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INFN - Roma 3

Workshop on ***Hadron Physics on the Lattice***

7th European Conference on ***EINN***

Milos (Greece), September 10th - 11th, 2007

Electric Dipole Moment of the Neutron

Outline

- EDM and CP violation: experimental status and perspectives
- nEDM in the Standard Model and beyond
- what is being calculated and what should be calculated by lattice QCD

* definition of the neutron EDM: first moment of the neutron charge distribution

$$\vec{d}_n \equiv \int d^3x \, \vec{x} \, \langle n | J^0(x) | n \rangle$$

J^0 = time component of the e.m. current operator

- one has $\vec{d}_n \propto \vec{S}_n$, where $\vec{S}_n$ is the neutron spin, and the same holds for its magnetic moment $\vec{\mu}_n \propto \vec{S}_n$, so both moments have the same behavior under discrete space-time symmetries

- electric and magnetic dipole moments describe the response to an external e.m. field:

$$-\vec{d} \cdot \vec{E} \quad \text{and} \quad -\vec{\mu} \cdot \vec{B}$$

time reversal (T):	$\vec{S} \to -\vec{S}$	$\vec{E} \to \vec{E}$	$\vec{B} \to -\vec{B}$	$\Rightarrow$	$\vec{d} \cdot \vec{E} \to -\vec{d} \cdot \vec{E}$	$\vec{\mu} \cdot \vec{B} \to \vec{\mu} \cdot \vec{B}$
parity (P):	$\vec{S} \to \vec{S}$	$\vec{E} \to -\vec{E}$	$\vec{B} \to \vec{B}$	$\Rightarrow$	$\vec{d} \cdot \vec{E} \to -\vec{d} \cdot \vec{E}$	$\vec{\mu} \cdot \vec{B} \to \vec{\mu} \cdot \vec{B}$

* a non-vanishing EDM signals the breaking of P and T symmetries (Landau '57), while a non-vanishing magnetic moment does not

* P-violation is very well known because of the weak interaction (the V-A structure)

* CP-violation (or T-violation assuming the CPT theorem) is also known to occur in nature:
  - kaon (‘64), B-meson (‘01) and D-meson (‘07) decays (direct and indirect CP-violations);
  - baryon-antibaryon asymmetry.

* The celebrated KM mechanism (the phase in the CKM matrix) may account for the observed CP-violations in mesons, but it cannot explain the baryon-antibaryon asymmetry

the search for a non-vanishing nEDM is of particular relevance to unravel the mechanism(s) of CP violation

* Experimental methods to measure the nEDM:

1) direct way: storing of ultracold polarized neutrons;
2) indirect way: measurement of the nuclear Schiff moment (via atomic EDMs).

the EDM is measured by determining the Larmor frequency ν of the spin-precession process under uniform **E** and **B** fields:

$$h\nu = |2\mu B \pm 2dE| \qquad \pm \;\Rightarrow\; B \text{ and } E \;\; \uparrow\uparrow \text{ or } \uparrow\downarrow$$

* **present upper bound** (ILL of Grenoble, Baker et al. PRL '06) **:**

$$|d_n| < 2.9 \times 10^{-26}\, e \cdot cm \qquad (90\%\ \text{C.L.})$$

* the bound turns out to be enough small to act as an important constraint on CP-violations in the SM as well as in New Physics models

* **future perspective**: the nEDM experiment (Cooper and Lamoreaux, spokespersons) at ORNL

  - it is expected to lower the upper bound on d_n by two orders of magnitude;
  - it is expected to be operative in 2013.

* the determination of the **electron EDM** is also of great importance:

$$SM\ :\ |d_e| < 10^{-38}\, e \cdot cm$$ [Barr ('93)]

$$\text{exp.}:\ |d_e| < 4 \cdot 10^{-27}\, e \cdot cm$$ [Commins et al. ('94)]

NP effects are needed !

Sources of CP violation

* Electroweak Lagrangian in the SM:

- leptons: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L; \qquad \mathrm{e}_R, \quad \mu_R, \quad \tau_R$

- quarks: $\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b \end{pmatrix}_L; \qquad \mathrm{u}_R, \quad d_R, \quad c_R, \quad s_R, \quad t_R, \quad b_R$

- gauge bosons: $(\vec{W}, B)$

- scalars: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad \Phi^\dagger = \begin{pmatrix} \phi^- \\ \phi^{0*} \end{pmatrix}$

$$L_{ew} = L_G(f, W, B) + L_H(f, \Phi)$$

L_G: interaction of fermions with gauge bosons

L_H: interaction of fermions with Higgs scalars

* after spontaneous breaking of the $SU_w(2) \otimes U_Y(1)$ symmetry and through the Higgs mechanism, both fermions and gauge bosons (as well as the neutral Higgs scalar) acquire mass

- quark sector: $$L_H(f,\Phi) \xrightarrow[SSB]{} -\sum_{i,j=1}^{3} \bar{u}_L^i M_{ij}^U u_R^j - \sum_{i,j=1}^{3} \bar{d}_L^i M_{ij}^D d_R^j + h.c. + \ldots$$

$$u_L^{phys} = V_L^{up} u_L \to V_L^{up} \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix}, \quad d_L^{phys} = V_L^{down} d_L \to V_L^{down} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}, \quad u_R^{phys} = V_R^{up} u_R, \quad d_R^{phys} = V_R^{down} d_R$$

$$L_H(f,\Phi) \xrightarrow[SSB]{} -\sum_{j=1}^{6} m_j e^{i\zeta} \bar{q}_L^j q_R^j + h.c. + \ldots$$

(L_{NC} is flavor diagonal according to GIM)

$$L_G(f,W,B) \xrightarrow[SSB]{} L_{NC} + L_{CC} \qquad \begin{cases} L_{NC} = e\sum_{i=1}^{6} \bar{q}_i \left[Q_i \gamma^\mu A_\mu + \dfrac{1}{\cos\theta_W \sin\theta_W} \left(T_{3i} - Q_i \sin^2\theta_W \right) \gamma^\mu Z_\mu \right] q_i \\ L_{CC} = \dfrac{e}{\sqrt{2}\sin\theta_W} J_{CC}^\mu W_\mu^+ + h.c. \end{cases}$$

$$\begin{cases} T_{3i} = \pm 1/2 \text{ for L doublets} \\ T_{3i} = 0 \text{ for R fermions} \\ Q_i = \text{electric charge} \end{cases}$$

$$J_{CC}^\mu = \left(\bar{u}\ \bar{c}\ \bar{t} \right) \gamma^\mu V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \qquad V_{CKM} = V_L^{up} \left(V_L^{down} \right)^\dagger$$

