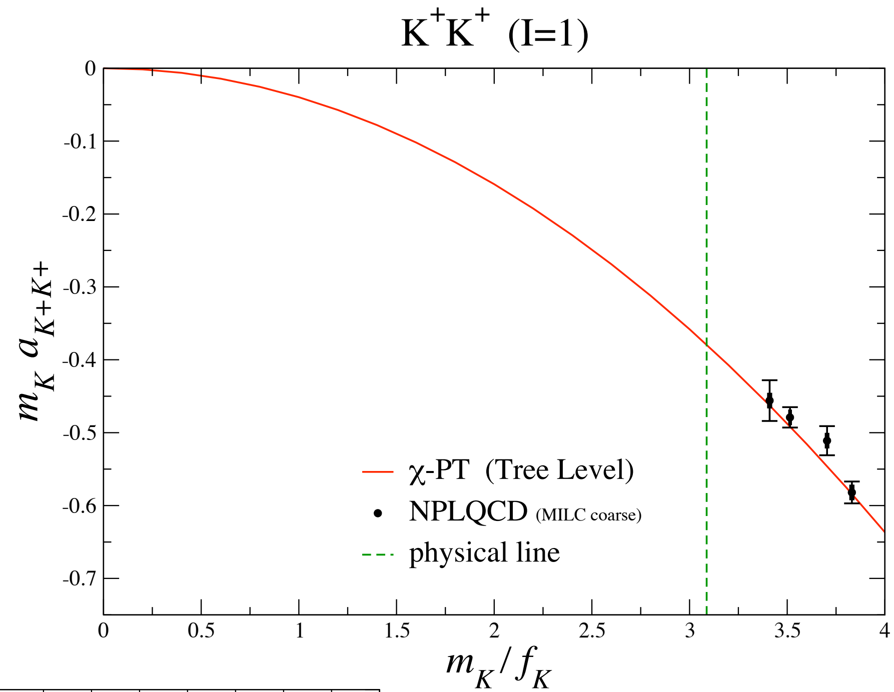
$L_{\pi\pi}^{I=2}$	$I = 2 \ \pi\pi$	PRD 73 (2006)
L_5	f_K / f_π	PRD 75 (2007)
$L_{\pi\pi}^{I=2}$ L_5	$I = 3/2 \ \pi K$	PRD 74 (2006)

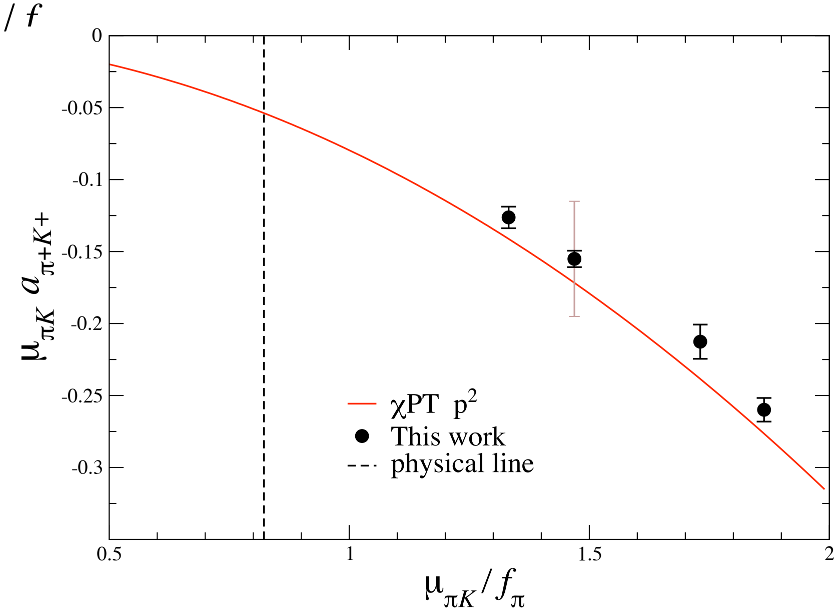
Applications: Counter Terms



Applications: Two Meson Scattering







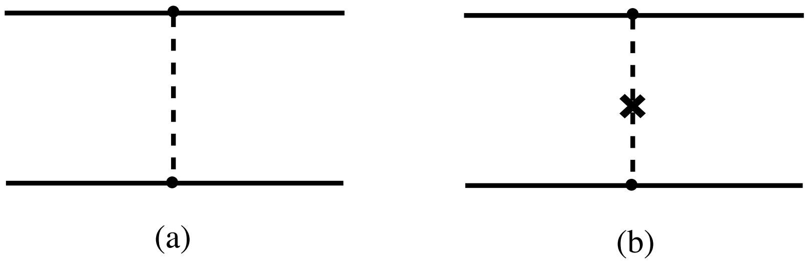
Nucleon-Nucleon

PRL 97 (2006)

