Hadron Structure from Lattice QCD

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Hadronic Physics on the Lattice Workshop
Electromagnetic Interactions with Nucleons and Nuclei

Milos September 10, 2007

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Outline

- Introduction
- Understanding quark structure of hadrons from QCD
  - Deep inelastic scattering
  - Moments of quark distributions
  - Form factors
  - Generalized form factors
    - Transverse structure
    - Origin of nucleon spin
- Understanding gluon structure of hadrons
  - Gluon contribution to the pion mass and momentum
- Insight into how QCD works
- Summary and future challenges

QCD

- How do hadrons arise from QCD?

- Lagrangian constrained by Lorentz invariance, gauge invariance and renormalizability:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}^2$$

$$D_\mu = \partial_\mu - igA_\mu \qquad F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$$

- Deceptively simple Lagrangian produces amazingly rich and complex structure of strongly interacting matter in our universe

Goals

- Quantitative calculation of hadron observables from first principles
  - Agreement with experiment
  - Credibility for predictions and guiding experiment
- Insight into how QCD works
  - Mechanisms
    - Paths that dominate action - instantons
    - Variational wave functions
    - Diquark correlations
  - Dependence on parameters
    - N_c, N_f, gauge group, m_q

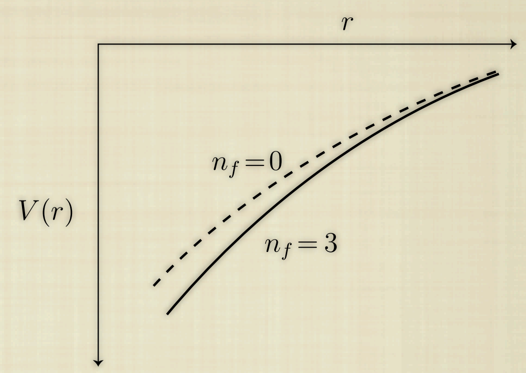
Computational Issues

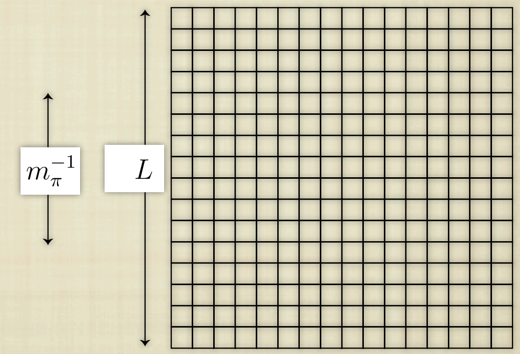
- Fermion determinant - Full QCD
- Small lattice spacing
- Small quark mass
- Large lattice volume

$$\frac{1}{m_\pi} \leq \frac{L}{4}$$

L(fm)	m_π (Mev)
1.6	500
4.0	200
5.7	140

- Cost ~ $(m_\pi)^{-7}$ - $(m_\pi)^{-9}$





Current Status

- Include fermion determinant - Full QCD
- Precision results in heavy quark systems
- $(m_\pi)^{-7}$ - $(m_\pi)^{-9}$ limited past nucleon structure to "heavy pion world" - $m_\pi \geq$ 500 MeV
- Beginning to explore physical "light pion world" $m_\pi \geq$ 300 MeV - role of chiral symmetry

Resources

- US 2006
  - DOE NP, HEP, ASCR Partnership: 8 sustained Tflop
  - NERSC, ORNL, ANL, LLNL
  - NSF centers
- 2006 world sustained Teraflops for lattice
  - USLQCD 8
  - Europe + UK 20 - 25
  - Japan 14 - 18

Hadron structure revealed by high energy scattering

- High energy scattering measures correlation functions along light cone
  - Asymptotic freedom: reaction theory perturbative
  - Unambiguous measurement of operators in light cone frame
  - Must think about physics on light cone
- Parton distribution q(x) gives longitudinal momentum distribution of light-cone wave function
- Generalized parton distribution $q(x, r_\perp)$ gives transverse spatial structure of light-cone wave function

Parton and generalized parton distributions

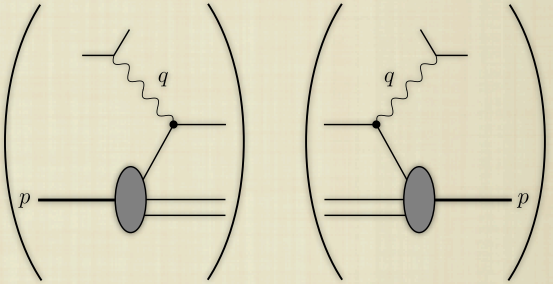
High energy scattering: light-cone correlation function $(\lambda = p^+ x^-)$

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n) \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$$

Deep inelastic scattering: diagonal matrix element

$$\langle P|\mathcal{O}(x)|P\rangle = q(x)$$

$$[\not{n} \to \not{n}\gamma_5 : \quad \Delta q(x)]$$

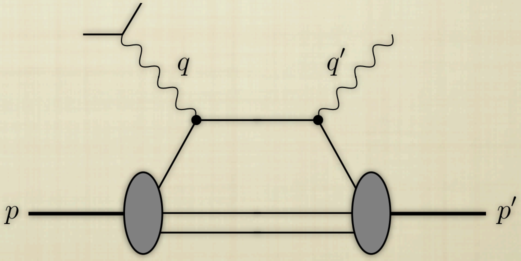


Deeply virtual Compton scattering: off-diagonal matrix element

$$\langle P'|\mathcal{O}(x)|P\rangle = \langle\gamma\rangle H(x,\xi,t) + \tfrac{i\Delta}{2m}\langle\sigma\rangle E(x,\xi,t)$$

$$\Delta = P' - P, \quad t = \Delta^2, \quad \xi = -n\cdot\Delta/2$$

$$[\not{n} \to \not{n}\gamma_5 : \quad \tilde{E}(x,\xi,t), \tilde{H}(x,\xi,t)]$$



Moments of parton distributions

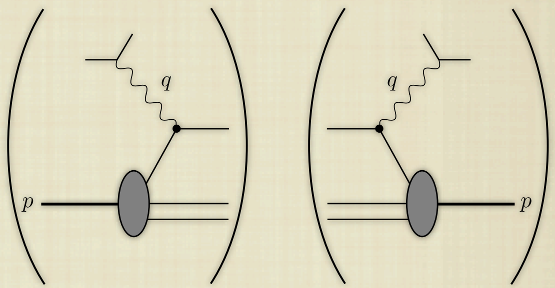
Expansion of $$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n) \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$$

Generates tower of twist-2 operators

$$\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \overline{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}} \psi_q$$

Diagonal matrix element

$$\langle P|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle \sim \int dx\, x^{n-1} q(x)$$



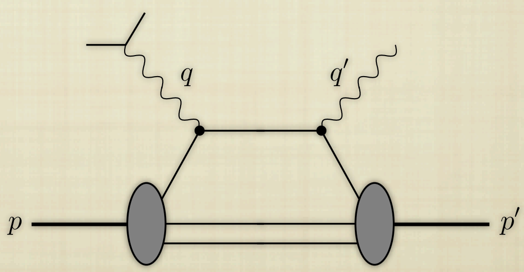
Off-diagonal matrix element

$$\langle P'|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle \to A_{ni}(t), B_{ni}(t), C_{n0}(t)$$

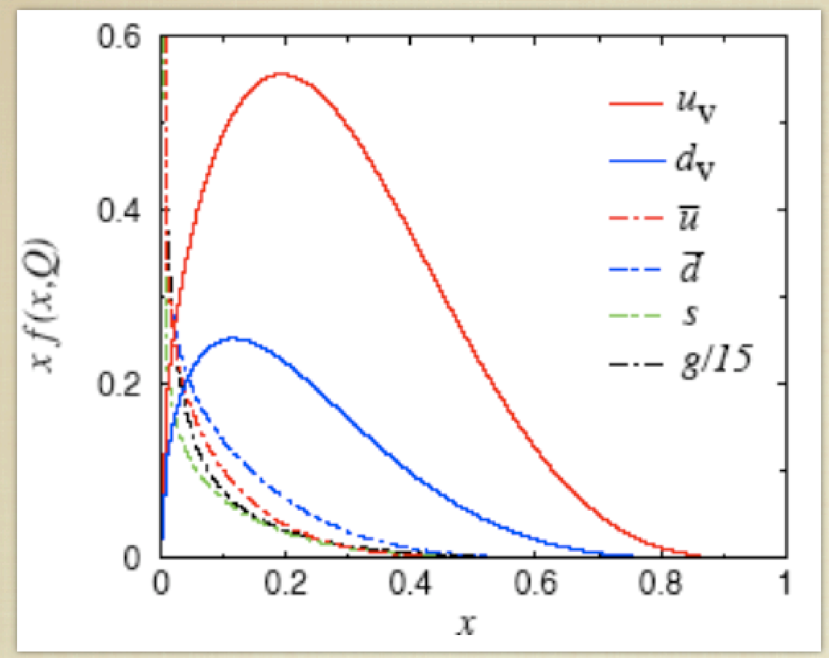
$$\int dx\, x^{n-1} H(x,\xi,t) \sim \sum \xi^i A_{ni}(t) + \xi^n C_{n0}(t)$$

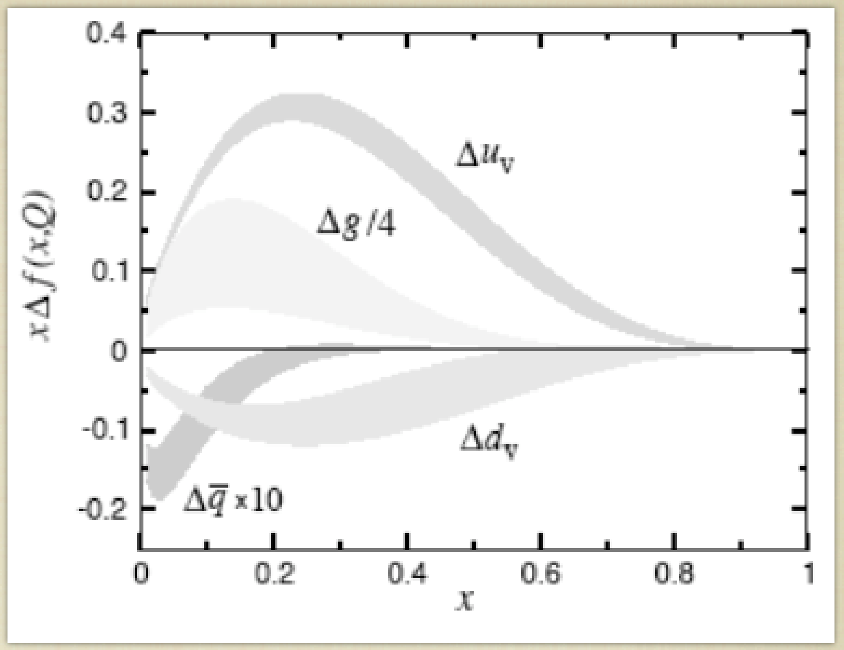
$$\int dx\, x^{n-1} E(x,\xi,t) \sim \sum \xi^i B_{ni}(t) - \xi^n C_{n0}(t)$$

$[\not{n} \to \not{n}\gamma_5 : \quad \tilde{A}_{ni}(t), \tilde{B}_{ni}(t)]$



Moments of parton distributions





$$\langle p|\bar{\psi}\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_q \quad = \int_0^1 dx x^n[q(x)+(-1)^{(n+1)}\bar{q}(x)]$$

$$\langle p|\bar{\psi}\gamma_5\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_{\Delta q} \quad = \int_0^1 dx x^n[\Delta q(x)+(-1)^{(n)}\Delta\bar{q}(x)]$$

$$\langle p|\bar{\psi}\gamma_5\sigma_{\mu\nu} D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_{\delta q} \quad = \int_0^1 dx x^n[\delta q(x)+(-1)^{(n+1)}\delta\bar{q}(x)]$$

where $\quad q = q_\uparrow + q_\downarrow, \quad \Delta q = q_\uparrow - q_\downarrow, \quad \delta q = q_\top + q_\perp,$

Lattice operators: irreducible representations of hypercubic group with minimal operator mixing and minimal non-zero momentum components

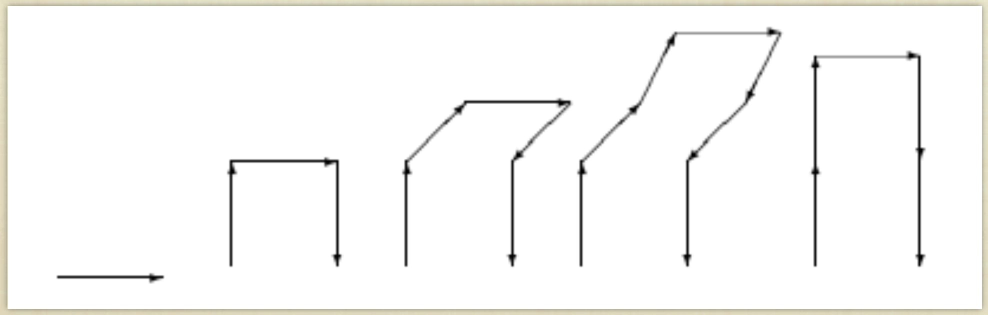
$\langle x \rangle_q^{(a)}$	6_3^+	$\bar{\psi}\gamma_{\{1}\overleftrightarrow{D}_{4\}}\psi$
$\langle x \rangle_q^{(b)}$	3_1^+	$\bar{\psi}\gamma_4\overleftrightarrow{D}_4\psi - \frac{1}{3}\sum_{i=1}^{3}\bar{\psi}\gamma_i\overleftrightarrow{D}_i\psi$
$\langle x^2 \rangle_q$	8_1^-	$\bar{\psi}\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_{4\}}\psi - \frac{1}{2}\sum_{i=2}^{3}\gamma_{\{i}\overleftrightarrow{D}_i\overleftrightarrow{D}_{4\}}\psi$
$\langle x^3 \rangle_q$	2_1^+	$\bar{\psi}\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_4\overleftrightarrow{D}_{4\}}\psi + \bar{\psi}\gamma_{\{2}\overleftrightarrow{D}_2\overleftrightarrow{D}_3\overleftrightarrow{D}_{3\}}\psi - \{3 \leftrightarrow 4\}$
$\langle 1 \rangle_{\Delta q}$	4_4^+	$\bar{\psi}\gamma^5\gamma_3\psi$
$\langle x \rangle_{\Delta q}^{(a)}$	6_3^-	$\bar{\psi}\gamma^5\gamma_{\{1}\overleftrightarrow{D}_{3\}}\psi$
$\langle x \rangle_{\Delta q}^{(b)}$	6_3^-	$\bar{\psi}\gamma^5\gamma_{\{3}\overleftrightarrow{D}_{4\}}\psi$
$\langle x^2 \rangle_{\Delta q}$	4_2^+	$\bar{\psi}\gamma^5\gamma_{\{1}\overleftrightarrow{D}_3\overleftrightarrow{D}_{4\}}\psi$
$\langle 1 \rangle_{\delta q}$	6_1^+	$\bar{\psi}\gamma^5\sigma_{34}\psi$
$\langle x \rangle_{\delta q}$	8_1^-	$\bar{\psi}\gamma^5\sigma_{3\{4}\overleftrightarrow{D}_{1\}}\psi$
d_1	6_1^+	$\bar{\psi}\gamma^5\gamma_{[3}\overleftrightarrow{D}_{4]}\psi$
d_2	8_1^-	$\bar{\psi}\gamma^5\gamma_{[1}\overleftrightarrow{D}_{\{3]}\overleftrightarrow{D}_{4\}}\psi$

Domain wall quarks on a staggered sea

- Improved staggered sea quarks (MILC)
  - Economical - lattices with large L, small m_π , several a
  - Fourth root appears manageable
    - RG indicates coefficient of nonlocal term → 0
    - Partially quenched staggered XPT accounts well for ugly properties
  - Order a^2 improved
- Domain wall valence quarks
  - Chiral symmetry avoids operator mixing
  - Order a^2
  - Conserved 5-d axial current facilitates renormalization
- Hybrid ChPT available
  - One-loop results have simple chiral behavior

Asqtad Action: $O(a^2)$ perturbatively improved

- Symansik improved glue
  - $S_g(U) = C_0 W^{1\times1} + C_1 W^{1\times2} + C_2 W^{cube}$
- Smeared staggered fermions $S_f(V,U)$
  - Fat links remove taste changing gluons
  - Tadpole improved



HYP Smearing

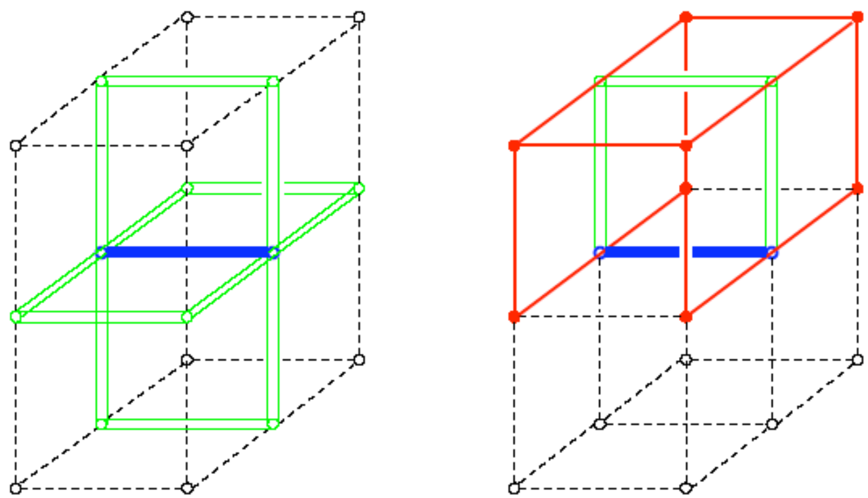
- Three levels of SU(3) projected blocking within hypercube

- Minimize dislocations - important for DW fermions

$$V_{i,\mu} = Proj_{SU(3)}[(1-\alpha_1)U_{i,\mu} + \frac{\alpha_1}{6}\sum_{\pm\nu\neq\mu} \tilde{V}_{i,\nu;\mu}\tilde{V}_{i+\hat{\nu},\mu;\nu}\tilde{V}^{\dagger}_{i+\hat{\mu},\nu;\mu}]\,,$$

$$\tilde{V}_{i,\mu;\nu} = Proj_{SU(3)}[(1-\alpha_2)U_{i,\mu} + \frac{\alpha_2}{4}\sum_{\pm\rho\neq\nu,\mu} \bar{V}_{i,\rho;\nu\mu}\bar{V}_{i+\hat{\rho},\mu;\rho\nu}\bar{V}^{\dagger}_{i+\hat{\mu},\rho;\nu\mu}]\,,$$

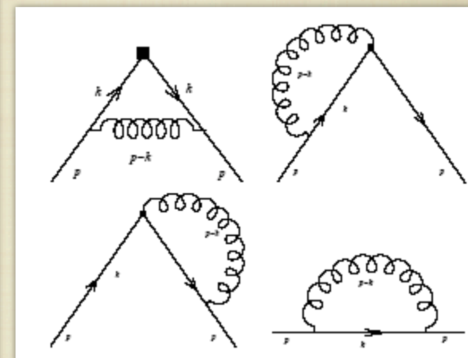
$$\bar{V}_{i,\mu;\nu\rho} = Proj_{SU(3)}[(1-\alpha_3)U_{i,\mu} + \frac{\alpha_3}{2}\sum_{\pm\eta\neq\rho,\nu,\mu} U_{i,\eta}U_{i+\hat{\eta},\mu}U^{\dagger}_{i+\hat{\mu},\eta}]\,.$$

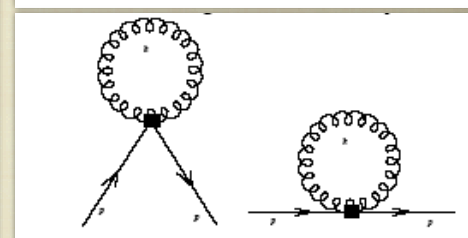


Perturbative renormalization

HYP smeared domain wall fermions - B. Bistrovic

operator	$H(4)$	NOS	HYP	APE
$\bar{q}[\gamma_5]q$	$1_1^{\pm}$	0.792	0.981	1.046
$\bar{q}[\gamma_5]\gamma_\mu q$	$4_4^{\mp}$	0.847	0.976	0.994
$\bar{q}[\gamma_5]\sigma_{\mu\nu} q$	$6_1^{\mp}$	0.883	0.992	0.993
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$6_3^{\pm}$	0.991	0.979	0.954
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$3_1^{\pm}$	0.982	0.975	0.951
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$8_1^{\mp}$	1.134	0.988	0.934
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	mixing	5.71×10^{-3}	1.88×10^{-3}	8.21×10^{-4}
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$4_2^{\mp}$	1.124	0.987	0.934
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_\alpha D_{\beta\}}q$	$2_1^{\pm}$	1.244	0.993	0.919
$\bar{q}[\gamma_5]\sigma_{\mu\{\nu}D_{\alpha\}}q$	$8_1^{\pm}$	1.011	0.994	0.964
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\nu]}q$	$6_1^{\mp}$	0.979	0.982	0.989
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\{\nu]}D_{\alpha\}}q$	$8_1^{\pm}$	0.955	0.959	0.965





$$O_i^{\overline{MS}}(Q^2) = \sum_j \left(\delta_{ij} + \frac{g_0^2}{16\pi^2}\frac{N_c^2-1}{2N_c}\left(\gamma_{ij}^{\overline{MS}}\log(Q^2a^2) - (B_{ij}^{LATT} - B_{ij}^{\overline{MS}})\right)\right)\cdot O_j^{LATT}(a^2)$$

Numerical calculations

- Improved staggered sea quarks (MILC configurations)
  - $N_F = 3$, a=0.125 fm
- Domain wall valence quarks
  - $L_S = 16$, M = 1.7
  - Masses and volumes:

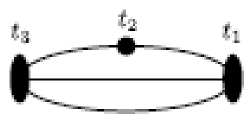
m_π	configs	Vol	L (fm)
761	425	20^3	2.5
693	350	20^3	2.5
544	564	20^3	2.5
486	498	20^3	2.5
354	655	20^3	2.5
354	270	28^3	3.5

Matrix elements on the lattice

$J^\dagger$: Current with quantum numbers of proton

$|\psi_J\rangle = J^\dagger|\Omega\rangle$ Trial function

$$\langle T J(t_3)\,\mathcal{O}(t_2)\,J^\dagger(t_1)\rangle = \sum_{m,n}\langle\psi_J|n\rangle\langle n|\mathcal{O}|m\rangle\langle m|\psi_J\rangle e^{-E_n(t_3-t_2)-E_m(t_2-t_1)}$$



$$\xrightarrow[\substack{t_3-t_2\gg 1\\ t_2-t_1\gg 1}]{} \left|\langle\psi_J|0\rangle\right|^2\langle 0|\mathcal{O}|0\rangle e^{-E_0(t_3-t_1)}$$

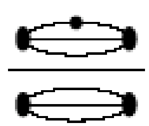
Normalize:

$$\langle T J(t_3)\,J^\dagger(t_1)\rangle = \sum_n\left|\langle\psi_J|n\rangle\right|^2 e^{-E_n(t_3-t_1)}$$

$$\xrightarrow[t_3-t_1\gg 1]{} \left|\langle\psi_J|0\rangle\right|^2 e^{-E_0(t_3-t_1)}$$