CP conservation ⇒ $$\begin{cases} \zeta = 0 \\ V_{CKM} = \left(V_{CKM} \right)^* \end{cases}$$

(L_{NC} conserves CP)

Cabibbo angles: n (n-1) / 2
phases: (n-1) (n-2) / 2

- for two quark families V_{CKM} can be made real ⇒ no CP violation

- for three (or more) families V_{CKM} may contain a phase and therefore it may not be real:

$$V_{CKM} = R_{23}(\theta_{23},0)\, R_{13}(\theta_{13},\delta_{CKM})\, R_{12}(\theta_{12},0)$$

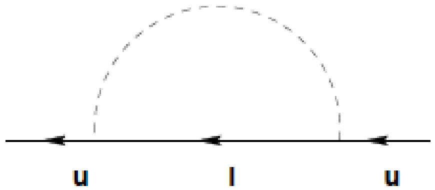
PDG ('07)

$$(V_{CKM})_{Wolfenstein} = \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + O(\lambda^4)$$

$\lambda \simeq \sin\theta_C$

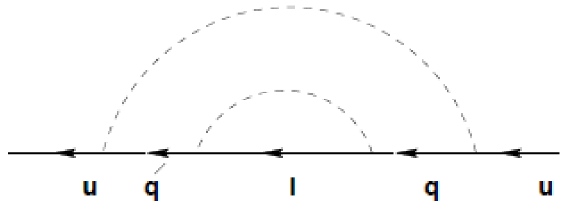
* the phase δ_{CKM} can generate an EDM for **free quarks** via loop effects:

1-loop



self-conjugate ⇒ no EDM

2-loop



$V_{CKM} (V_{CKM})^{\dagger} = 1$ ⇒ no EDM

[Shabalin ('80)]

* the CKM mechanism for CP violation can generate an EDM for **free quarks** only at three loops:

$$\begin{cases} d_d \simeq -10^{-34}\ e\cdot cm & \left(@\ m_d = 10\ MeV\right) \\ d_u \simeq -10^{-35}\ e\cdot cm & \left(@\ m_u = 5\ MeV\right) \end{cases}$$

Czarnecki & Krause ('97)

assuming a non-relativistic SU(6) wave function
for the three valence quarks in the neutron

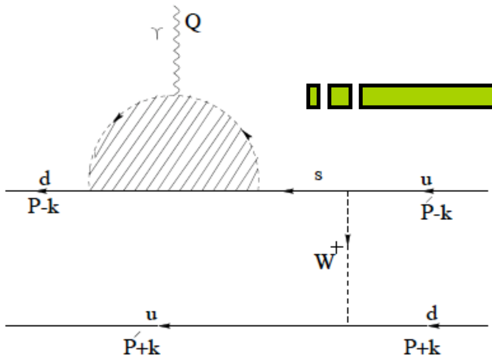
$$d_n = \frac{4}{3} d_d - \frac{1}{3} d_u$$

$$\delta_{CKM} \neq 0 \quad \Rightarrow \quad |d_n| \approx 10^{-34}\ e\cdot cm$$

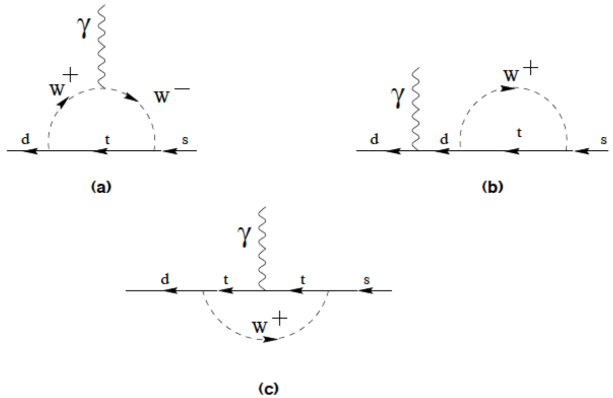
($K_{S,L} \to 2\pi$)

8 orders of magnitude below
the exp. upper bound !

* for **bound quarks** there is a 2-loop contribution [Nanopoulos et al. ('79)] :



$$\propto V_{td}^{*} V_{ts} V_{us}^{*} V_{ud}$$



$$|d_n| \approx 10^{-32}\ e\cdot cm$$

still 6 orders of magnitude
below the exp. upper bound !

*** electroweak θ-term:**

$$L_H(f,\Phi) \xrightarrow[SSB]{} -\sum_{j=1}^{N_f} m_j e^{i\zeta}\, \bar{q}_L^j q_R^j + h.c. \qquad \zeta = \frac{1}{N_f} Arg\ Det[M]$$

- the phase ζ can be eliminated by a singlet chiral rotations of the quark fields

$$\begin{cases} q(x) \to q'(x) = \left(1 + i\frac{\alpha}{2}\gamma_5\right) q(x) \\ \bar{q}(x) \to \bar{q}'(x) = \bar{q}(x)\left(1 + i\frac{\alpha}{2}\gamma_5\right) \end{cases} \Longrightarrow \begin{cases} q'_{L,R}(x) = \left(1 \mp i\frac{\alpha}{2}\right) q_{L,R}(x) \\ \bar{q}'_{L,R}(x) = \bar{q}_{L,R}(x)\left(1 \pm i\frac{\alpha}{2}\right) \end{cases} \qquad (\alpha = -\zeta)$$

- the measure of path integrals is not invariant [Fujikawa ('79)]

$$Dq'\, D\bar{q}' \to Dq\, D\bar{q}\, e^{iN_f\alpha \int d^4x \frac{g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}}$$

$G^a_{\mu\nu}$ = gluon field tensor

$\tilde{G}^{a,\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma}$

- real quark mass matrix, but presence of a θ-term:

$$L(\zeta) \to L(\zeta = 0) - \theta_{ew} \frac{g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}$$

→ it violates CP and P

$$\theta_{ew} = N_f \zeta = Arg\ Det[M]$$

* **strong θ-term:** $L_{QCD}\left(\theta_{QCD}\right) = L_{QCD}\left(\theta_{QCD}=0\right) + \theta_{QCD}\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}$

- equations of motion do not change because $\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} = \partial_\mu K^\mu$ (K^μ gauge-variant)