Hyperon-Nucleon

hep-lat/0612026

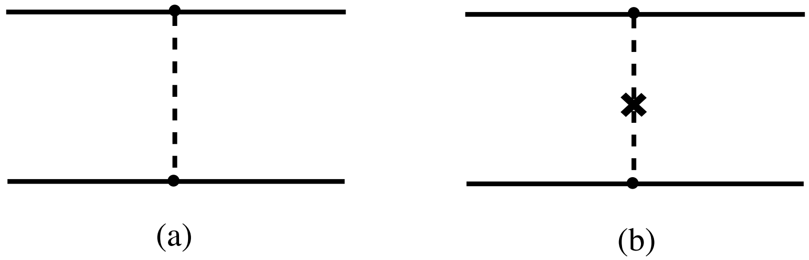
Applications: Nucleon-Nucleon and Hyperon-Nucleon Interactions



$$V_{OPE}^{PQ}(r) = \frac{1}{8\pi f_\pi^2} \vec{\sigma}_1 \cdot \vec{\nabla} \vec{\sigma}_2 \cdot \vec{\nabla} \left[g_A^2 \frac{\vec{\tau}_1 \cdot \vec{\tau}_2}{r} - (g_A + g_1)^2 \frac{\Delta_{ju}^2}{2m_\pi} \right] e^{-m_\pi r}$$

Beane and Savage PLB 535(2002)

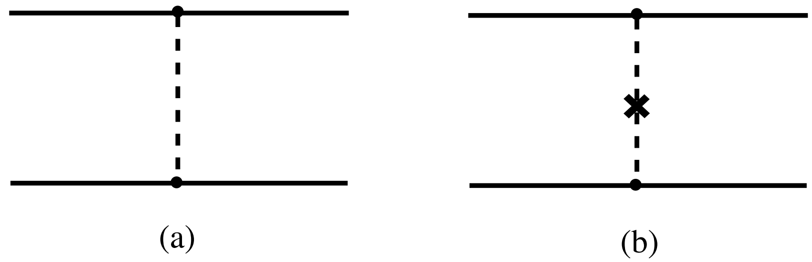
Applications: Nucleon-Nucleon and Hyperon-Nucleon Interactions



$$V_{OPE}^{MA}(r) = \frac{1}{8\pi f_\pi^2} \vec{\sigma}_1 \cdot \vec{\nabla} \vec{\sigma}_2 \cdot \vec{\nabla} \left[g_A^2 \frac{\vec{\tau}_1 \cdot \vec{\tau}_2}{r} - (g_A + g_1)^2 \frac{\tilde{\Delta}_{ju}^2}{2m_\pi} \right] e^{-m_\pi r}$$

Beane and Savage PLB 535(2002)

Applications: Nucleon-Nucleon and Hyperon-Nucleon Interactions



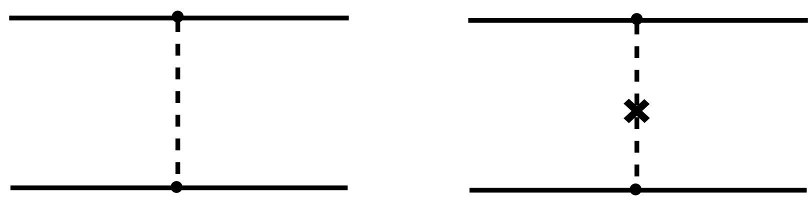
$$V_{OPE}^{MA}(r) = \frac{1}{8\pi f_\pi^2}\vec{\sigma}_1 \cdot \vec{\nabla}\vec{\sigma}_2 \cdot \vec{\nabla}\left[g_A^2 \frac{\vec{\tau}_1 \cdot \vec{\tau}_2}{r} - (g_A + g_1)^2 \frac{\tilde{\Delta}_{ju}^2}{2m_\pi}\right] e^{-m_\pi r}$$