$\Longrightarrow$

$$\langle 0|\mathcal{O}|0\rangle = \frac{\langle J\mathcal{O}J^\dagger\rangle}{\langle JJ^\dagger\rangle} =$$



Overdetermined system for form factors

Calculate ratio

$$R_{\mathcal{O}}(\tau,P',P)=\frac{C^{3\mathrm{pt}}_{\mathcal{O}}(\tau,P',P)}{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}},P')}\left[\frac{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}}-\tau+\tau_{\mathrm{src}},P)\,C^{2\mathrm{pt}}(\tau,P')\,C^{2\mathrm{pt}}(\tau_{\mathrm{snk}},P')}{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}}-\tau+\tau_{\mathrm{src}},P')\,C^{2\mathrm{pt}}(\tau,P)\,C^{2\mathrm{pt}}(\tau_{\mathrm{snk}},P)}\right]^{1/2}$$

Perturbative renormalization

$$\mathcal{O}_i^{\overline{\mathrm{MS}}}(\mu)=\sum_j Z_{ij}(\mu,a)\mathcal{O}_j^{\mathrm{lat}}(a)$$

$$\langle P'|\,\mathcal{O}_i^{\overline{\mathrm{MS}}}\,|P\rangle=\sqrt{E(P')E(P)}\sum_j Z_{ij}\overline{R}_j$$

$$\langle P'|\,\mathcal{O}^q_{\{\mu_1\mu_2\dots\mu_n\}}\,|P\rangle=\sum_i a_i A^q_{ni}+\sum_j b_j B^q_{nj}+cC^q_n$$

Schematic form

$$\begin{aligned}\langle\mathcal{O}_i^{cont}\rangle &= \sum_j a_{ij}\mathcal{F}_j\\ \langle\mathcal{O}_i^{cont}\rangle &= \sqrt{E'E}\sum_j Z_{ij}\overline{R}_j\\ \overline{R}_i &= \frac{1}{\sqrt{E'E}}\sum_{jk} Z^{-1}_{ij}a_{jk}\mathcal{F}_k\\ &\equiv \sum_j a'_{ij}\mathcal{F}_j\,.\end{aligned}$$

Nucleon axial charge in full lattice QCD

- Why g_A?
- Matrix element of axial current $A_\mu = \bar{q}\gamma_\mu\gamma_5\frac{\vec{\tau}}{2}q$

$$\langle N(p+q)|A_\mu|N(p)\rangle = \bar{u}(p+q)\frac{\vec{\tau}}{2}\left[g_A(q^2)\gamma_\mu\gamma_5 + g_P(q^2)q_\mu\gamma_5\right]u(p)$$

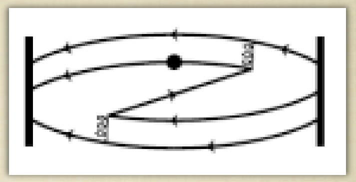
$$g_A(0) = 1.2695 \pm 0.0029$$

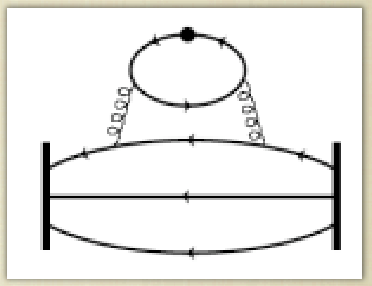
- Adler Weisberger $g_A^2 - 1 \sim \int(\sigma_{\pi^+p} - \sigma_{\pi^-p})$
- Goldberger Treiman $g_A \to f_\pi g_{\pi NN}/M_N$
- Spin content $\langle 1\rangle_{\Delta q} = \int_0^1 dx[\Delta q(x) + \Delta\bar{q}(x)]$

$$g_A = \langle 1\rangle_{\Delta u} - \langle 1\rangle_{\Delta d} \qquad \Sigma = \langle 1\rangle_{\Delta u} + \langle 1\rangle_{\Delta d} + \langle 1\rangle_{\Delta s}$$

Nucleon axial charge

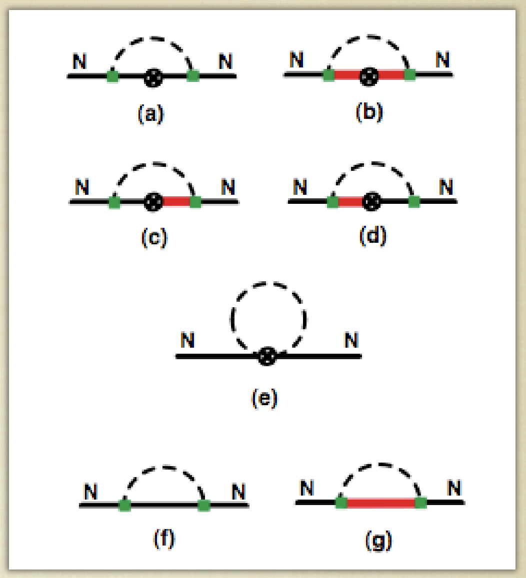
- Gold-Plated observable
  - Accurately measured
  - No disconnected diagrams
  - Chiral perturbation theory for $g_A(m_\pi^2, V)$
  - Renormalization - 5-d conserved current
- hep-lat/0510062





Nucleon Axial Charge

- Chiral perturbation theory $g_A(m_\pi^2, V)$
  - Beane and Savage hep-ph/0404131
  - Detmold and Lin hep-lat/0501007
- 1-loop theory has 6 parameters
  - Fix $f_\pi,\, m_\Delta - m_N,\, g_{\Delta N}$ (0.3% error)
  - Fit $g_A,\, g_{\Delta\Delta},\, C$
  - Result $g_A(m_\pi = 140) = 1.212 \pm 0.084$



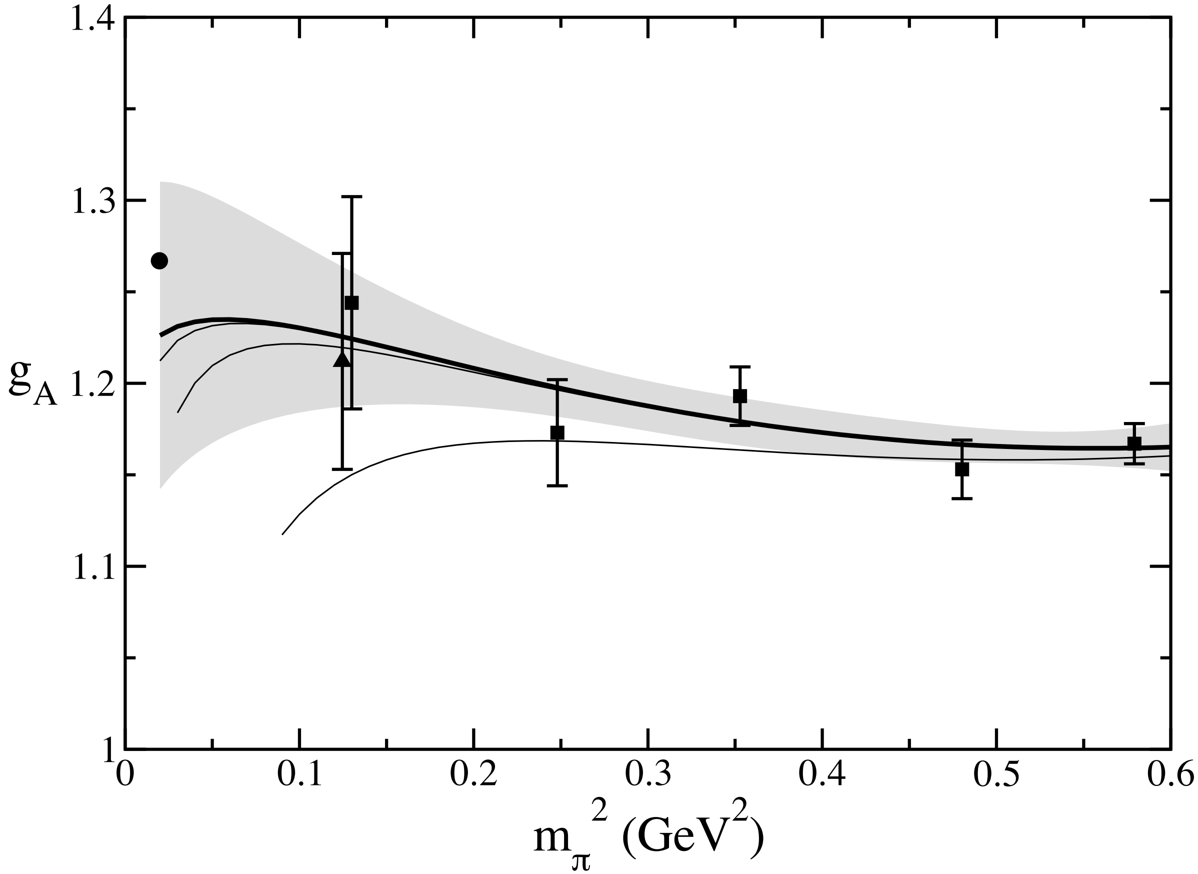
Chiral expansion of axial charge

Beane and Savage hep-ph/0404131

$$\begin{aligned}
\Gamma_{NN} = g_A \quad & - \quad i\frac{4}{3f^2}[4g_A^3 J_1(m_\pi, 0, \mu) \\
& + \quad 4(g_{\Delta N}^2 g_A + \frac{25}{81} g_{\Delta N}^2 g_{\Delta\Delta}) J_1(m_\pi, \Delta, \mu) \\
& + \quad \frac{3}{2} g_A R_1(m_\pi, \mu) \\
& - \quad \frac{32}{9} g_{\Delta N} g_A N_1(m_\pi, \Delta, \mu)] \\
& + \quad C m_\pi^2
\end{aligned}$$

$$\begin{aligned}
J_1(m, \Delta, \mu) &= -\frac{3}{4}\frac{i}{16\pi^2}\left[(m^2 - 2\Delta^2)\log\frac{m^2}{\mu^2} + 2\Delta F(m, \Delta)\right] \\
R_1(m, \mu) &= \frac{i}{16\pi^2} m^2 \left[\Gamma(\epsilon) + 1 - \log\frac{m^2}{\mu^2}\right] \\
N_1(m, \Delta, \mu) &= -\frac{3}{4}\frac{i}{16\pi^2}\left[(m^2 - \frac{2}{3}\Delta^2)\log\frac{m^2}{\mu^2} + \frac{2}{3}\Delta F(m, \Delta) + \frac{2}{3}\frac{m^2}{\Delta}[\pi m - F(m, \Delta)]\right] \\
f(m, \Delta) &= \sqrt{\Delta^2 - m^2 - i\epsilon}\,\log\left(\frac{\Delta - \sqrt{\Delta^2 - m^2 - i\epsilon}}{\Delta + \sqrt{\Delta^2 - m^2 - i\epsilon}}\right)
\end{aligned}$$

Nucleon axial charge g_A $\langle 1 \rangle_{\Delta q}^{u-d}$



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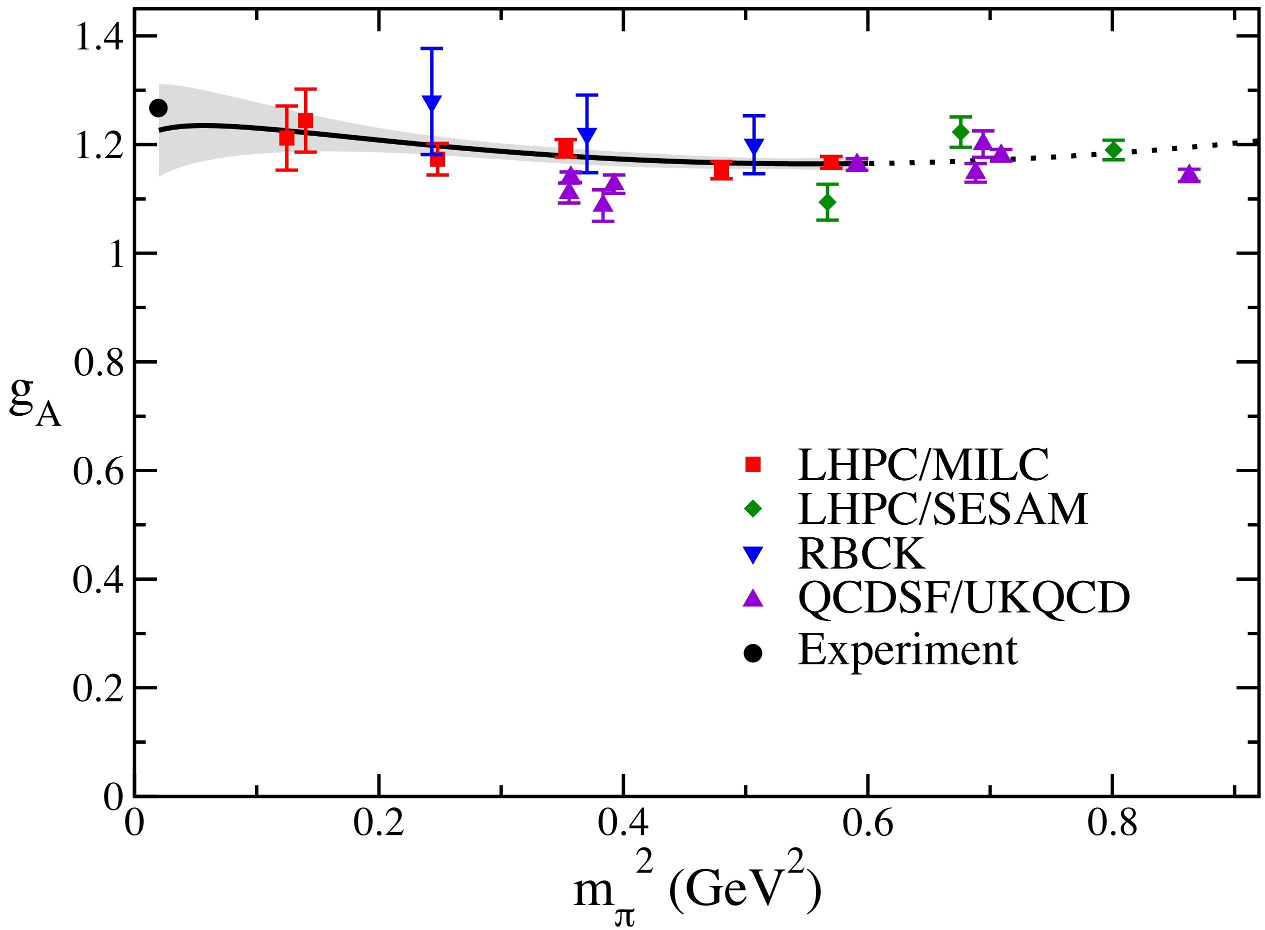
Nucleon axial charge g_A $\langle 1 \rangle_{\Delta q}^{u-d}$



Chiral Perturbation Theory

Self-consistently improved 1-loop ChPT

Heavy Baryon ChPT expand in $\frac{p}{\Lambda_\chi}, \frac{m_\pi}{\Lambda_\chi}, \frac{p}{M_N}, \frac{m_\pi}{M_N}$

Covariant BaryonChPT sum all powers $\left(\frac{1}{M_N}\right)^n$

ChPT with finite range regulators

GFF	HBChPT	CBChPT	expected dependence on m_π, t
A_{20}^{u-d}	$\mathcal{O}(p^2)$	$\mathcal{O}(p^2)$	non-analytic in m_π, $\approx$ linear in t
B_{20}^{u-d}		$\mathcal{O}(p^2)$ + corr. of $\mathcal{O}(p^3)$	non-analytic in m_π, $\approx$ linear in t
C_{20}^{u-d}		$\mathcal{O}(p^2)$ + corr. of $\mathcal{O}(p^3)$	non-analytic in m_π, $\approx$ linear in t
A_{20}^{u+d}		$\mathcal{O}(p^2)$ + corr. of $\mathcal{O}(p^3)$	non-analytic in m_π and t
B_{20}^{u+d}	$\mathcal{O}(p^2)$	$\mathcal{O}(p^2) + \mathcal{O}(p^3)$-CTs	non-analytic in m_π and t
C_{20}^{u+d}		$\mathcal{O}(p^2)$ + corr. of $\mathcal{O}(p^{3,4})$	non-analytic in m_π and t
$J^{u+d} = 1/2(A+B)_{20}^{u+d}$		$\mathcal{O}(p^2)$ + corr. of $\mathcal{O}(p^3)$	
$E_{20}^{u+d} = (A + t/(4m_N)^2 B)_{20}^{u+d}$	$\mathcal{O}(p^2)$		linear in m_π^2 and t
$M_{20}^{u+d} = (A+B)_{20}^{u+d}$	$\mathcal{O}(p^2)$		non-analytic in m_π and t
C_{20}^{u+d}	$\mathcal{O}(p^2)$		non-analytic in m_π and t
$J^{u+d} = 1/2(A+B)_{20}^{u+d}$	$\mathcal{O}(p^2)$		
$J^{u+d} = 1/2(A+B)_{20}^{u+d}$	$\mathcal{O}(p^2)$ with Δ		

Chiral Extrapolation of Moments

- for example, unpolarized moments

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_{A,0}^2 + 1)}{(4\pi f_{\pi,0})^2} m_\pi^2 \ln\left(\frac{m_\pi^2}{\mu^2}\right)\right) + b_n'(\mu) m_\pi^2$$

- choose $\mu = f_{\pi,0}$, and at one loop $g_{A,0} \to g_{A,m_\pi}$ and $f_{\pi,0} \to f_{\pi,m_\pi}$

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_{A,m_\pi}^2 + 1)}{(4\pi)^2} \frac{m_\pi^2}{f_{\pi,m_\pi}^2} \ln\left(\frac{m_\pi^2}{f_{\pi,m_\pi}^2}\right)\right) + b_n \frac{m_\pi^2}{f_{\pi,m_\pi}^2}$$

- self consistently $g_A \to g_{A,\text{lat}}$, $f_\pi \to f_{\pi,\text{lat}}$, $m_\pi \to m_{\pi,\text{lat}}$

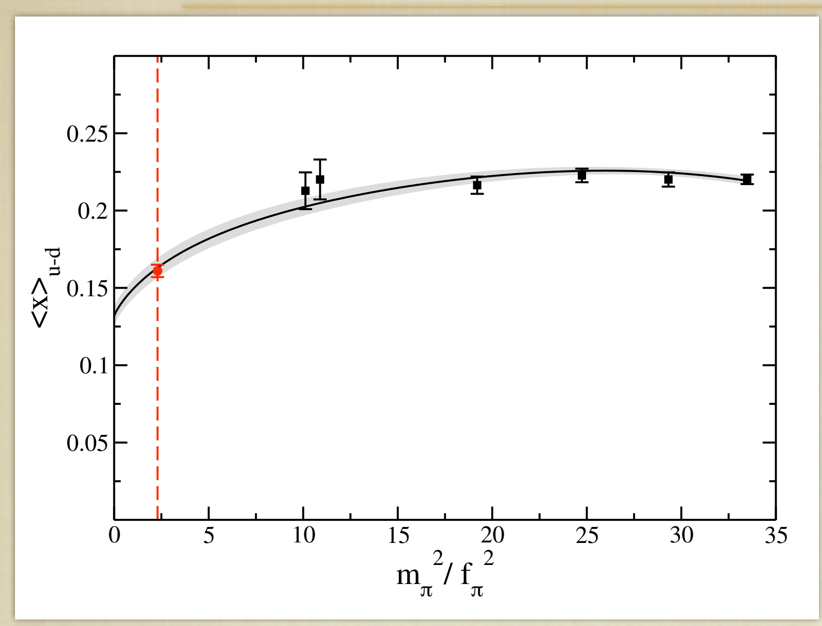
$$\langle x^n \rangle_{u-d} \;=\; a_n \left(1 - \frac{(3g_{A,\text{lat}}^2 + 1)}{(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln\left(\frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}\right)\right) + b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

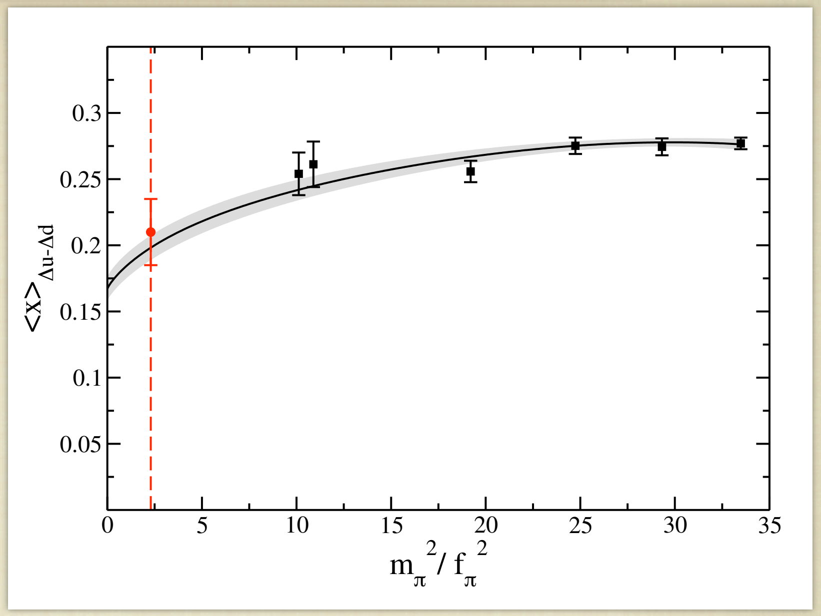
- similarly for the helicity and transversity moments

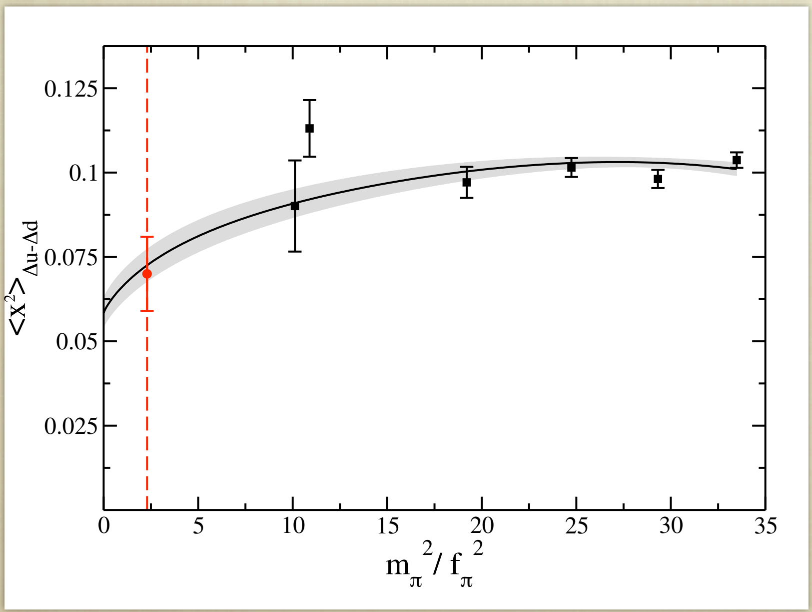
$$\langle x^n \rangle_{\Delta u-\Delta d} \;=\; \Delta a_n \left(1 - \frac{(2g_{A,\text{lat}}^2 + 1)}{(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln\left(\frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}\right)\right) + \Delta b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

$$\langle x^n \rangle_{\delta u-\delta d} \;=\; \delta a_n \left(1 - \frac{(4g_{A,\text{lat}}^2 + 1)}{2(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln\left(\frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}\right)\right) + \delta b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