- however $\int d^4x\,\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} \neq 0$ because of non-trivial gauge configurations

⇓

physical effect: CP-violation ⇒ nEDM ≠ 0

- topological charge: $Q \equiv \int d^4x\,\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} \xrightarrow[\text{Index Theorem}]{} \nu_+ - \nu_-$

$\nu_\pm$= number of zero modes of the Dirac operator with ± chiralities

- addition in the action of the θ-term, $\boldsymbol{\theta Q}$, with $\theta = \theta_{QCD} - \theta_{ew} = \theta_{QCD} - ArgDet\left[M\right]$

$$\left|vacuum\right\rangle = \left|\theta\right\rangle = \sum_{Q=0,\pm1,\pm2,\ldots} e^{iQ\theta}\left|Q\right\rangle$$

the value of θ is not determined by the theory

- using (rough) estimates from quark model [Baluni ('79)] and ChPT [Crewther et al. ('79)], one has:

$\left|d_n\right| < 2.9\times10^{-26}\,e\cdot cm$ ⇒ $\theta < 10^{-10}$

<u>**the strong CP problem**</u>

* possible proposed solutions for the strong CP problem:

- $m_u = 0$ [Kaplan and Manohar ('86)]
  - inconsistent with low energy spectrum of QCD [Leutwyler ('96)]

- additional U(1) chiral symmetry spontaneously broken: axions [Peccei and Quinn ('77)]
  - not yet detected by experiments

- CP is a symmetry spontaneously broken at some scale
  - at a low scale (ew) presence of large FCNC
  - at a large scale (Plank) elusive effects at the hadron scale

- …

* **CP violation in NP models**

- general form of the CP-violating effective Hamiltonian

$$H_{eff} = \sum_{i=1}^{N_f} C_e^i(\mu) O_e^i(\mu) + \sum_{i=1}^{N_f} C_c^i(\mu) O_c^i(\mu) + C_G(\mu) O_G(\mu) + \ldots$$

→ possible four fermion operators suppressed by tanβ

electric: $O_e^i = -\frac{i}{2} e Q_i m_i \bar{q}_i \sigma^{\mu\nu} \gamma_5 q_i F_{\mu\nu}$

chromo-electric: $O_c^i = -\frac{i}{2} m_i \bar{q}_i \sigma^{\mu\nu} \gamma_5 t^a q_i G_{\mu\nu}^a$

gluonic: $O_G = -\frac{1}{3} f^{abc} G_{\mu\lambda}^a G_{\nu}^{b,\lambda c}{}_{\mu\nu} \tilde{G}^{c,\mu\nu}$ [Weinberg (‘89)]

* contributions to C_e, C_c and C_G at the high scale μ may start at **1-loop level**

- RGE’s for the Wilson coefficients: $$\frac{d\vec{C}}{d\ln\mu} = \frac{\alpha_s(\mu)}{4\pi} \gamma^{(0)} \vec{C}$$

$\gamma^{(0)}$ → anomalous dimensions

- evolution of O_e and O_c similar to the one of O_7 and O_8 in $b \to s\gamma$

- evolution of O_G calculated by Braaten et al. (‘90)

* LO anomalous dimensions

$$\gamma^{(0)} = \begin{pmatrix} \gamma_e & 0 & 0 \\ \gamma_{ce} & \gamma_c & 0 \\ 0 & \gamma_{Gc} & \gamma_G \end{pmatrix} = \begin{pmatrix} 8C_F & 0 & 0 \\ 8C_F & 16C_F - N_c & 0 \\ 0 & -2N_c & N_c + 2N_f + 2\beta_0 \end{pmatrix}$$

$$\begin{cases} C_F = \frac{4}{3} \\ \beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_f \end{cases}$$

- the Wilson coefficients C_e, C_c and C_G are calculable in the given NP model at a high scale μ and then evolved down to the hadronic scale μ_H

$$C_e(\mu_H) = \eta^{\gamma_e/2\beta_0} C_e(\mu) + \frac{\gamma_{ce}}{\gamma_e - \gamma_c}\left(\eta^{\gamma_e/2\beta_0} - \eta^{\gamma_c/2\beta_0}\right)\eta^{-1/2}\frac{C_c(\mu)}{g_s(\mu)} +$$
$$+\gamma_{Gc}\gamma_{ce}\left[\frac{\eta^{\gamma_e/2\beta_0}}{(\gamma_e-\gamma_c)(\gamma_e-\gamma_G)} + \frac{\eta^{\gamma_c/2\beta_0}}{(\gamma_e-\gamma_c)(\gamma_G-\gamma_c)} + \frac{\eta^{\gamma_G/2\beta_0}}{(\gamma_e-\gamma_G)(\gamma_c-\gamma_G)}\right]\eta^{-1/2}\frac{C_G(\mu)}{g_s(\mu)}$$

$$C_c(\mu_H) = \eta^{\gamma_c/2\beta_0} C_c(\mu) + \frac{\gamma_{Gc}}{\gamma_c - \gamma_G}\left(\eta^{\gamma_c/2\beta_0} - \eta^{\gamma_G/2\beta_0}\right) C_G(\mu)$$

$$C_c(\mu_H) = \eta^{\gamma_G/2\beta_0} C_c(\mu)$$

$$\eta \equiv \frac{\alpha_s(\mu)}{\alpha_s(\mu_H)}$$

* γ_e, γ_c and $\gamma_G > 0$ $\Longrightarrow$ the CP-violating effects are suppressed from high to low scales

[Degrassi et al. ('05)]

- what should be calculated non-perturbatively:

$$\langle n|O_e^i(\mu_H)|n\rangle, \quad \langle n|O_c^i(\mu_H)|n\rangle, \quad \langle n|O_G(\mu_H)|n\rangle$$

- estimates only from QCD sum rules [Pospelov et al. ('01), ('03)], chiral Lagrangians [Pich and de Rafael ('91), Hisano and Shimizu ('04)] and quark model [Faessler et al. ('06)] exist to date

- many NP analyses still adopts the simple SU(6) model for the nucleon

$$d_n = \frac{4}{3}d_d - \frac{1}{3}d_u$$

(effects of d_s are neglected)

- naïve dimensional analysis [Manohar and Georgi ('84)]

electric: $d_n^e = -\frac{2}{9}e\left[m_u(\mu_H)C_e^u(\mu_H) + 2m_d(\mu_H)C_e^d(\mu_H)\right]$

chromo-electric: $d_n^c = \frac{e}{4\pi}\left[m_u(\mu_H)C_c^u(\mu_H) + m_d(\mu_H)C_c^d(\mu_H)\right]$

gluonic: $d_n^G = \frac{e}{4\pi}\Lambda_\chi C_G(\mu_H)$ $\quad\left[\Lambda_\chi \simeq 1\,GeV\right]$

Brief Summary

* the electroweak sector in the SM and in NP models contains sources of CP violation:
  - the phase in the CKM matrix is able to account for the observed CP-violation in mesons, but it produces contributions to the electric dipole moment of the neutron and the electron which are several orders of magnitude less than present experimental bounds;
  - the contributions from NP starts at one-loop level and may be quite large, so that present experimental bounds on the neutron and electron EDM represent important constraints on the NP parameters.

