Computing quark masses with lattice QCD

ALPHA Collaboration

Rainer Sommer

DESY, Zeuthen

SFB TR9

EINN workshop, Milos, September 2007

TODO

I have 40+5'

- clean finite_vol_scheme.tex, add mbar plot
- strange.tex
- mb_equ.tex

based on

a lot of work of the ALPHA Collaboration in particular

Precision computation of the strange quark's mass in quenched QCD
J. Garden, J. Heitger, R. S. and H. Wittig, Nucl. Phys. B **571**, 237 (2000)

A precise determination of the charm quark's mass in quenched QCD
J. Rolf and S. Sint, JHEP **0212** (2002) 007

and more recently:

Computation of the strong coupling in QCD with two dynamical flavours
M. Della Morte, R. Frezzotti, J. Heitger, J. Rolf, R. S. and U. Wolff
Nucl. Phys. B **713** (2005) 378

Non-perturbative quark mass renormalization in two-flavor QCD
M. Della Morte, R. Hoffmann, F. Knechtli, J. Rolf, R. S., I. Wetzorke
and U. Wolff, Nucl. Phys. B **729** (2005) 117

Non-perturbative Heavy Quark Effective Theory, J. Heitger & R.S., JHEP 0402:022

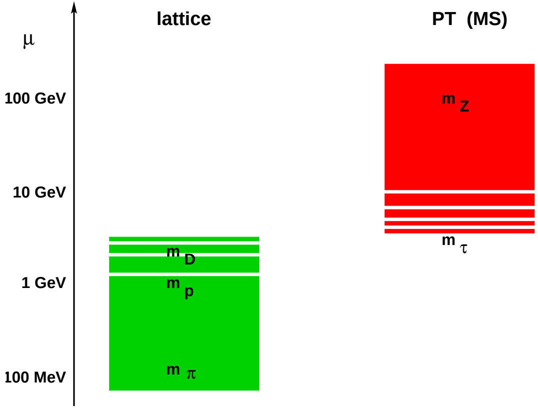
On lattice actions for static quarks
M. Della Morte, A. Shindler, R. S., JHEP **0508** (2005) 051

Heavy quark effective theory computation of the mass of the bottom quark
M. Della Morte, N. Garron, R. S. and M. Papinutto,
JHEP **0701** (2007) 007

What do we want

$$\overbrace{\begin{bmatrix} F_\pi \\ m_\pi \\ m_\mathrm{K} \\ \mathbf{m_D} \\ \mathbf{m_B} \end{bmatrix}}^{\text{Experiment}} \quad \overset{\mathcal{L}_\mathrm{QCD}(g_0, m_f)}{\Longrightarrow} \quad \overbrace{\begin{bmatrix} \Lambda_\mathrm{QCD} \\ \hat{M} = (M_\mathrm{u} + M_\mathrm{d})/2 \\ M_\mathrm{s} \\ \mathbf{M_c} \\ \mathbf{M_b} \end{bmatrix}}^{\text{QCD parameters (RGI)}} \quad + \quad \overbrace{\begin{bmatrix} \xi \\ F_B \\ B_B \\ \dots \end{bmatrix}}^{\textbf{Predictions}}$$

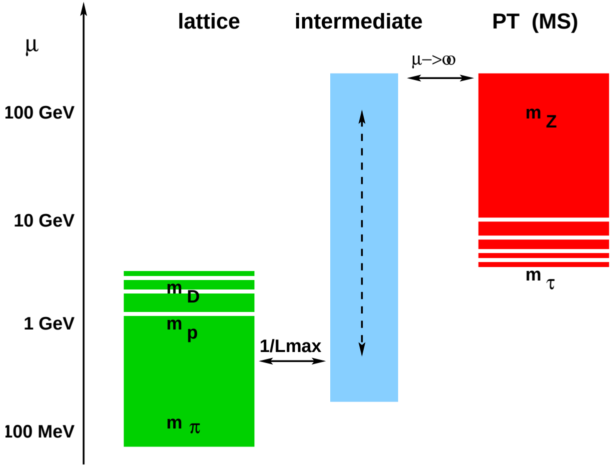
Scale problem and strategy



$1/L \ll \mu, E, |\mathbf{p}| \ll 1/a$

$\mu_{\rm pert} \gg \Lambda_{\rm QCD}$

Scale problem and strategy



- intermediate: Schrödinger functional scheme, finite volume

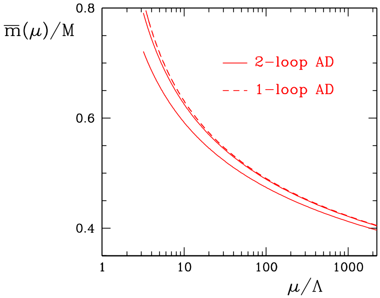
RGI quark masses

$$M = \lim_{\mu\to\infty} \overline{m}(\mu)[2b_0\bar{g}(\mu)^2]^{-d_0/2b_0}$$

$$\mu\frac{\partial\overline{m}}{\partial\mu} = \tau(\bar{g})$$

$$\tau(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^2\left\{d_0 + \bar{g}^2 d_1 + \ldots\right\} ,$$