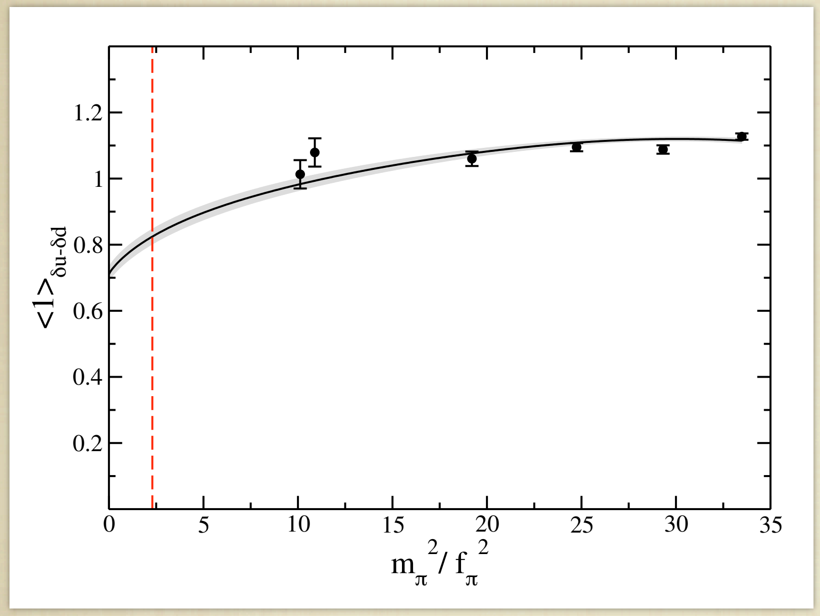
Chiral Extrapolation of Moments

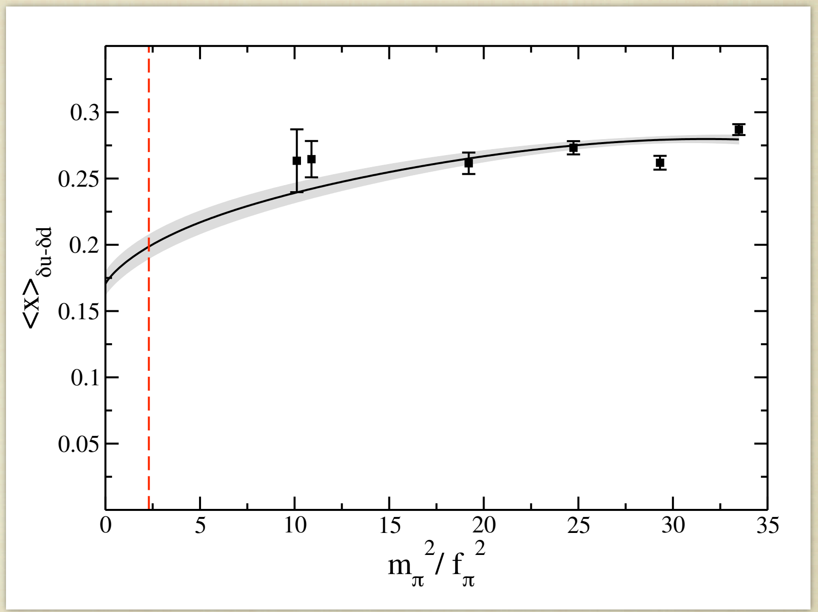




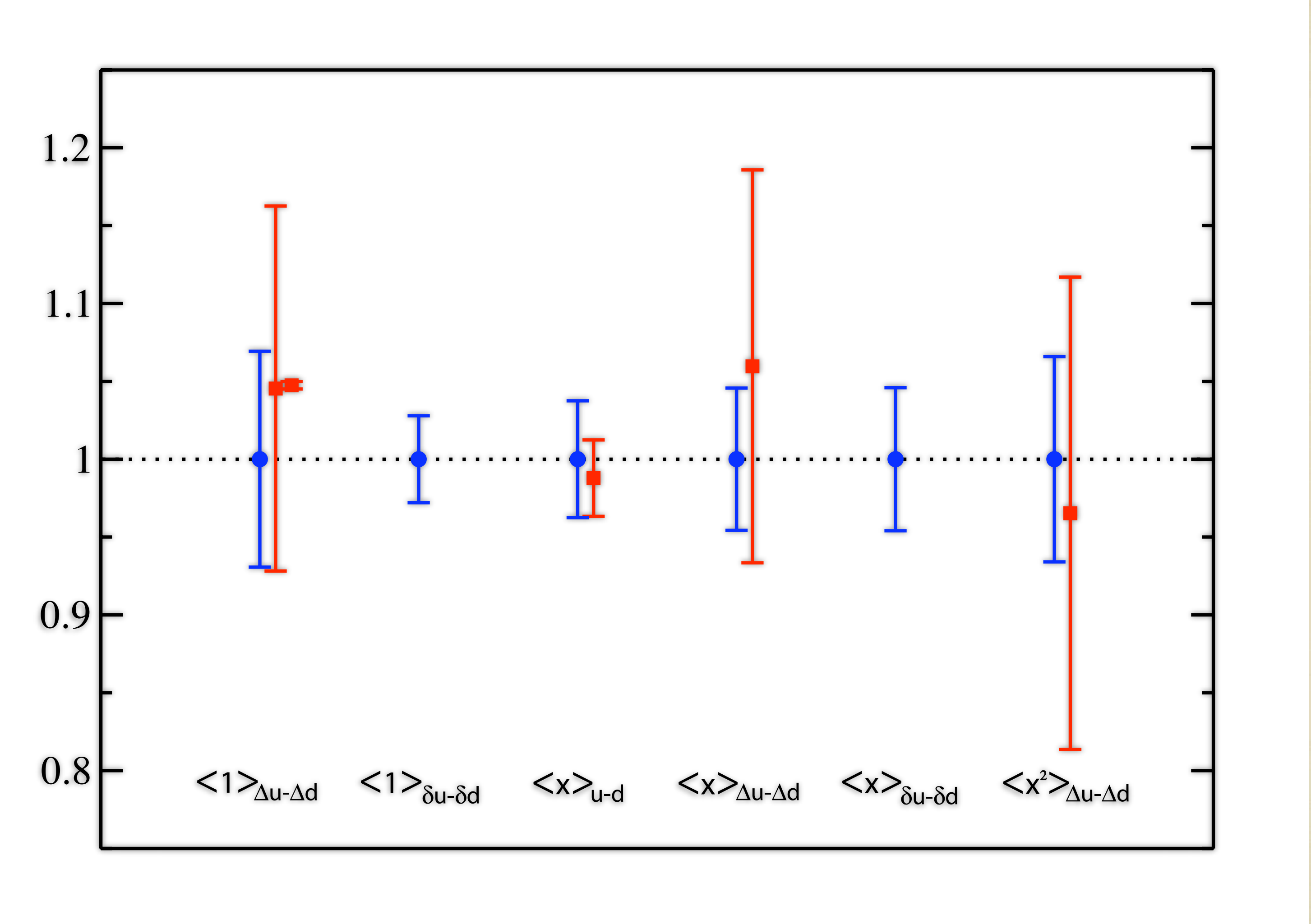


Chiral Extrapolation of Moments





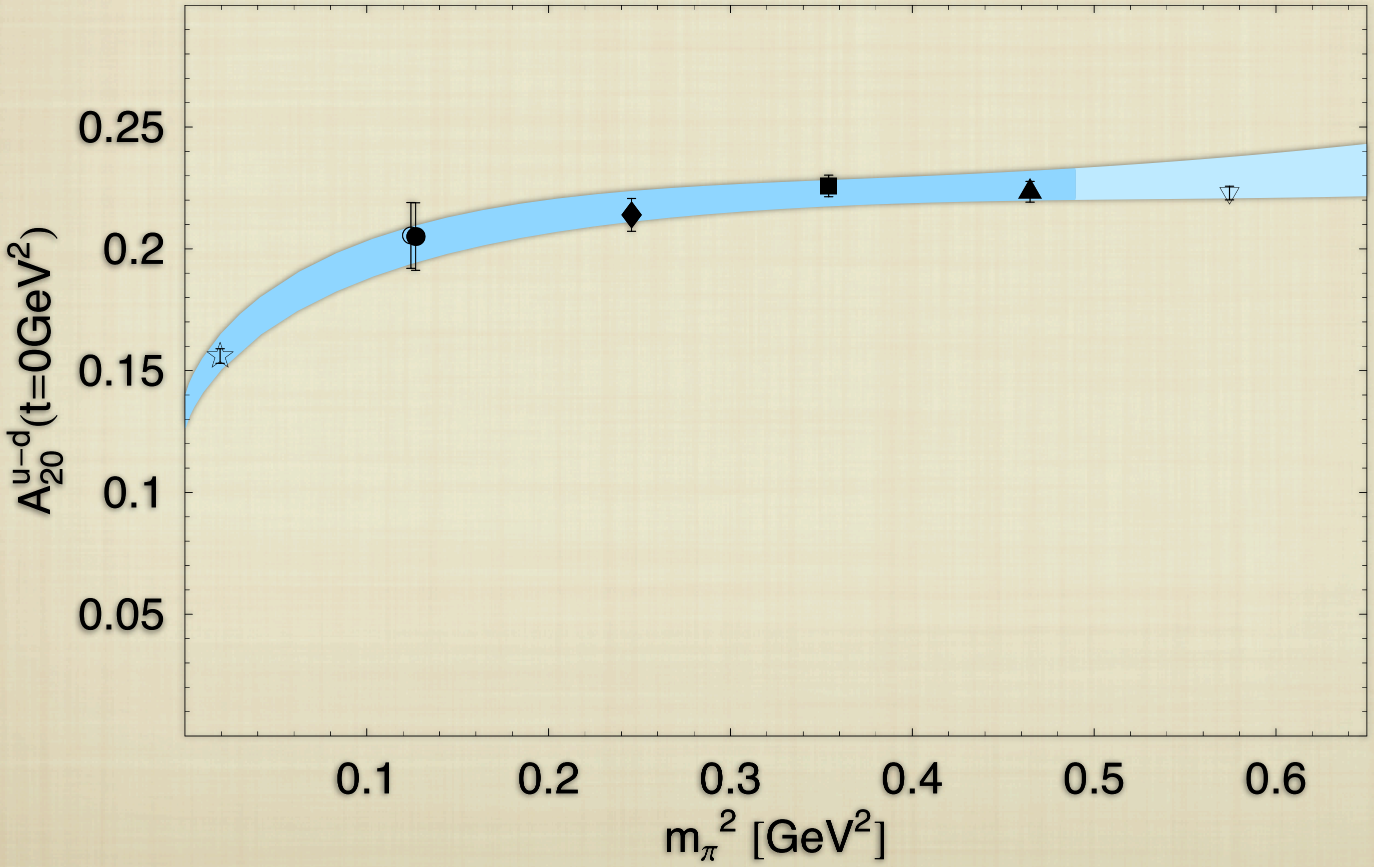
Chiral Extrapolation of Moments



Chiral extrapolation of $\langle x \rangle_q^{u-d} = A_{20}^{u-d}(t=0)$

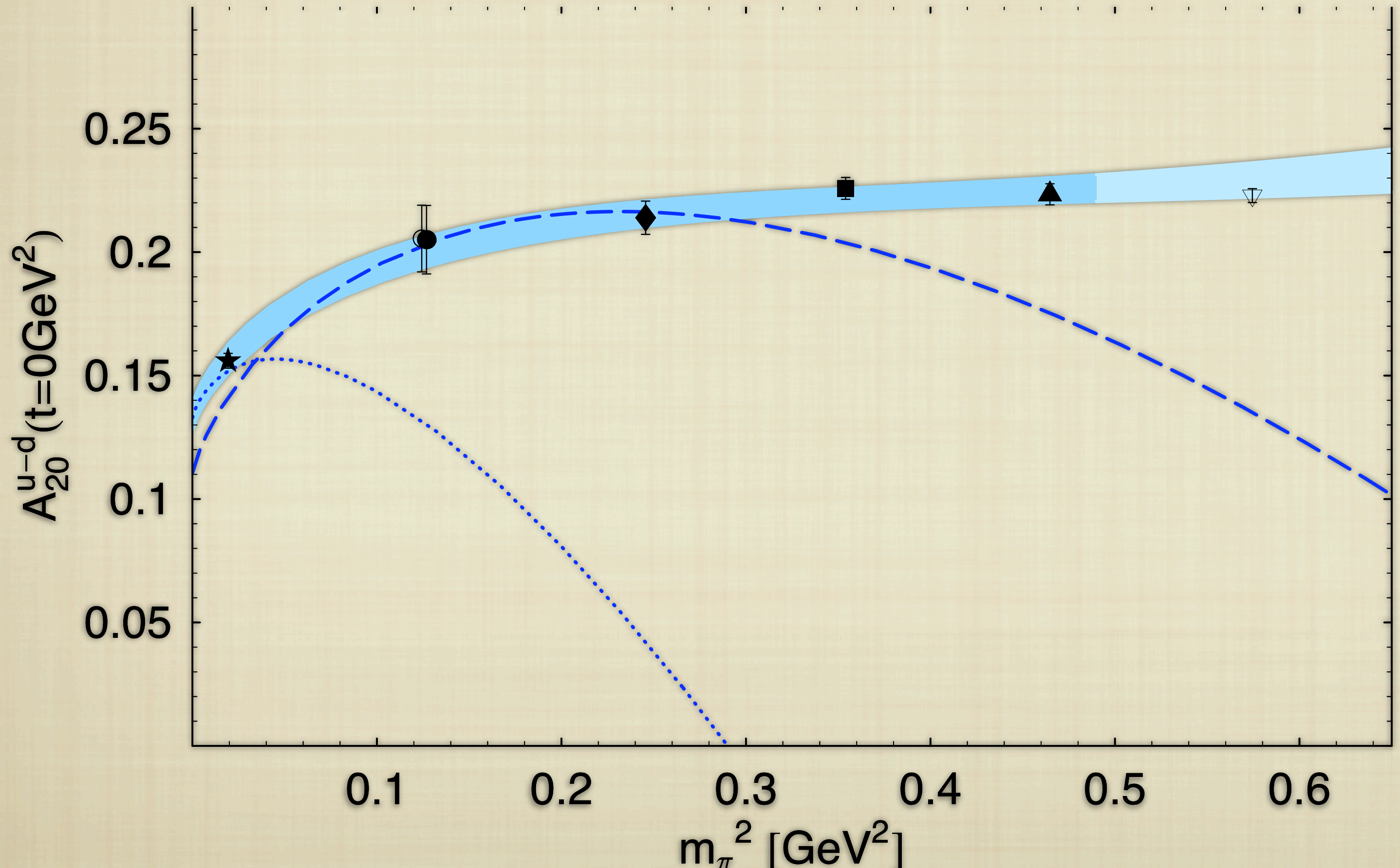
Chiral extrapolation O(p^2) covariant ChPT (Dorati, Hemmert, et. al.)

$$A_{20}^{u-d}(t,m_\pi) = A_{20}^{0,u-d}\left(f_A^{u-d}(m_\pi)+\frac{g_A^2}{192\pi^2 f_\pi^2}h_A(t,m_\pi)\right)+\widetilde{A}_{20}^{0,u-d}\dot{j}_A^{u-d}(m_\pi)+A_{20}^{m_\pi,u-d}m_\pi^2+A_{20}^{t,u-d}t$$



Chiral extrapolation of $\langle x \rangle_q^{u-d} = A_{20}^{u-d}(t=0)$

Chiral extrapolation O(p^2) covariant BChPT
Heavy baryon limit (dotted curve)
HBChPT fit: $t < 0.3$ GeV^2, $m_\pi < 0.5$ GeV (dashed curve)

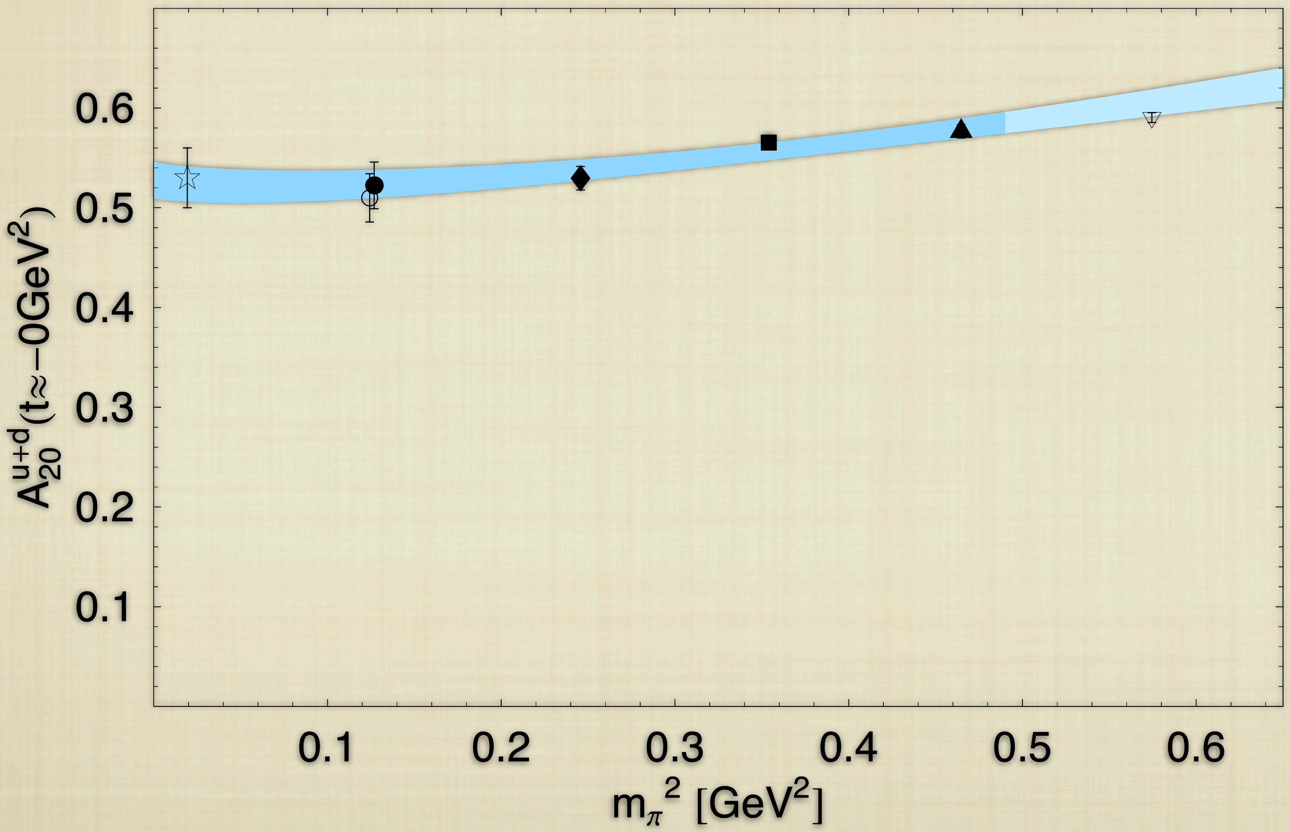


Chiral extrapolation of $\langle x \rangle_q^{u+d} = A_{20}^{u+d}(t=0)$

Chiral extrapolation O(p^2) relativistic ChPT (Dorati, Hemmert, et. al.)

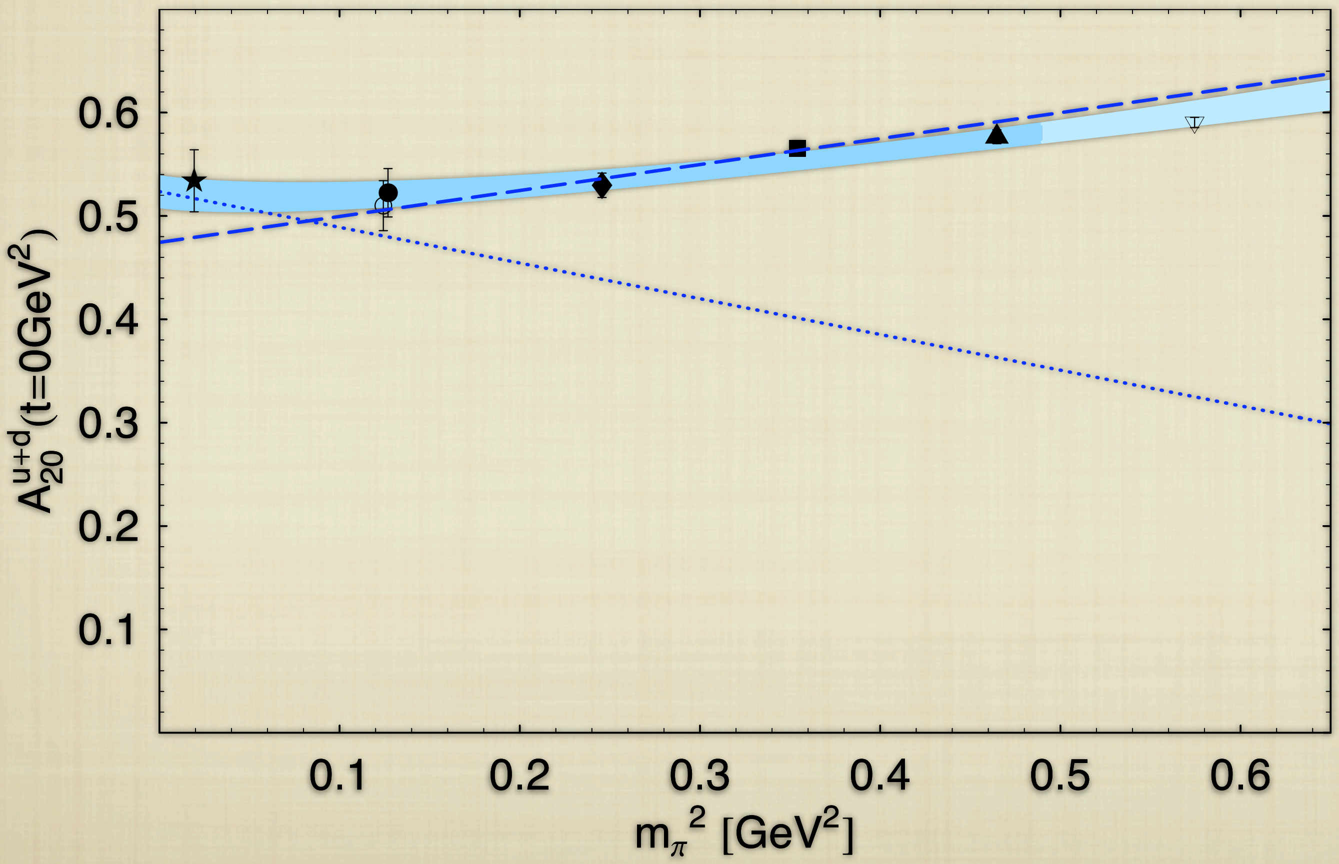
Note: connected diagrams only

$$A_{20}^{u+d}(t,m_\pi) = A_{20}^{0,u+d}\left(f_A^{u+d}(m_\pi) - \frac{g_A^2}{64\pi^2 f_\pi^2} h_A(t,m_\pi)\right) + A_{20}^{m_\pi,u+d} m_\pi^2 + A_{20}^{t,u+d} t + \Delta A_{20}^{u+d}(t,m_\pi) + \mathcal{O}(p^3)$$