Beane and Savage PLB 535(2002)

$$\Delta_{ju}^2 = 2B_0(m_j - m_u) + \ldots$$

$$\tilde{\Delta}_{ju}^2 = 2B_0(m_j - m_u) + b^2 \Delta_{sea} + \ldots$$

Applications: Nucleon-Nucleon and Hyperon-Nucleon Interactions



$$V_{OPE}^{MA}(r) = \frac{1}{8\pi f_\pi^2}\vec{\sigma}_1 \cdot \vec{\nabla}\vec{\sigma}_2 \cdot \vec{\nabla}\left[g_A^2 \frac{\vec{\tau}_1 \cdot \vec{\tau}_2}{r} - (g_A + g_1)^2 \frac{\tilde{\Delta}_{ju}^2}{2m_\pi}\right] e^{-m_\pi r}$$

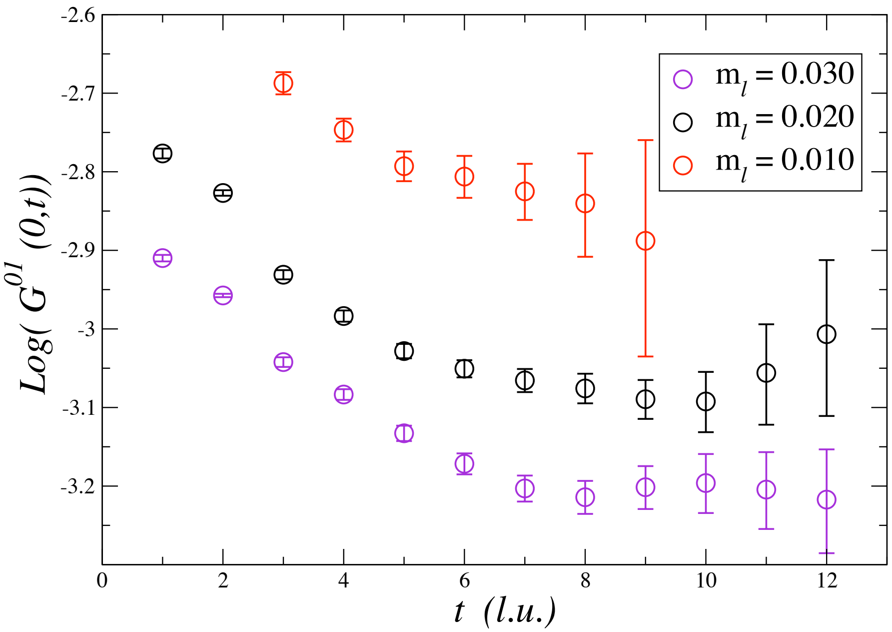
$$\frac{1}{a^{(^1S_0)}} = \gamma - \frac{M_N}{4\pi}(\mu-\gamma)^2 D_2^{(^1S_0)}(\mu)\, m_\pi^2 + \frac{g_A^2 M_N}{8\pi f_\pi^2}\left[m_\pi^2 \ln\left(\frac{\mu}{m_\pi}\right) + (m_\pi^2-\gamma)^2 - (\mu-\gamma)^2\right]$$
$$- \left(\Delta_{ju}^2 D_{2B}^{(^1S_0)}(\mu) + b^2 D_{2b}^{(^1S_0)}(\mu)\right)\frac{M_N}{4\pi}(\mu-\gamma)^2 + \tilde{\Delta}_{ju}^2 \frac{g_0^2 M_N}{8\pi f_\pi^2}\left[\ln\left(\frac{\mu}{m_\pi}\right) + \frac{1}{2} - \frac{\gamma}{m_\pi}\right]$$

Beane and Savage PRD 67(2003)

A similar analysis holds for Hyper-Nuclear interactions. Additionally, the lattice spacing dependent couterterms are flavor-blind, so all the baryon-baryon scattering processes share only 2 unphysical counterterms.

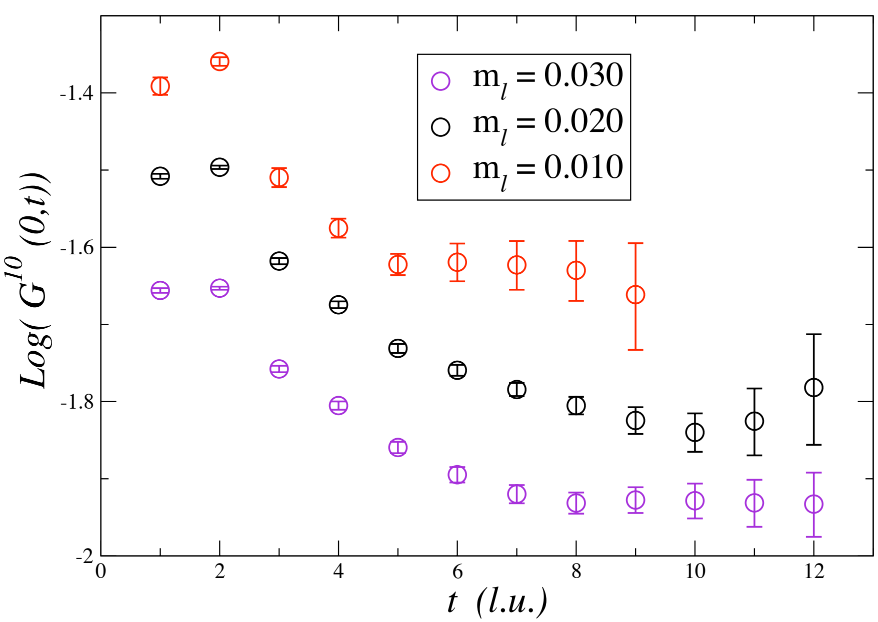
Applications: Numerical Results NPLQCD PRL 97 (2006)

