Towards the QCD spectrum using a space-time lattice

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Lattice Hadron Physics Collaboration

- charge from Nathan Isgur to use Monte Carlo method to extract the spectrum of baryon resonances (Hall B at JLab)
- formed the Lattice Hadron Physics Collaboration (LHPC)
- acquired funding through DOE SciDAC to build large computing cluster at JLab (also at Fermilab and Brookhaven), develop software
- LHPC has several broad goals
  - compute QCD spectrum (baryons, mesons,…)
  - hadron structure (form factors, structure functions,…)
  - hadron-hadron interactions
- current members of spectroscopy effort:
  - John Bulava, Robert Edwards, George Fleming, Justin Foley, Jimmy Juge, Adam Lichtl, CM, David Richards, Steve Wallace

LHPC spectroscopy efforts

- extracting spectrum of resonances is big challenge!!
  - need sets of extended operators (correlator matrices)
  - multi-hadron operators needed too
  - deduce resonances from finite-box energies
  - anisotropic lattices $(a_t < a_s)$
  - inclusion of light-quark loops at realistically light quark mass
- long-term project
- this talk is a brief status report
  - operator construction
    - smearing and pruning, selection of operators
  - initial results
  - use of stochastic all-to-all quark propagators

Energies from correlation functions

- stationary state energies extracted from asymptotic decay rate of temporal correlations of the fields (imaginary time formalism)
- evolution in Heisenberg picture $\phi(t) = e^{Ht}\ \phi(0)\ \ e^{-Ht}$ ($H =$Hamiltonian)
- spectral representation of a simple correlation function
  - assume transfer matrix, ignore temporal boundary conditions
  - focus only on one time ordering

insert complete set of energy eigenstates (discrete and continuous)

$$\langle 0|\,\phi(t)\phi(0)\,|0\rangle = \sum_n \langle 0|e^{Ht}\phi(0)\,e^{-Ht}\,|n\rangle\langle n|\phi(0)|0\rangle$$
$$= \sum_n \left|\langle n|\phi(0)\,|0\rangle\right|^2 e^{-(E_n-E_0)t} = \sum_n A_n e^{-(E_n-E_0)t}$$

- extract A_1 and $E_1 - E_0$ as $t \to \infty$

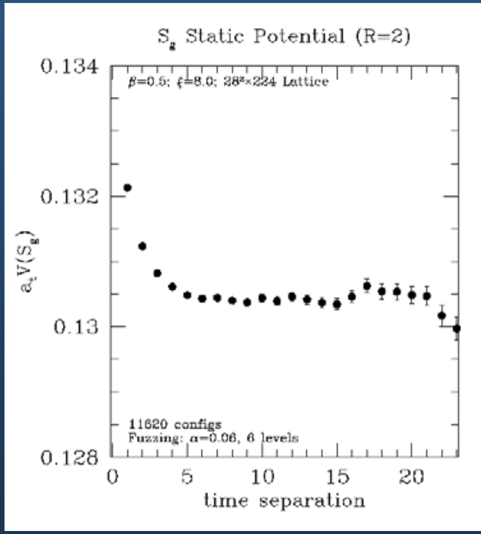
  (assuming $\langle 0|\phi(0)\,|0\rangle = 0$ and $\langle 1|\phi(0)\,|0\rangle \neq 0$)

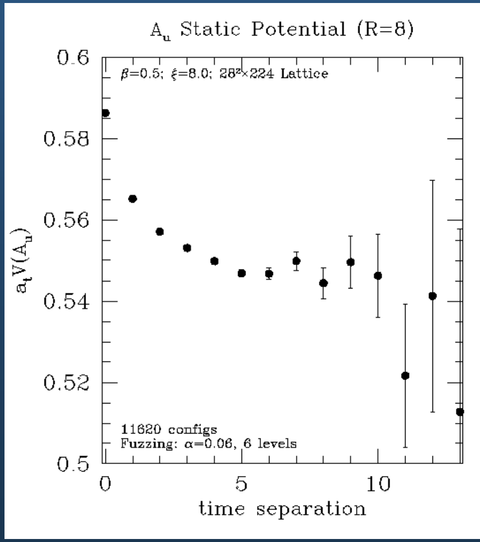
Effective mass

- the "effective mass" is given by $m_{\mathrm{eff}}(t) = \ln\left(\frac{C(t)}{C(t+1)}\right)$
- notice that (take $E_0 = 0$)

$$\lim_{t\to\infty} m_{\mathrm{eff}}(t) = \ln\left(\frac{A_1 e^{-E_1 t} + A_2 e^{-E_2 t} + \cdots}{A_1 e^{-E_1(t+1)} + \cdots}\right) \to \ln e^{E_1} = E_1$$

- effective mass tends to the actual mass (energy) asymptotically
- effective mass plot is convenient visual tool to see signal extraction
  - seen as a plateau
- plateau sets in quickly for good operator
- excited-state contamination before plateau





Reducing contamination

- statistical noise generally increases with temporal separation t
- effective masses associated with correlation functions of simple local fields often do not reach a plateau before noise swamps the signal
  - need better operators
  - better operators have reduced couplings with higher-lying contaminating states
- recipe for making better operators
  - crucial to construct operators using *smeared* fields
    - link variable smearing
    - quark field smearing
  - spatially extended operators
  - use large *set* of operators (variational coefficients)

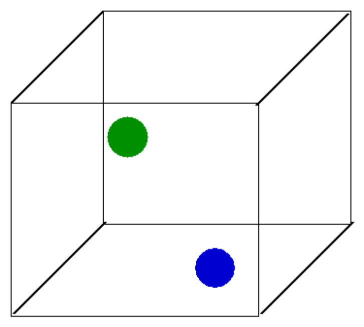
Principal correlators

- extracting excited-state energies described in
  - C. Michael, NPB **259**, 58 (1985)
  - Luscher and Wolff, NPB **339**, 222 (1990)
- can be viewed as exploiting the variational method
- for a given $N \times N$ correlator matrix $C_{\alpha\beta}(t) = \langle 0 | O_\alpha(t) O_\beta^+(0) | 0 \rangle$ one defines the N *principal correlators* $\lambda_\alpha(t, t_0)$ as the eigenvalues of

$$C(t_0)^{-1/2} C(t)\, C(t_0)^{-1/2}$$

  where t_0 (the time defining the "metric") is small
- can show that $\lim_{t\to\infty} \lambda_\alpha(t, t_0) = e^{-(t-t_0)E_\alpha}(1 + e^{-t\Delta E_\alpha})$
- N principal effective masses defined by $m_\alpha^{\text{eff}}(t) = \ln\left(\frac{\lambda_\alpha(t, t_0)}{\lambda_\alpha(t+1, t_0)}\right)$ now tend (plateau) to the N lowest-lying stationary-state energies

Unstable particles (resonances)



- our computations done in a periodic box
  - momenta quantized
  - discrete energy spectrum of stationary states → single hadron, 2 hadron, …
- scattering phase shifts → resonance masses, widths (in principle) deduced from finite-box spectrum
  - B. DeWitt, PR **103**, 1565 (1956) (sphere)
  - M. Luscher, NPB**364**, 237 (1991) (cube)
- more modest goal: “ferret” out resonances from scattering states
  - must differentiate resonances from multi-hadron states
  - avoided level crossings, different volume dependences
  - know masses of decay products → placement and pattern of multi-particle states known
  - resonances show up as extra states with little volume dependence

Operator design issues

- must facilitate spin identification
  - shun the usual method of operator construction which relies on cumbersome continuum space-time constructions
  - focus on constructing operators which transform irreducibly under the symmetries of the lattice
- one eye on maximizing overlaps with states of interest
- other eye on minimizing number of quark-propagator sources
- use building blocks useful for baryons, mesons, multi-hadron operators

Three stage approach (PRD72:094506,2005)

- concentrate on baryons at rest (zero momentum)
- operators classified according to the irreps of O_h

$$G_{1g}, G_{1u}, G_{2g}, G_{2u}, H_g, H_u$$

- (1) basic building blocks: smeared, covariant-displaced quark fields

$$(\widetilde{D}_j^{(p)}\widetilde{\psi}(x))_{Aa\alpha} \qquad p\text{-link displacement } (j=0,\pm1,\pm2,\pm3)$$

- (2) construct elemental operators (translationally invariant)

$$B^F(x) = \phi^F_{ABC}\varepsilon_{abc}(\tilde{D}_i^{(p)}\tilde{\psi}(x))_{Aa\alpha}(\tilde{D}_j^{(p)}\tilde{\psi}(x))_{Bb\beta}(\tilde{D}_k^{(p)}\tilde{\psi}(x))_{Cc\gamma}$$

  - flavor structure from isospin, color structure from gauge invariance

- (3) group-theoretical projections onto irreps of O_h

$$B_i^{\Lambda\lambda F}(t) = \frac{d_\Lambda}{g_{O_h^D}} \sum_{R\in O_h^D} D_{\lambda\lambda}^{(\Lambda)}(R)^* U_R B_i^F(t) U_R^+$$

  - wrote Grassmann package in Maple to do these calculations

Three-quark elemental operators

- three-quark operator

$$\Phi^{ABC}_{\alpha\beta\gamma,ijk}(t)=\sum_{\vec{x}}\varepsilon_{abc}(\tilde{D}^{(p)}_i\tilde{\psi}(\vec{x},t))^A_{a\alpha}\ (\tilde{D}^{(p)}_j\tilde{\psi}(\vec{x},t))^B_{b\beta}\ (\tilde{D}^{(p)}_k\tilde{\psi}(\vec{x},t))^C_{c\gamma}$$

- covariant displacements

$$\tilde{D}^{(p)}_j(x,x')=\tilde{U}_j(x)\,\tilde{U}_j(x+\hat{j})\cdots\tilde{U}_j(x+(p-1)\,\hat{j})\,\delta_{x',x+p\hat{j}}\qquad(j=\pm1,\pm2,\pm3)$$

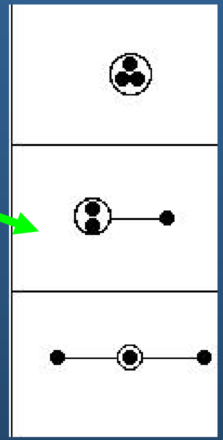
$$\tilde{D}^{(p)}_0(x,x')=\delta_{x',x}$$

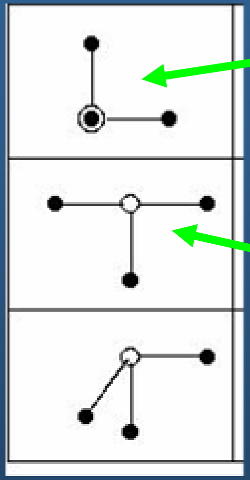
Baryon	Operator
Δ^{++}	$\Phi^{uuu}_{\alpha\beta\gamma,ijk}$
Σ^{+}	$\Phi^{uus}_{\alpha\beta\gamma,ijk}$
N^{+}	$\Phi^{uud}_{\alpha\beta\gamma,ijk}-\Phi^{duu}_{\alpha\beta\gamma,ijk}$
Ξ^{0}	$\Phi^{ssu}_{\alpha\beta\gamma,ijk}$
Λ^{0}	$\Phi^{uds}_{\alpha\beta\gamma,ijk}-\Phi^{dus}_{\alpha\beta\gamma,ijk}$
Ω^{-}	$\Phi^{sss}_{\alpha\beta\gamma,ijk}$

Incorporating orbital and radial structure

- displacements of different lengths build up radial structure
- displacements in different directions build up orbital structure

mock up quark-diquark





- operator design minimizes number of sources for quark propagators
- useful for mesons, tetraquarks, pentaquarks even!
- can even incorporate hybrid meson operators

Enumerating the three-quark operators

- lots of operators (too many!)