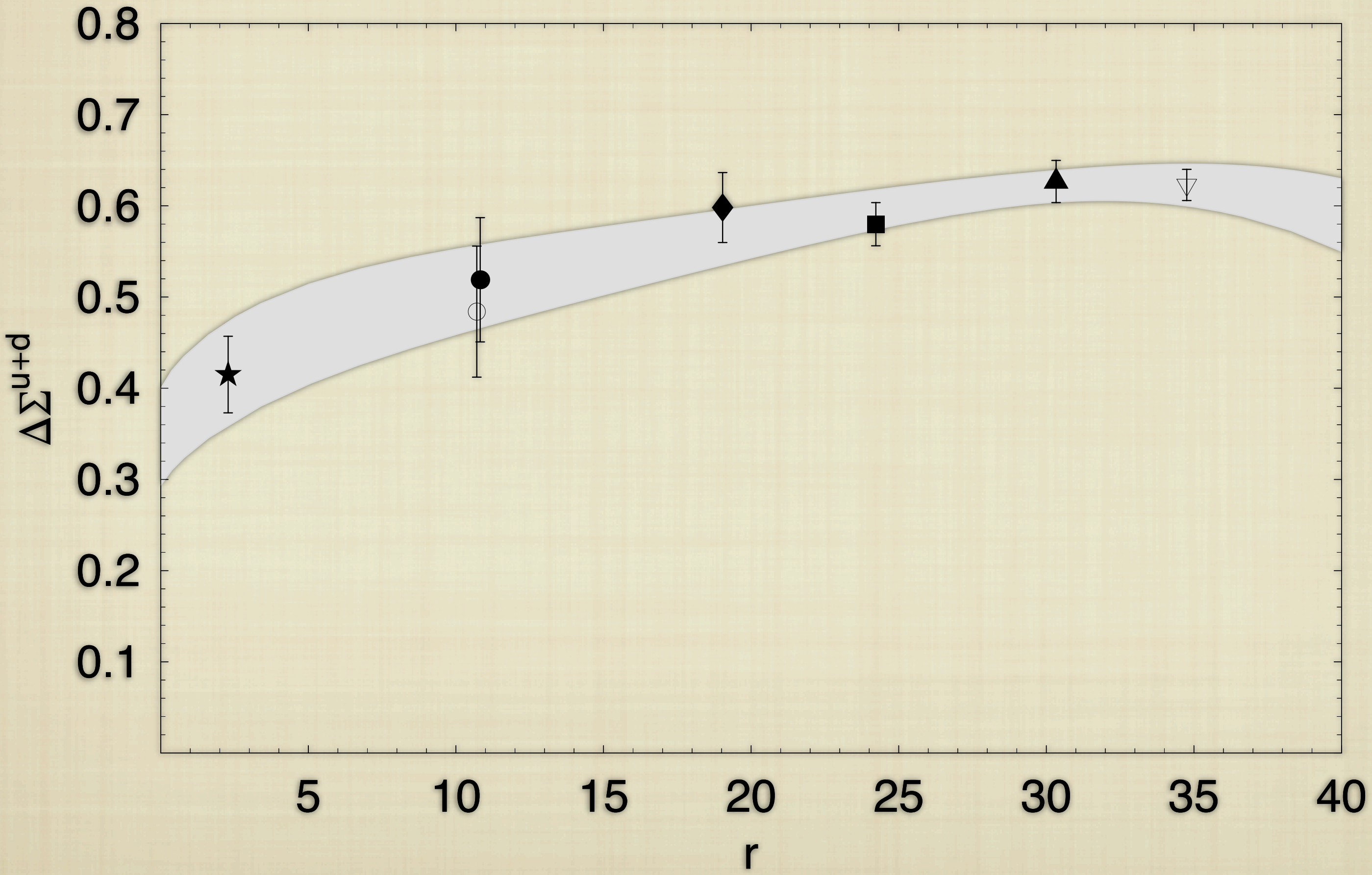
Chiral extrapolation of $\langle x \rangle_q^{u+d} = A_{20}^{u+d}(t=0)$

Chiral extrapolation O(p^2) covariant BChPT
Heavy baryon limit (dotted line)
HBChPT fit: $t < 0.3$ GeV^2, $m_\pi < 0.5$ GeV (dashed line)



Quark spin contribution to Nucleon Spin

$$\Delta\Sigma = \langle 1 \rangle_{\Delta u} + \langle 1 \rangle_{\Delta d}$$



Electromagnetic form factors

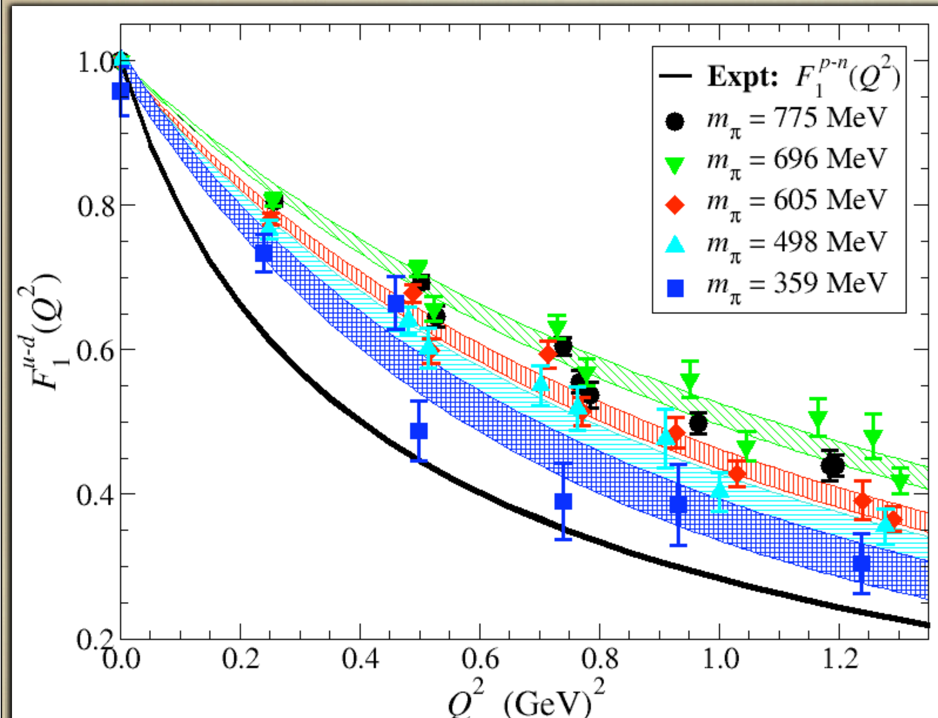
- Simplest off-diagonal matrix element

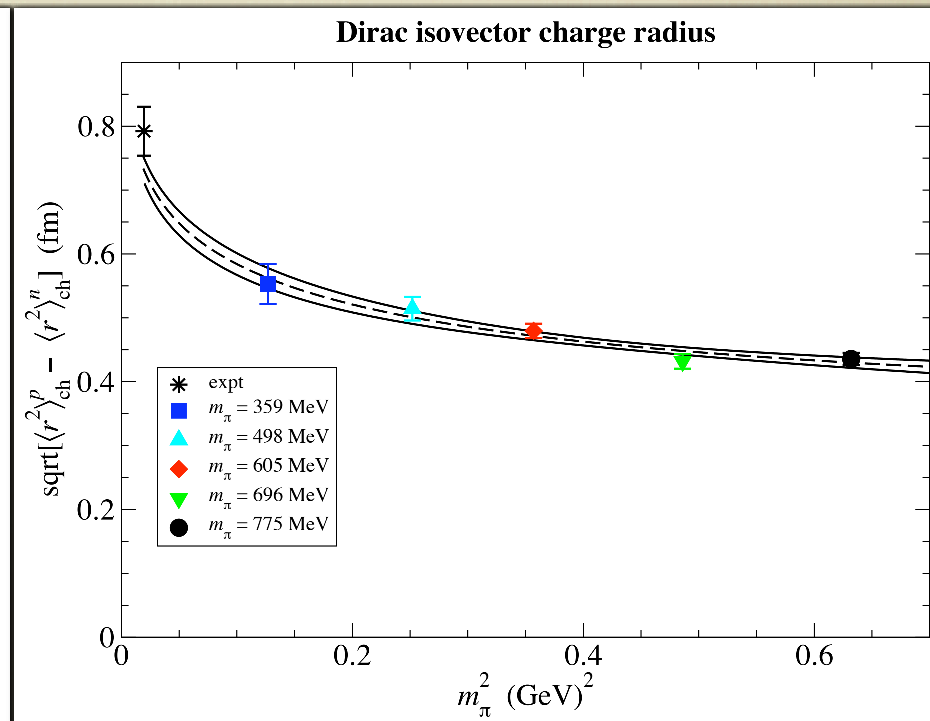
$$\langle p|\bar{\psi}\gamma^\mu\psi|p'\rangle = \bar{u}(p)[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}]u(p')$$

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

- Fourier transform of charge density if $\mathrm{L}_{\mathrm{system}} \gg \mathrm{L}_{\mathrm{wavepacket}} \gg \frac{1}{m}$
  - Pb: 5 fm >> 10^{-3} fm, Proton: 0.8 fm ~ 0.2 fm: marginal
  - For transverse Fourier transform of light cone w. f., m → p_+ ~ ∞
- Large q^2: ability of one quark to share q^2 with other constituents to remain in ground state - q^2 counting rules

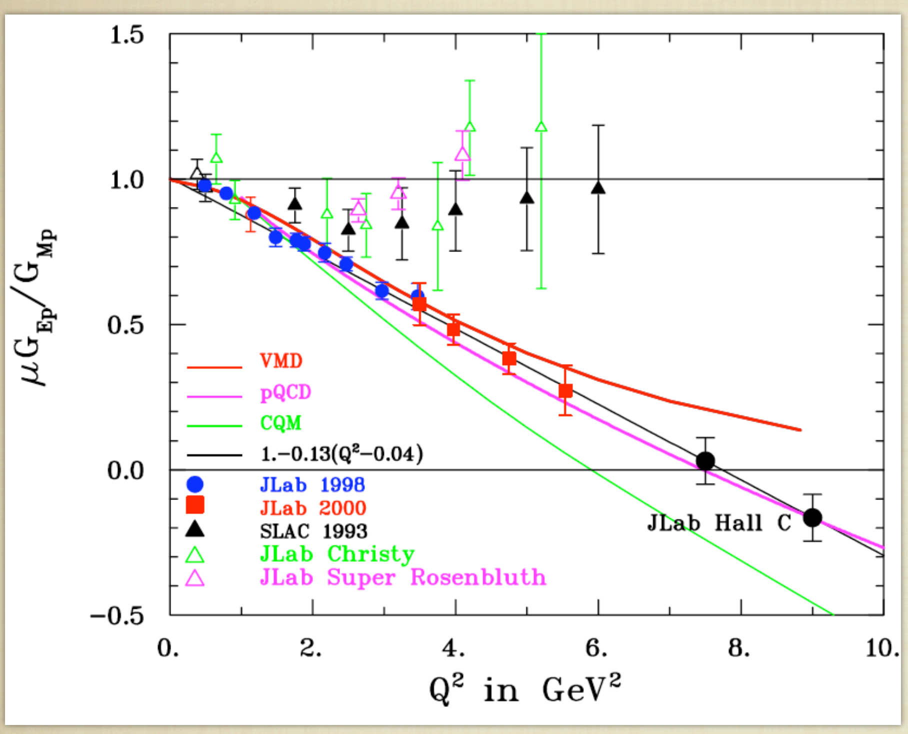
F_1 Isovector Form Factor



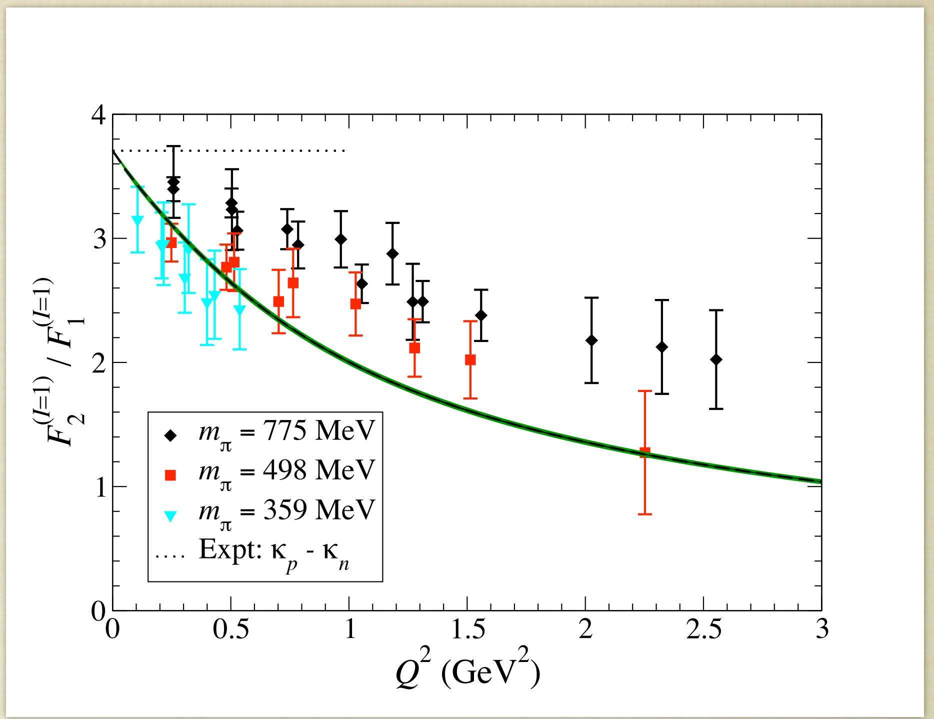


$$\langle r^2 \rangle^{u-d} = a_0 - \frac{(1+5g_A^2)}{(4\pi f_\pi)^2} \log\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)$$

Form factor ratio: F_2 / F_1



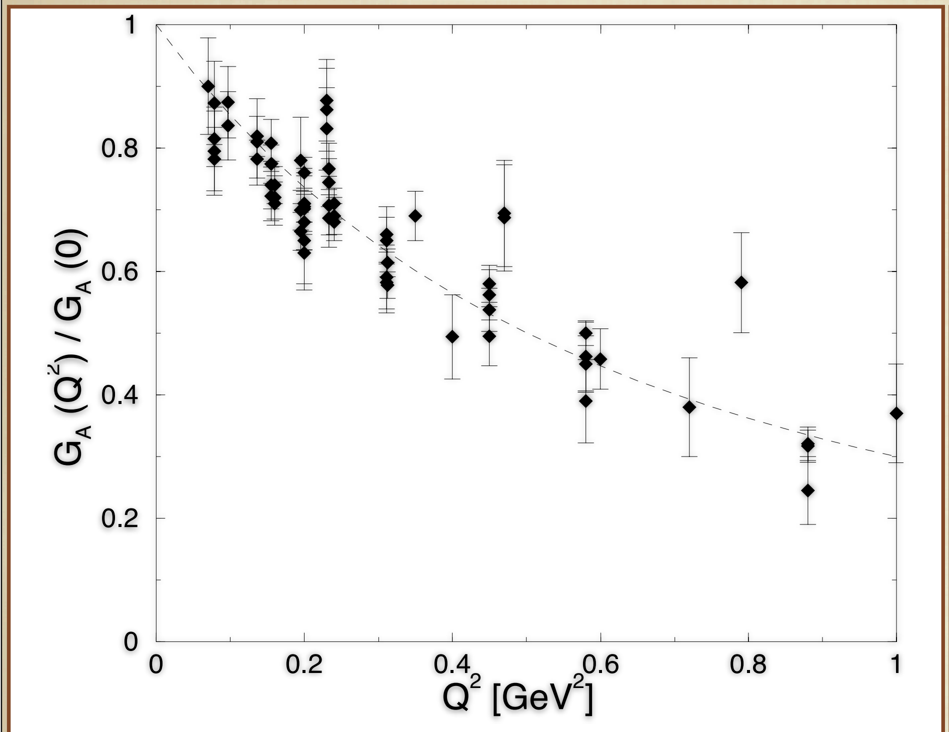
Polarization transfer at JLab

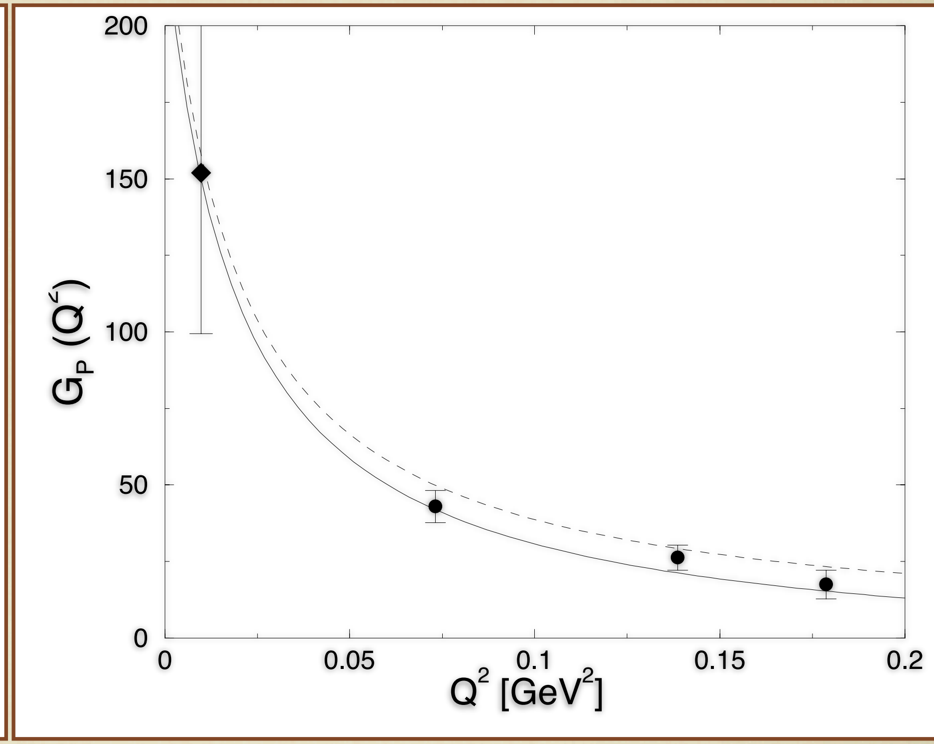


Lattice results

Polarized Nucleon Form Factors G_A and G_P

$$\langle p|\bar{\psi}\gamma^{\mu}\gamma_5\psi|p'\rangle = \bar{u}(p)[G_A(q^2)\gamma^{\mu}\gamma_5 + q^{\mu}\gamma_5 G_P(q^2) + \sigma^{\mu\nu}\gamma_5 q_{\nu} G_M(q^2)]u(p')$$