* at variance with the case of the electron, the neutron EDM may receive an important contribution from the so-called **θ-term**

LATTICE QCD

- it is a non-perturbative approach based on our fundamental field theory of the strong interaction

- three limits of the results of lattice simulations should be performed:
  1) the continuum limit (lattice spacing $\rightarrow 0$);
  2) the infinite volume limit;
  3) the chiral limit in the u and d quark masses.

- full QCD simulations has to be unquenched (inclusion of sea quark effects)

- at the last Lattice Conference in Regensburg the results presented by several collaborations have been very impressive:
  1) simulations with 2 and 2+1 dynamical quarks;
  2) $m_{u,d}$ down to $\sim m_s / 6 \Rightarrow$ pion masses down to 250 - 300 MeV;
  3) large volumes up to $L \sim 3$ fm and lattice spacings fine as ~ 0.08 - 0.1 fm;
  4) uncertainties almost dominated by systematical and not by statistical errors.

- as for nEDM the tasks for the lattice community is well defined and apparently simple:

1) the matrix element of the quark electric dipole operator: $\langle \mathrm{n}|\bar{q}\sigma^{\mu\nu}\gamma^5 q|n\rangle F_{\mu\nu}$

2) the matrix element of the quark chromo-electric dipole operator: $\langle \mathrm{n}|\bar{q}\sigma^{\mu\nu}\gamma^5 t^a G^a_{\mu\nu} q|n\rangle$

3) the matrix element of the Weinberg operator: $\langle \mathrm{n}| f^{abc} G^a_{\mu\lambda} G^{b,\lambda}_{\nu} \tilde{G}^{c,\mu\nu}|n\rangle$

4) the nEDM generated by a θ-term in the action.

- no lattice results for the first three tasks are available (in particular unquenched results)

msg for the lattice community: start calculating the matrix elements of the three bare operators and start thinking how to perform a non-perturbative renormalization

* lattice strategies to calculate the nEDM induced by the θ-term:

1) measure the energy difference between spin-up and spin-down neutrons in presence of a uniform and static external electric field;

2) measure the CP-odd e.m. form factor $F_3(q^2)$;

3) evaluate disconnected insertions of the singlet pseudo-scalar density.

* the first strategy involves the calculations of 2-point correlation functions at zero momentum:

$$G^N_{\alpha\beta}(t) = \sum_{\vec{x}} \langle N_\alpha(\vec{x},t)\,\bar{N}_\beta(0)\rangle \xrightarrow[t\to\infty]{} Z^N e^{-m_N t}(1+\gamma_4)_{\alpha\beta}$$

$$N_\alpha(x) = \text{ nucleon interpolating field } \to \varepsilon^{abc}\left(d^{aT} C\gamma_5 u^b\right) d^c_\alpha$$

α, β = Dirac indexes

- application of a static and uniform external electric field in Minkowski space [Shintani et al. ('07)]

it corresponds in Euclidean space-time to a redefinition of the gauge links

$$U_i(x) \to \tilde{U}_i(x; E_i) = e^{e_q E_i t}\, U_i(x)$$

- turning on the θ-term:

$$\tilde{G}^N_{\alpha\beta}(t; \vec{E}; \theta) = \sum_{\vec{x}} \langle N_\alpha(\vec{x},t)\,\bar{N}_\beta(0)\, e^{i\theta Q}\rangle_{U\to\tilde{U}(\vec{E})}$$

reweighting factor
O.K. for θ small

- extraction of d_N:

$$m^\uparrow_N(\theta) - m^\downarrow_N(\theta) = 2\vec{d}_N \cdot \vec{E} + O(E^3)$$

(O.K. for small E)

* <u>lattice evaluation of the topological charge</u>

- fermionic definition: $Q = \nu_+ - \nu_-$, applicable for fermions with exact chiral symmetry (like, overlap fermions)

- gluonic definition:

$$Q \equiv \int d^4x \, \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

$$\xrightarrow[\text{lattice}]{} \frac{5}{3} Q_{\text{plaquette}} - \frac{1}{12} Q_{\text{rectangular}} + O(a^2 g^2, a^3)$$

E. SHINTANI *et al.*

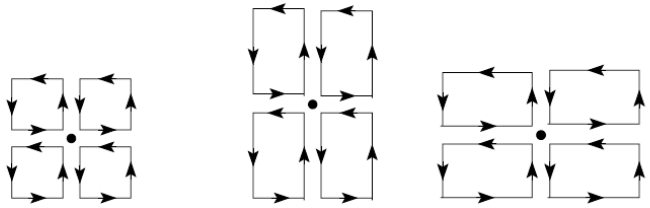


FIG. 1. Plaquette clover (left) and rectangular clovers (middle and right).

* cooling method [Teper ('85)]:

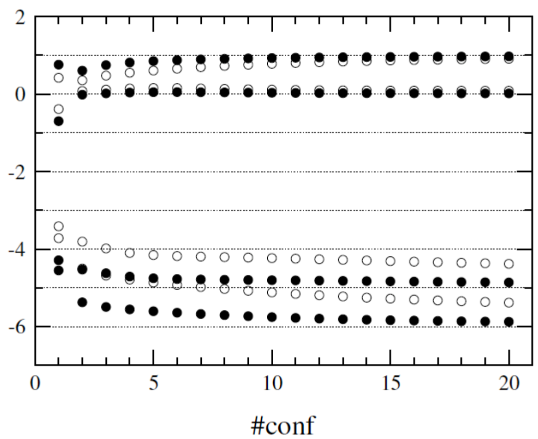


FIG. 4. Topological charge as a function of the number of cooling steps. Open symbols represent the naive plaquette definition and solid symbols the improved definition.