	Δ^{++}, Ω^{-}	Σ^{+}, Ξ^{0}	N^{+}	Λ^{0}
Single-site	20	40	20	24
Singly-displaced	240	624	384	528
Doubly-displaced-I	192	572	384	576
Doubly-displaced-L	768	2304	1536	2304
Triply-displaced-T	768	2304	1536	2304
Triply-displaced-O	512	1536	1024	1536

Spin identification and other remarks

- spin identification possible by pattern matching

J	$n^J_{G_1}$	$n^J_{G_2}$	n^J_H
$\frac{1}{2}$	1	0	0
$\frac{3}{2}$	0	0	1
$\frac{5}{2}$	0	1	1
$\frac{7}{2}$	1	1	1
$\frac{9}{2}$	1	0	2
$\frac{11}{2}$	1	1	2
$\frac{13}{2}$	1	2	2
$\frac{15}{2}$	1	1	3
$\frac{17}{2}$	2	1	3

total numbers of operators assuming two different displacement lengths

Irrep	Δ,Ω	N	Σ,Ξ	Λ
G_{1g}	221	443	664	656
G_{1u}	221	443	664	656
G_{2g}	188	376	564	556
G_{2u}	188	376	564	556
H_g	418	809	1227	1209
H_u	418	809	1227	1209

- total numbers of operators is huge → uncharted territory
- ultimately must face two-hadron scattering states

Single-site operators

- choose Dirac-Pauli convention for γ-matrices

- 20 independent single-site Δ^{++} elemental operators:

$$\Delta_{\alpha\beta\gamma} = \epsilon_{abc}\, \bar{u}_{a\alpha}\, \bar{u}_{b\beta}\, \bar{u}_{c\gamma}, \qquad (\alpha \leq \beta \leq \gamma)$$

- 20 independent single-site N^{+} elemental operators:

$$N_{\alpha\beta\gamma} = \epsilon^{abc} \left(\bar{u}_{a\alpha}\, \bar{u}_{b\beta}\, \bar{d}_{c\gamma} - \bar{d}_{a\alpha}\, \bar{u}_{b\beta}\, \bar{u}_{c\gamma} \right), \qquad (\alpha \leq \beta,\ \alpha < \gamma)$$

- 40 independent single-site Σ^{+} elemental operators:

$$\Sigma_{\alpha\beta\gamma} = \epsilon_{abc}\, \bar{u}_{a\alpha}\, \bar{u}_{b\beta}\, \bar{s}_{c\gamma} \qquad (\alpha \leq \beta)$$

- 24 independent single-site Λ^{0} elemental operators:

$$\Lambda_{\alpha\beta\gamma} = \epsilon_{abc} \left(\bar{u}_{a\alpha}\, \bar{d}_{b\beta}\, \bar{s}_{c\gamma} - \bar{d}_{a\alpha}\, \bar{u}_{b\beta}\, \bar{s}_{c\gamma} \right) \qquad (\alpha < \beta)$$

Δ++ single-site operators

Irrep	Row	DP Operators
G_{1g}	1	$\Delta_{144} - \Delta_{234}$
G_{1g}	2	$-\Delta_{134} + \Delta_{233}$
G_{1u}	1	$\Delta_{124} - \Delta_{223}$
G_{1u}	2	$-\Delta_{114} + \Delta_{123}$
H_g	1	Δ_{222}
H_g	2	$-\sqrt{3}\,\Delta_{122}$
H_g	3	$\sqrt{3}\,\Delta_{112}$
H_g	4	$-\Delta_{111}$
H_g	1	$\sqrt{3}\,\Delta_{244}$
H_g	2	$-\Delta_{144} - 2\Delta_{234}$
H_g	3	$2\Delta_{134} + \Delta_{233}$
H_g	4	$-\sqrt{3}\,\Delta_{133}$

Irrep	Row	DP Operators
H_u	1	$\sqrt{3}\,\Delta_{224}$
H_u	2	$-2\Delta_{124} - \Delta_{223}$
H_u	3	$\Delta_{114} + 2\Delta_{123}$
H_u	4	$-\sqrt{3}\,\Delta_{113}$
H_u	1	Δ_{444}
H_u	2	$-\sqrt{3}\,\Delta_{344}$
H_u	3	$\sqrt{3}\,\Delta_{334}$
H_u	4	$-\Delta_{333}$

Single-site N+ operators

Irrep	Row	DP Operators
G_{1g}	1	N_{122}
G_{1g}	2	$-N_{112}$
G_{1g}	1	$N_{144} - N_{243}$
G_{1g}	2	$-N_{134} + N_{233}$
G_{1g}	1	$N_{144} - 2N_{234} + N_{243}$
G_{1g}	2	$N_{134} - 2N_{143} + N_{233}$
G_{1u}	1	N_{142}
G_{1u}	2	$-N_{132}$
G_{1u}	1	N_{344}
G_{1u}	2	$-N_{334}$
G_{1u}	1	$2N_{124} - N_{142} - 2N_{223}$
G_{1u}	2	$-2N_{114} + 2N_{123} - N_{132}$

Irrep	Row	DP Operators
H_g	1	$\sqrt{3}\, N_{244}$
H_g	2	$-N_{144} - N_{234} - N_{243}$
H_g	3	$N_{134} + N_{143} + N_{233}$
H_g	4	$-\sqrt{3}\, N_{133}$
H_u	1	$\sqrt{3}\, N_{224}$
H_u	2	$-2N_{124} + N_{142} - N_{223}$
H_u	3	$N_{114} + 2N_{123} - N_{132}$
H_u	4	$-\sqrt{3}\, N_{113}$

Testing the three-quark operators

- Next step: smearing optimization and operator pruning
  - optimize link-variable and quark-field smearings
  - remove dynamically redundant operators
  - remove ineffectual operators
  - low statistics runs in quenched approx on small lattices

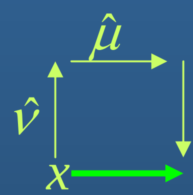
Quark- and gauge-field smearing

- smeared quark and gluon fields fields → dramatically reduced coupling with short wavelength modes
- link-variable smearing (stout links PRD**69**, 054501 (2004))
  - define $C_\mu(x) = \sum_{\pm(\nu\neq\mu)} \rho_{\mu\nu} U_\nu(x) U_\mu(x+\hat{\nu}) U_\nu^{+}(x+\hat{\mu})$
  - spatially isotropic $\rho_{jk} = \rho, \quad \rho_{4k} = \rho_{k4} = 0$
  - exponentiate traceless Hermitian matrix

$$\Omega_\mu = C_\mu U_\mu^{+} \qquad Q_\mu = \frac{i}{2}\left(\Omega_\mu^{+} - \Omega_\mu\right) - \frac{i}{2N}\mathrm{Tr}\left(\Omega_\mu^{+} - \Omega_\mu\right)$$

  - iterate $U_\mu^{(n+1)} = \exp\left(iQ_\mu^{(n)}\right) U_\mu^{(n)}$

$$U_\mu \to U_\mu^{(1)} \to \cdots \to U_\mu^{(n)} \equiv \widetilde{U}_\mu$$



- quark-field smearing (covariant Laplacian uses smeared gauge field)

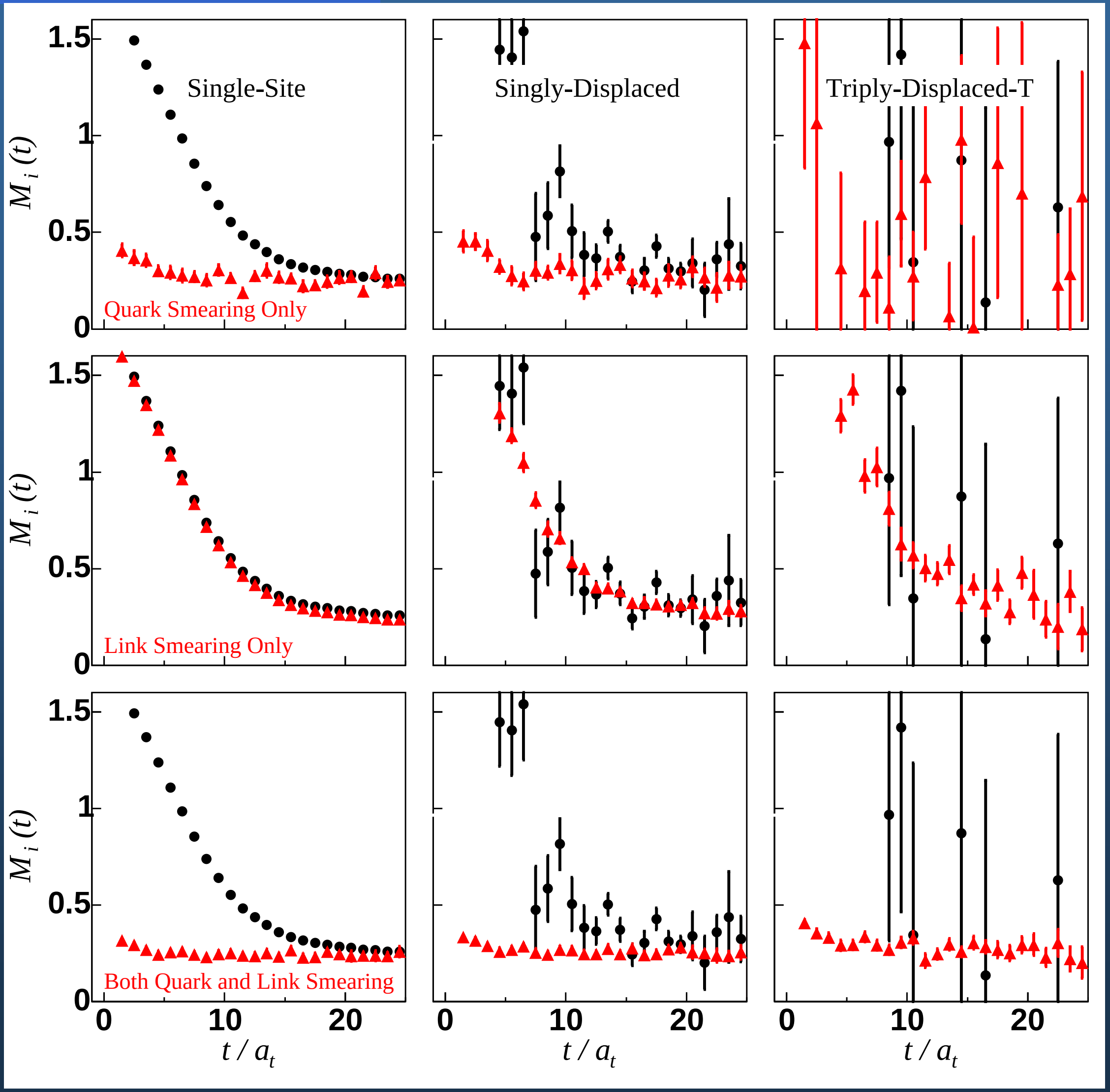
$$\tilde{\psi}(x) = \left(1 + \frac{\sigma_s}{4n_\sigma}\tilde{\Delta}^2\right)^{n_\sigma} \psi(x)$$

Importance of smearing

- Nucleon G_{1g} channel
- effective masses of 3 selected operators
- noise reduction from link variable smearing, especially for displaced operators
- quark-field smearing reduces couplings to high-lying states

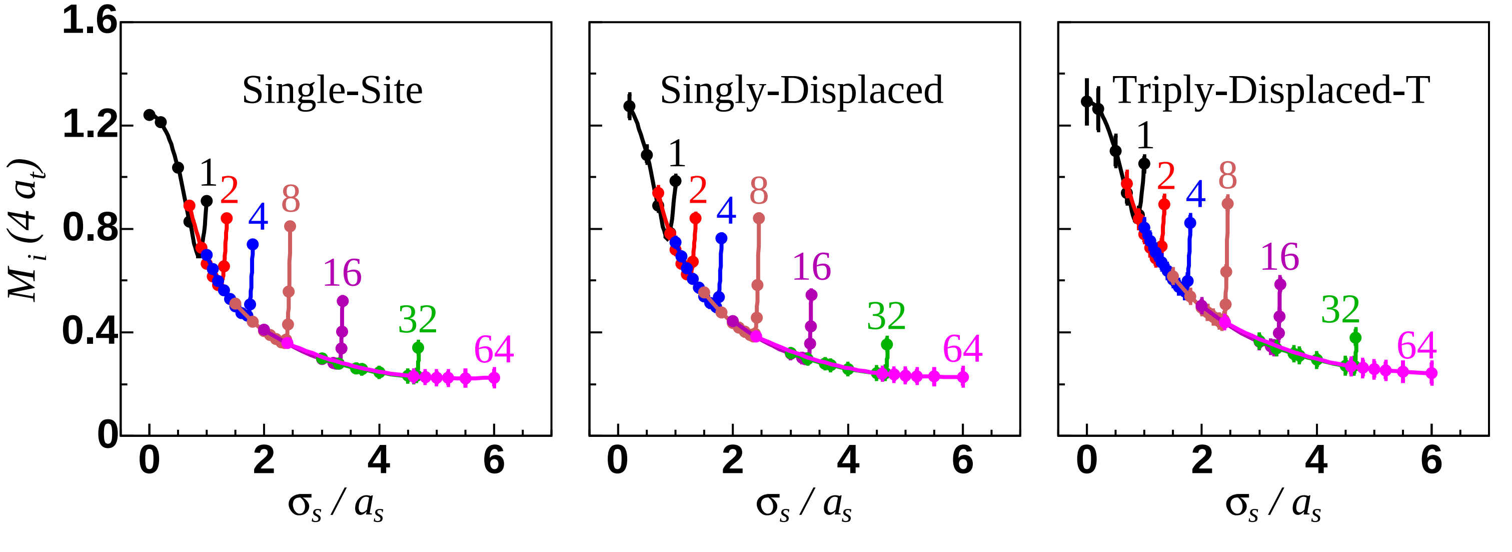
$$\sigma_s = 4.0, \quad n_\sigma = 32$$
$$n_\rho \rho = 2.5, \quad n_\rho = 16$$