$$d_0 = \frac{8}{(4\pi)^2}$$



- M scheme & scale independent: (schemes S, S')

$$\frac{\overline{m}_S(\mu)}{\overline{m}_{S'}(\mu)} = 1 + \mathrm{O}(\alpha(\mu))$$

$$\overset{\mu\to\infty}{\longrightarrow} 1 \quad\to\quad M_S = M_{S'} = M$$

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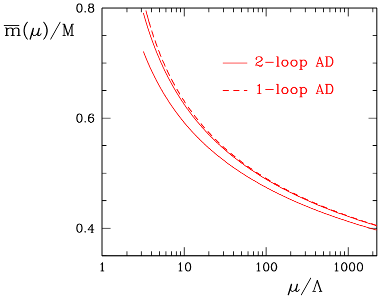

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- We can choose a convenient scheme for computing M

Running mass, definition

- from the PCAC relation:

$$\begin{aligned} A_\mu^{su} &= \bar{s}\,\gamma_\mu\gamma_5\,u\,,\;\; P^{su} = \bar{s}\,\gamma_5\,u \\ \partial_\mu A_\mu^{su} &= (m_\mathrm{s} + m_\mathrm{u})\,P^{su} \end{aligned}$$

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- precisely

$g_0 \leftrightarrow a$

$$\underbrace{\overline{m}_\mathrm{s}(\mu) + \overline{m}_\mathrm{u}(\mu)}_{\text{renormalized, running}} = \frac{Z_\mathrm{A}(g_0)}{Z_\mathrm{P}(\mu, g_0)} \underbrace{\frac{\langle K^-(\mathbf{p}=0)|\,\partial_\mu A_\mu^{su}|0\rangle}{\langle K^-(\mathbf{p}=0)|\,P^{su}|0\rangle}}$$

$\Uparrow$

$$m_\mathrm{s} + m_\mathrm{u} = \frac{m_\mathrm{K}^2 F_\mathrm{K}}{G_\mathrm{K}}: \text{ bare, PCAC}$$

scheme+scale dependence from Z_P

(Z_A from current algebra $\leftrightarrow$ Ward identity)

- and for charm etc. ...

$$\underbrace{\overline{m}_\mathrm{s}(\mu)+\overline{m}_\mathrm{c}(\mu)}_{\text{renormalized, running}} = \frac{Z_\mathrm{A}(g_0)}{Z_\mathrm{P}(\mu,g_0)}\underbrace{\frac{\langle D^+(\mathbf{p}=0)|\,\partial_\mu A_\mu^{cs}|0\rangle}{\langle D^+(\mathbf{p}=0)|\,P^{cs}|0\rangle}}_{\substack{\Uparrow \\ m_\mathrm{s}+m_\mathrm{c}:\ \text{bare, PCAC}}}$$

same Z_P (mass-independent renormalization)

The basic equation for the RGI mass

$$\overline{m}_{\rm s}(\mu) = \frac{Z_{\rm A}(g_0)}{Z_{\rm P}(\mu,g_0)} \underbrace{m_{\rm s}(g_0)}_{\text{bare, PCAC}} \qquad \text{remember } g_0 \leftrightarrow a$$

$$M_{\rm s} = \frac{M}{\overline{m}(\mu)}\overline{m}_{\rm s}(\mu) = Z_{\rm M}(g_0) \underbrace{m_{\rm s}(g_0)}_{\text{bare, PCAC}} \quad ,$$

$$\begin{aligned} Z_{\rm M}(g_0) &= \frac{M}{\overline{m}(\mu)} \frac{Z_{\rm A}(g_0)}{Z_{\rm P}(\mu,g_0)} \\ &= \underbrace{\frac{M}{\overline{m}(\mu_{\rm pert})}}_{\text{pert. theory}} \underbrace{\frac{\overline{m}(\mu_{\rm pert})}{\overline{m}(\mu_{\rm had})}}_{\text{NP}} \underbrace{\frac{Z_{\rm A}(g_0)}{Z_{\rm P}(\mu_{\rm had},g_0)}}_{\text{"easy"}} \end{aligned}$$