np



$^1S_0 - {}^3D_1$

pp



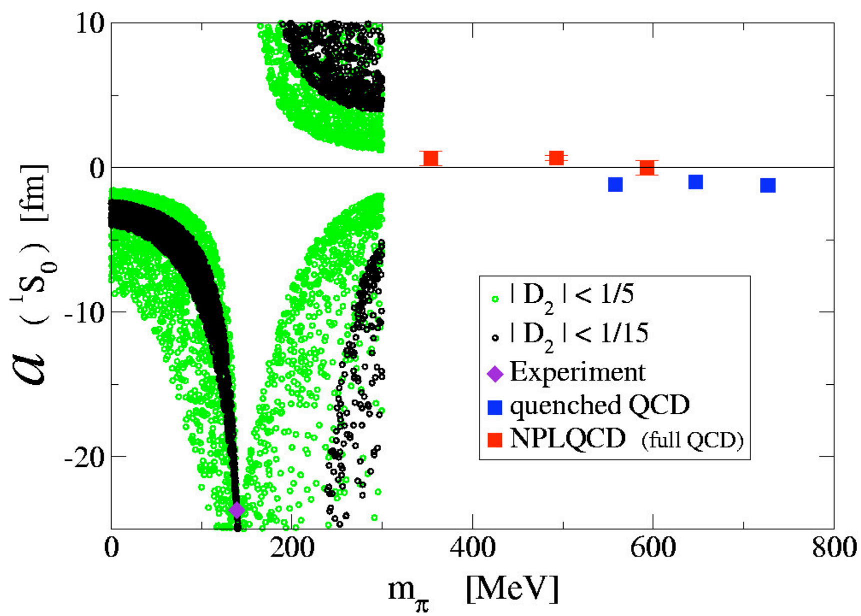
$^1S_0$

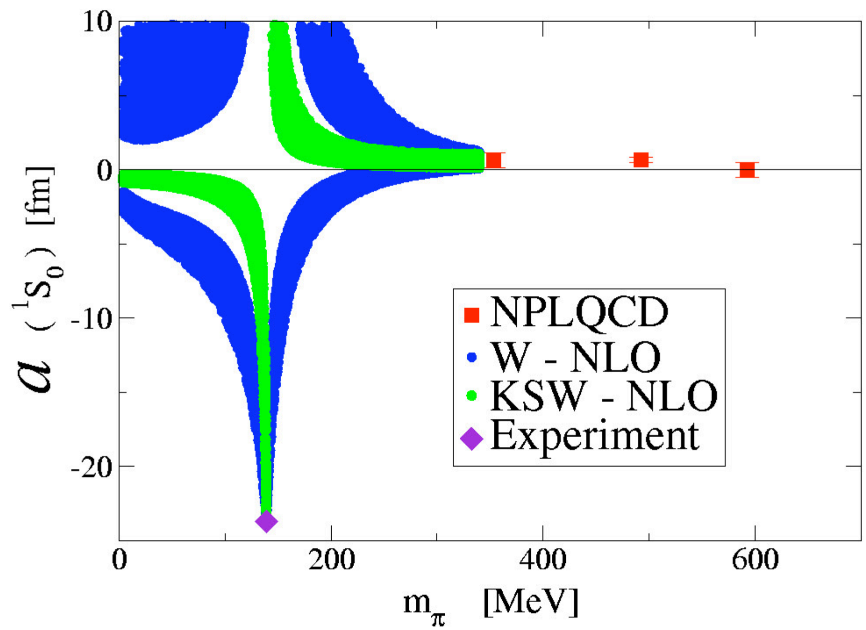
$$\text{signal/noise} \sim \sqrt{N_{cfg}}\, e^{-(2M_N - 3m_\pi)t}$$