- less noise in excited states using $\sigma_s = 3.0$

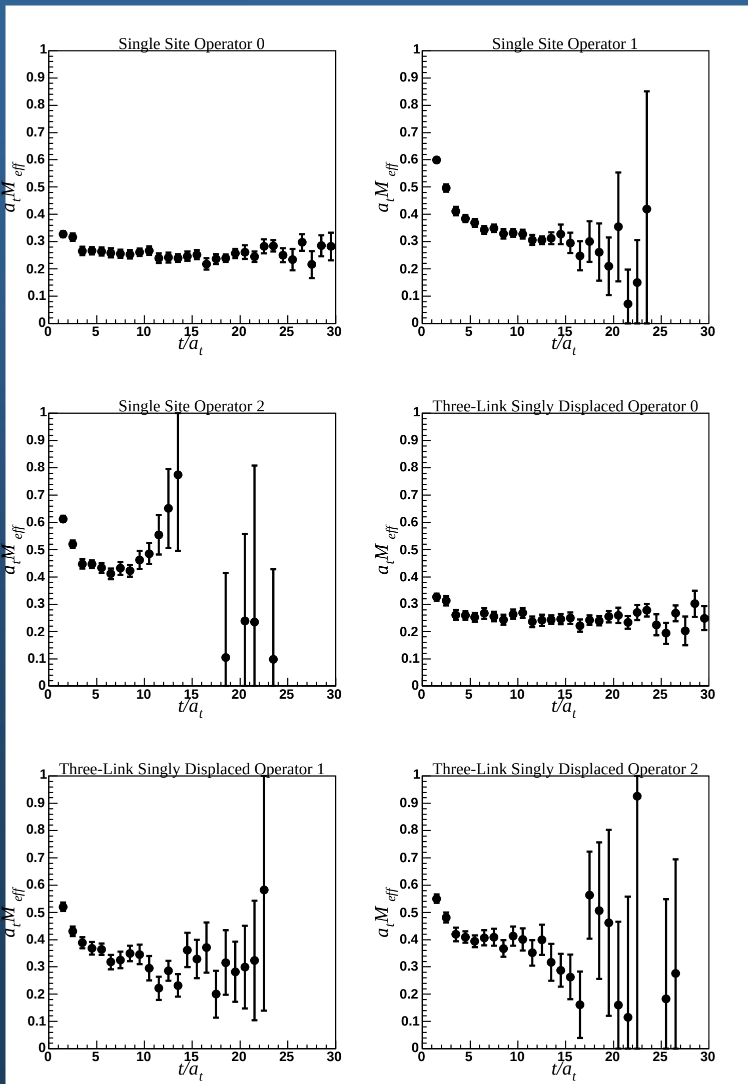


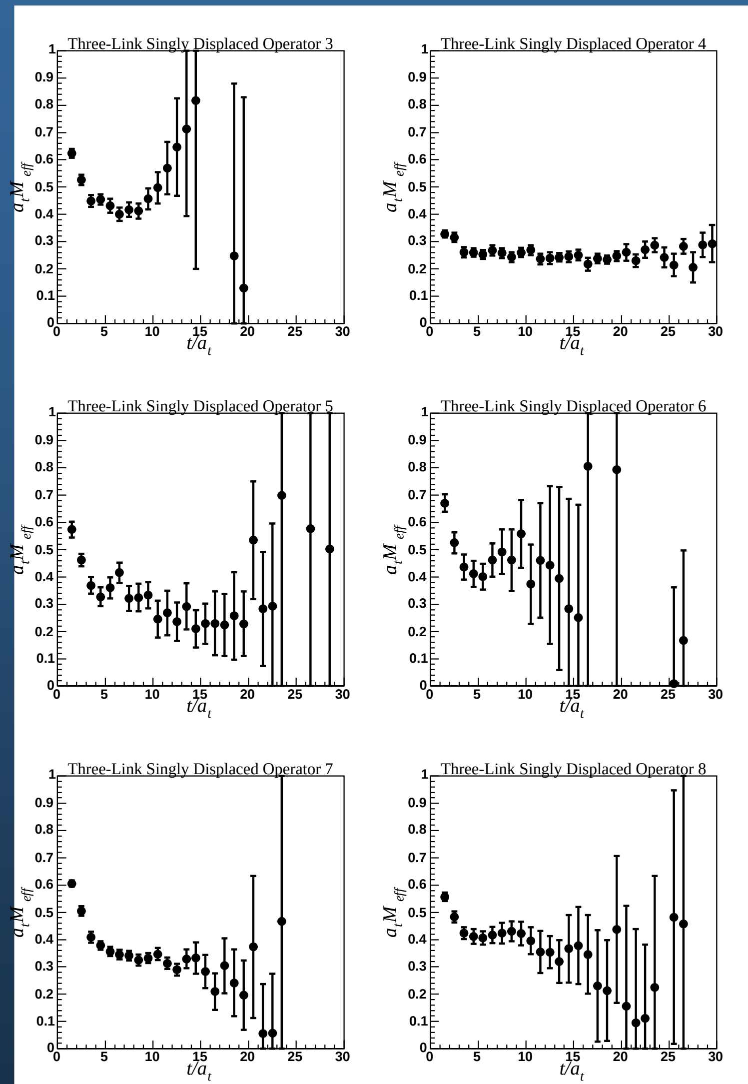
Tuning the smearing

- the effective mass at $t = 4a_t$ for three specific nucleon operators for different quark-field smearings (link smearing same as last slide)

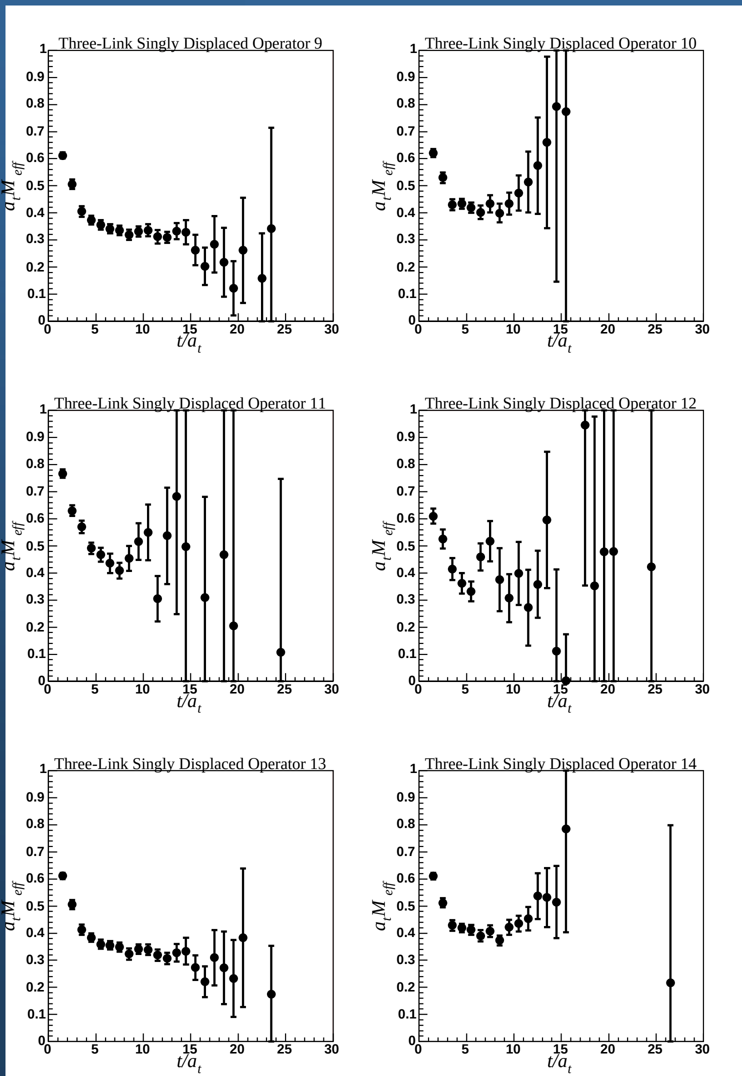


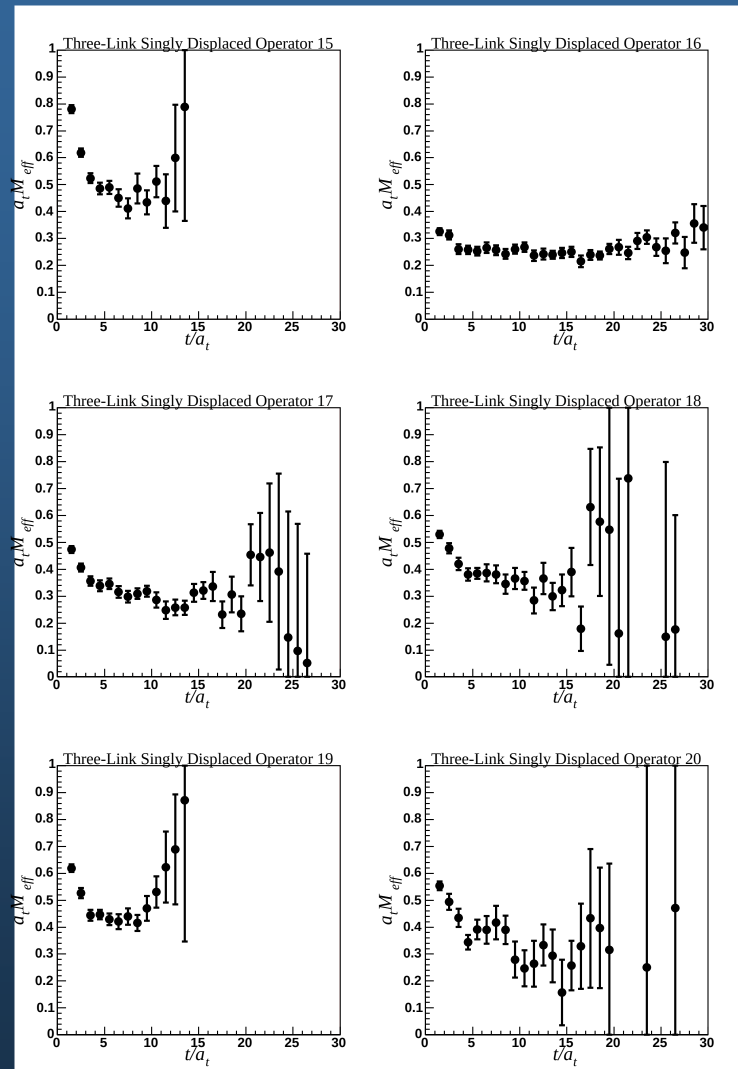
Operator plethora (G_{1g} Nucleon)