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- physics input: bare strange mass in the Lagrangian s.t. $m_\mathrm{K}/F_\mathrm{K} =$ experiment

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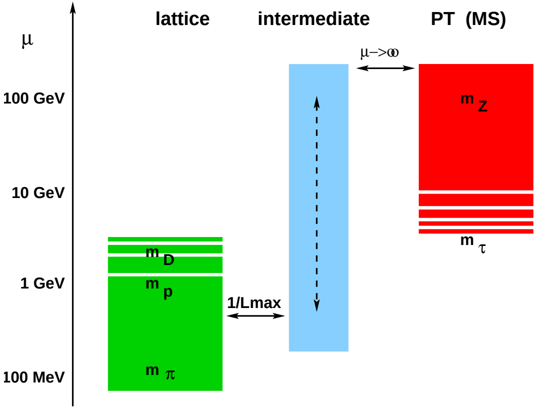
- remains NP computation of

$$\frac{\overline{m}(\mu_\mathrm{pert})}{\overline{m}(\mu_\mathrm{had})}$$



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NP running from finite volume schemes

need $\Lambda_{\mathrm{QCD}} \ll \mu_{\mathrm{pert}} \ll a^{-1} = \Lambda_{\mathrm{cut}}$
trick [Lüscher, Weisz & Wolff] $\mu = 1/L$: finite volume scheme

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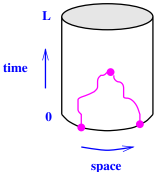

NP definition of $Z_{\rm P}$: $(O, P$ at a distance $L/2)$

$$Z_O Z_{\rm P} \langle\, O\ P \rangle = \langle\, O\ P \rangle_{\text{tree level}}$$

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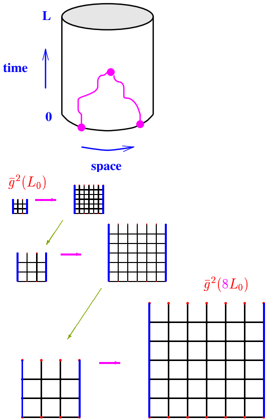

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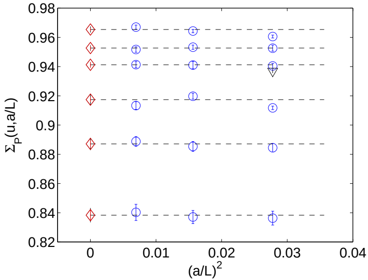
recursively: $L_0 \to L_1 = 2L_0 \to ...2^n L_0$

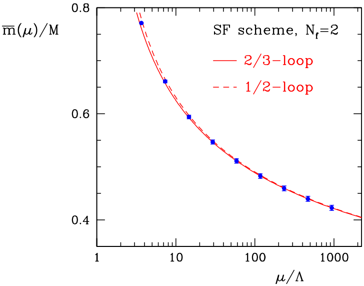
- coupling: $\bar{g}^2(2L) = \sigma(\bar{g}^2(L))$,
  $\sigma(\bar{g}^2(L)) = \lim_{a/L\to 0} \Sigma(\bar{g}^2(L), a/L)$
- quark mass $(Z_{\rm P})$:

$$\overline{m}(1/2L) = \overline{m}(1/L)\,/\,\sigma_{\rm P}(\bar{g}^2(L))$$
$$\sigma_{\rm P}(\bar{g}^2(L)) = \lim_{a/L\to 0} Z_{\rm P}(\tfrac{1}{2L}, g_0)/Z_{\rm P}(\tfrac{1}{L}, g_0)$$

The running of the quark mass: $N_\mathrm{f} = 2$



Continuum limit



reconstructed running

Strange quark mass (quenched, 1999) (with a small cheat)

remember:

$$M_\mathrm{s} + M_\mathrm{u} = Z_\mathrm{M}\, m_\mathrm{K}^2 \underbrace{\frac{F_\mathrm{K}}{G_\mathrm{K}}}_{\equiv R}$$

set the physical scale by fixing the static quark potential to phenomenology:
$r_0 = 0.5\,\mathrm{fm}$

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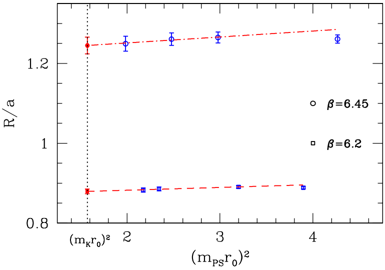