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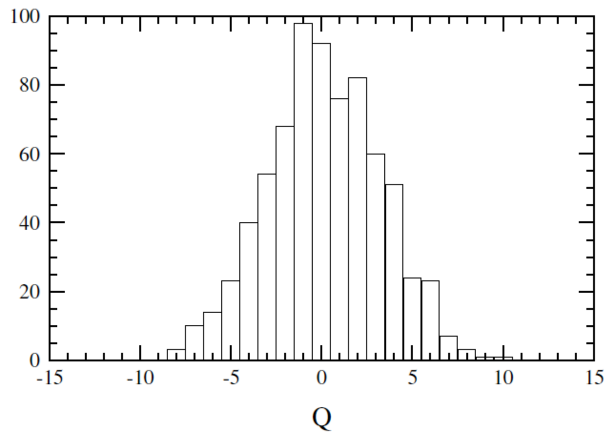


FIG. 6. Histogram of the topological charge.

* quark mass dependence of d_n

- ChPT at NLO [Crewther et al. ('79), O'Connell and Savage ('06)]:

$$L_{\pi NN} = \vec{\pi}\cdot\bar{N}\vec{\tau}\left[i\gamma^5 g_{\pi NN} + \bar{g}_{\pi NN}\right]N \qquad \begin{cases} \bar{g}_{\pi NN} \propto \theta\,\bar{m} \\ \dfrac{1}{\bar{m}} = \dfrac{1}{m_u} + \dfrac{1}{m_d} + \dfrac{1}{m_s} \end{cases}$$

- leading non-analytic term: $d_n \propto \theta\,\bar{m}\,\ln(m_u + m_d) \xrightarrow[m_{u,d}\to 0]{} 0$ (full QCD)

- partially quenched QCD: $d_n \propto \theta\, m_{sea}\, \ln m_{valence} \xrightarrow[m_{valence}\to 0 \text{ at } m_{sea} \text{ fixed}]{} \infty$

- quenched approx.: $d_n \propto \bar{m}^{-p}$

Instanton Liquid Model →

P. FACCIOLI, D. GUADAGNOLI, AND S. SIMULA ('06)

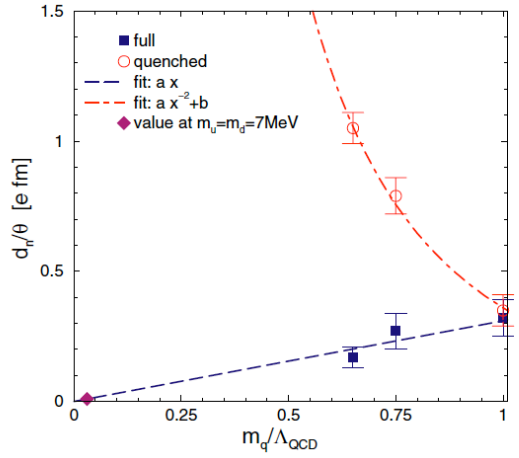


FIG. 3 (color online). ILM results obtained at different values of the quark masses with quenched (circles) and unquenched (squares) simulations. The behavior of the unquenched and quenched calculations for small quark masses is consistent with Eqs. (4.9) and (4.10), respectively.

huge differences between the chiral behavior of quenched and full simulations

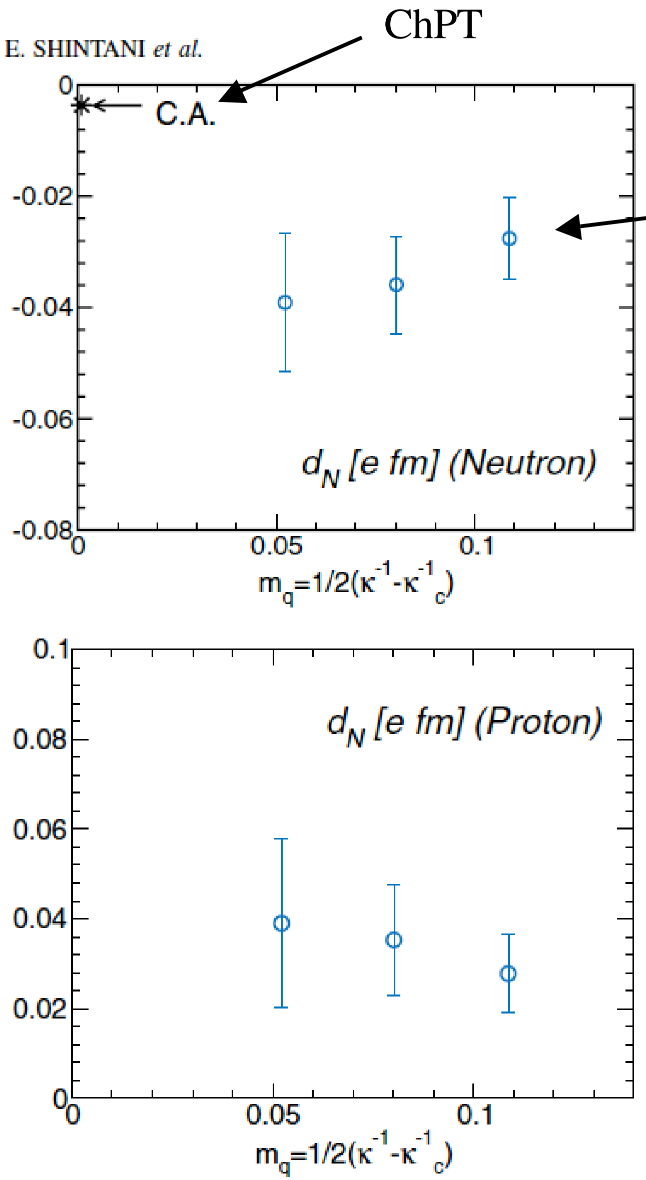


FIG. 17 (color online). The mass dependence of the EDM factor with clover fermion. In the top figure the star symbol shows the prediction from the current algebra in [5]. (Top) the neutron case, (bottom) the proton case.

quenched QCD:

Clover fermions

$a^{-1} \sim 1.9$ GeV ($a \sim 0.1$ fm)