G_{1g} nucleon operators





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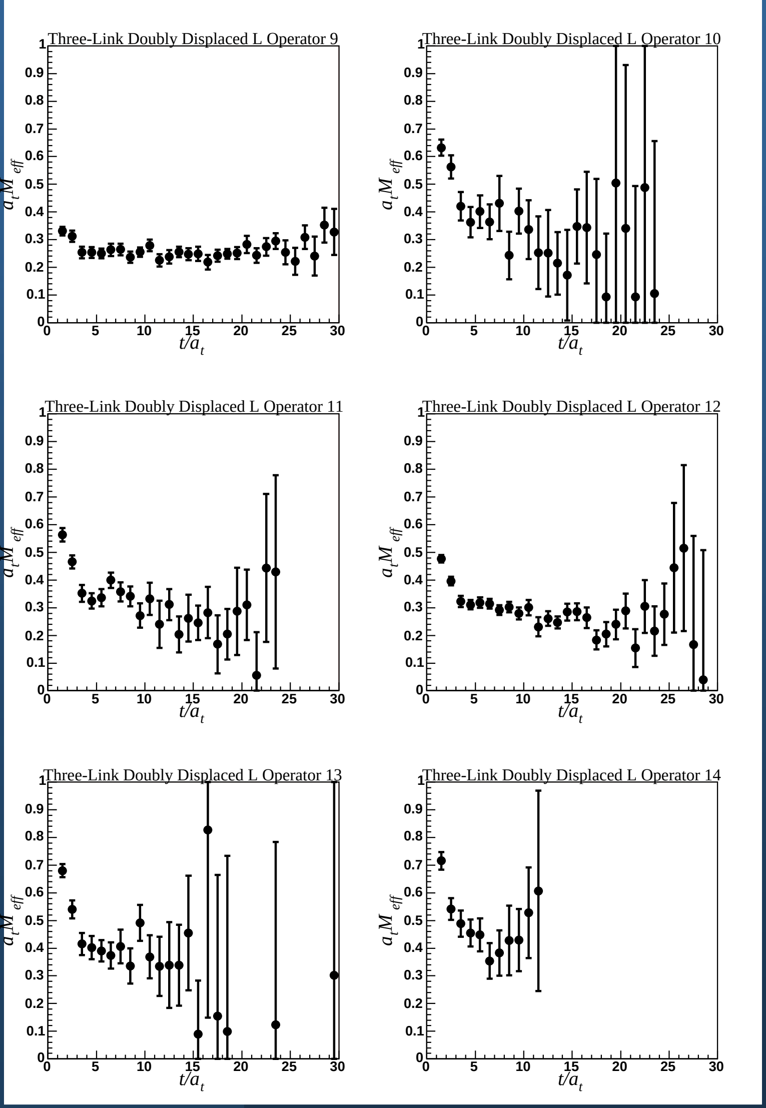
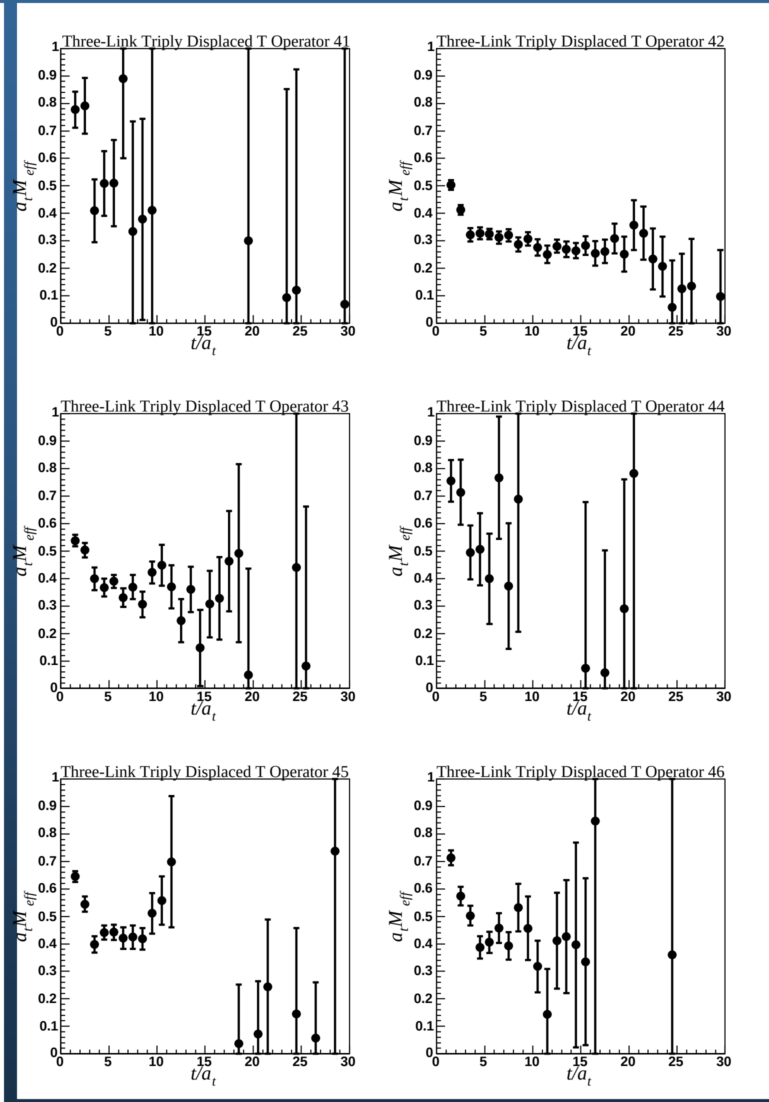
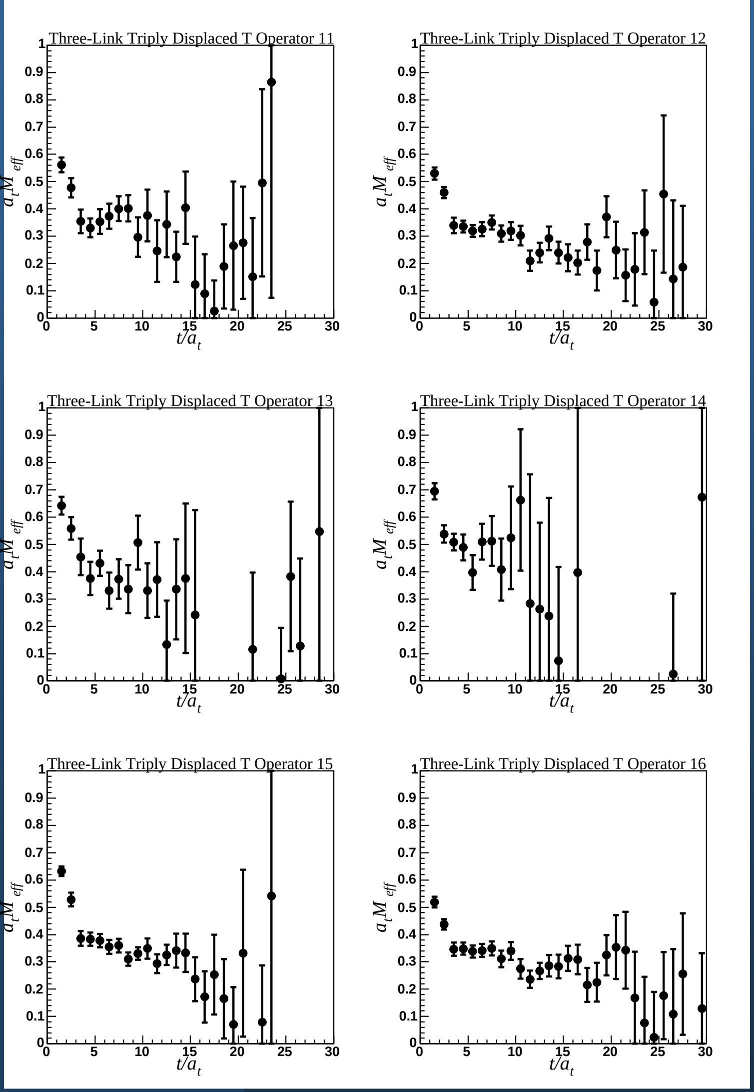
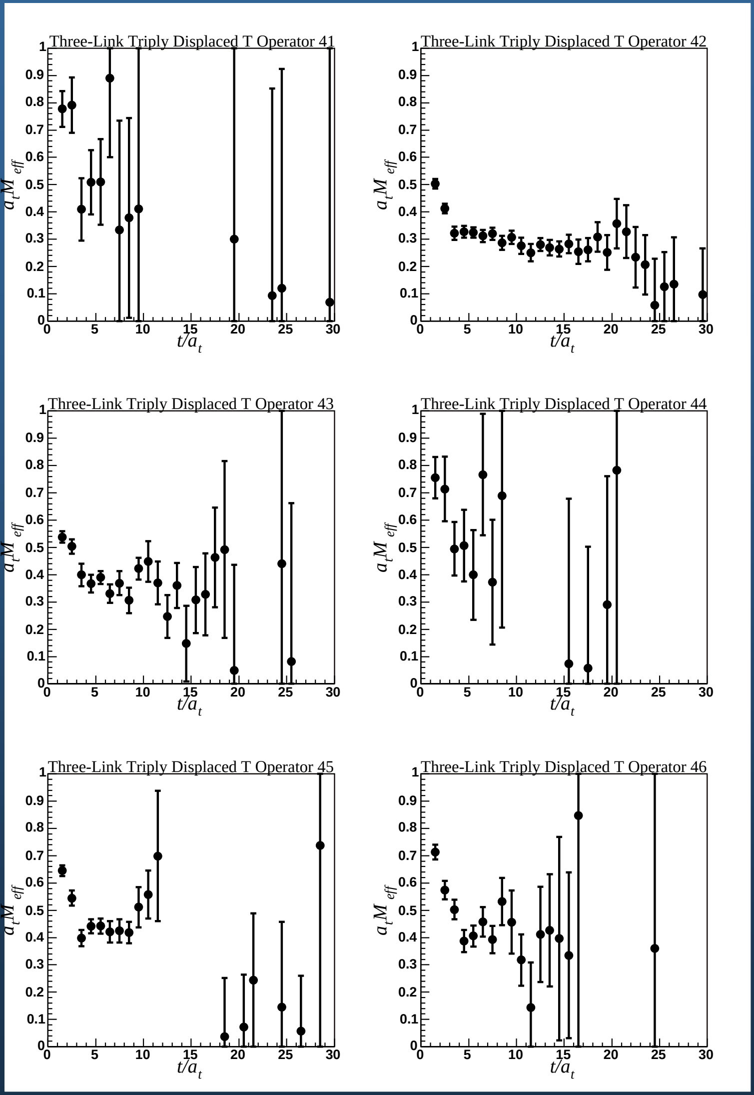