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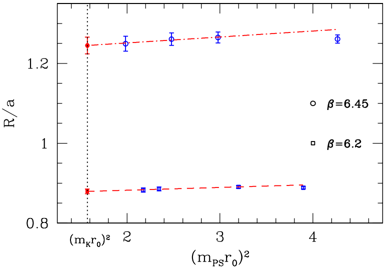
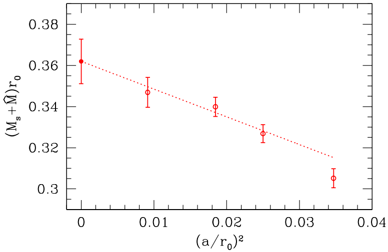


Continuum extrapolation

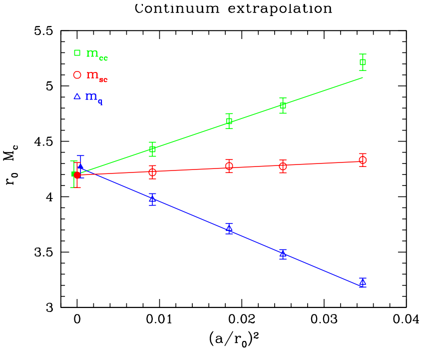


Charm quark mass (quenched, 2002)

- remove $O(am_c)$ effects non-perturbatively [ALPHA Collaboration 2000]

quenched [Sint & Rolf]

charm just doable



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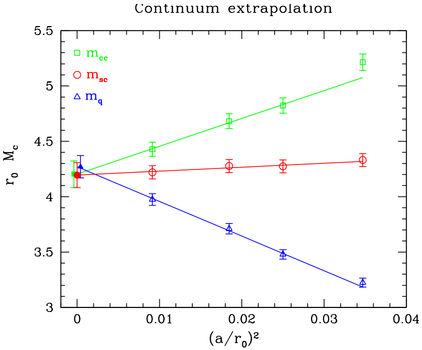

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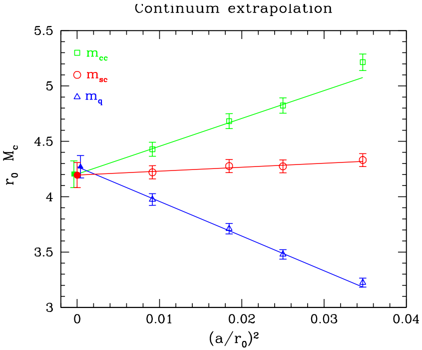

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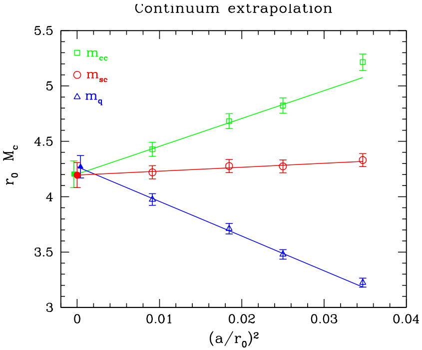

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- $aM_\mathrm{b} \approx 4aM_\mathrm{c}$!

  for b-quarks $a \rightarrow 0$ can't be controlled in this way $\rightarrow$ eff. theory

Effective theory for b-quark in heavy-light systems: HQET

[Caswell & Lepage ... Eichten & Hill ... ALPHA Collaboration]

$$\bar{b}(x)[\gamma_\mu D_\mu + m_\mathrm{b}]b(x) \quad \rightarrow$$

$$\mathcal{L}_\mathrm{HQET} = a^4 \sum_x \{\overline{\psi}_\mathrm{h}(x)[D_0 + \delta m]\psi_\mathrm{h}(x) + \underbrace{\omega_\mathrm{spin}}_{\sim 1/2m_\mathrm{b}} \overline{\psi}_\mathrm{h}(-\sigma \cdot \mathbf{B})\psi_\mathrm{h} + \underbrace{\omega_\mathrm{kin}}_{\sim 1/2m_\mathrm{b}} \overline{\psi}_\mathrm{h}(-\tfrac{1}{2}\mathbf{D}^2)\psi_\mathrm{h} + \ldots\}$$

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non-trivial matching problem

$$\omega_{\rm kin}\,,\ \omega_{\rm spin}\,,\ \delta m \quad \Leftrightarrow \quad M_{\rm b}$$

Mass renormalization in HQET

- static: $\overline{\psi}_\mathrm{h}(x)[D_0 + \delta m]\psi_\mathrm{h}(x)$

$$m_\mathrm{b}^{\overline{\mathrm{MS}}} = Z_{\overline{\mathrm{MS}},\mathrm{pole}}\, m_\mathrm{pole}\,, \qquad m_\mathrm{pole} = \underbrace{m_\mathrm{B}}_{\mathrm{exp.}} - \underbrace{E_\mathrm{stat}}_{\text{“binding energy”}} - \underbrace{\delta m}_{\Uparrow}$$

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no continuum limit

- need non-perturbative $e(g_0)$

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- $1/m$ corrections make it worse: $a^{-1} \to a^{-2}$

Non-perturbative matching

- $\omega_{\rm kin}, \omega_{\rm spin}, \delta m + m_{\rm bare}$ from QCD [Heitger & S.]