Bernard, Elouadrhiri, Meissner, J. Phys. G Nucl. Part. Phys. 2002, R1

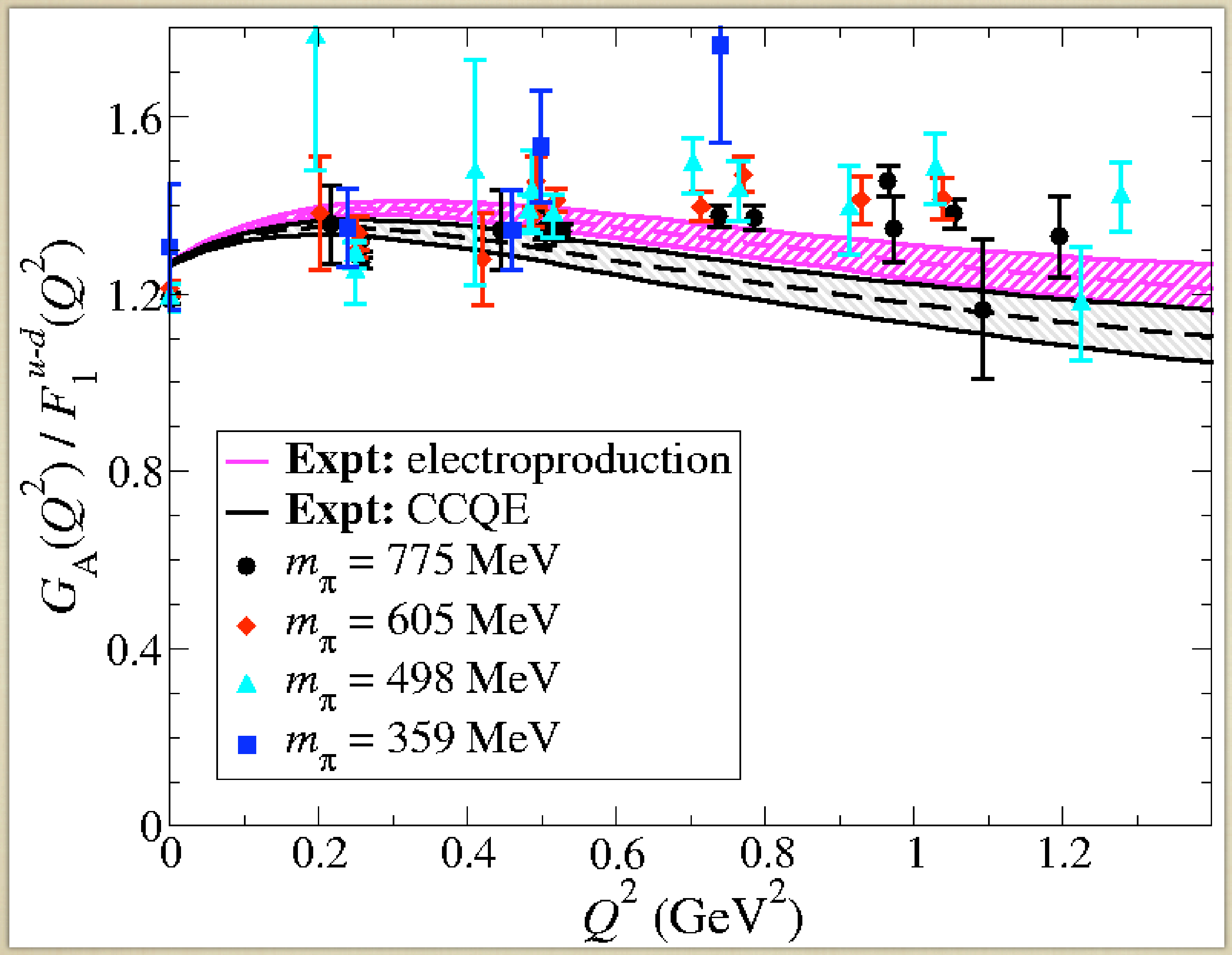
pion electroproduction ◆

$\nu_\mu\, n \rightarrow \mu^-\, p$

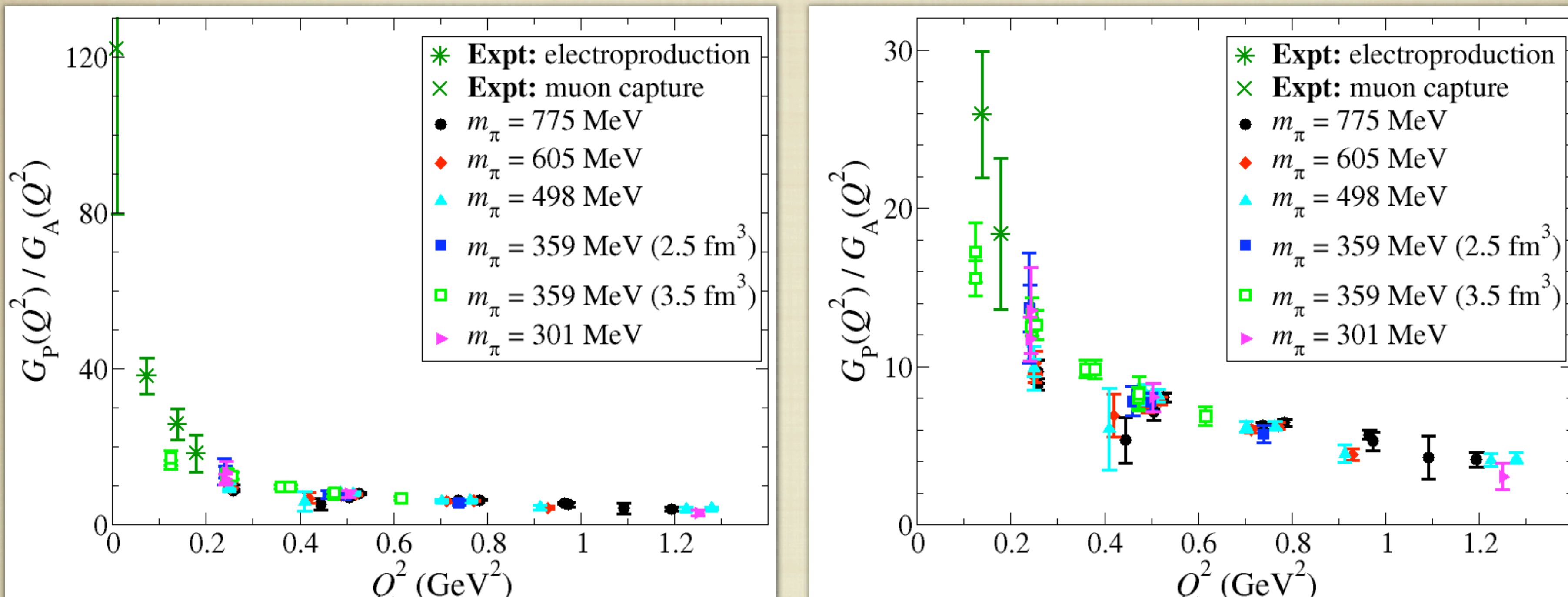
pion electroproduction ●

$\mu^-\, p \rightarrow \nu_\mu\, n$ ◆

Form factor ratio: G_A/F_1

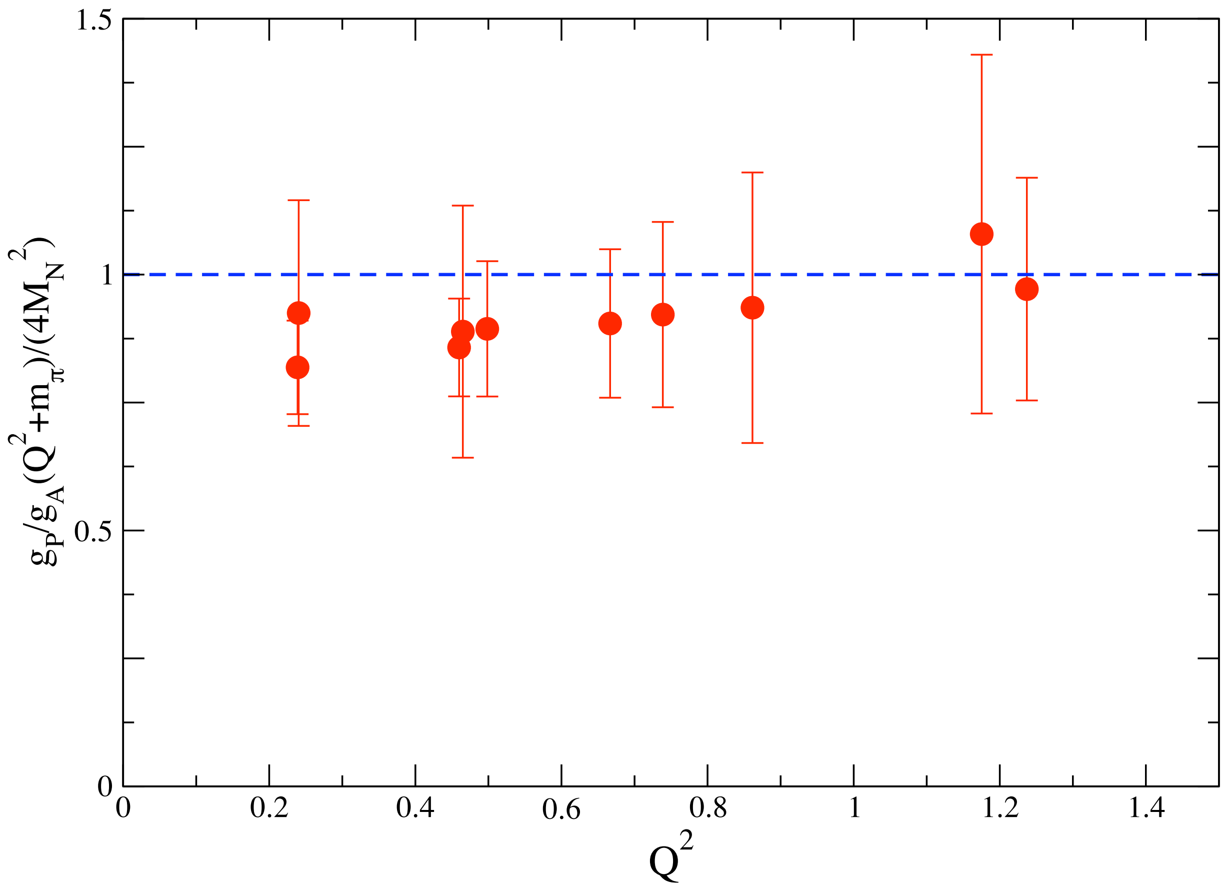


Form factor ratio: G_P/G_A



soft pion pole: $$G_P(q^2) \sim \frac{4M^2 G_A(q^2)}{q^2 - m_\pi^2}$$

Form factor ratio: G_P/G_A



Generalized Parton Distributions

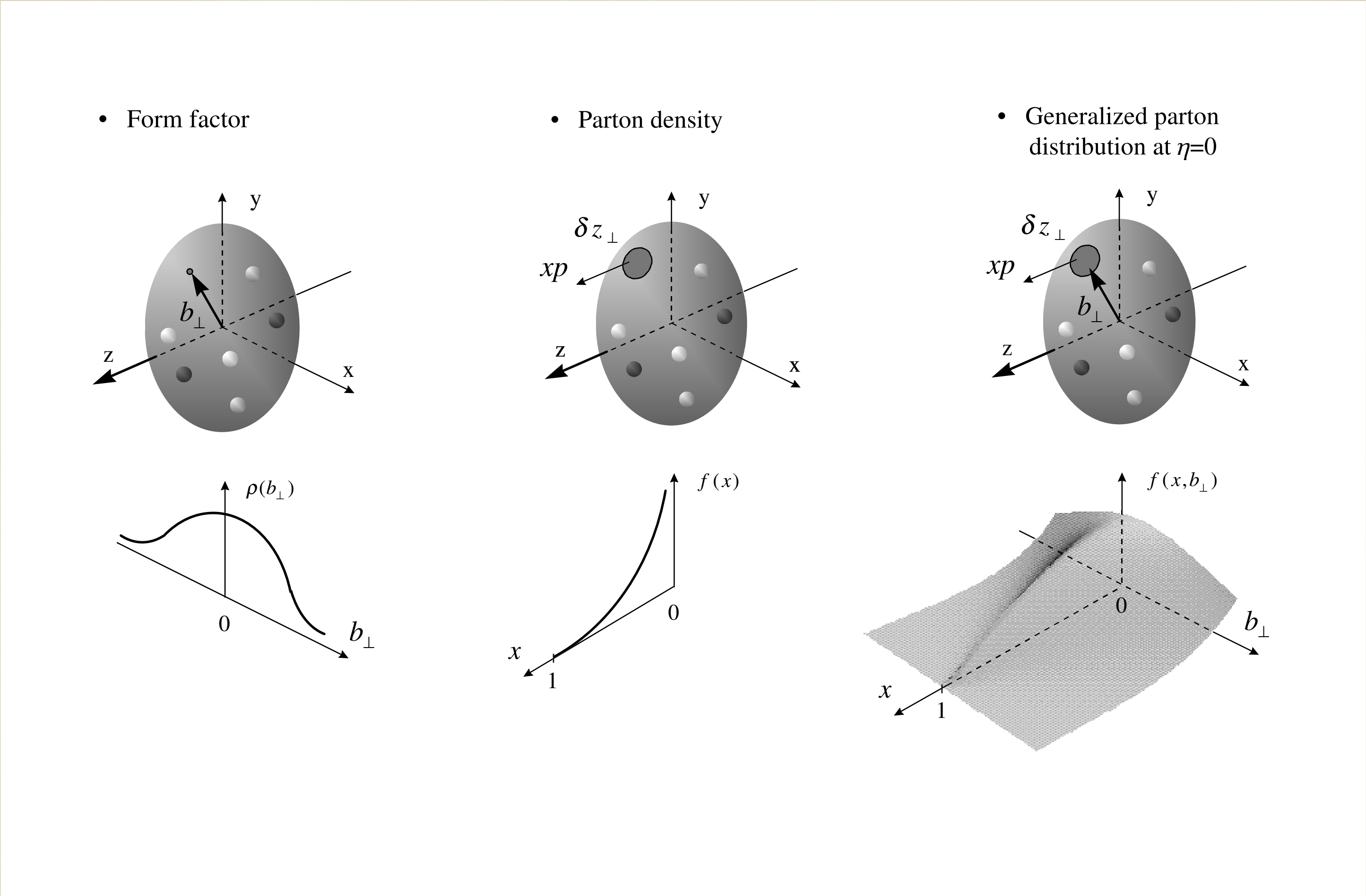


Fig. from G. Schierholz

Generalized form factors

$$\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \overline{\psi}_q\gamma^{\{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi_q$$

$$\bar{P} = \tfrac{1}{2}(P' + P)$$

$$\Delta = P' - P$$

$$t = \Delta^2$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\mu_1}|P\rangle &= \langle\langle\gamma^{\mu_1}\rangle\rangle A_{10}(t) \\
&+ \frac{i}{2m}\langle\langle\sigma^{\mu_1\alpha}\rangle\rangle\Delta_\alpha B_{10}(t)\,,
\end{aligned}$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\}}|P\rangle &= \bar{P}^{\{\mu_1}\langle\langle\gamma^{\mu_2\}}\rangle\rangle A_{20}(t) \\
&+ \frac{i}{2m}\bar{P}^{\{\mu_1}\langle\langle\sigma^{\mu_2\}\alpha}\rangle\rangle\Delta_\alpha B_{20}(t) \\
&+ \frac{1}{m}\Delta^{\{\mu_1}\Delta^{\mu_2\}}C_2(t)\,,
\end{aligned}$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\mu_3\}}|P\rangle &= \bar{P}^{\{\mu_1}\bar{P}^{\mu_2}\langle\langle\gamma^{\mu_3\}}\rangle\rangle A_{30}(t) \\
&+ \frac{i}{2m}\bar{P}^{\{\mu_1}\bar{P}^{\mu_2}\langle\langle\sigma^{\mu_3\}\alpha}\rangle\rangle\Delta_\alpha B_{30}(t) \\
&+ \Delta^{\{\mu_1}\Delta^{\mu_2}\langle\langle\gamma^{\mu_3\}}\rangle\rangle A_{32}(t) \\
&+ \frac{i}{2m}\Delta^{\{\mu_1}\Delta^{\mu_2}\langle\langle\sigma^{\mu_3\}\alpha}\rangle\rangle\Delta_\alpha B_{32}(t),
\end{aligned}$$

Limits of generalized form factors

- Moments of parton distributions $t \to 0$

$$A_{n0} = \int dx x^{n-1} q(x)$$

- Electromagnetic form factors

$$A_{10} = F_1(t), \quad B_{10} = F_2(t)$$

- Total quark angular momentum

$$J_q = \frac{1}{2}[A(0)_{20} + B(0)_{20}]$$

Sum Rules

- Momentum sum rule

$$1 = A_{20,q}(0) + A_{20,g}(0) = \langle x \rangle_q + \langle x \rangle_g$$

- Nucleon spin sum rule

$$\begin{aligned} \frac{1}{2} &= \frac{1}{2}\big(A_{20,q}(0) + A_{20,g}(0) + B_{20,q}(0) + B_{20,g}(0)\big) \\ &= \frac{1}{2}\Delta\Sigma_q + L_q + J_g \end{aligned}$$

- Vanishing of anomalous gravitomagnetic moment

$$0 = B_{20,q}(0) + B_{20,g}(0)$$

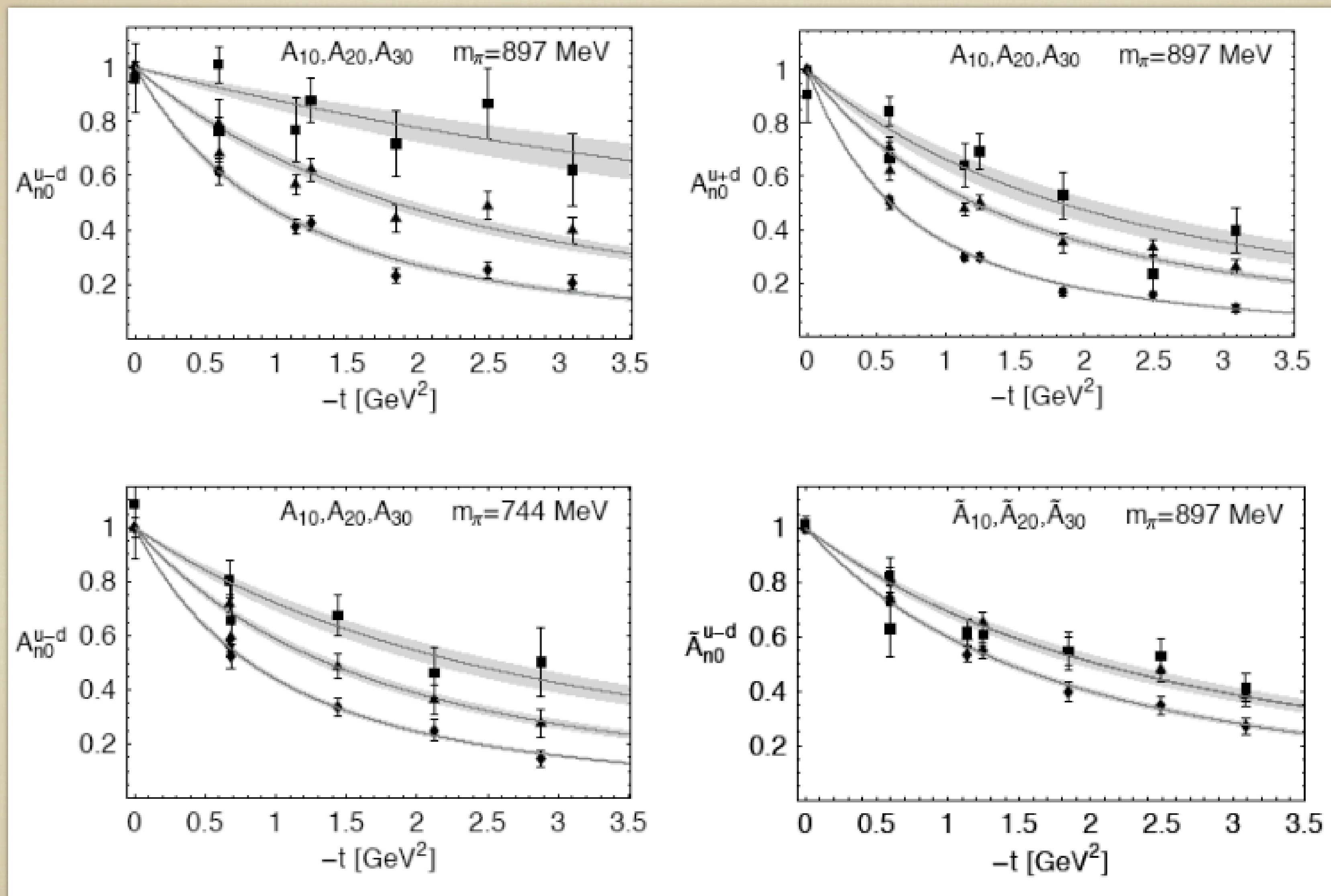
Transverse structure of nucleon

$H(x, 0, -\Delta^2_\perp)$ is transverse Fourier transform of light cone quark distribution $q(x,r_\perp)$ at momentum fraction x

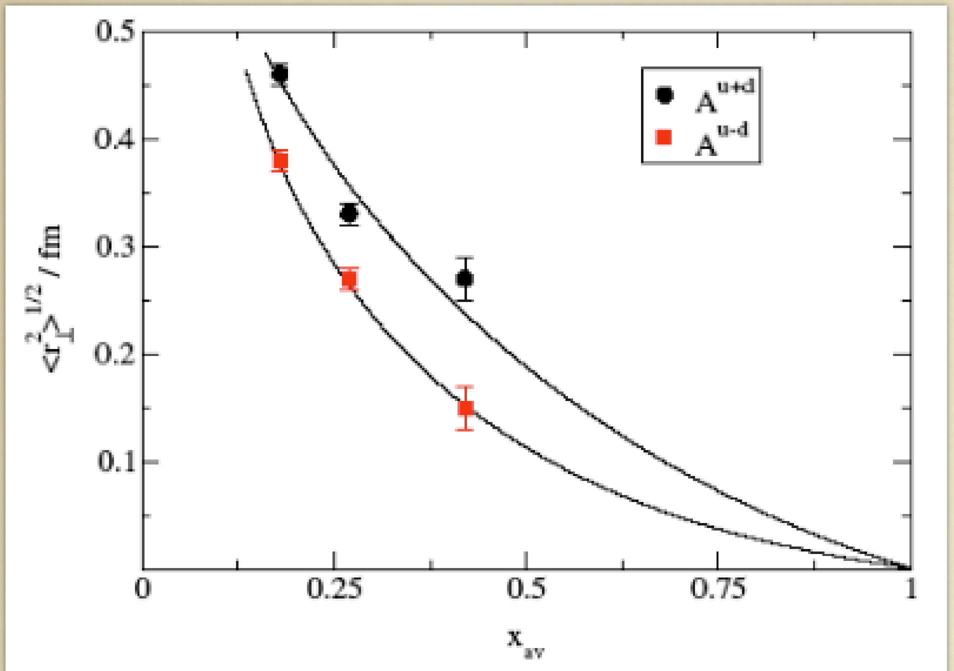
$$
\begin{aligned}
q(x, r_\perp) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta^2_\perp)\, e^{-ir_\perp\Delta_\perp} \\
\int dx x^{n-1} q(x, r_\perp) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} A(-\Delta^2_\perp)\, e^{-ir_\perp\Delta_\perp}
\end{aligned}
$$

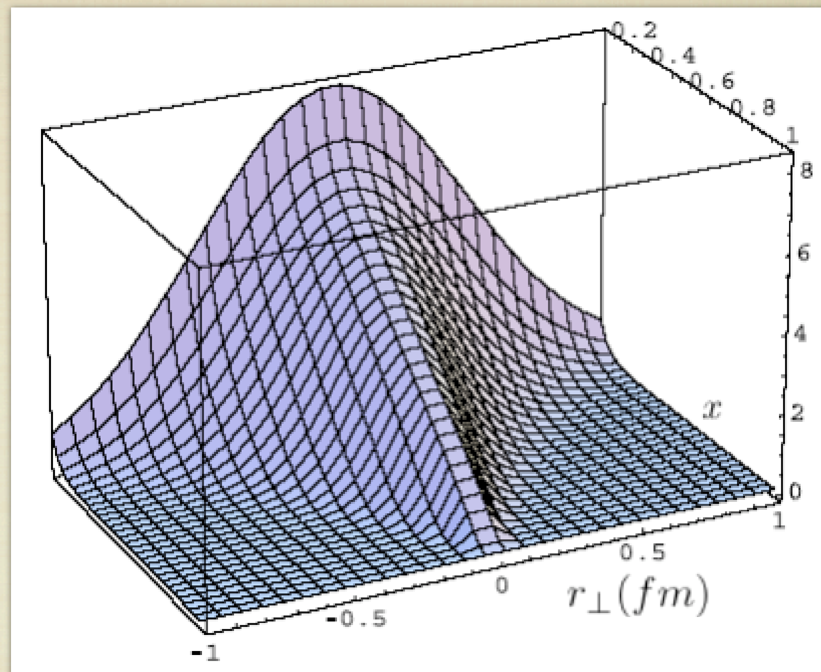
- x → 1: Single Fock space component ⇒ slope → 0
- x ≠ 1: Transverse structure ⇒ slope steeper