Applications: Nucleon-Nucleon

NPLQCD

$^1S_0$ of NN





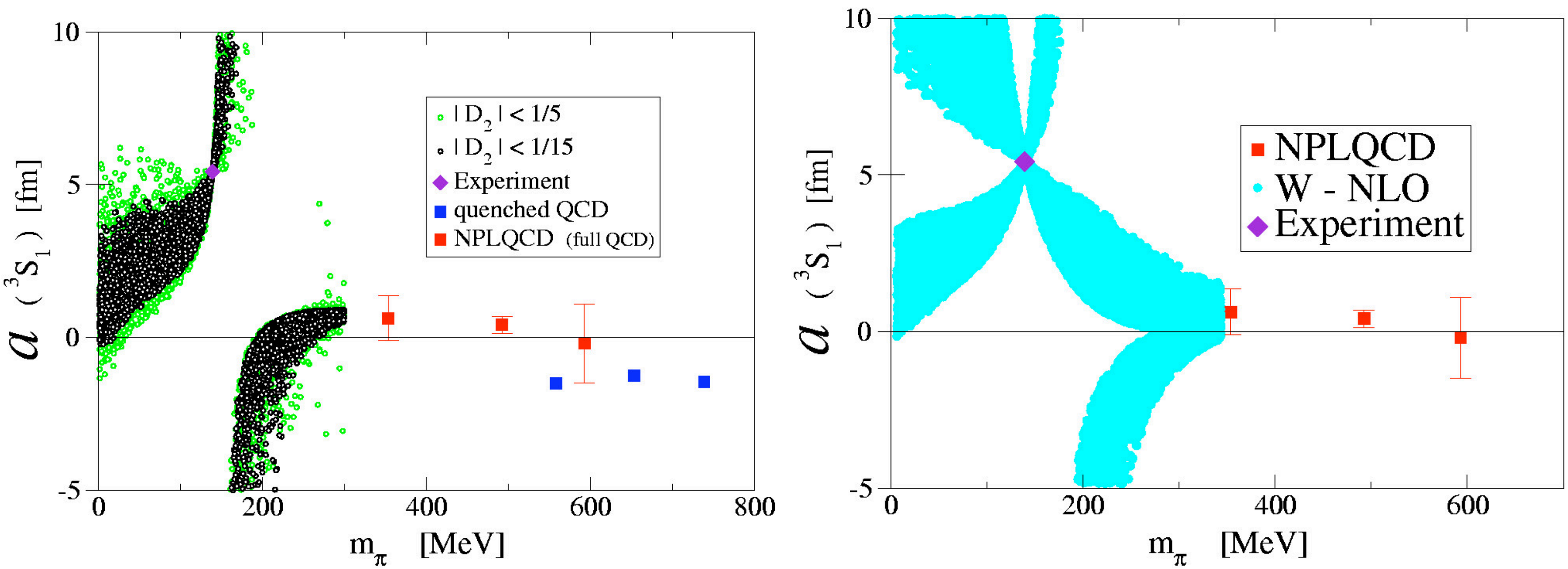
m_π (MeV)	$a^{(^1S_0)}$ (fm)
353.7 ± 2.1	$0.63 \pm 0.50 \pm 0.2$
492.5 ± 1.1	$0.65 \pm 0.18 \pm 0.2$
593.0 ± 1.6	$0.0 \pm 0.5 \pm 0.2$