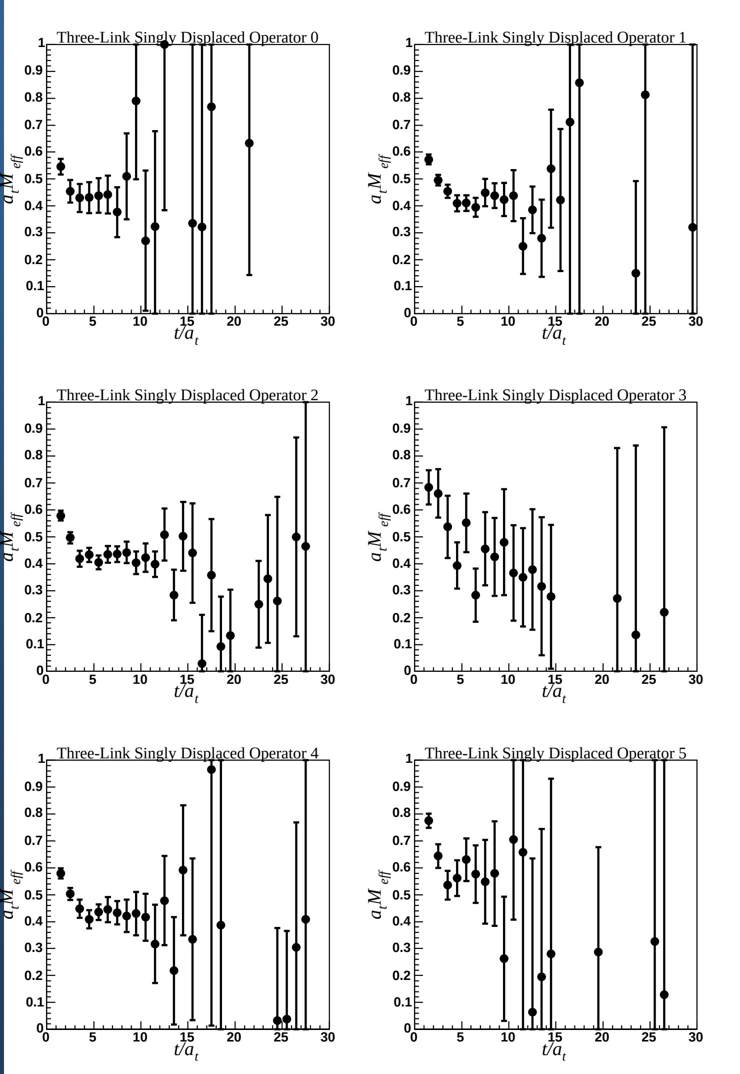


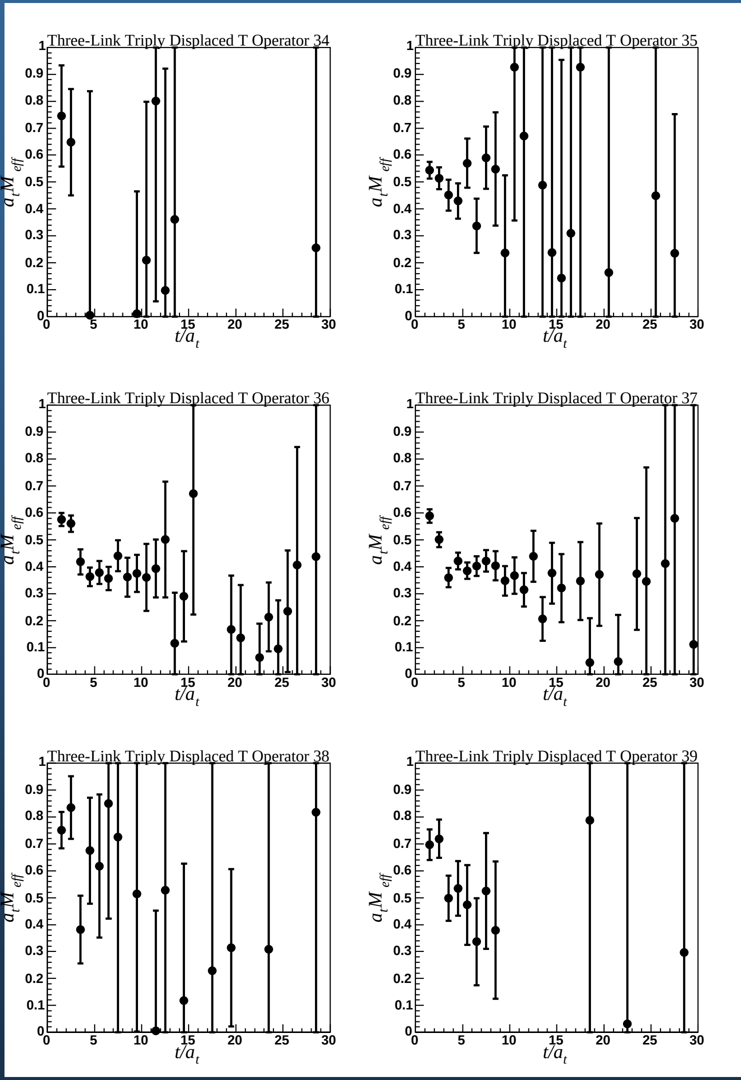
G_{1g} nucleon operators



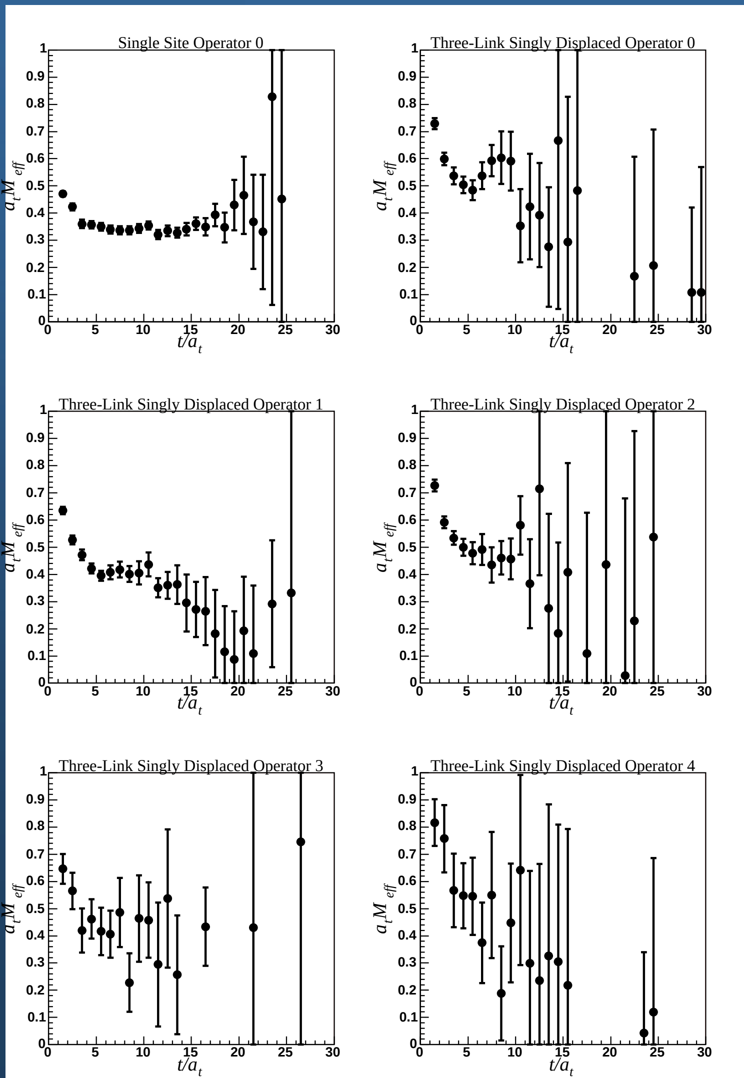


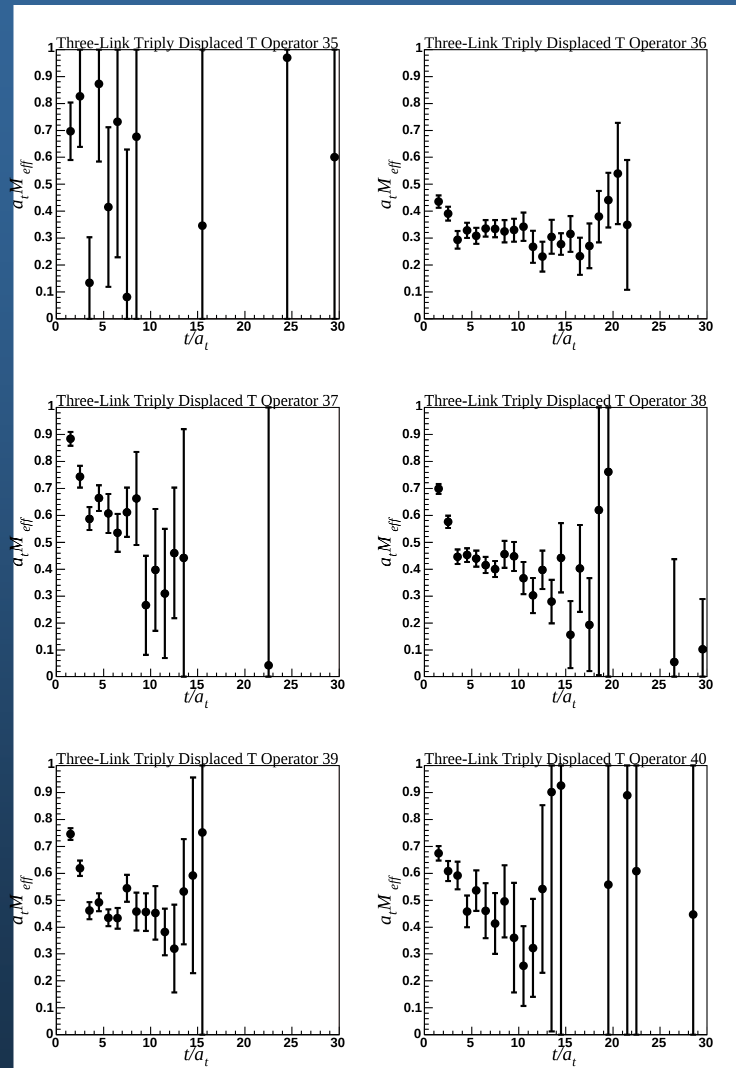
G_{2g} nucleon operators





H_u nucleon operators





Operator selection

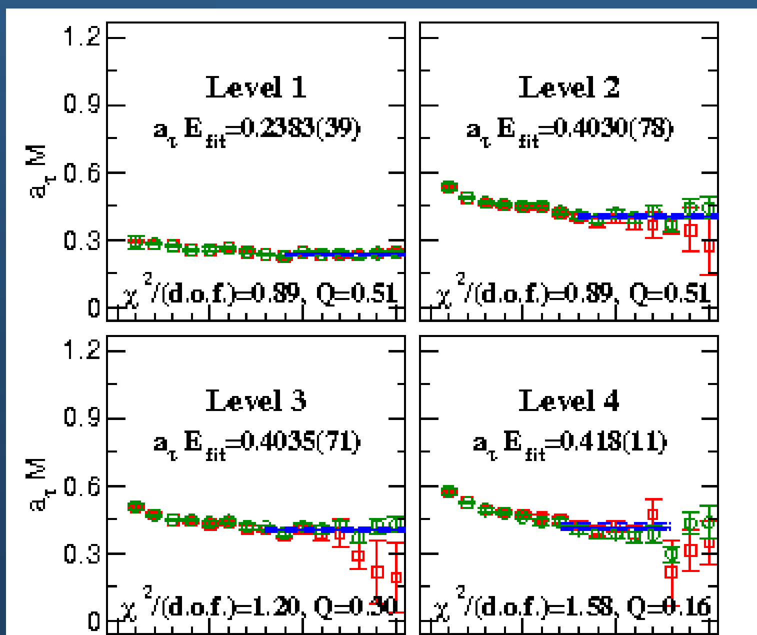
- Do we need all of these operators?
- If not, how many? How do we choose?
- over six months of experimentation led us to the following rules of thumb:
  - noise is the enemy!
  - prune first using intrinsic noise (diagonal correlators)
  - prune next within operator *types* (single-site, singly-displaced, *etc.*) based on condition number of
  - prune across all operators based on condition number

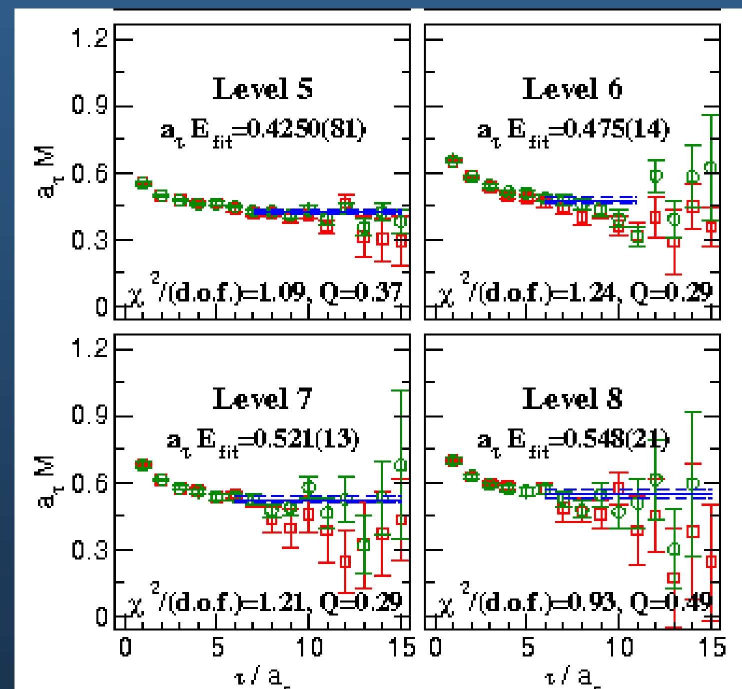
$$\hat{C}_{ij}(t) = \frac{C_{ij}(t)}{\sqrt{C_{ii}(t)C_{jj}(t)}}, \qquad t = 1$$

- best to keep a variety of different types of operators, as long as condition numbers maintained
- low lying spectrum robust if noise minimized, good operator variety
- typically use 16 operators to get 8 lowest lying levels