Generalized form factors from lattice



Transverse size of light-cone wave function

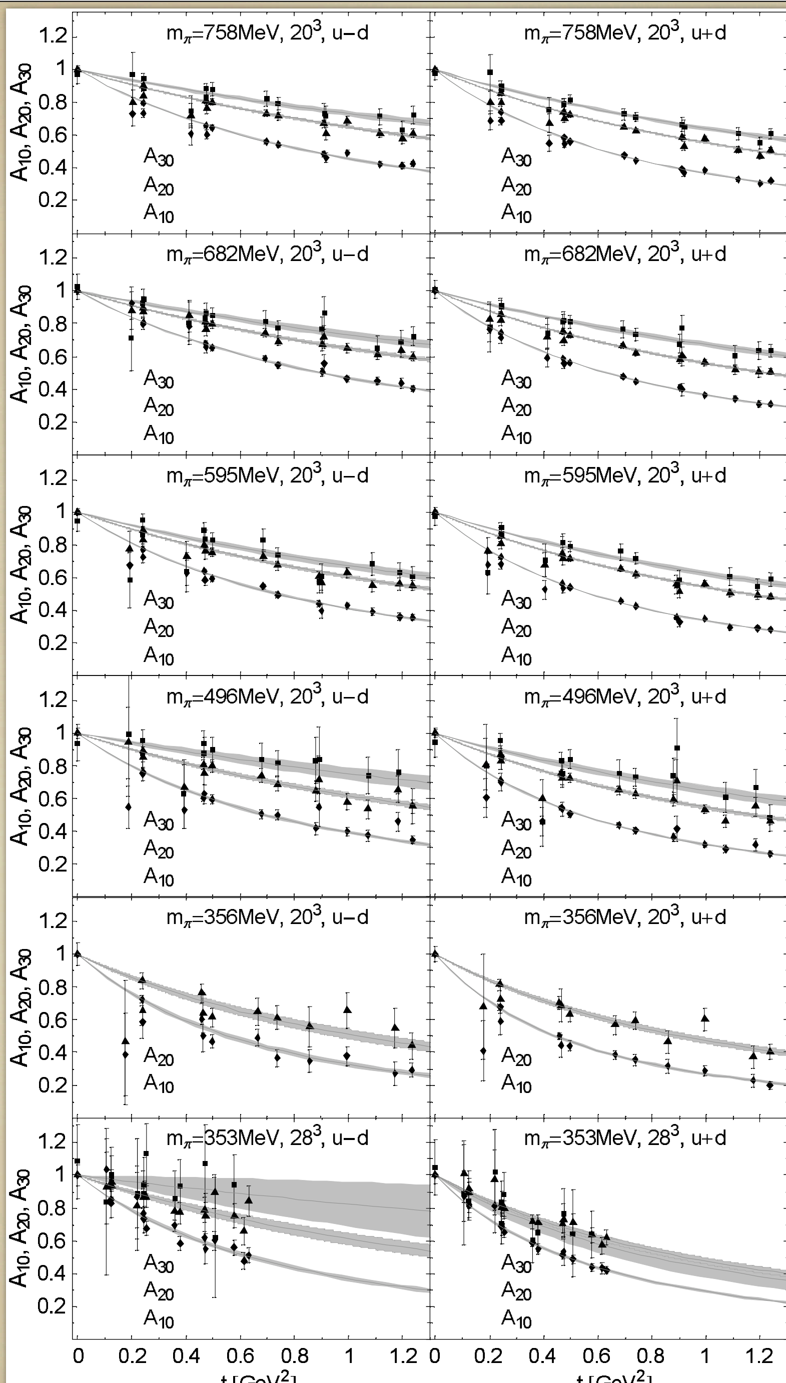


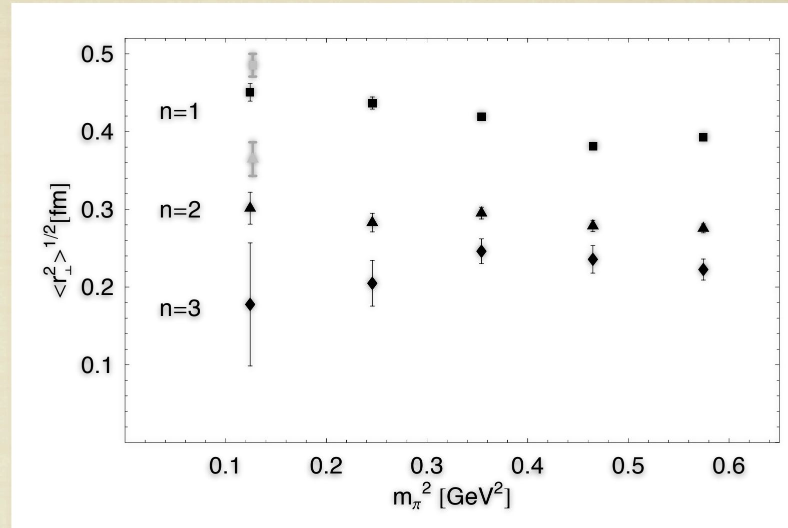


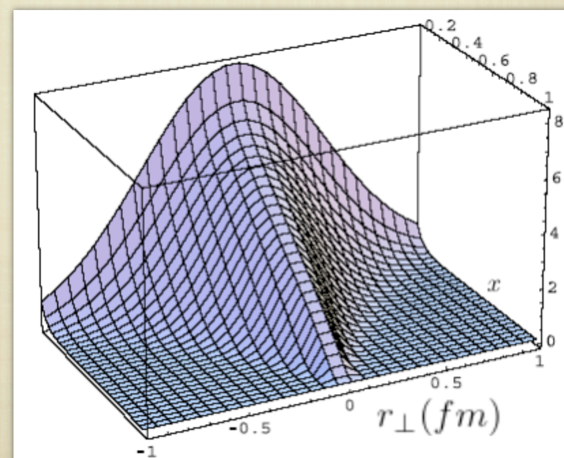
$$x_{\rm av}^n = \frac{\int d^2 r_\perp \int dx\, x \cdot x^{n-1} q(x, \vec{r}_\perp)}{\int d^2 r_\perp \int dx x^{n-1} q(x, \vec{r}_\perp)}$$

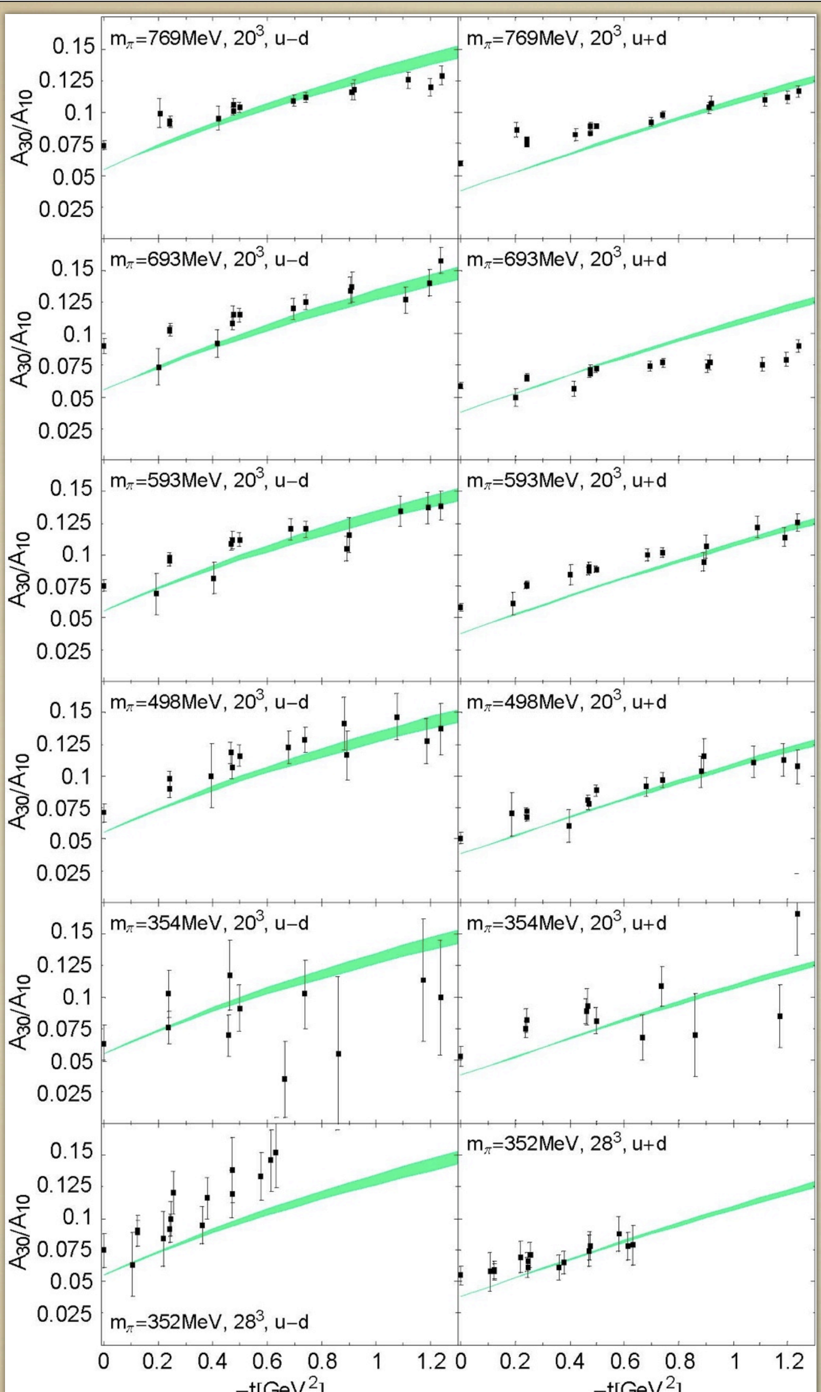
$q(x, \vec{r}_\perp)$ **model** (Burkardt hep-ph/0207047)

Generalized form factors A_{10}, A_{20}, A_{30}



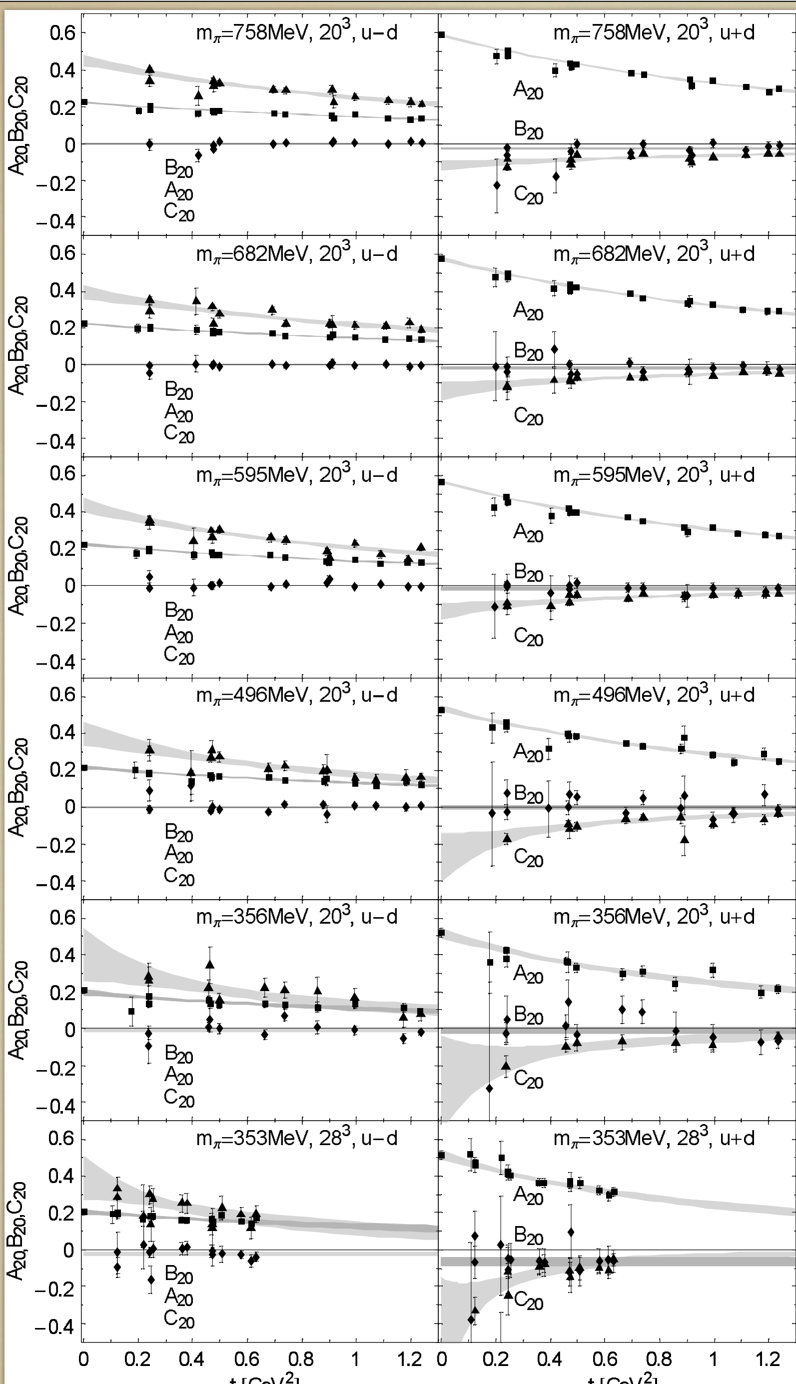






Generalized form factor ratios A_{30} / A_{10}

GPD parameterization:
Nucleon form factors,
CTEQ parton distributions,
Regge behavior,
Ansatz
Diehl, Feldmann, Jakob, Kroll EPJC 2005



First x moments:

$$A_{20}, B_{20}, C_{20}$$

Consistent with large N behavior [Goeke et. al.]

$$|A_{20}^{u+d}| > |A_{20}^{u-d}|$$
$$|B_{20}^{u-d}| > |B_{20}^{u+d}|$$
$$|C_{20}^{u+d}| > |C_{20}^{u-d}|$$

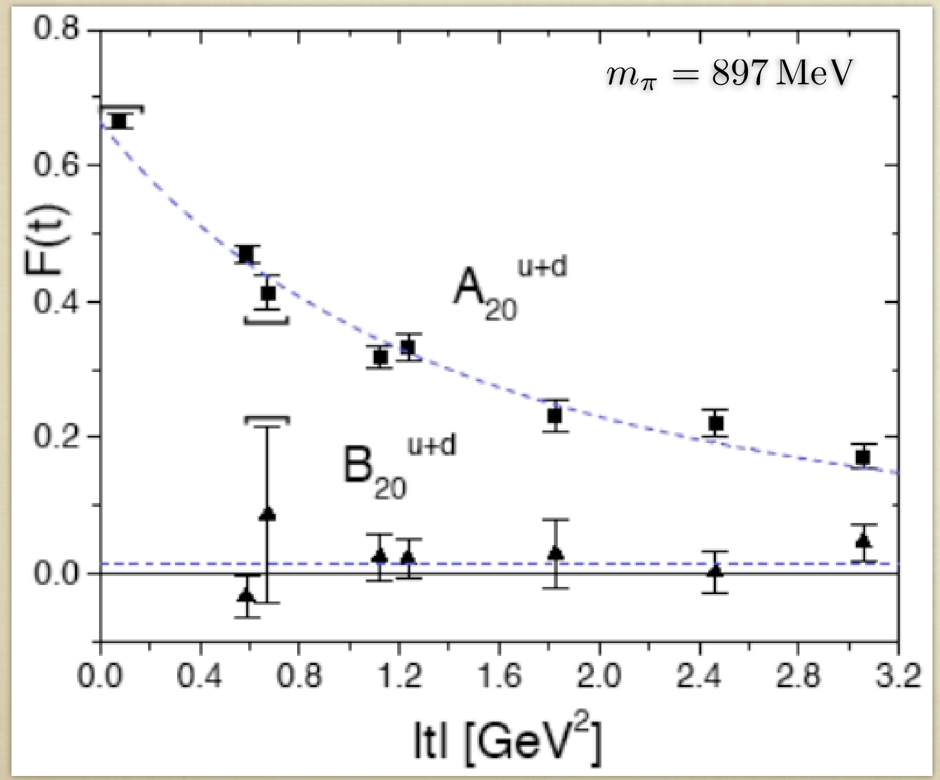
Origin of nucleon spin

"Spin crisis" - only ~ 30% arises from quark spins

quark spin contribution $\frac{1}{2}\Delta\Sigma = \frac{1}{2}\langle 1\rangle_{\Delta u+\Delta d} \quad \sim \frac{1}{2}0.682(18)$

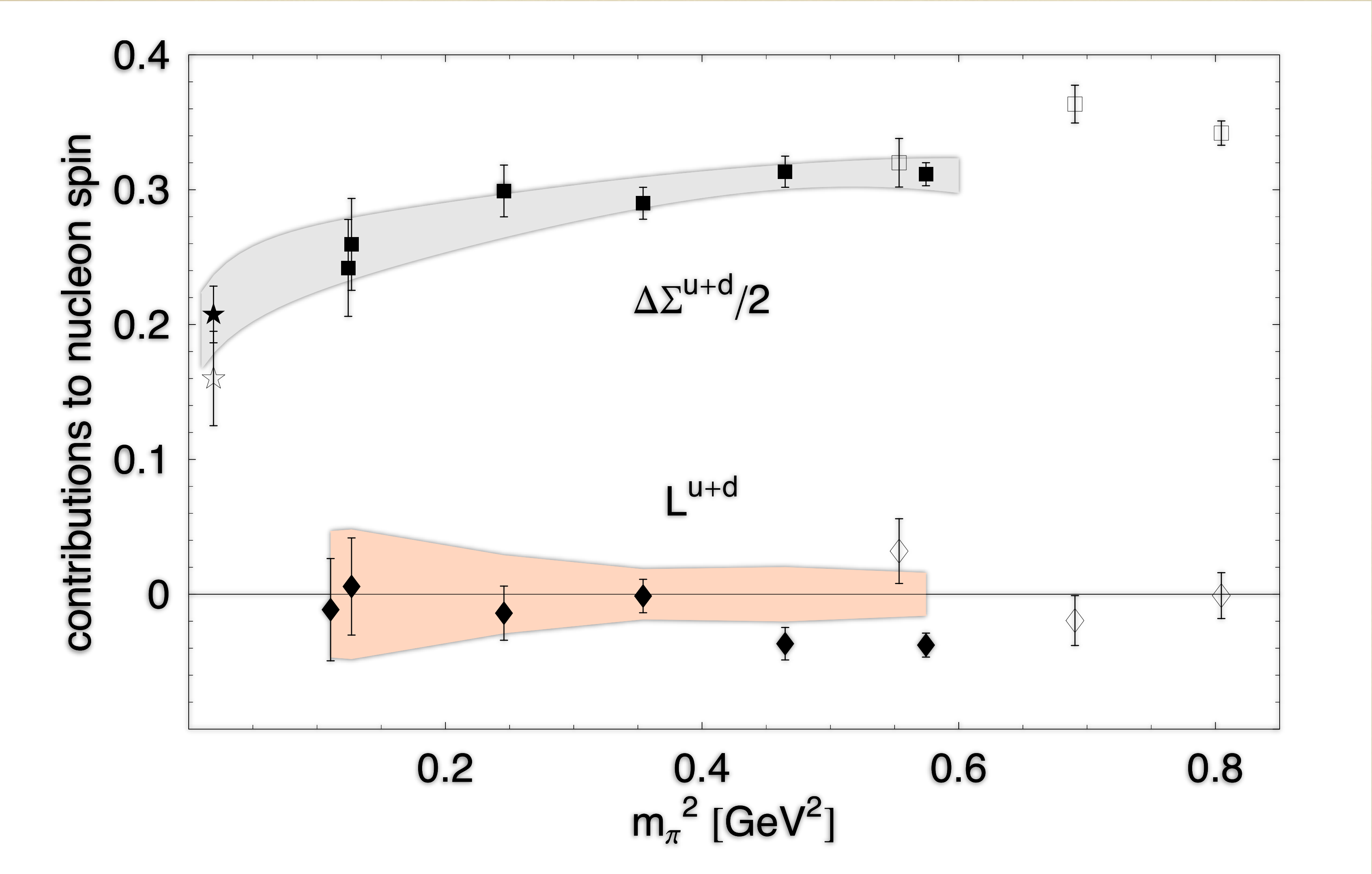
total quark contribution (spin plus orbital)

$$J_q = \frac{1}{2}[A_{20}^{u+d}(0) + B_{20}^{u+d}(0)] = \frac{1}{2}[\langle x\rangle_{u+d} + B_{20}^{u+d}(0)] \quad \sim \frac{1}{2}0.675(7)$$



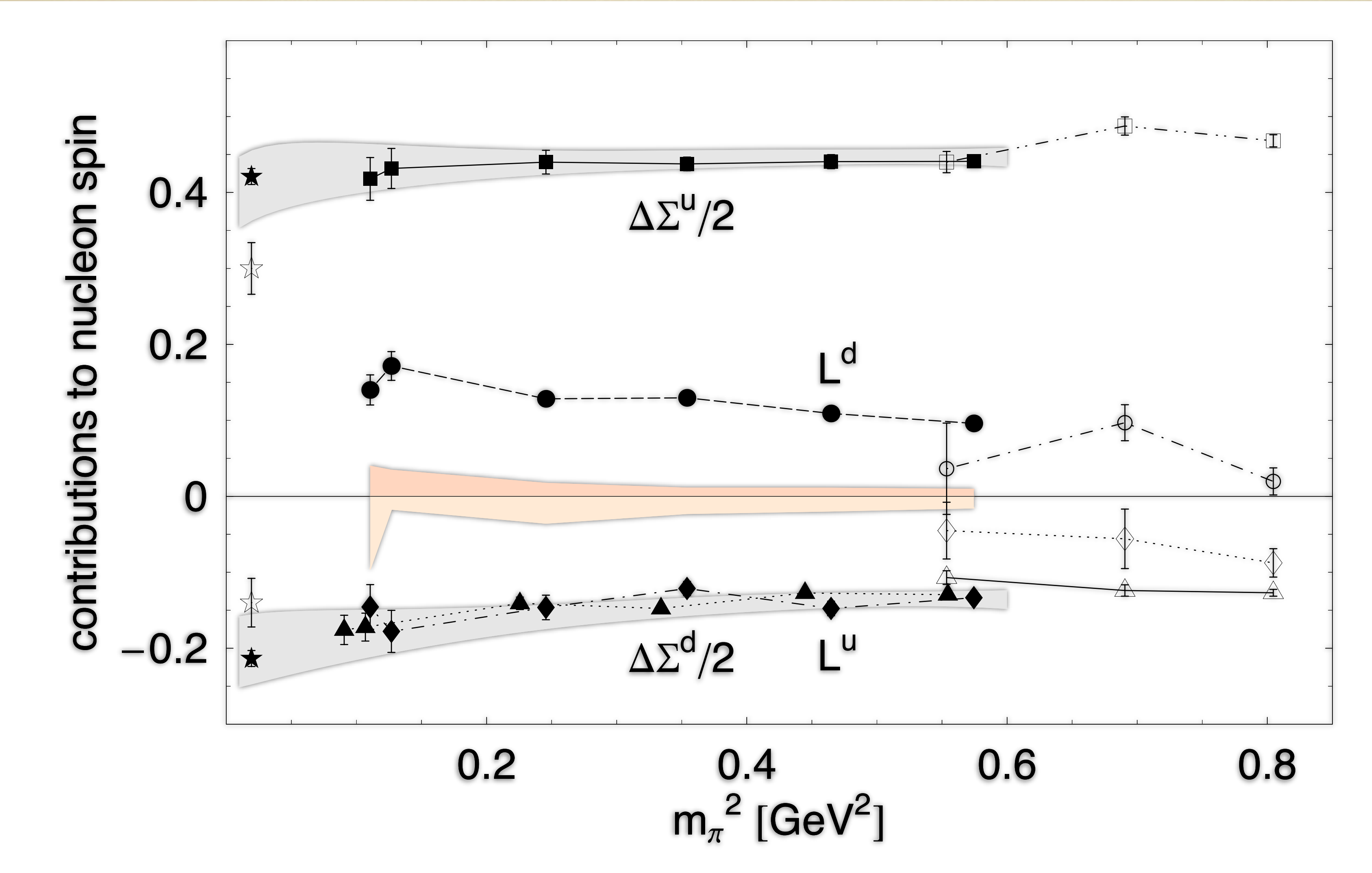
Spin Inventory
68% quark spin
0% quark orbital
32% gluons