  $m_{\rm pole} + \delta m \leftrightarrow M_{\rm b}$

Non-perturbative matching

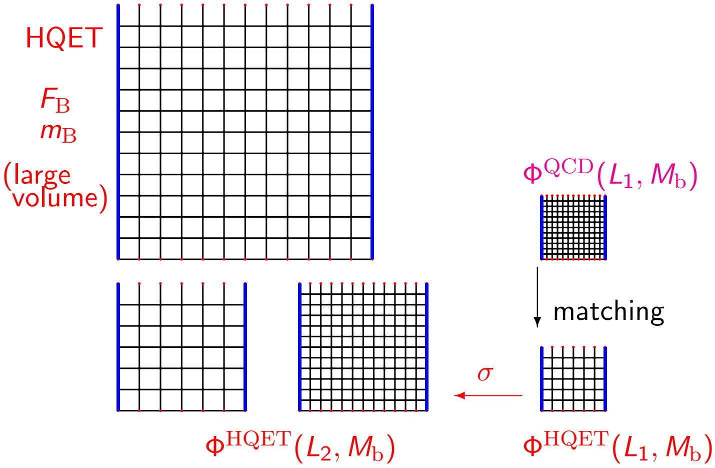
- $\omega_{\rm kin}, \omega_{\rm spin}, \delta m + m_{\rm bare}$ from QCD [Heitger & S.]

  $m_{\rm pole} + \delta m \leftrightarrow M_{\rm b}$

- match three observables

$$\Phi_i^{\rm QCD}(L, M_{\rm b}) = \Phi_i^{\rm HQET}(L, M_{\rm b})\,, \quad i = 1, 2, 3$$

The full strategy



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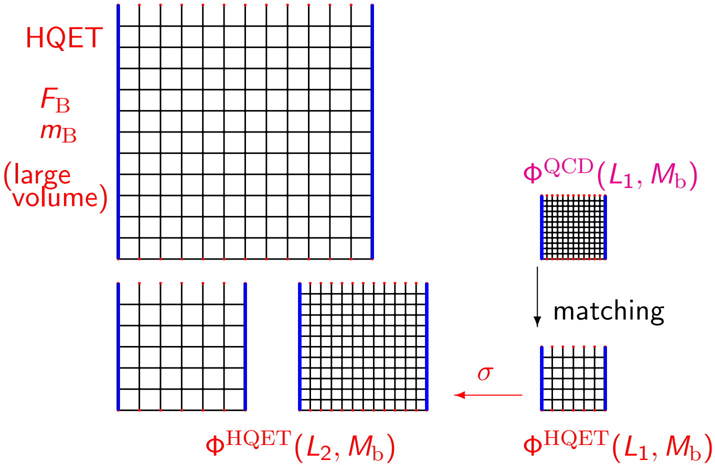

- continuum limit can be taken in all steps

Equation for $M_{\rm b}$ (static)

- pert:

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- NP:

$$m_{\rm B} \quad = \quad \underbrace{E_{\rm stat} - E_{\rm stat}^{\rm sub}(M_{\rm b})}_{E_{\rm stat}-E_{\rm stat}(L_2)+E_{\rm stat}(L_2)-E_{\rm stat}(L_1)} + \underbrace{E_{\rm stat}^{\rm sub}(M_{\rm b})}_{=E(L_1,M_{\rm b})}$$

HQET QCD

$$L_1 \approx 0.4\,{\rm fm}\,, \quad L_2 = 2L_1$$

Example for continuum extrapolation (quenched)

$$m_{\rm B} = \underbrace{E_{\rm stat} - E_{\rm stat}(L_2) + E_{\rm stat}(L_2) - E_{\rm stat}(L_1)}_{a\to 0 \text{ in HQET}} + \underbrace{E(L_1, M_{\rm b})}_{\equiv \Phi_2/L_1}$$