Nucleon G_{1g} effective masses

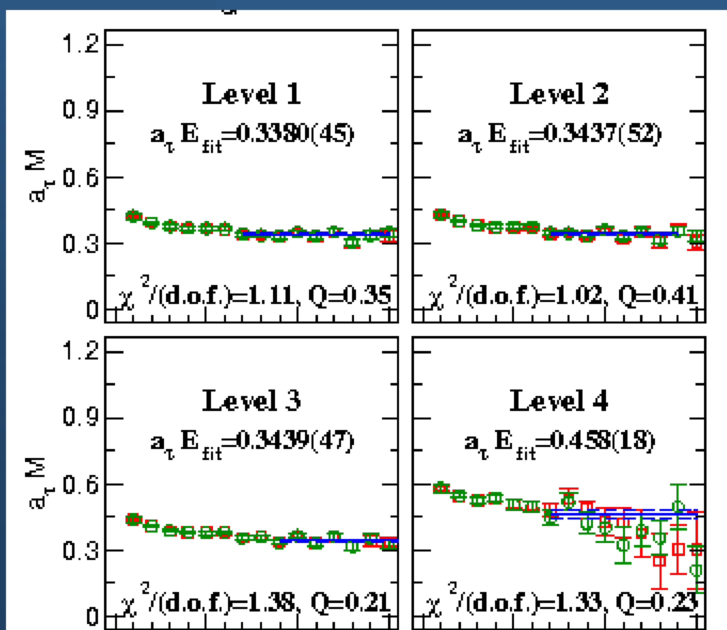
- 200 quenched configs, $12^3 \times 48$ anisotropic Wilson lattice, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3$, $m_\pi \sim 700$ MeV
- nucleon G_{1g} channel
- green=fixed coefficients, red=principal

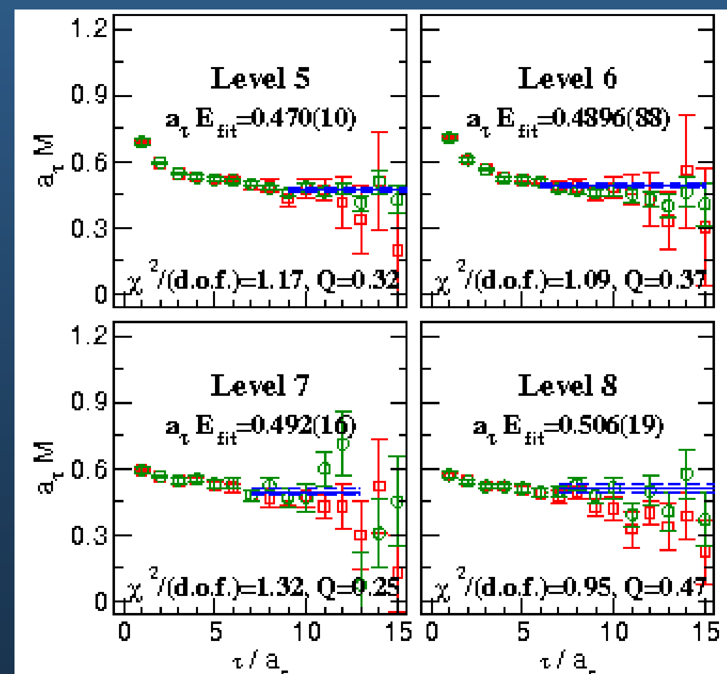




Nucleon H_u effective masses

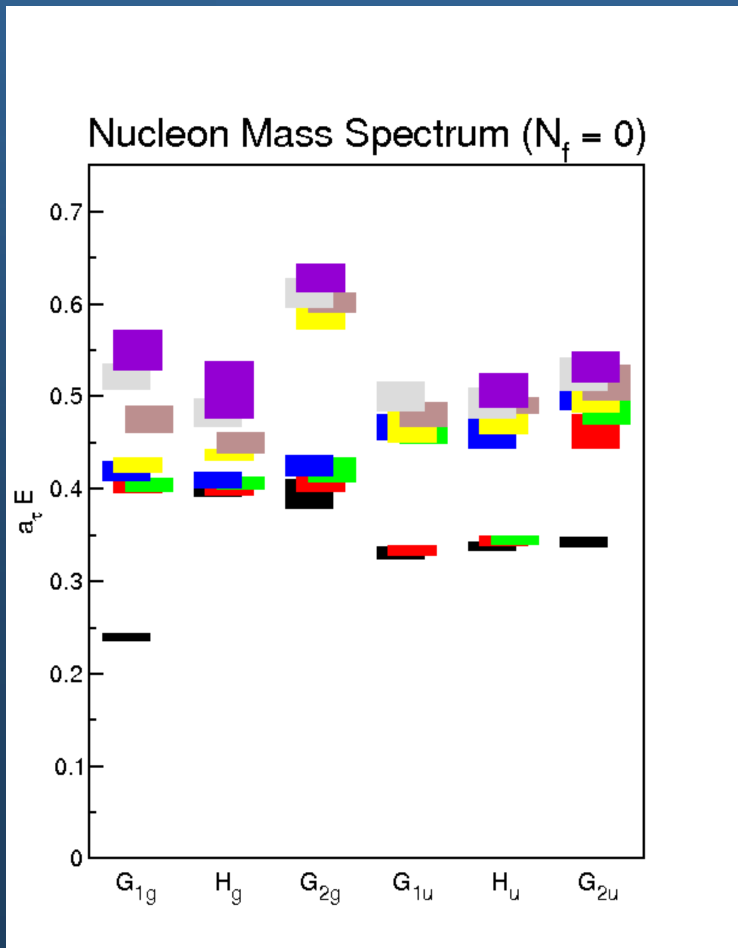
- 200 quenched configs, $12^3 \times 48$ anisotropic Wilson lattice, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3$, $m_\pi \sim 700$ MeV
- nucleon H_u channel
- green=fixed coefficients, red=principal

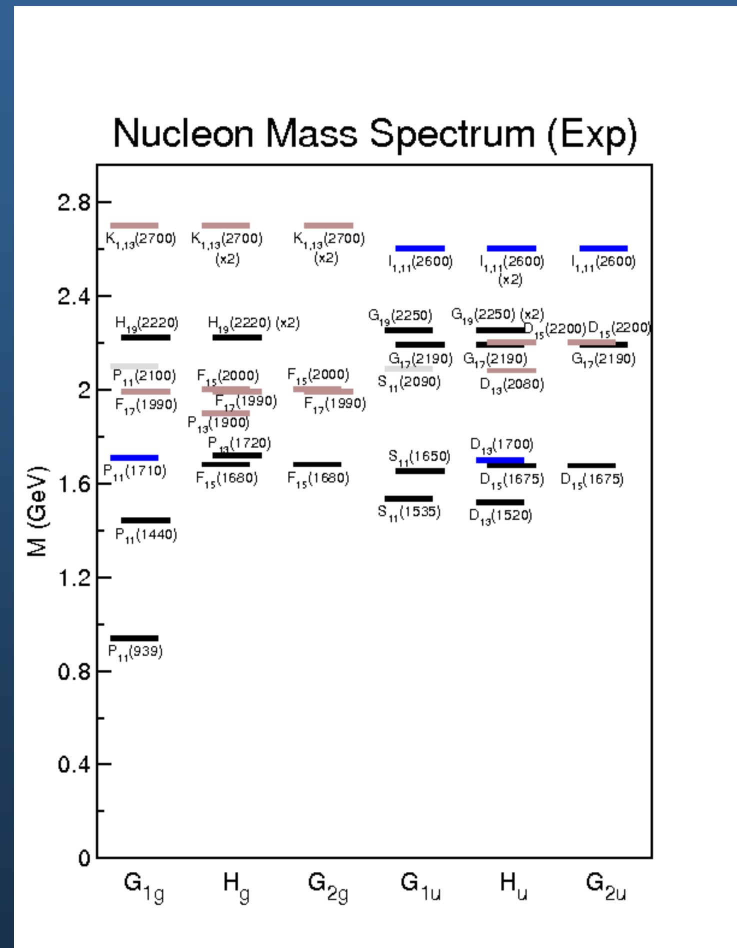




Nucleon spectrum

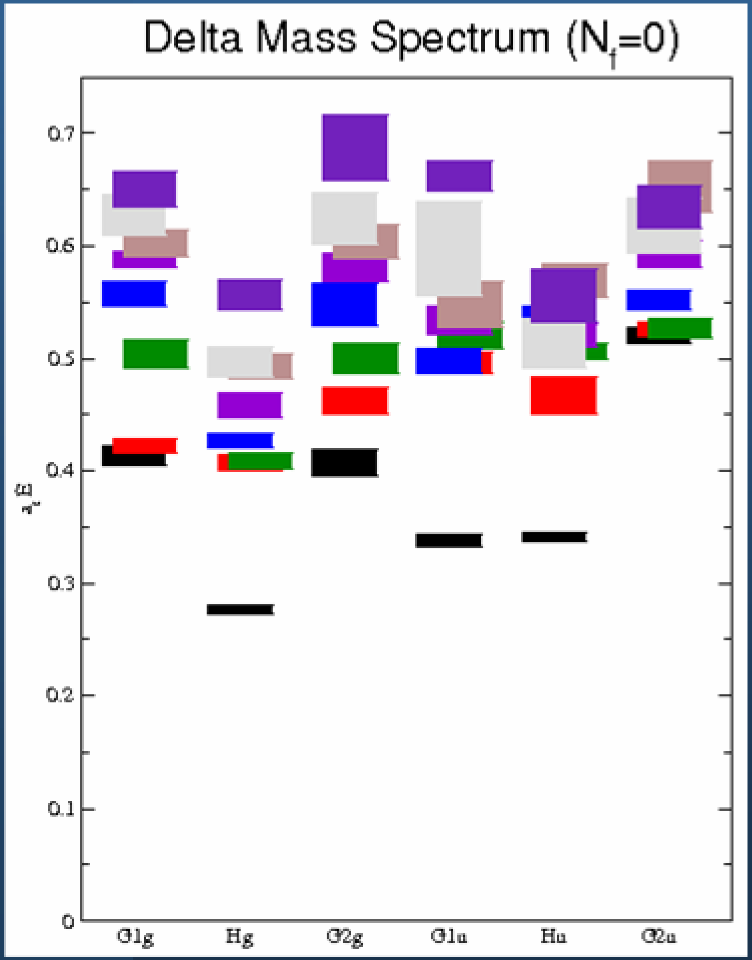
- 200 quenched configs, $12^3 \times 48$ anisotropic Wilson lattice, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3$, $m_\pi \sim 700$ MeV

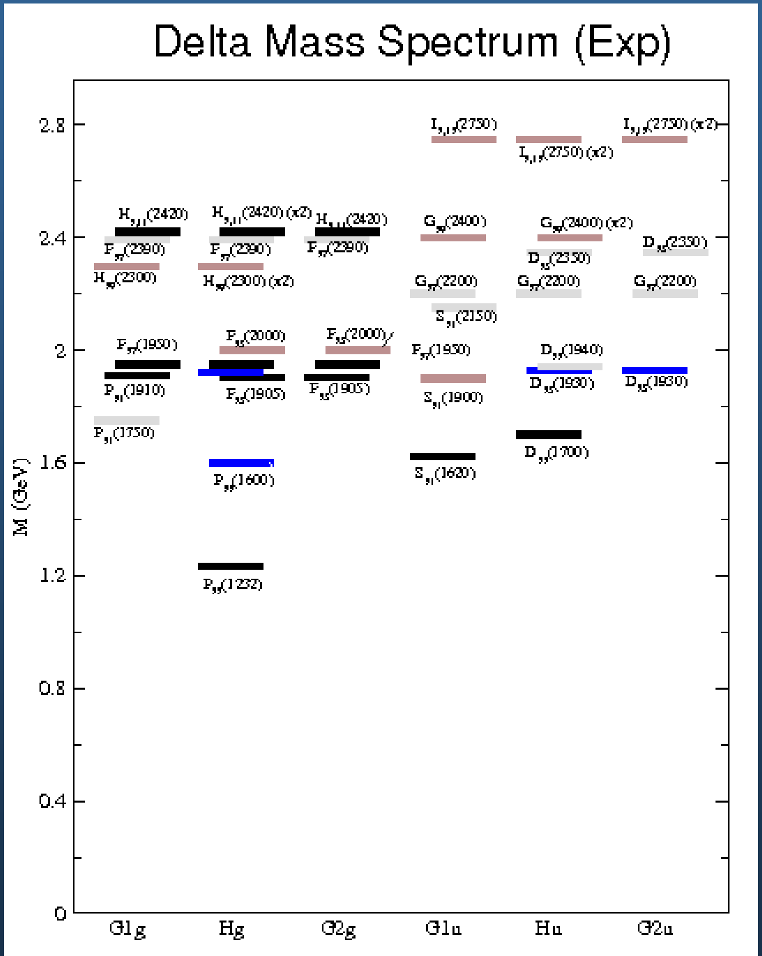




Delta spectrum

- 200 quenched configs, $12^3 \times 48$ anisotropic Wilson lattice, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3$, $m_\pi \sim 700$ MeV