Nucleon spin decomposition



LHPC hep-lat/0705.4295

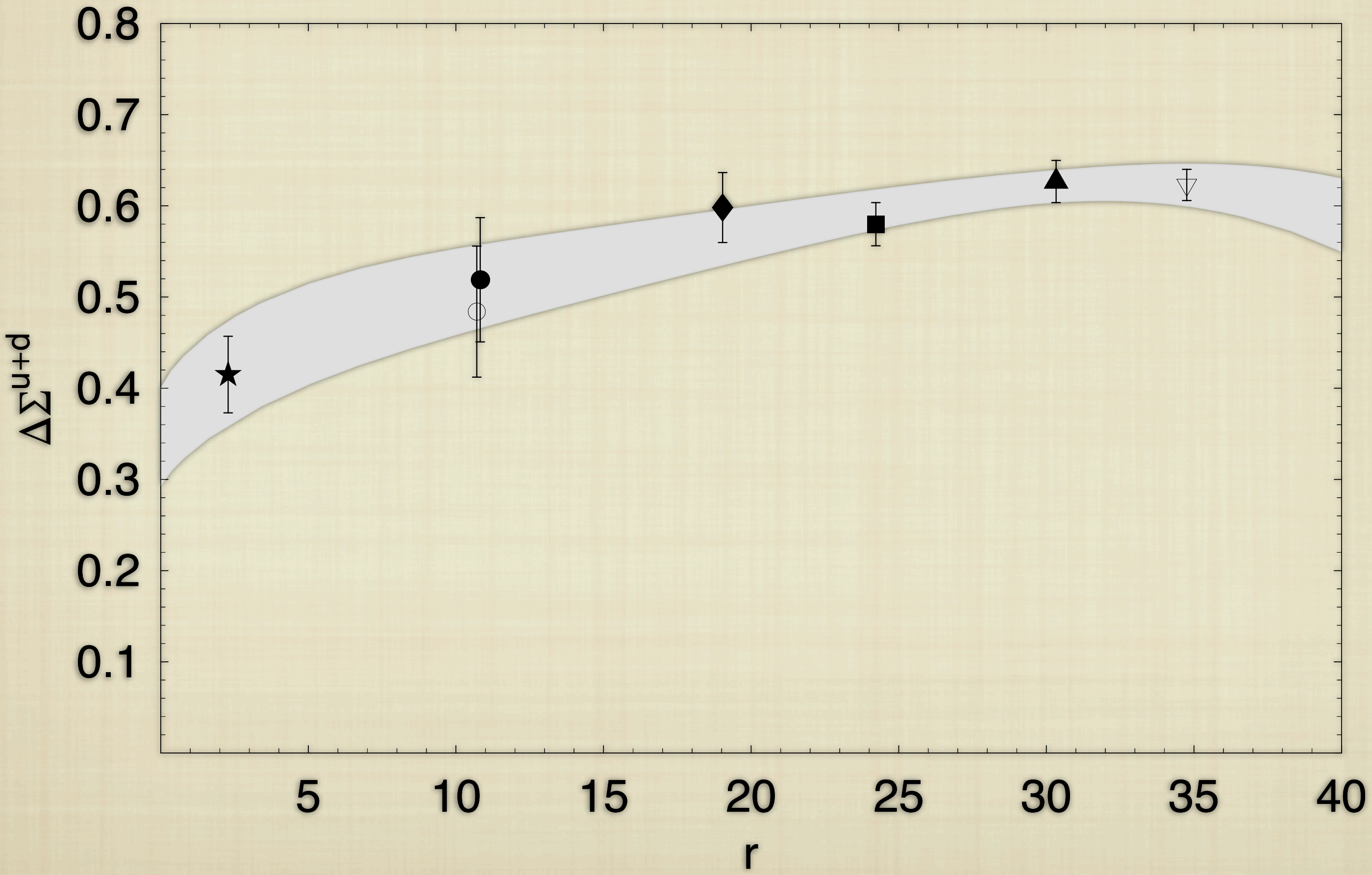
Nucleon spin decomposition



LHPC hep-lat/0705.4295

Quark spin contribution to Nucleon Spin

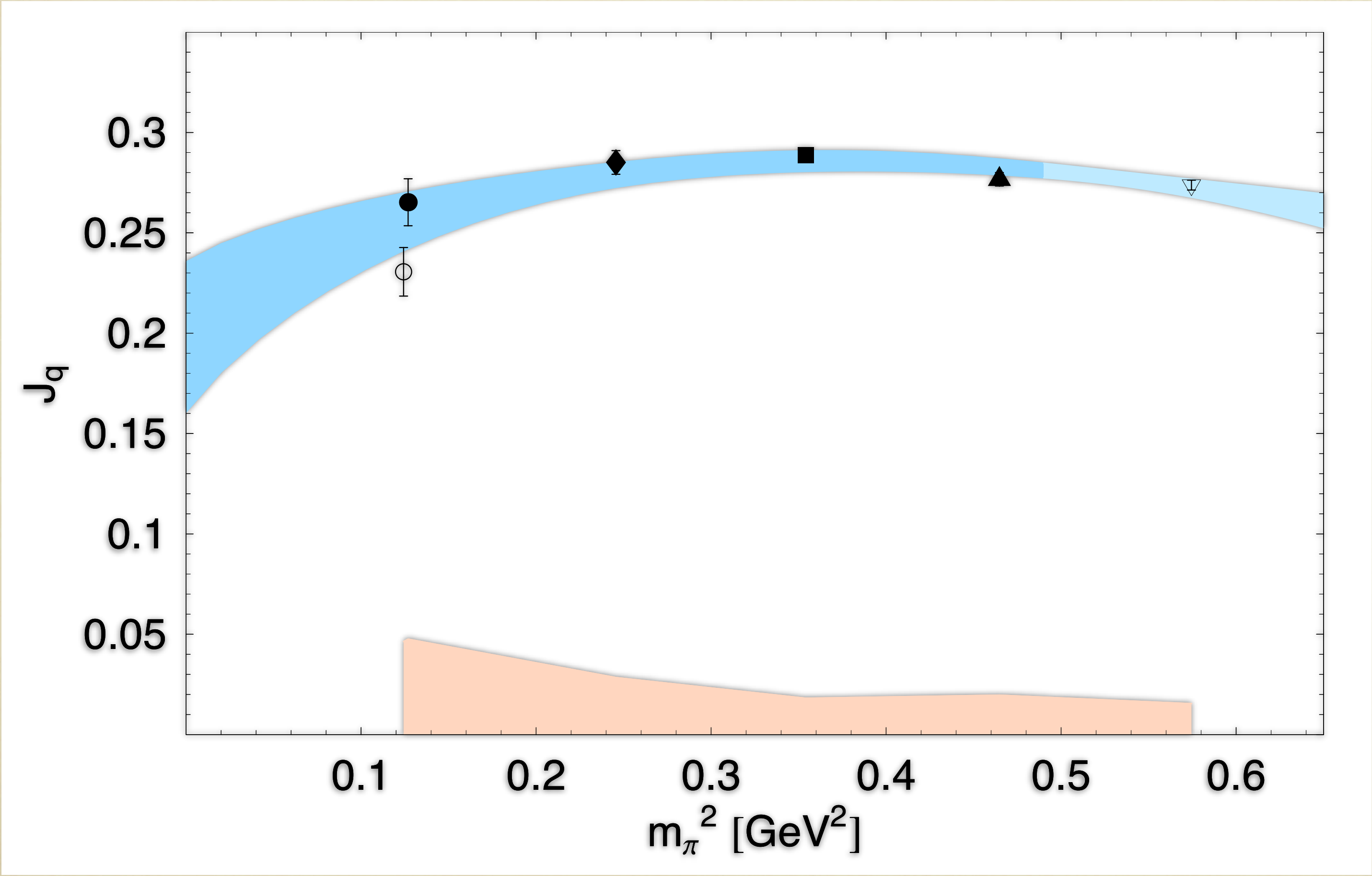
$$\Delta\Sigma = \langle 1 \rangle_{\Delta u} + \langle 1 \rangle_{\Delta d}$$



Chiral extrapolation of $J_q = \frac{1}{2}\left(A_{20}^{u+d}(0) + B_{20}^{u+d}(0)\right)$

ChPT including Delta (Chen and Ji)

$$J_q(m_\pi;\Delta) = J_q(m_\pi) - \frac{1}{2}\left(\frac{9}{2}b_{qN} + 3a_{q\pi} - \frac{15}{2}b_{q\Delta}\right)\frac{8g_{\pi N\Delta}^2}{9(4\pi f_\pi)^2}(m_\pi^2 - 2\Delta^2)\ln\left(\frac{m_\pi^2}{\Lambda_\chi^2}\right) + 2\Delta\sqrt{\Delta^2 - m_\pi^2}\ln\left(\frac{\Delta - \sqrt{\Delta^2 - m_\pi^2}}{\Delta + \sqrt{\Delta^2 - m_\pi^2}}\right)$$



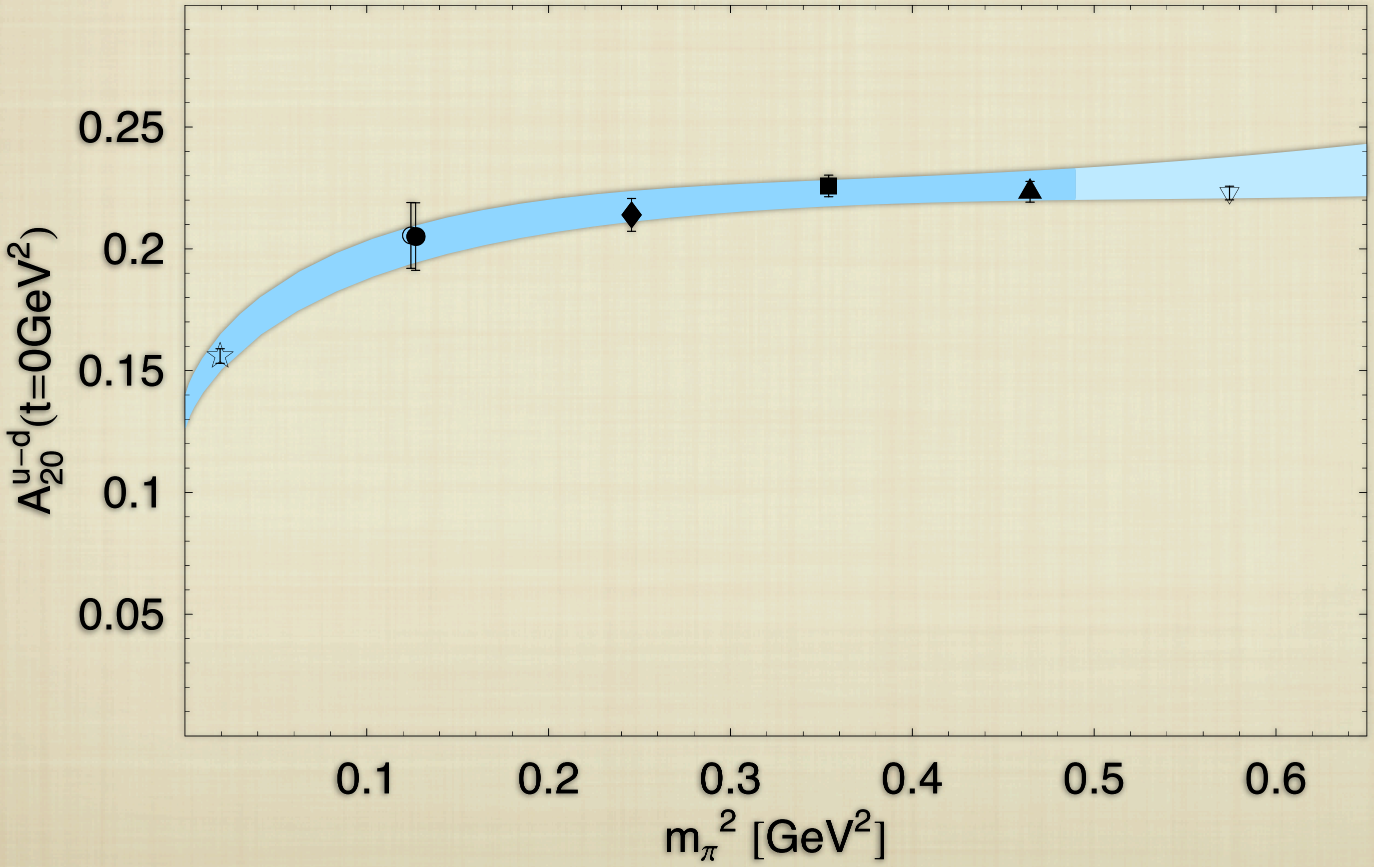
Summary of Nucleon Spin

- HERMES - Fraction of spin from quark spin
  - $\Sigma^u = .84 \pm .01$ $\Sigma^d = -0.43 \pm .01$ $\Sigma^{u+d} = 0.42 \pm .02$
- Lattice - Connected Diagrams
  - $\Sigma^u \sim .8$ $\Sigma^d \sim -.4$ $\Sigma^{u+d} = 0.41 \pm .06$
  - $2L^u \sim .3$ $2L^u \sim -.3$ $2L^{u+d} \sim 0$
  - $2J^{u+d} = 0.42 \pm .06$

Chiral extrapolation of $\langle x \rangle_q^{u-d} = A_{20}^{u-d}(t=0)$

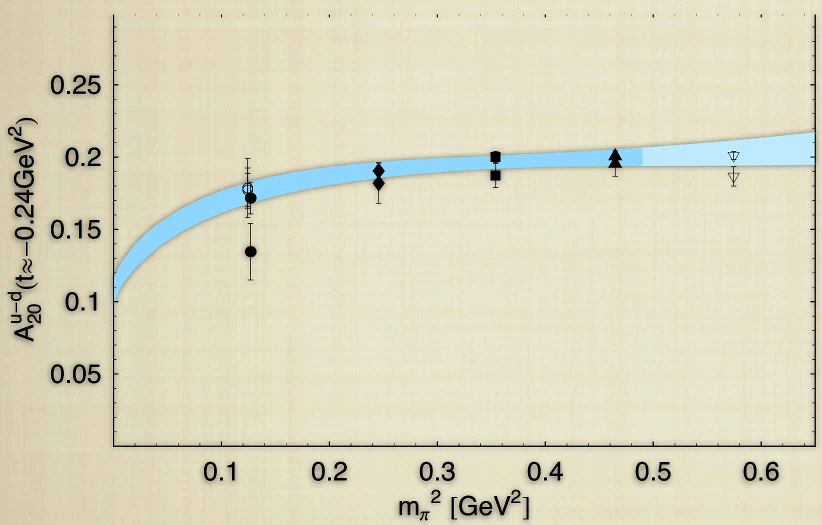
Chiral extrapolation O(p²) covariant ChPT (Dorati, Hemmert, et. al.)

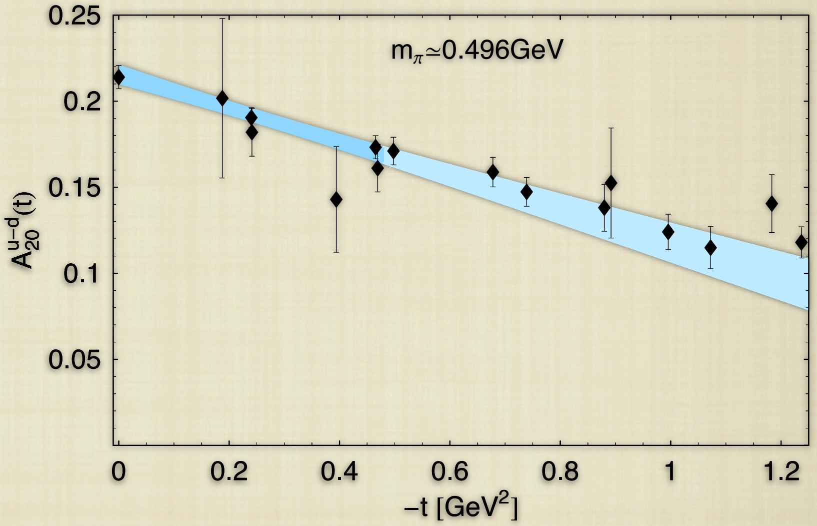
$$A_{20}^{u-d}(t, m_\pi) = A_{20}^{0,u-d}\left(f_A^{u-d}(m_\pi) + \frac{g_A^2}{192\pi^2 f_\pi^2} h_A(t, m_\pi)\right) + \widetilde{A}_{20}^{0,u-d} j_A^{u-d}(m_\pi) + A_{20}^{m_\pi,u-d} m_\pi^2 + A_{20}^{t,u-d} t$$



Chiral extrapolation

Chiral extrapolation $O(p^2)$ covariant BChPT
t and m_π dependence



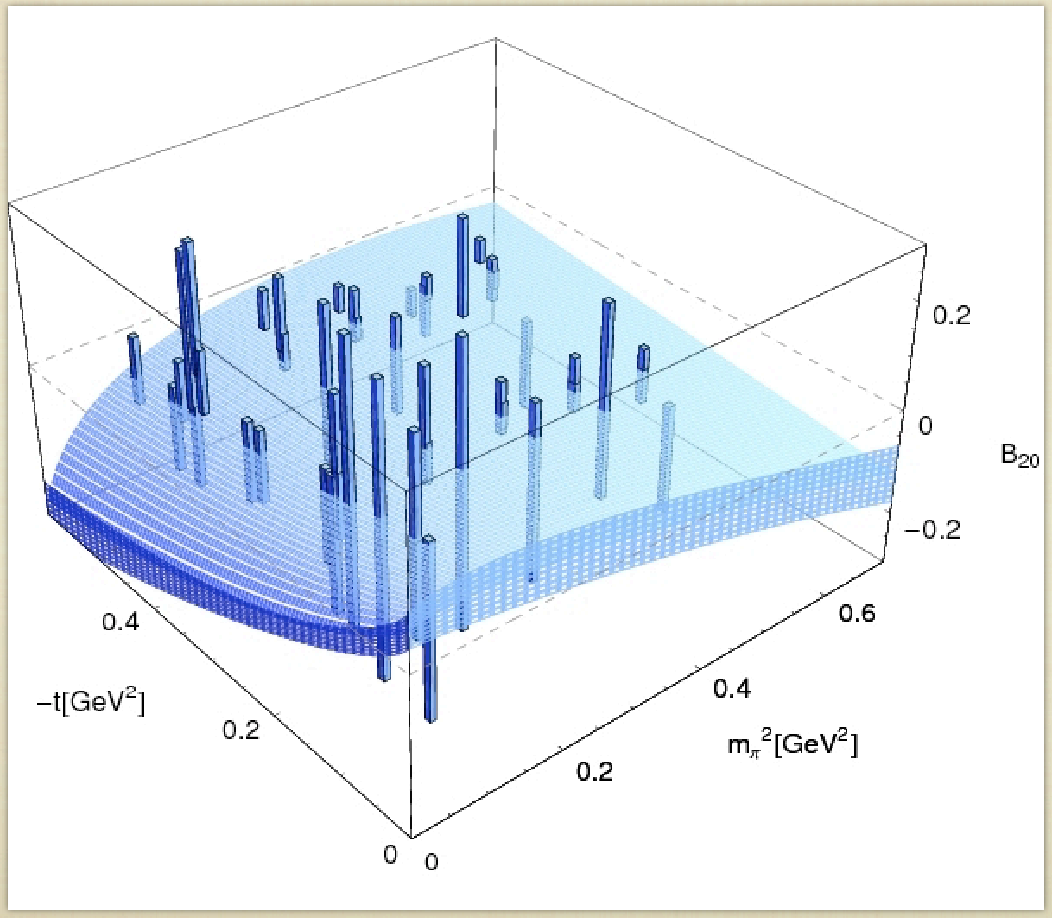


Chiral Extrapolation of $B_{20}^{u+d}(t, m_\pi)$

Chiral extrapolation O(p⁴) relativistic ChPT O(p⁵) corrections

Note: connected diagrams only (Dorati, Hemmert, et. al.)

$$B_{20}^{u-d}(t, m_\pi) = \frac{m_N(m_\pi)}{m_N}\left\{B_{20}^{0,u-d} + A_{20}^{0,u-d} g_B(t, m_\pi) + \delta_B^t\, t + \delta_B^{m_\pi}\, m_\pi^2\right\}$$

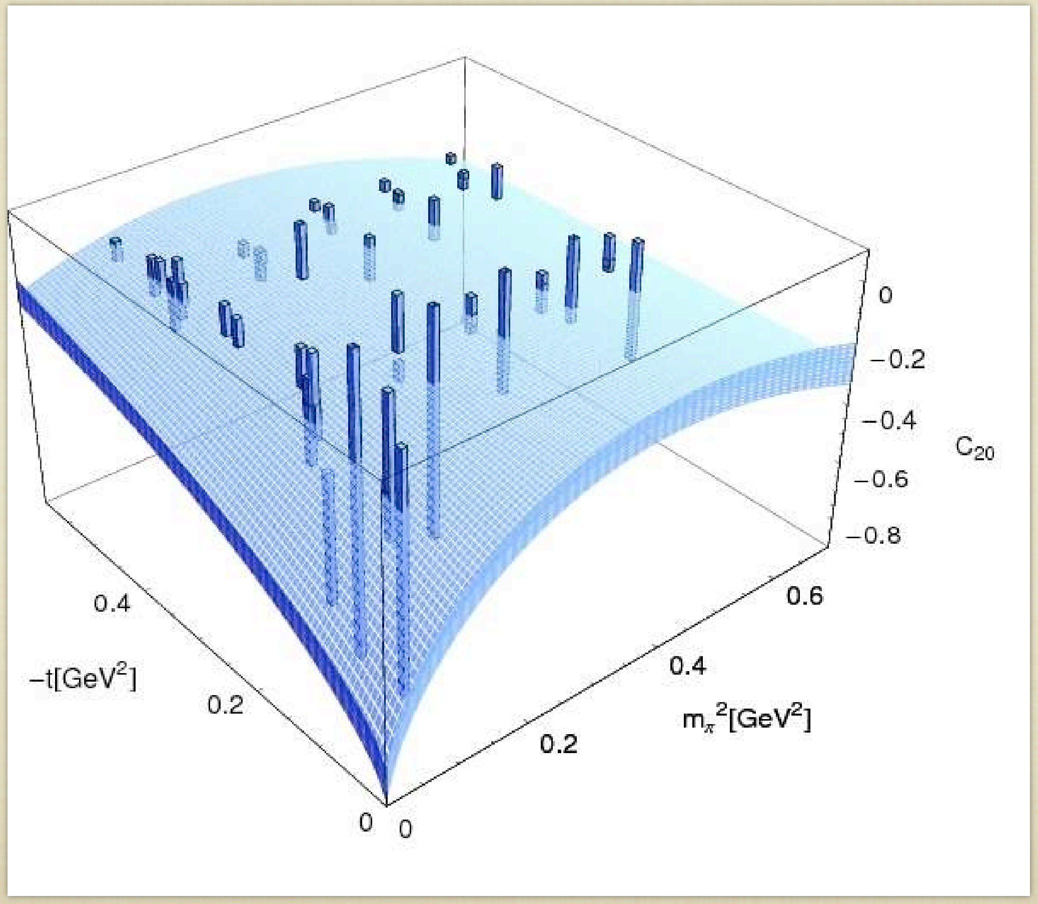


Chiral Extrapolation of $C_{20}^{u+d}(t, m_\pi)$

Chiral extrapolation O(p[4]) relativistic ChPT O(p[5]) corrections

Note: connected diagrams only (Dorati, Hemmert, et. al.)

$$C_{20}^{u-d}(t, m_\pi) = \frac{m_N(m_\pi)}{m_N}\left\{C_{20}^{0,u-d} + A_{20}^{0,u-d} g_C(t, m_\pi) + \delta_C^t\, t + \delta_C^{m_\pi}\, m_\pi^2\right\}$$