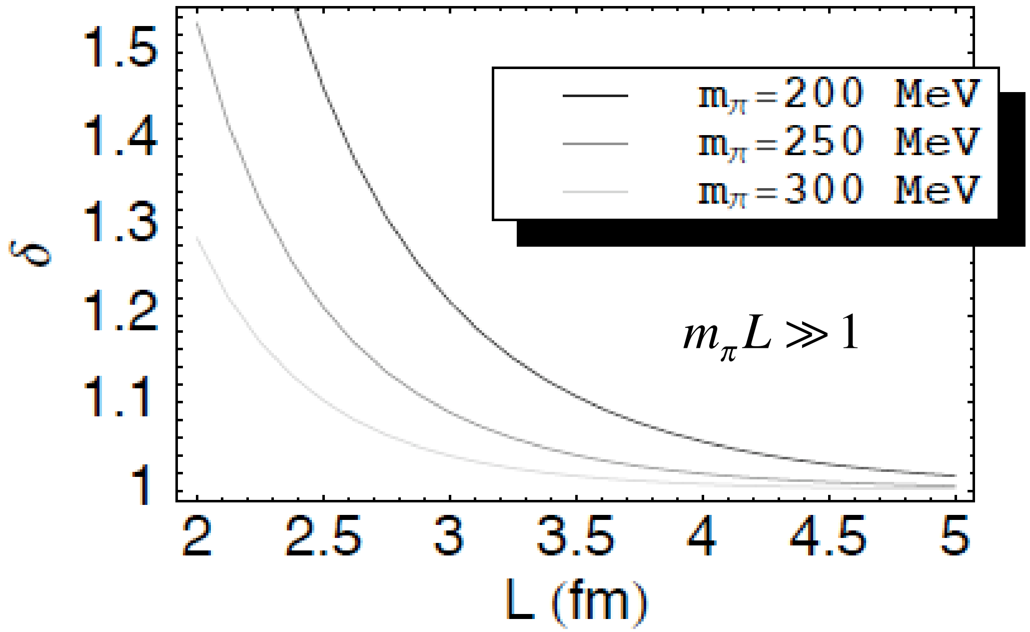
$V*T = 24^3*64$

$1.5 < M_n$ (GeV) < 2

quenched QCD is unreliable for the nEDM

* finite volume effects in ChPT at NLO [O'Connell and Savage ('06)]:

$$\delta = \frac{d_n^L}{d_n^\infty}$$



- large volume effects: $a \sim 0.1, \quad V \cdot T = 32^3 \cdot 64 \quad \rightarrow \quad L \sim 3\ fm$

$$\delta \sim 10\% \quad @\ m_\pi \sim 250\ MeV$$

* <u>determination of the CP-odd form factor</u> $F_3(q^2)$ [Shintani et al. ('05), Berruto et al. ('06)]

- matrix elements of the nucleon e.m. current: $\langle p',s'|J^{\mu}_{e.m.}|p,s\rangle = \bar{u}(p',s')\Gamma^{\mu}u(p,s)$

$$\Gamma^{\mu} = \gamma^{\mu}F_1(q^2) + \frac{i}{2M}\sigma^{\mu\nu}q_{\nu}F_2(q^2) + \\ +(\gamma^{\mu}q^2 - \gamma\cdot q\, q^{\mu})\gamma^5 F_A(q^2) + \\ +\frac{1}{2M}\sigma^{\mu\nu}q_{\nu}\gamma^5 F_3(q^2)$$

Lorentz, gauge invariance and CPT

F_1, F_2 = Dirac and Pauli form factors (P-even and T-even)

F_A = anapole form factor (P-odd)

F_3 = pseudo-tensor form factor (T-odd)

$$d_N = \frac{e}{2M}F_3(q^2=0)$$

- the point at $q^2 = 0$ is not directly accessible: $F_3(0)$ should be obtained by extrapolation of the q^2-dependence of $F_3(q^2)$;
- calculations of 2- and 3-point correlation functions at various nucleon momenta is required;

- in the absence of the θ-term:

$$V^{\mu}_{\alpha\beta}(t_x,t_y;\vec{p},\vec{p}') = \sum_{\vec{x},\vec{y}}\langle N_{\alpha}(\vec{y},t_y)\, J^{\mu}_{e.m.}(\vec{x},t_x)\bar{N}_{\beta}(0)\rangle e^{-i(\vec{p}-\vec{p}')\cdot\vec{x}}e^{-i\vec{p}'\cdot\vec{y}}$$

$$\xrightarrow[t_x,(t_y-t_x)\to\infty]{} Z^N e^{-E't_x}e^{-E(t_y-t_x)}\left(\frac{ip'\cdot\gamma+M}{2E'}\right)_{\alpha\rho}(\Gamma^{\mu})_{\rho\sigma}\left(\frac{ip\cdot\gamma+M}{2E}\right)_{\sigma\beta}$$

- at first order in θ: $\tilde{V}^{\mu}_{\alpha\beta}\left(t_x,t_y;\vec{p},\vec{p}'\right)=\sum_{\vec{x},\vec{y}}\left\langle N_\alpha\left(\vec{y},t_y\right)J^{\mu}_{e.m.}\left(\vec{x},t_x\right)\bar{N}_\beta(0)\,i\theta Q\right\rangle e^{-i(\vec{p}-\vec{p}')\cdot\vec{x}}e^{-i\vec{p}'\cdot\vec{y}}$

$\tilde{V}^{\mu}$ depends on all the four form factors ⇒ delicate subtraction procedure to extract F_3

Berruto et al. (‘06)

$N_f = 2$ dynamical flavors
domain-wall fermions
$a^{-1} \sim 1.7$ GeV ($a \sim 0.12$ fm)
$V*T = 16^3*32$ ($L_s = 12$)
$M_n \sim 1.5$ GeV

periodic boundary conditions

$$p=\frac{2\pi}{L}n \quad \Rightarrow \quad \left|q^2\right| \geq 0.4\ GeV^2$$

$$\left|d_n\right| < 0.02\ e\cdot fm \sim 4\left|d_n\right|_{ChPT}$$

encouraging result to be improved

CALCULATION OF THE NEUTRON ELECTRIC DIPOLE ...