All-to-all stochastic quark propagators

- consider a temporal correlator of a three-quark Σ (*uus*) baryon:

$$C^{(\Sigma\Lambda\lambda)}_{lp|\overline{lp}}(t) = \frac{1}{N_t}\sum_{t_0} c^{(\Sigma\Lambda\lambda)(l)}_{\alpha\beta\gamma;ijk}\, c^{(\Sigma\Lambda\lambda)(\overline{l})*}_{\overline{\alpha}\overline{\beta}\overline{\gamma};\overline{i}\overline{j}\overline{k}} \sum_{\boldsymbol{x}} \varepsilon_{abc} \sum_{\overline{\boldsymbol{x}}} \varepsilon_{\overline{a}\overline{b}\overline{c}}\, \gamma^4_{\overline{\gamma}'\overline{\gamma}}\gamma^4_{\overline{\beta}'\overline{\beta}}\gamma^4_{\overline{\alpha}'\overline{\alpha}}$$

$$\times \Big\langle Q^{(u)}_{a\alpha ip;\overline{a}\overline{\alpha}'\overline{ip}}\big(\boldsymbol{x},t+t_0;\overline{\boldsymbol{x}},t_0\,|\,U\big)\, Q^{(u)}_{b\beta jp;\overline{b}\overline{\beta}'\overline{jp}}\big(\boldsymbol{x},t+t_0;\overline{\boldsymbol{x}},t_0\,|\,U\big)$$

$$Q^{(s)}_{c\gamma kp;\overline{c}\overline{\gamma}'\overline{kp}}\big(\boldsymbol{x},t+t_0;\overline{\boldsymbol{x}},t_0\,|\,U\big) - Q^{(u)}_{a\alpha ip;\overline{b}\overline{\beta}'\overline{jp}}\big(\boldsymbol{x},t+t_0;\overline{\boldsymbol{x}},t_0\,|\,U\big)$$

$$Q^{(u)}_{b\beta jp;\overline{a}\overline{\alpha}'\overline{ip}}\big(\boldsymbol{x},t+t_0;\overline{\boldsymbol{x}},t_0\,|\,U\big)\, Q^{(s)}_{c\gamma kp;\overline{c}\overline{\gamma}'\overline{kp}}\big(\boldsymbol{x},t+t_0;\overline{\boldsymbol{x}},t_0\,|\,U\big)\Big\rangle_U$$

- above expression needs quark propagators from *all* spatial sites $\overline{x}$ on time slice t_0 to all spatial sites $\boldsymbol{x}$ on time slice $t+t_0$

All-to-all stochastic quark propagators (2)

- computing all elements of propagators exactly not feasible
- translational invariance can limit summation over source site to a single site for local operators
- cannot limit source to single site for multi-hadron operators
- disconnected diagrams (scalar mesons) will also need many-to-many quark propagators
- *stochastic estimates* of all quark propagator elements are needed!

Matrix inversion

- quark propagator is just inverse of Dirac matrix M
- noise vectors η satisfying $E(\eta_i)=0$ and $E(\eta_i\eta_j^*)=\delta_{ij}$ are useful for stochastic estimates of inverse of a matrix M
- Z_4 noise is used $\{1, i, -1, -i\}$
- define $X(\eta)=M^{-1}\eta$ then

$$E(X_i\eta_j^*) = E\left(\sum_k M_{ik}^{-1}\eta_k\eta_j^*\right) = \sum_k M_{ik}^{-1}E\left(\eta_k\eta_j^*\right) = \sum_k M_{ik}^{-1}\delta_{kj} = M_{ij}^{-1}$$

- if can solve $M X^{(r)}= \eta^{(r)}$ for each of N_R noise vectors $\eta^{(r)}$ then we have a Monte Carlo estimate of all elements of M^{-1}:

$$M_{ij}^{-1} \approx \frac{1}{N_R}\sum_{r=1}^{N_R} X_i^{(r)}\eta_j^{(r)*}$$

- variances in above estimates usually unacceptably large
- introduce variance reduction using source *dilution*

Source dilution for single matrix inverse

- dilution introduces a complete set of projections:

$$P^{(a)}P^{(b)} = \delta^{ab}P^{(a)}, \qquad \sum_a P^{(a)} = 1, \qquad P^{(a)\dagger} = P^{(a)}$$

- observe that

$$M_{ij}^{-1} = M_{ik}^{-1}\delta_{kj} = \sum_a M_{ik}^{-1}P_{kj}^{(a)} = \sum_a M_{ik}^{-1}P_{kk'}^{(a)}\delta_{k'j'}P_{j'j}^{(a)}$$

$$= \sum_a M_{ik}^{-1}P_{kk'}^{(a)}E\left(\eta_{k'}\eta_{j'}^*\right)P_{j'j}^{(a)} = \sum_a M_{ik}^{-1}E\left(P_{kk'}^{(a)}\eta_{k'}\eta_{j'}^*P_{j'j}^{(a)}\right)$$

- define $\eta_k^{[a]} = P_{kk'}^{(a)}\eta_{k'}, \qquad \eta_j^{[a]*} = \eta_{j'}^*P_{j'j}^{(a)}, \qquad X_k^{[a]} = M_{kj}^{-1}\eta_j^{[a]}$

  so that $M_{ij}^{-1} = \sum_a E\left(X_i^{[a]}\eta_j^{[a]*}\right)$

- Monte Carlo estimate is now

$$M_{ij}^{-1} \approx \frac{1}{N_R}\sum_{r=1}^{N_R}\sum_a X_i^{(r)[a]}\eta_j^{(r)[a]*}$$

- $\sum_a \eta_i^{[a]}\eta_j^{[a]*}$ has same expected value as $\eta_i\eta_j^*$, but reduced variance (statistical zeros → exact)

Dilution for products of matrix inverses

- in baryon correlators, need estimates of $M^{-1}_{ij}M^{-1}_{kl}M^{-1}_{mn}$
- introduce independent noise vectors for each quark line for unbiased estimate

$$M^{-1}_{ij}M^{-1}_{kl}M^{-1}_{mn} \approx \frac{1}{N_R}\sum_{r=1}^{N_R}\sum_{abc} X_i^{(1,r)[a]}\eta_j^{(1,r)[a]*}\ X_k^{(2,r)[b]}\eta_l^{(2,r)[b]*}\ X_m^{(3,r)[c]}\eta_n^{(3,r)[c]*}$$

- take average of permutations of quark line indices 123 for increased statistics

Source-sink factorization

- baryon correlator has form

$$C_{l\bar{l}} = c_{ijk}^{(l)} c_{\bar{i}\bar{j}\bar{k}}^{(\bar{l})*} Q_{i\bar{i}}^{A} Q_{j\bar{j}}^{B} Q_{k\bar{k}}^{C}$$

- stochastic estimates with dilution

$$C_{l\bar{l}} \approx \frac{1}{N_R} \sum_{r} \sum_{d_A d_B d_C} c_{ijk}^{(l)} c_{\bar{i}\bar{j}\bar{k}}^{(\bar{l})*} \left(\varphi_i^{(Ar)[d_A]} \eta_{\bar{i}}^{(Ar)[d_A]*} \right) \times \left(\varphi_j^{(Br)[d_B]} \eta_{\bar{j}}^{(Br)[d_B]*} \right) \left(\varphi_k^{(Cr)[d_C]} \eta_{\bar{k}}^{(Cr)[d_C]*} \right)$$

- define

$$\Gamma_l^{(r)[d_A d_B d_C]} = c_{ijk}^{(l)} \, \varphi_i^{(Ar)[d_A]} \varphi_j^{(Br)[d_B]} \varphi_k^{(Cr)[d_C]}$$

$$\Omega_l^{(r)[d_A d_B d_C]} = c_{ijk}^{(l)} \, \eta_i^{(Ar)[d_A]} \eta_j^{(Br)[d_B]} \eta_k^{(Cr)[d_C]}$$

- correlator becomes dot product of source vector with sink vector

$$C_{l\bar{l}} \approx \frac{1}{N_R} \sum_{r} \sum_{d_A d_B d_C} \Gamma_l^{(r)[d_A d_B d_C]} \, \Omega_{\bar{l}}^{(r)[d_A d_B d_C]*}$$

- store *ABC* permutations to handle Wick orderings

Dilution schemes for spectroscopy

- Time dilution (particularly effective)

$$P^{(B)}_{a\alpha;b\beta}\left(\vec{x},t;\vec{y},t'\right)=\delta_{ab}\delta_{\alpha\beta}\delta\left(\vec{x},\vec{y}\right)\delta_{Bt}\delta_{Bt'},\qquad B=0,1,\ldots,N_t-1$$

- Spin dilution

$$P^{(B)}_{a\alpha;b\beta}\left(\vec{x},t;\vec{y},t'\right)=\delta_{ab}\delta_{B\alpha}\delta_{B\beta}\delta\left(\vec{x},\vec{y}\right)\delta_{tt'},\qquad B=0,1,2,3$$

- Color dilution

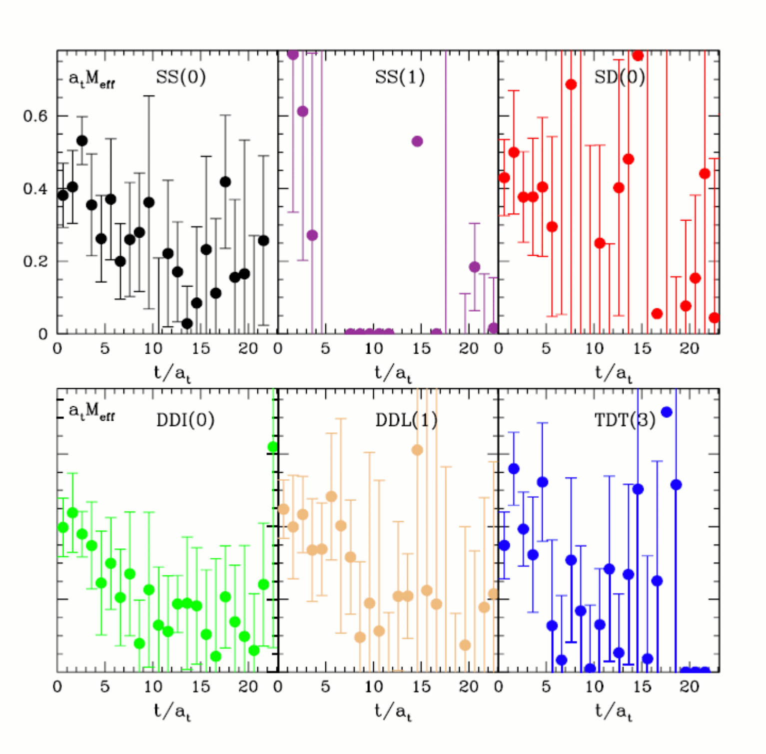
$$P^{(B)}_{a\alpha;b\beta}\left(\vec{x},t;\vec{y},t'\right)=\delta_{Ba}\delta_{Bb}\delta_{\alpha\beta}\delta\left(\vec{x},\vec{y}\right)\delta_{tt'},\qquad B=0,1,2$$

- Spatial dilutions?