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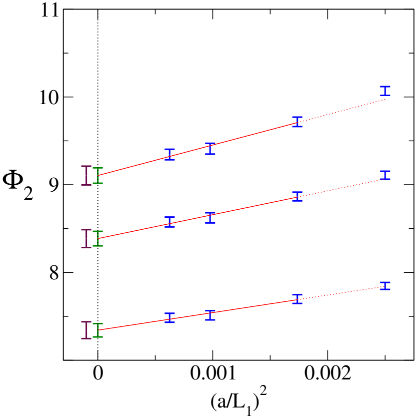
3 values of $M_{\rm b}$ example:

($M_{\rm b}$ from bare mass as before: $Z_{\rm M}$)



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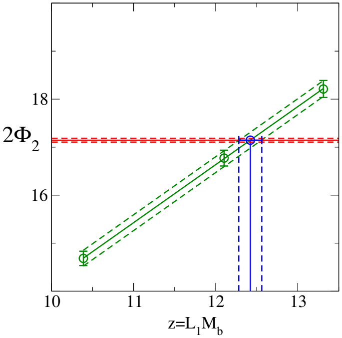
in static approximation
express everything in units of $r_0 \approx 0.5\,\mathrm{fm}$ and solve:

$$\underbrace{r_0 m_\mathrm{B}}_{\text{experiment}} \quad - r_0[E_\mathrm{stat} - E_\mathrm{stat}(L_2) + r_0[E_\mathrm{stat}(L_2) - E_\mathrm{stat}(L_1)] = \frac{r_0}{L_1}\Phi^\mathrm{QCD}(L_1, M_\mathrm{b})$$



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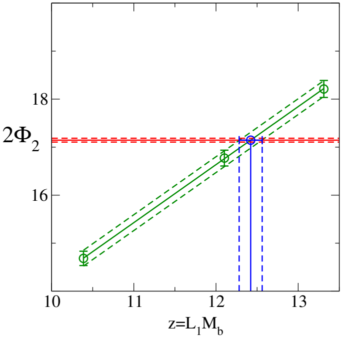

with $r_0 = 0.5\,\mathrm{fm}$

$$M_b^\mathrm{stat} = \begin{cases} 6771 \pm 99 \text{ MeV (HYP2)} \\ 6757 \pm 99 \text{ MeV (HYP1)} \end{cases}$$

and obtain the slope

$$\begin{aligned} S &= \frac{1}{L_1}\frac{\partial \Phi^\mathrm{QCD}(L_1, M)}{\partial M} \\ &= 0.61(5) \end{aligned}$$

error is dominated by the one on Z_M.

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- related strategy, HQET only at lowest order, $1/m_{\rm b}$ corrections from QCD interpolations $\overline{m}_{\rm b}^{\overline{\rm MS}}(\overline{m}_{\rm b}) = 4.421(67){\rm GeV}$ [Guazzini, S., Tantalo]
  talk by Damiano Guazzini

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- **very important: light up and down quarks, large volumes, chiral extrapolation → talk by G. Colangelo**

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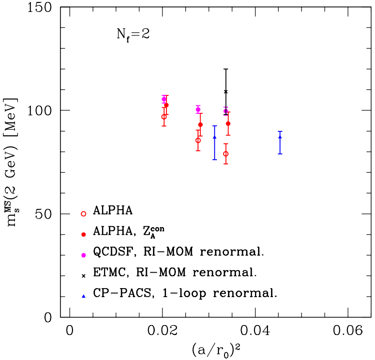
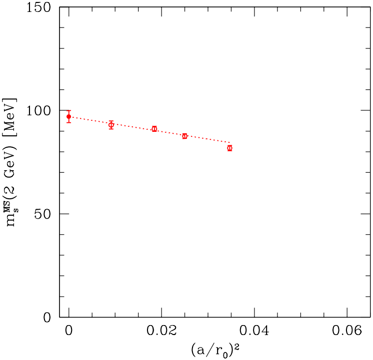
Status of M_s: a rough idea

$N_f = 2$, lattice 2007





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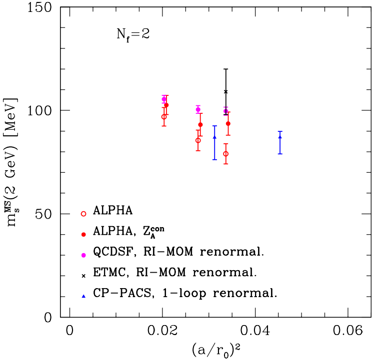
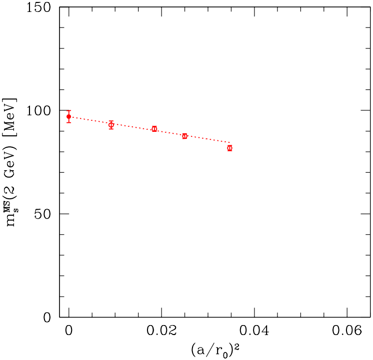


quenched for comparison (1999)