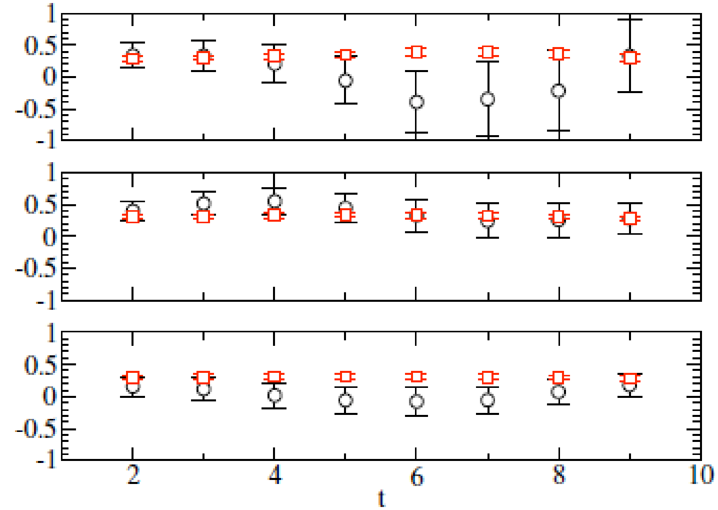


FIG. 15 (color online). The ratio $(F_3(q^2)/2m)/G_E(q^2)$ [Eq. (45)]. q^2 increases from bottom to top panel. In the limit $q^2 \to 0$ this ratio yields the electric dipole moment of the neutron. Also shown is the subtraction term (squares) in Eq. (45). $m_{\rm sea} = 0.03$.

non-periodic boundary conditions
on the fermion fields ⇒ $\left|q^2\right| \sim 0.05\ GeV^2$

lattice evaluation of $F_\pi(q^2)$
by the ETM Collaboration
(S.S. in Proc. of Lattice ‘07)

* <u>method of disconnected insertions of the singlet pseudo-scalar density</u> [Guadagnoli et al. ('04)]

- treating the θ-term as a perturbation at first order, the nEDM in Euclidean space reads as

$$\vec{d}_n = -i\theta \frac{g_s^2}{32\pi^2} \int d^3y\, \vec{y}\, \langle N | J^0_{e.m.}(y) \int d^4x\, G\tilde{G}(x) | N \rangle$$

($|N\rangle$ stands for $|\mathrm{N}\rangle_{\theta=0}$)

- the idea is to work out the equivalence under a singlet chiral rotation between the insertion of the θ-term (a gluonic operator) and that of the singlet PS density (a fermion operator)

$$\vec{d}_n \xrightarrow[?]{} i\theta\bar{m} \int d^3y\, \vec{y}\, \langle N | J^0_{e.m.}(y) \int d^4x\, P_S(x) | N \rangle$$

$$\begin{cases} \dfrac{1}{\bar{m}} = \dfrac{1}{3}\left(\dfrac{1}{m_u} + \dfrac{1}{m_d} + \dfrac{1}{m_s}\right) \\ P_S = \dfrac{1}{3}\left(\bar{u}\gamma^5 u + \bar{d}\gamma^5 d + \bar{s}\gamma^5 s\right) \end{cases}$$

connected insertions   disconnected insertions

○ γ_5 insertion

Figure 1: Connected (a) and disconnected (b) insertions of the flavor-singlet pseudoscalar density operator. The insertions are represented by open dots, while solid lines are quark propagators. The hatched ovals are the operators which create and destroy the neutron. Gluon lines as well as extra quark loops are not shown.

(a) = 0

[Aoki et al. ('90)]

- vacuum expectation value of a generic operator O (in Euclidean space)

$$\left\langle O\left(x_1,\ldots,x_n\right)\right\rangle = \frac{1}{Z}\int d\left[G\right]d\left[\psi\right]d\left[\bar{\psi}\right]O\left(x_1,\ldots,x_n\right)e^{-S}$$

$Z = \int d\left[G\right]d\left[\psi\right]d\left[\bar{\psi}\right]e^{-S}$

$S =$ QCD action

- singlet chiral rotation:

$$\begin{cases}\psi'(x) = \left(1 + i\alpha(x)\gamma_5\right)\psi(x)\\ \bar{\psi}'(x) = \bar{\psi}(x)\left(1 + i\alpha(x)\gamma_5\right)\end{cases}$$

- axial WI:

$$\left\langle O\frac{\delta S}{\delta\left(i\alpha(x)\right)}\right\rangle = \left\langle\frac{\delta O}{\delta\left(i\alpha(x)\right)}\right\rangle$$

$$\left\langle O\frac{\delta S}{\delta\left(i\alpha(x)\right)}\right\rangle = -\partial_\mu\left\langle OA_S^\mu(x)\right\rangle + \left\langle O\left\{M,\lambda_0\right\}\bar{\psi}(x)\gamma^5\psi(x)\right\rangle + 2N_f\frac{g_s^2}{32\pi^2}\left\langle OG\tilde{G}(x)\right\rangle$$

$$2N_f\frac{g_s^2}{32\pi^2}\int d^4x\left\langle OG\tilde{G}(x)\right\rangle = \left\langle\delta\left[O\right]\right\rangle - 2\bar{m}\int d^4x\left\langle OP_S(x)\right\rangle$$

- relevant operator:

$$O = N_\alpha(z)J^0_{e.m.}(y)N_\beta(0)$$

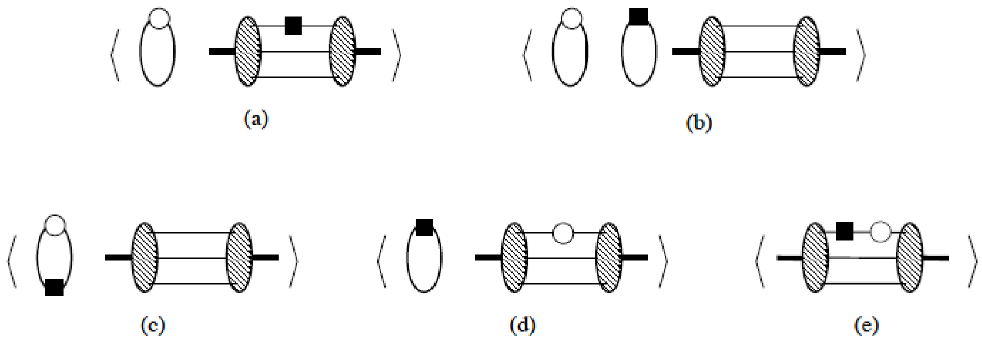
$$N_\alpha = \varepsilon^{abc}\left(d^{aT}C\gamma_5u^b\right)d^c_\alpha$$

$$J^0_{e.m.} = e_u\,\bar{u}\gamma^0u + e_d\,\bar{d}\gamma^0d + e_s\,\bar{s}\gamma^5s$$