S. Beane

Applications: Nucleon-Nucleon

NPLQCD

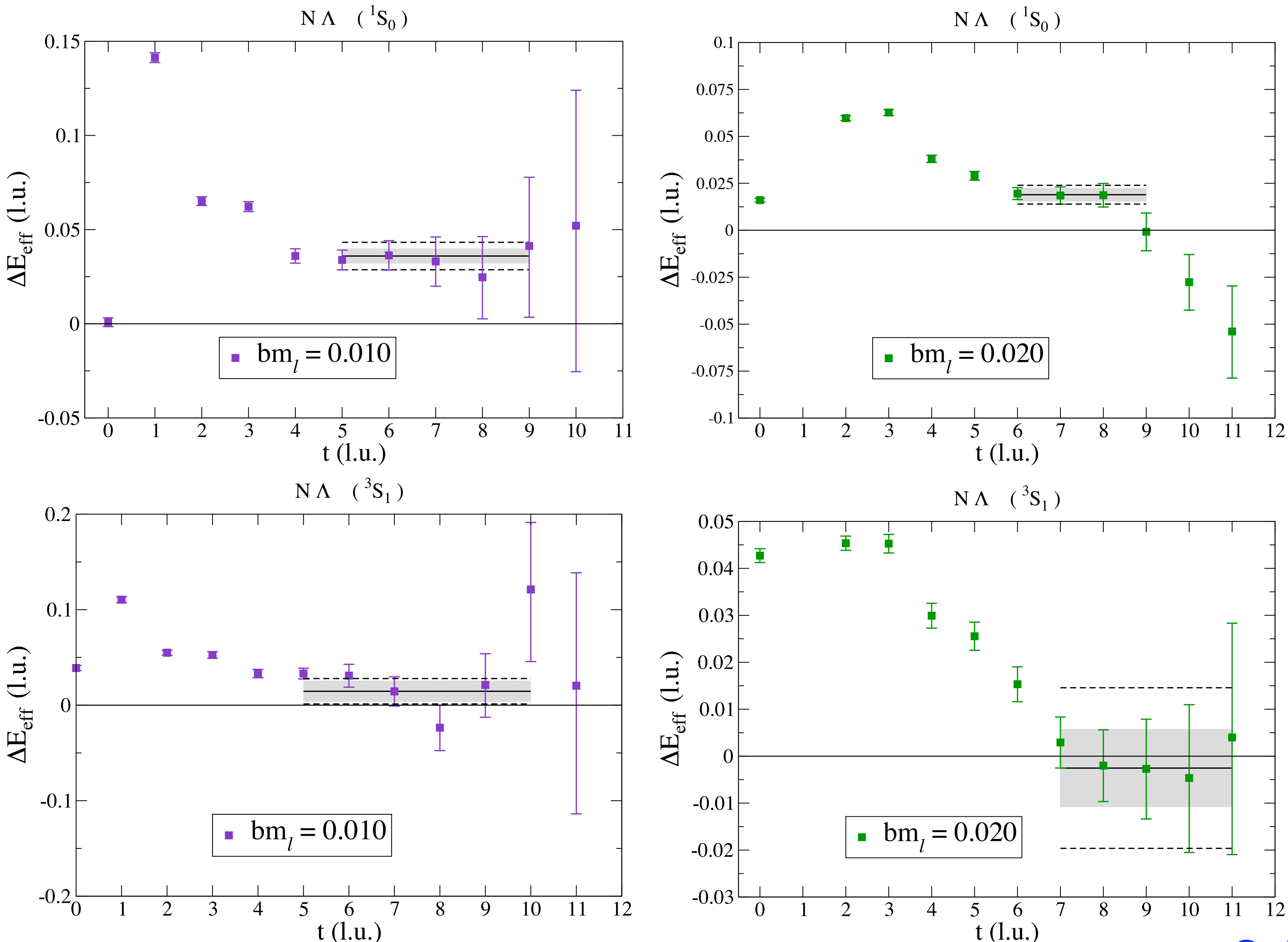
$^3S_1$ of NN



m_π (MeV)	$a^{(^3S_1)}$ (fm)
353.7 ± 2.1	$0.63 \pm 0.74 \pm 0.2$
492.5 ± 1.1	$0.41 \pm 0.28 \pm 0.2$
593.0 ± 1.6	$-0.2 \pm 1.3 \pm 0.2$

S. Beane

Applications: Hyperon-Nucleon   NPLQCD hep-lat/0612026



S. Beane

Restless Pions:

Orbifold boundary conditions and noise suppression in Lattice QCD

P.F. Bedaque, A.W-L arXiv:0708.0207

Restless Pions Signal-to-Noise Problem

Consider a nucleon two-point correlation function

P. Lepage
1989 TASI Lectures

$$C(t) = \langle q(t)q(t)q(t)\bar{q}(0)\bar{q}(0)\bar{q}(0)\rangle$$
$$\xrightarrow{t\to\infty} \quad Ae^{-Mt}$$

But we estimate this correlation function with a Monte-Carlo technique

$$C(t) \simeq \bar{C}(t) = \frac{1}{N}\sum_U S_U(t)S_U(t)S_U(t)$$

$$\sigma_C^2(t) = \frac{1}{N}\sum_U |S_U(t)S_U(t)S_U(t) - \bar{C}(t)|^2$$
$$= \langle S_U^3(t)S_U^{\dagger\,3}(t)\rangle - |\bar{C}(t)|^2$$