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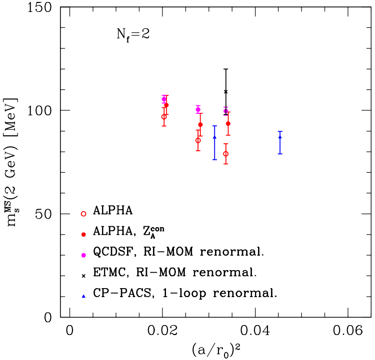

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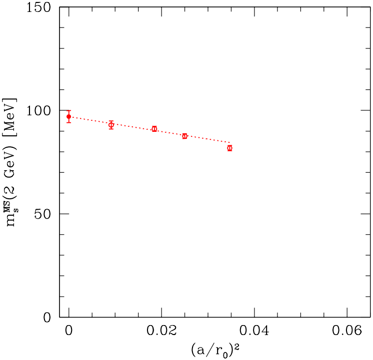

a bit early for a cont. limit

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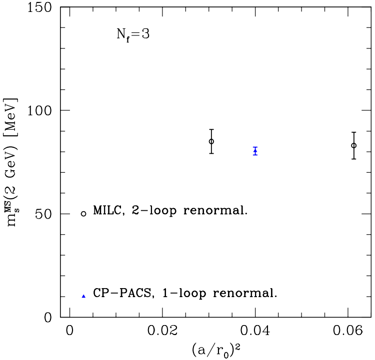

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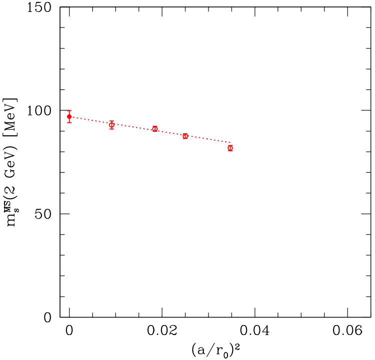
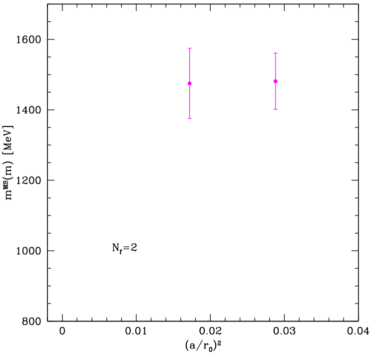


Status of M_c

$N_f = 2$, lattice 2007

[B. Blossier for ETMC]



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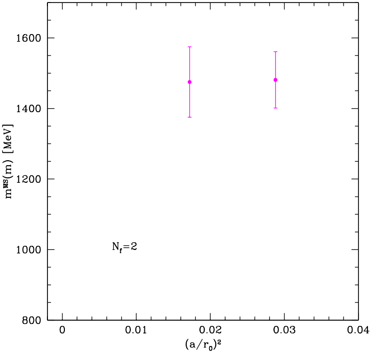

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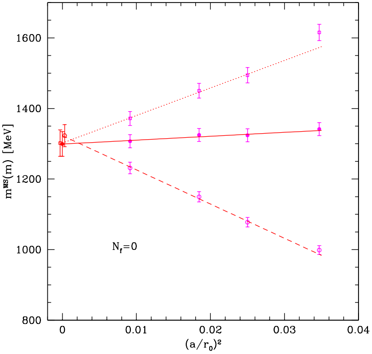

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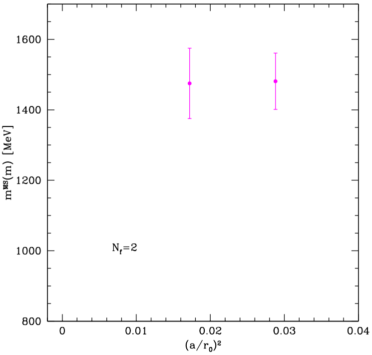

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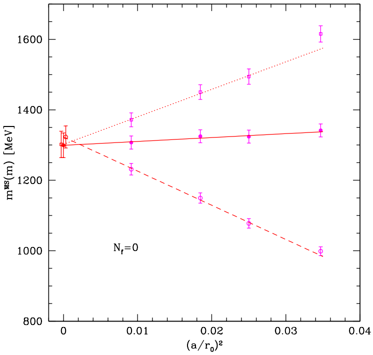