- diagrams generated by the chiral variation of O

$$\delta[O]=\delta[N_\alpha](z)J^0_{e.m.}(y)N_\beta(0)+N_\alpha(z)\delta\left[J^0_{e.m.}\right](y)N_\beta(0)+N_\alpha(z)J^0_{e.m.}(y)\delta\left[N_\beta\right](0)$$

- diagrams generated by the insertion of the P_S density



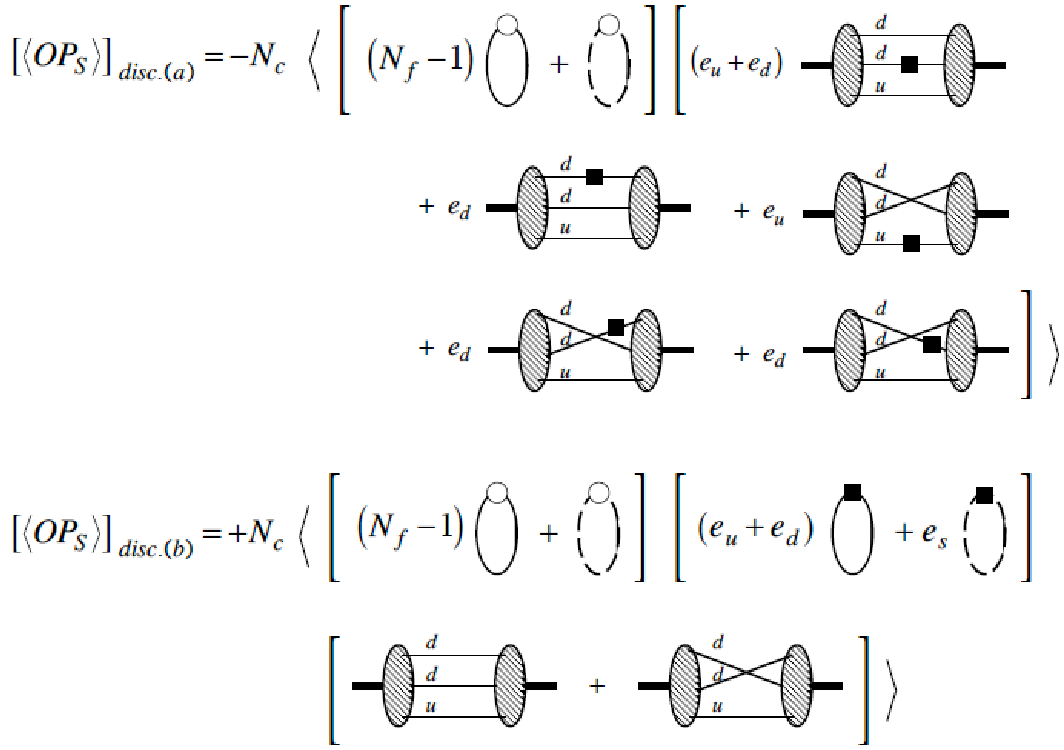
■ γ_0 insertion

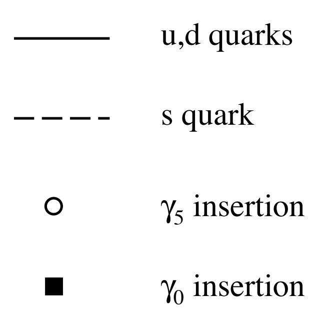
(c) = 0
(d) cancels out $\delta[O]$
(e) = 0

Figure 2: Possible types of disconnected diagrams (a)–(d) and an example of a connected diagram (e), involved in $\mathcal{O}P_S(x)$. The meaning of the vertices and lines is the same as in figure 1. Open dots and full squares represent the insertion of γ_5 (P_S) and γ_0 (J_0), respectively.

- final result:

$$2N_f\frac{g_s^2}{32\pi^2}\int d^4x\left\langle OG\tilde{G}(x)\right\rangle=-2\bar{m}\int d^4x\left\{\left[\left\langle OP_S(x)\right\rangle\right]_{disc.(a)}+\left[\left\langle OP_S(x)\right\rangle\right]_{disc.(b)}\right\}$$





(b) = 0

in the SU(3) limit

Figure 3: Disconnected diagrams contributing to the r.h.s. of eq. (3.6). The notation is the same as in figure 2, but solid lines are now u- and d-quark propagators, while dashed lines are s-quark propagators.

- disconnected diagrams are noisy on the lattice
- important, recent algorithm improvements [see C. Michael and C. McNeile in Proc. of Lattice '07]

CONCLUSIONS

* a non-vanishing value of the electric dipole moment of stable particles is a signal of breaking of parity and time-reversal symmetry, and therefore of CP symmetry assuming the CPT theorem

* the electroweak sector in the SM and in NP models contains sources of CP violation:
  - the phase in the CKM matrix is able to account for observed CP-violations in mesons, but it produces contributions to the electric dipole moment of the neutron and the electron which are several orders of magnitude less than present experimental bounds;
  - the contributions from NP starts at one-loop level and may be quite large, so that present experimental bounds on the neutron and electron EDM represent important constraints on the NP parameters;
  - the task for the lattice community is to start calculations of the matrix elements of three operators:
    1) the quark EDM: $\langle \mathrm{n} | \bar{q} \sigma^{\mu\nu} \gamma^5 q | n \rangle F_{\mu\nu}$
    2) the quark chromo-EDM: $\langle \mathrm{n} | \bar{q} \sigma^{\mu\nu} \gamma^5 t^a G^a_{\mu\nu} q | n \rangle$
    3) the Weinberg term: $\langle \mathrm{n} | f^{abc} G^a_{\mu\lambda} G^{b,\lambda}_{\nu} \tilde{G}^{c,\mu\nu} | n \rangle$

* at variance with the case of the electron, the neutron EDM may receive an important contribution from the so-called **θ-term**

$$\theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

* three strategies have been developed to calculate the nEDM induced by the θ-term on the lattice:

1) measure the energy difference between spin-up and spin-down neutrons in presence of a uniform and static external electric field;

2) measure the CP-odd e.m. form factor $F_3(q^2)$;

3) evaluate disconnected insertions of the singlet pseudo-scalar density.

* the results so far obtained are quite **encouraging**

* results at low pion masses (~ 250 ÷ 300 MeV) are expected to come in the next few years

* warnings from ChPT at NLO:

  - the quark mass dependence may be totally different between quenched, partially quenched and full QCD;
  - volume effects may be quite large.