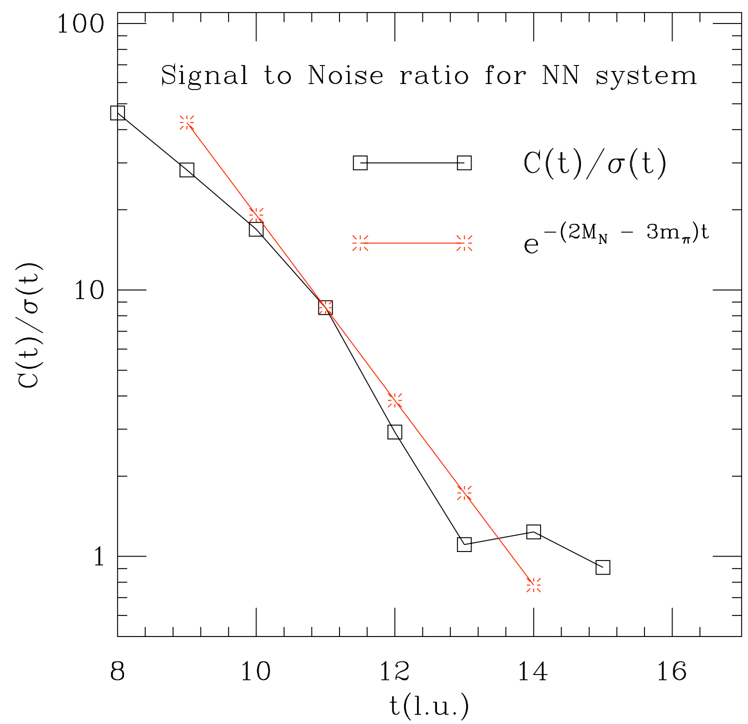
$$\langle S_U^3(t)S_U^{\dagger\,3}(t)\rangle = \langle q^3(t)\bar{Q}^3(t)\bar{q}^3(0)Q^3(0)\rangle$$
$$\xrightarrow{t\to\infty} Be^{-3m_\pi t}$$

Restless Pions Signal-to-Noise Problem

$$\frac{Sig.}{Noise} = \frac{\bar{C}(t)}{\sqrt{\frac{1}{N}\sigma_C^2(t)}} \xrightarrow{t\to\infty} A\sqrt{N}e^{-(M-3/2m_\pi)t}$$

Even worse for two-nucleon correlation functions

$$\frac{Sig.}{Noise} = \frac{\bar{C}_{NN}(t)}{\sqrt{\frac{1}{N}\sigma_{C_{NN}}^2(t)}} \xrightarrow{t\to\infty} A_{NN}\sqrt{N}e^{-(2M-3m_\pi)t}$$



Taken from NPLQCD

$m_\pi \sim 350$ MeV

Restless Pions parity-Orbifold condition

What if we could impose a boundary condition upon my quarks such that all pions were forbidden a zero momentum mode?

$$\langle S_U^3(t) S_U^{\dagger\,3}(t)\rangle = \langle q^3(t)\bar{Q}^3(t)\bar{q}^3(0)Q^3(0)\rangle$$

Then signal-to-noise $e^{-(M-3/2E_\pi)t}$

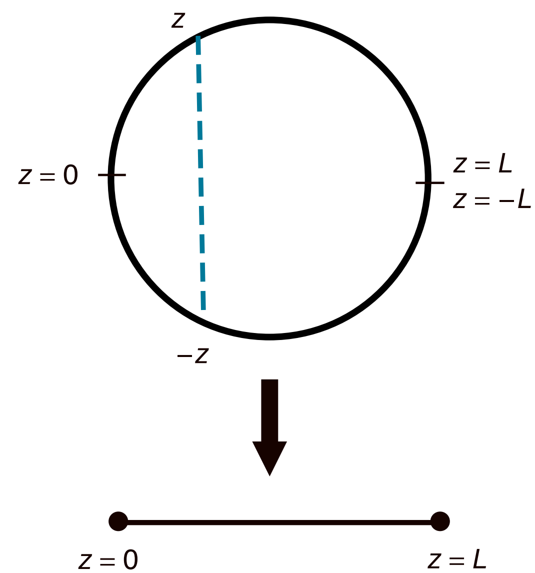
$$E_\pi = \sqrt{3\left(\frac{\pi}{L}\right)^2 + m_\pi^2}$$

$$m_\pi \sim 350 \text{ MeV} \quad , \quad L \sim 2.5 \text{ fm}$$

$$E_\pi \sim 550 \text{ MeV}$$

Restless Pions   parity-Orbifold condition

$$S_1/Z_2$$



Imagine doubling the size of the lattice in z-direction

$$q(t,x,y,-z) = \mathcal{P}_z\, q(t,x,y,z)$$
$$\bar{q}(t,x,y,-z) = \bar{q}(t,x,y,z)\,\mathcal{P}_z$$
$$A_\mu(t,x,y,-z) = (-)^{\delta_{\mu 3}} A_\mu(t,x,y,z)$$