Gluon contributions to the pion mass and light cone momentum fraction

Energy-momentum tensor

Harvey Meyer and J.N. arXiv 0707.3225

$$
\begin{aligned}
T_{\mu\nu} &\equiv \overline{T}^{\rm g}_{\mu\nu} + \overline{T}^{\rm f}_{\mu\nu} + \frac{1}{4}\delta_{\mu\nu}(S^{\rm g} + S^{\rm f}), \\
\overline{T}^{\rm g}_{\mu\nu} &= \tfrac{1}{4}\delta_{\mu\nu}F^a_{\rho\sigma}F^a_{\rho\sigma} - F^a_{\mu\alpha}F^a_{\nu\alpha}, \\
\overline{T}^{\rm f}_{\mu\nu} &= \frac{1}{4}\sum_f \bar{\psi}_f \overleftrightarrow{D}_\mu \gamma_\nu \psi_f + \bar{\psi}_f \overleftrightarrow{D}_\nu \gamma_\mu \psi_f - \tfrac{1}{2}\delta_{\mu\nu}\bar{\psi}_f \overleftrightarrow{D}_\rho \gamma_\rho \psi_f, \\
S^{\rm g} &= \beta(g)/(2g)\ F^a_{\rho\sigma}F^a_{\rho\sigma}, \quad S^{\rm f} = [1+\gamma_m(g)]\sum_f \bar{\psi}_f m \psi_f
\end{aligned}
$$

For on shell particle

$$
\begin{aligned}
\langle \Psi, \mathbf{p} | \int d^3\mathbf{z}\, \overline{T}^{\rm f,g}_{00}(z)\, | \Psi, \mathbf{p} \rangle &= [E_{\mathbf{p}} - \frac{1}{4}M^2/E_{\mathbf{p}}]\ \langle x \rangle_{\rm f,g}, \\
\langle \Psi, \mathbf{p} | \int d^3\mathbf{z}\, S^{\rm f,g}(z)\, | \Psi, \mathbf{p} \rangle &= (M^2/E_{\mathbf{p}})\ b_{\rm f,g}, \\
\langle x \rangle_{\rm f} + \langle x \rangle_{\rm g} &= b_{\rm f} + b_{\rm g} = 1,
\end{aligned}
$$

In infinite momentum frame $\langle x \rangle_{\rm g}$= momentum fraction

In rest frame, $\overline{T}^{\rm g}_{00}$ contributes $\frac{3}{4}\langle x \rangle_{\rm g} M$ to mass

$S^{\rm g}$ contributes $\frac{1}{4} b_{\rm g} M$ to mass (trace anomaly)

Evaluation of $\overline{T}^{\rm g}_{00} = \frac{1}{2}(-\mathbf{E}^a \cdot \mathbf{E}^a + \mathbf{B}^a \cdot \mathbf{B}^a)$

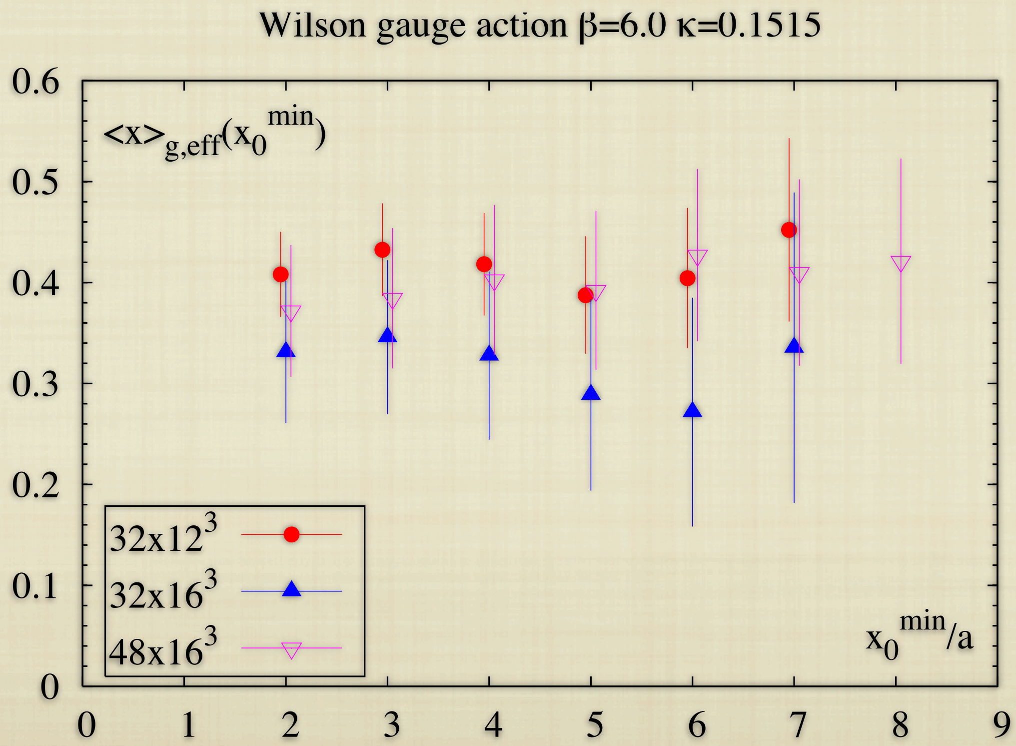
- Notoriously difficult: 5000 configurations - no signal
- Improved operator $E^2 - B^2$
- Evaluate with plaquett or clover
- Use bare or HYP smeared links
- Compare variance of entropy density at 1.26 T_C
- Normalize operator by ratio to known bare plaquette

		relative variance		normalization	
		bare	HYP	bare	HYP
$\overline{T}_{00}$	plaq.	26.4(71)	0.6518(43)	1	0.5489(68)
	clover	3.85(11)	0.3049(41)	2.184(67)	0.613(20)
S	plaq.	2.64 (12)	0.474(13)	1	0.9951(77)
	clover	1.180(39)	0.2975(72)	4.062(30)	1.410(13)

Calculation of $\langle x \rangle_{\rm g}^{\rm bare}$

Quenched Wilson fermions, β=6.0 m = 890 MeV 3066 configs

$\langle x \rangle_{\rm g}^{\rm bare} = 0.36(8)$



Renormalization

Renormalization in singlet sector

$$\begin{bmatrix} \overline{T}^{g}_{00}(\mu) \\ \overline{T}^{f}_{00}(\mu) \end{bmatrix} = \begin{bmatrix} Z_{gg} & 1-Z_{ff} \\ 1-Z_{gg} & Z_{ff} \end{bmatrix} \begin{bmatrix} \overline{T}^{g}_{00}(g_0) \\ \overline{T}^{f}_{00}(g_0) \end{bmatrix}$$

Quenched: $Z_{gg} = 1$

$$\begin{aligned} \langle x \rangle_g(\mu^2) &= \langle x \rangle_g + [1 - Z_{ff}(a\mu, g_0)] \, \langle x \rangle_f \\ \langle x \rangle_f(\mu^2) &= Z_{ff}(a\mu, g_0) \, \langle x \rangle_f \end{aligned}$$

Note:

$$\begin{aligned} \langle x \rangle_f &= Z_f(g_0) \langle x \rangle_f^{bare} \\ Z_f(g_0) &= 1.0(2) \\ Z_{ff}(a\mu, g_0) Z_f(g_0) &= 0.99(4) \end{aligned}$$

Guagnelli et al. hep-lat/0405027

Final result:

$$\begin{aligned} \langle x \rangle_g^{(\pi)}(\mu^2_{\overline{MS}} = 4\text{GeV}^2) &= 0.37(8)(12) \qquad (M_\pi = 890\text{MeV}) \\ \text{phenomenology} &= 0.38(5) \end{aligned}$$

Tests:

$$\begin{aligned} \langle x \rangle_g^{(\pi)} + \langle x \rangle_f^{(\pi),\text{lattice}} &= 0.99(8)(12) \\ \langle x \rangle_g^{G} &= 1.16(.18) \end{aligned}$$

Guagnelli et al. hep-lat/0405027

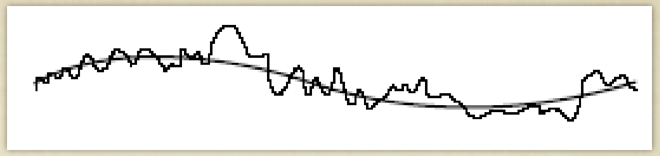
Trace anomaly contribution to mass

- $b_g \sim \langle E^2 + B^2 \rangle$ statistically accurate
- In absence of chiral symmetry, b_g acquires linearly divergent term from mixing with quarks.
- Strong mass dependence, since missing disconnected diagrams ~ 1/m
- Result: $b_g^{(\pi)(bare)} \sim 0.9(1)$ at largest mass Ji hep-ph/9410274

  $\sim b_g = 0.88(5)$ in proton
- Repeat with domain wall fermions

Insight into how QCD works: classical solutions

- Stationary phase approximation



$$\int D[A]e^{-\int d^4x S[A]} \sim [\det S'']^{-1} e^{-\int d^4 x S[A_{cl}]}$$

- Instanton solutions connect vacuua with different winding numbers

$$A^a_\mu(x) = \frac{2\eta_{a\mu\nu}x_\nu}{x^2+\rho^2}$$

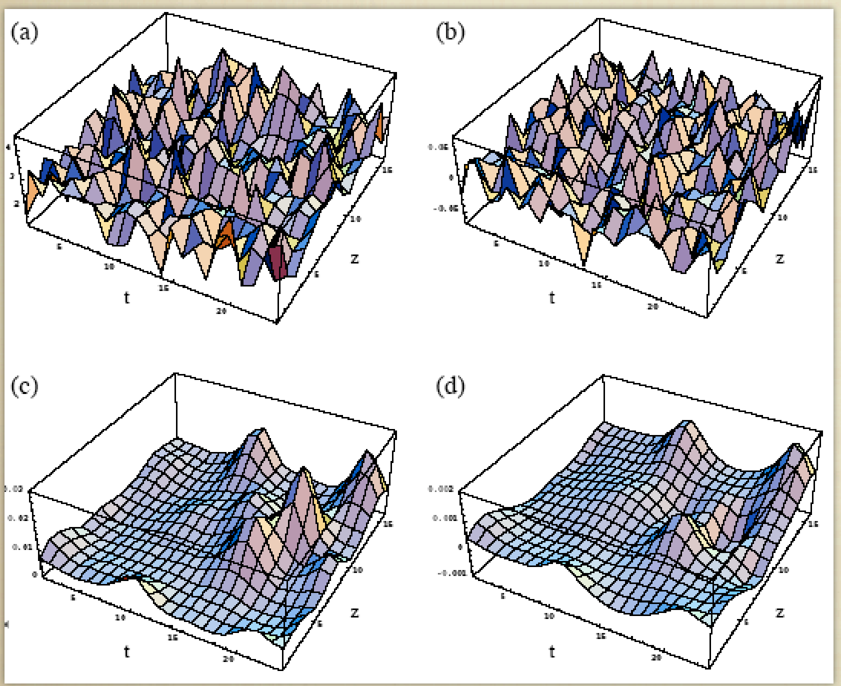
$$S = \frac{1}{4}\int F^2 = \frac{8\pi^2}{g^2}, \quad Q = \frac{g^2}{32\pi^2}\int F\tilde{F} = 1$$

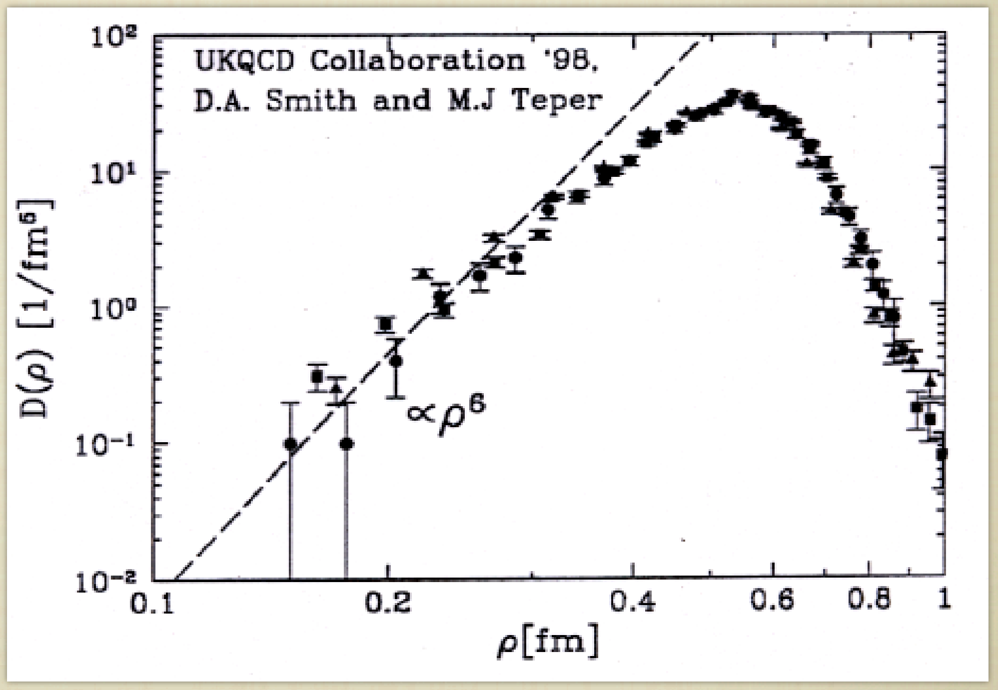
- To what extent are analytic expectations observed on lattice?

Instantons on the lattice

- Cooling (relaxation) reveals lumps with $S \sim \frac{8\pi^2}{g^2}$ and $Q \sim \pm 1$
- For small size ρ, distribution $\propto \rho^6$

s(x) q(x)

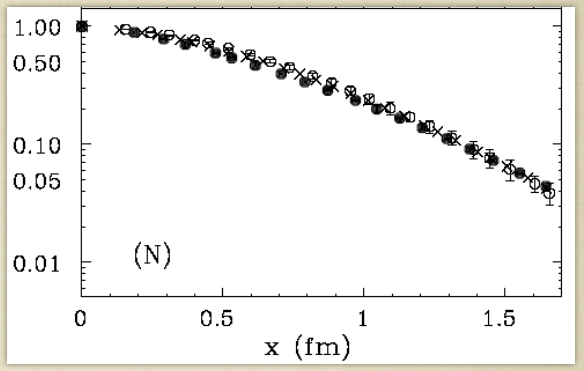




Chu, Grandy, Huang, JN hep-lat/9312071

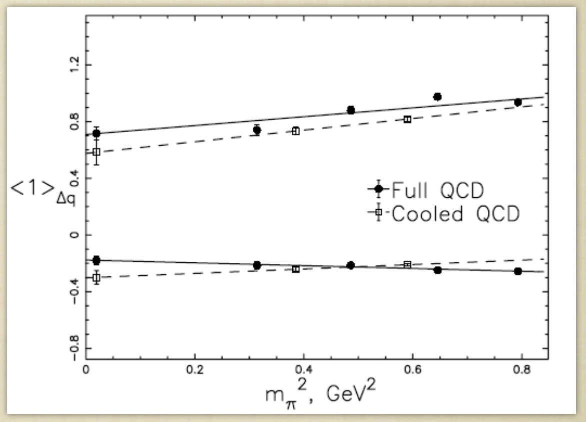
Instantons on the lattice

- Observables calculated with only instantons close to those including all gluons



- Observe quark zero modes localized at instantons

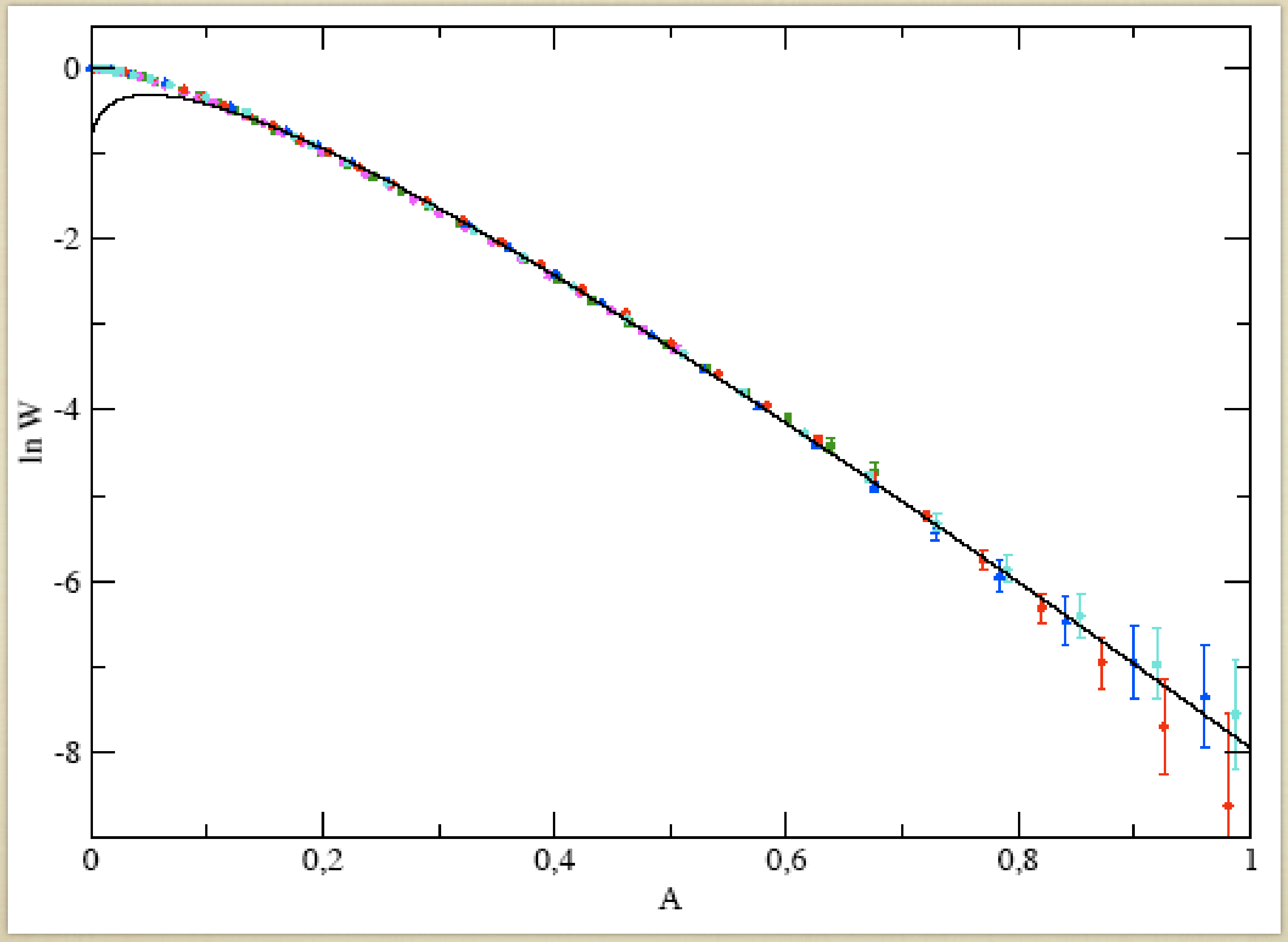
- Zero modes from instantons generate and dominate light quark propagators



- Topological susceptibility from instantons, Χ= (180MeV), yields η′ mass

Confinement from instantons

Ensemble of regular gauge instantons yields area law hep-th/0306105

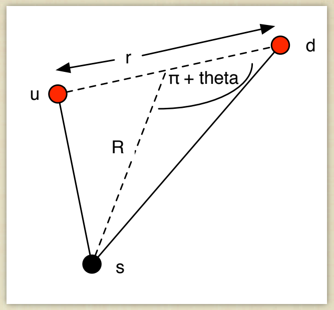


Diquark correlations in heavy light light baryon

Patrick Varilly - senior thesis

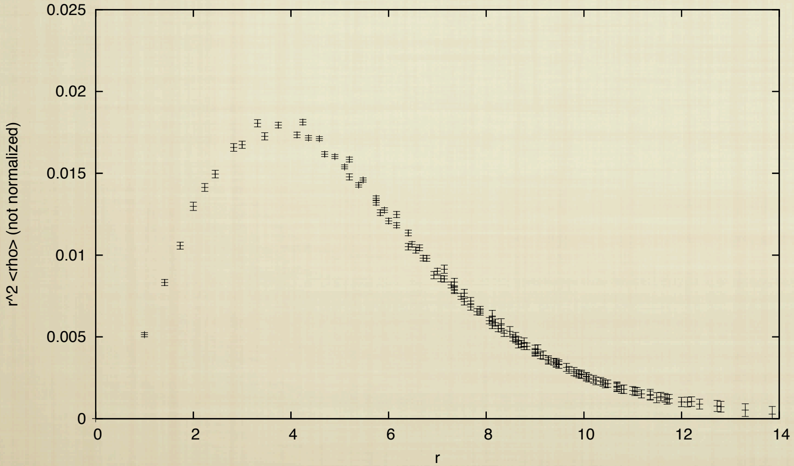
Good diquarks:
color antitriplet
flavor antisymmetric
spin singlet

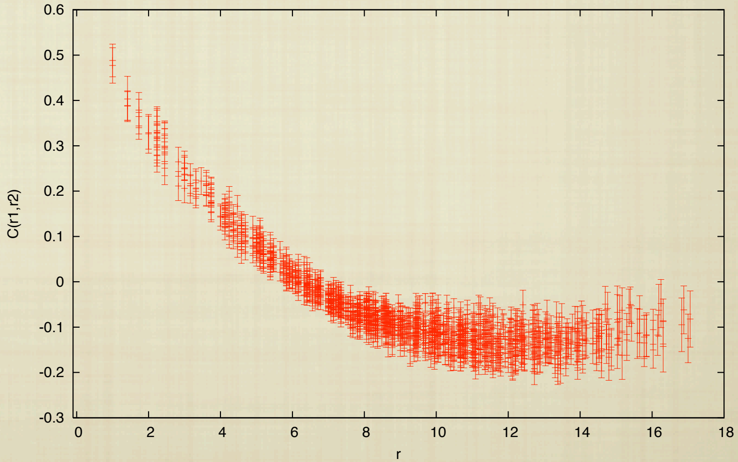
$$(uC\gamma_5 d)h$$



$$\langle \rho(r) \rangle$$

$$C(r_1, r_2) = \frac{\langle \rho(r_1)\rho(r_2)\rangle - \langle \rho(r_1)\rangle\langle \rho(r_2)\rangle}{\langle \rho(r_1)\rangle\langle \rho(r_2)\rangle}$$





Summary

- Entering era of quantitative solution in chiral regime
  - Moments of quark distributions
  - Form factors: F_1 , F_2 , G_A , G_P, N → Delta
  - Generalized form factors A B C
    - Transverse structure
    - Origin of nucleon spin
  - Beginning to calculate gluon observables
  - Insight: instantons, diquarks, dependence on parameters

Current effort and future work

- Full QCD with chiral fermions in chiral regime
  - LHPC/RBC/UKQCD collaboration
  - m_π = 360, 315, 260 MeV, a = 0.93 fm
  - 3.3 Tfyrs approved in 08, proposing 11 at ANL
  - Unprecedented precision
- Disconnected diagrams
  - Calculate proton and neutron separately, not just difference
  - Eigenmode expansion - deflation
- Gluon distributions
  - Nucleon momentum fraction
  - Total contribution of gluons to nucleon spin

MIT Blue Gene Computer