ETMC result:
$\overline{m}_c^{\overline{\mathrm{MS}}}(\overline{m}_c) = 1478(75)\mathrm{MeV}$

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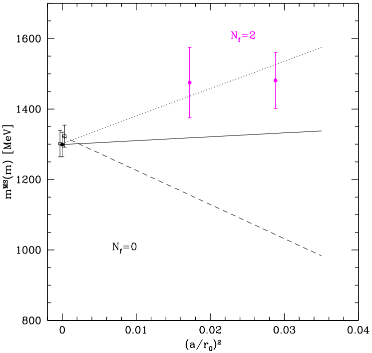

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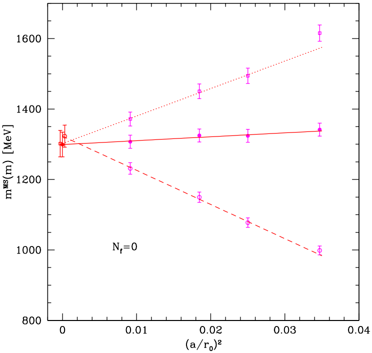

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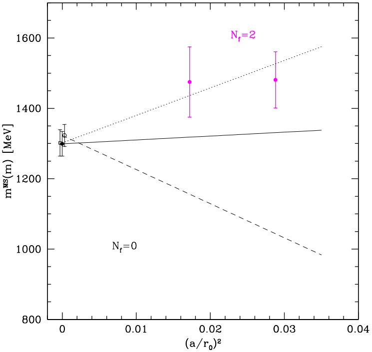

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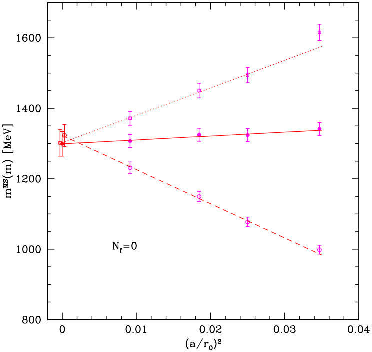

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- superb experimental input: $m_{\rm B},\ m_{\rm D},\ m_{\rm p},\ldots$
- no assumptions about (local) duality, "convergence" of the (divergent) pert. expansion, renormalon subtraction, ...

Significant progress does happen

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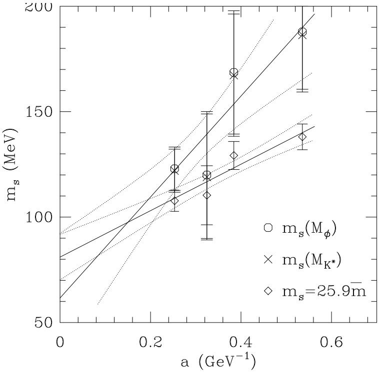

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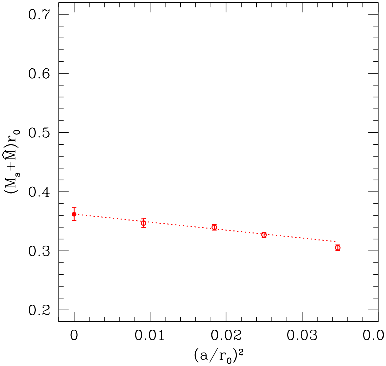

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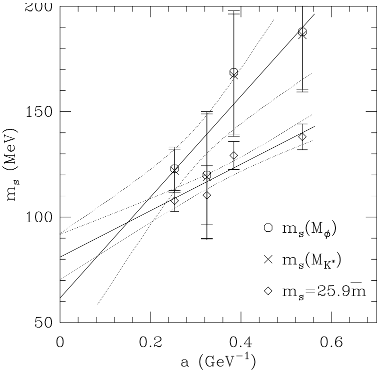

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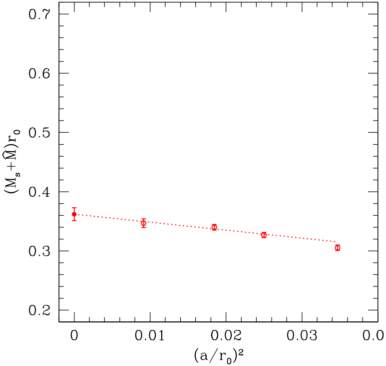

Significant progress in the next 3 years for $N_f = 2, 3$ (?)

Conclusion

- NP tools for renormalization are fully developed
- Quenched computations are under good control
- $N_\mathrm{f} \geq 2$: significant progress in recent years
  even more expected in the next 3 years