$\Rightarrow$ $$\pi(x) = \bar{q}(x)\gamma_5 q(x)$$

$$\pi(t,x,y,-z) = -\pi(t,x,y,z)$$

$$\mathcal{P}_z = \gamma_3\gamma_5$$

$\Rightarrow$ $$\pi(t,x,y,z) = \sum_{n=1}^{\infty} A_-^{(n)} \sin\left(\frac{n\pi z}{L}\right)$$

Restless Pions!!!   Boundary Conditions on “normal” lattice

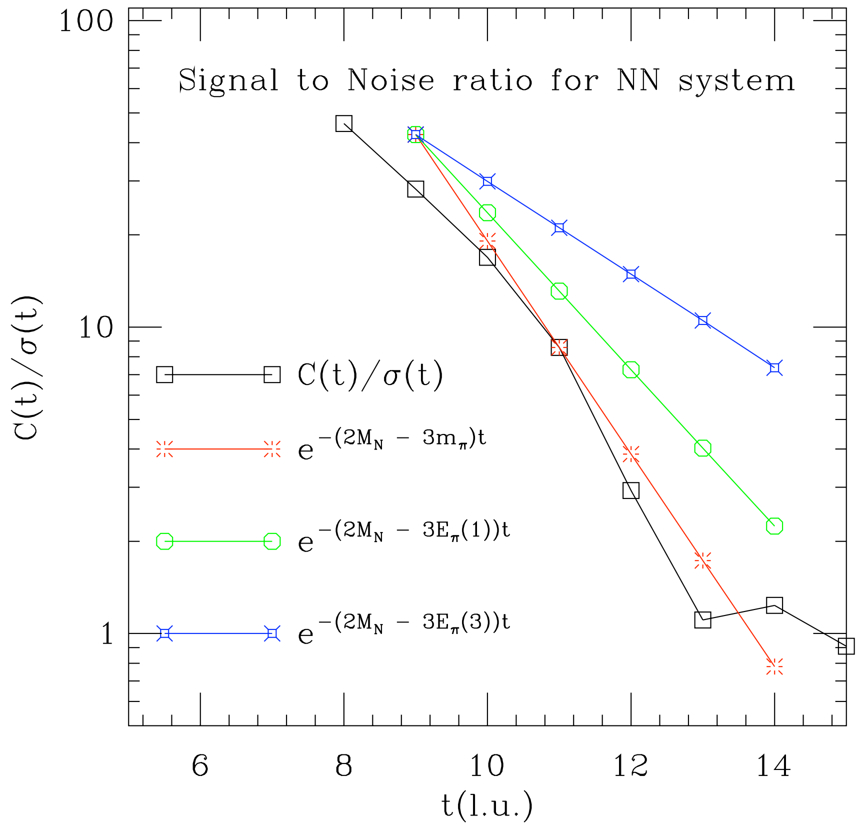
Restless Pions T_3/Z_2

Can apply a similar parity orbifolding to make pions *restless* in all three spatial directions

This method does not work for the sea-quarks - lose Gamma-5 Hermiticity

Numerical implementation of this method is currently underway

P.F. Bedaque, M.I. Buchoff, R. Edwards, K. Orginos, A.W-L



For further details see arXiv:0708.0207

Conclusions

- Two Meson scattering on the lattice is now in a precision age
  - Meson scattering lengths protected by chiral symmetry
  - Fermion discretization methods which (approximately) respect chiral symmetry can be used in the valence sector
  - Very well understood from an effective field theory view point: extrapolations in terms of lattice-physical quantities renormalizes most of lattice artifacts (through one-loop)
- Two-Nucleons are hard!!! but not inconceivable
  - relative to their rest mass, two-nucleon interaction energies are about an order of magnitude smaller than two-pion interaction energies with respect to their rest mass.
  - additionally, signal to noise problem is severe - just as effective mass plateaus, noise begins to wash out signal
  - Restless Pions boundary conditions may help with the signal to noise problem - under investigation
  - improved sources to couple to the deuteron better and clean up early time behavior?
  - In addition to high statistics, clever ideas are in high demand
- Two-Nucleons are hard but potential impact is great - especially in the hyperon sector where datum is extremely limited