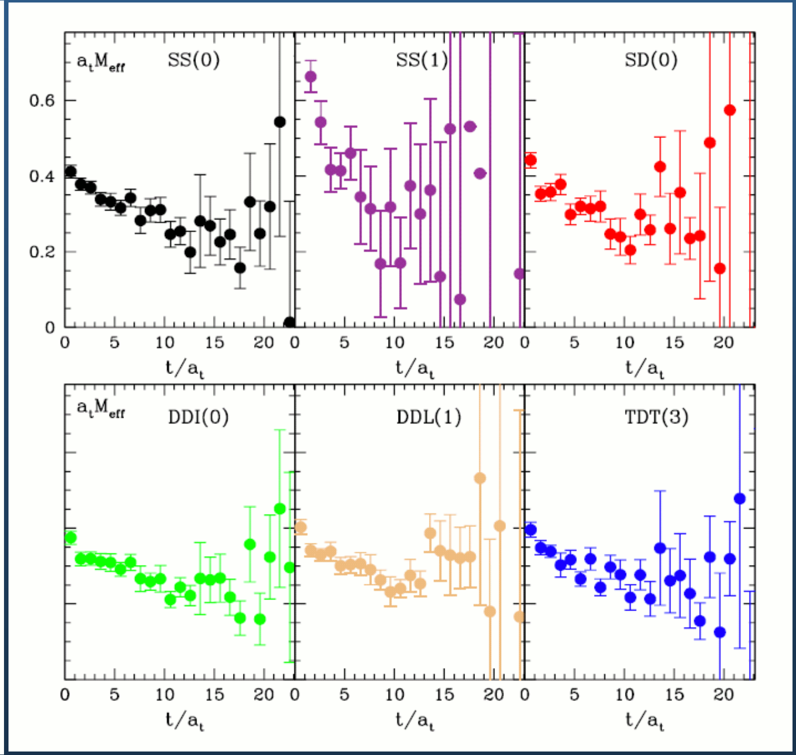
Dilution tests

- 20 quenched configs, $12^3 \times 48$ anisotropic Wilson lattice, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3$, $m_\pi \sim 700$ MeV (PRELIMINARY)
- nucleon G_{1g} channel

Time dilution



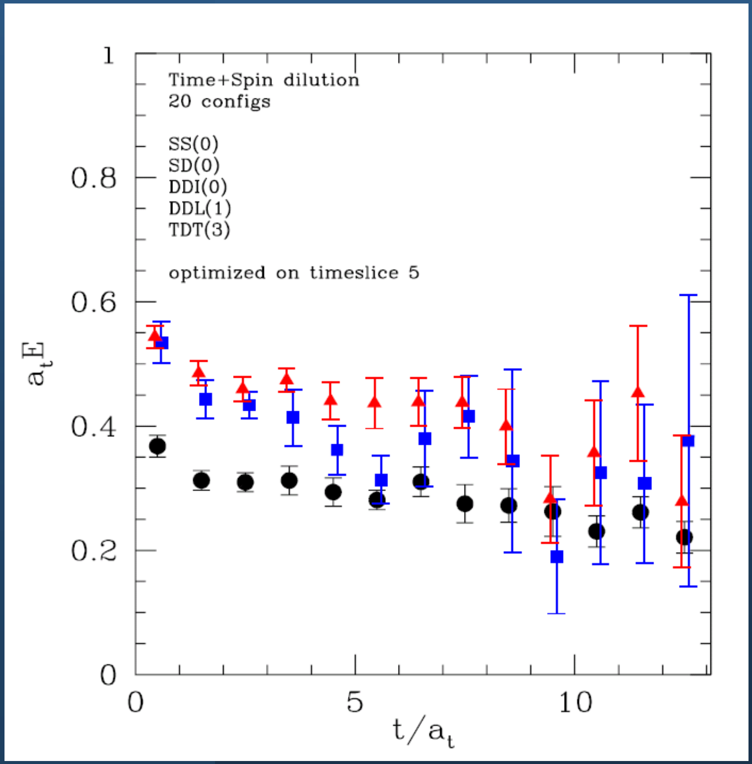
Time + spin dilution



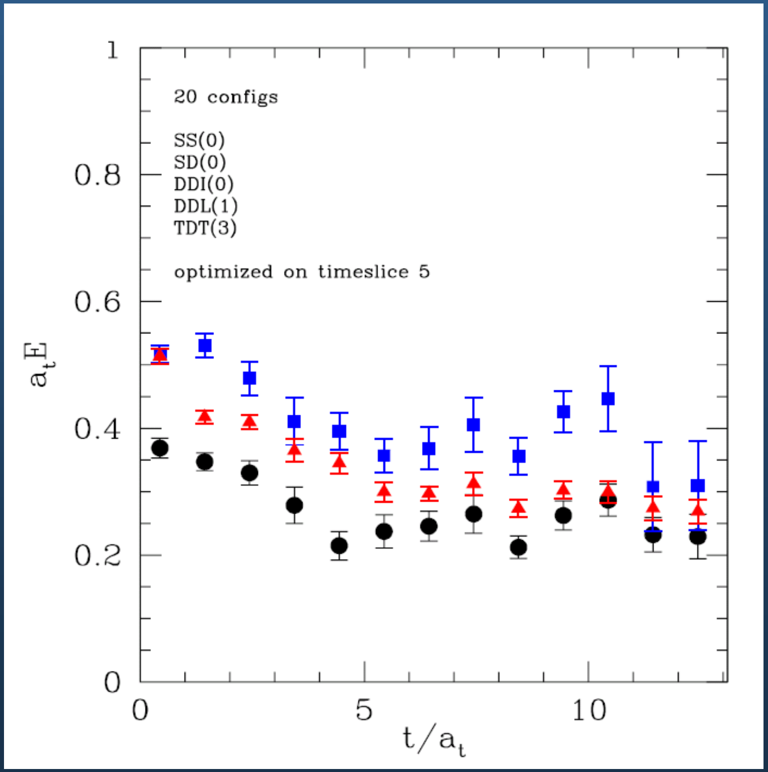
Dilution tests (2)

- 20 quenched configs, $12^3 \times 48$ anisotropic Wilson lattice, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3$, $m_\pi \sim 700$ MeV (PRELIMINARY)
- nucleon G_{1g} channel (lowest three principal effective masses)

Time + spin dilution



Point-to-all



Future work on dilutions

- Much work to do exploring stochastic quark propagators with dilutions
  - different dilution schemes
  - number of noise vectors
  - low-lying eigenmodes
  - dependence on lattice spacing, quark masses
- Software effort must be completed first! (soon!!)
- Study three-quark baryon and quark-antiquark meson operators first
- Multi-hadron operators important milestone
- Disconnected diagrams

Configuration generation

- Significant time on USQCD (DOE) and NSF computing resources
- Anisotropic clover fermion action (with stout links) and anisotropic improved gauge action
  - Tunings of couplings, aspect ratio, lattice spacing in progress
- Anisotropic Wilson action configurations generated during clover tuning
- Three lattice spacings: $a = 0.125$ fm, 0.10 fm, 0.08 fm
- Three volumes: $V = (3.2\text{ fm})^4, (4.0\text{ fm})^4, (5.0\text{ fm})^4$
- 2+1 flavors, $m_\pi \sim 350$ MeV, 220 MeV, 180 MeV
- USQCD Chroma software suite

Summary

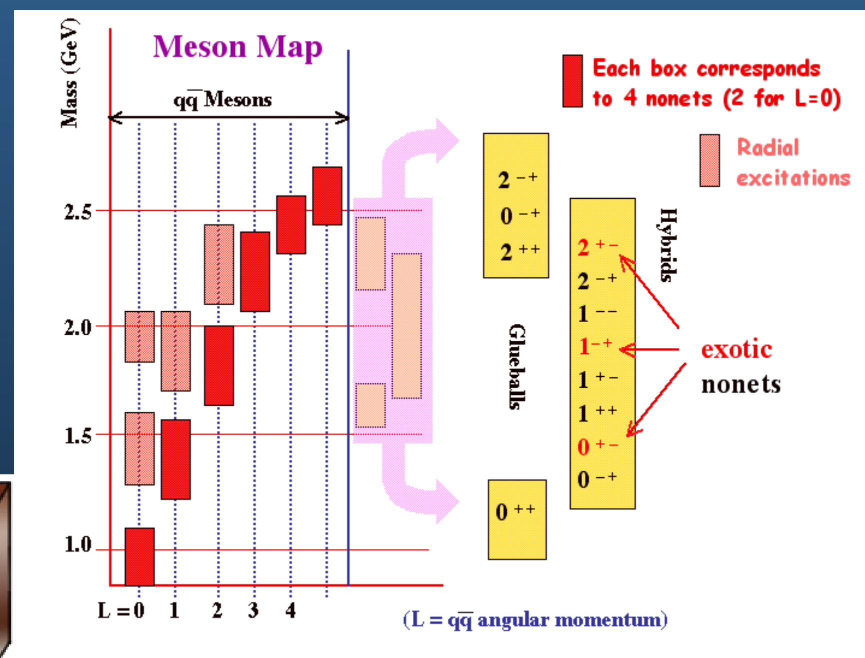
- outlined ongoing efforts of LHPC to extract baryon spectrum using Monte Carlo methods on a space-time lattice
  - mesons (and hybrids), tetraquarks, …to be studied as well
- emphasized need for correlation matrices to extract spectrum
- spin identification must be addressed
- as light-quark mass decreases, inclusion of multi-hadron operators will become important
- very challenging calculations
- …to be continued

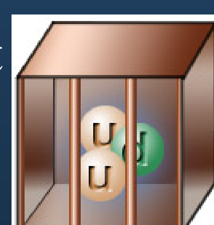
Review of exotics (briefly)

- Gluonic excitations
- Exotic quantum numbers
- Results from 2003 and earlier
- Focus on three new studies
  - Dudek, Edwards, Mathur, Richards hep-lat/0611006
  - Hedditch et al., Phys.Rev.D**72**, 114507 (2005)
  - McNeile et al., Phys.Rev.D**73**, 074506 (2006)

Gluonic excitations (new form of matter)

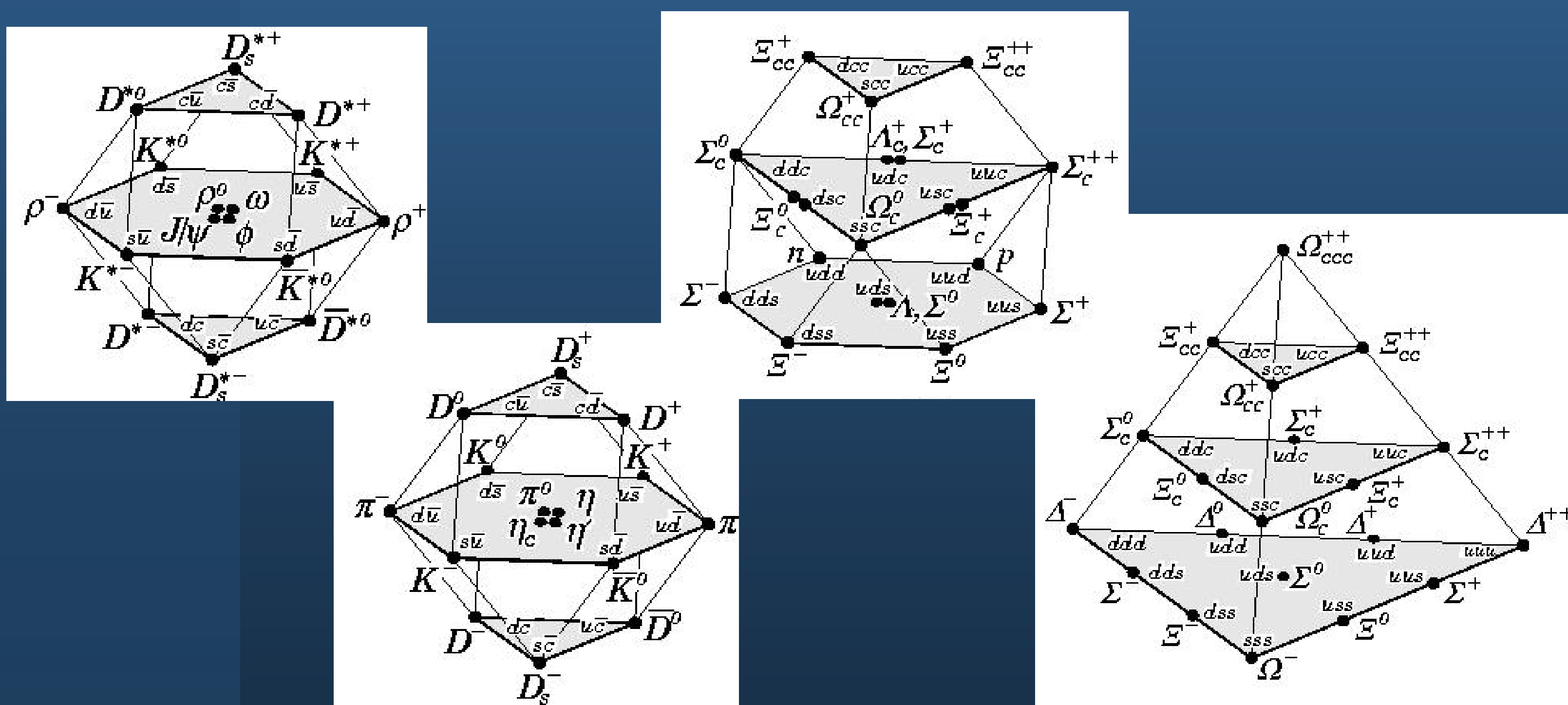
- QCD suggests existence of states in which *gluon* field is excited
  - glueballs (*excited glue*)
  - hybrid mesons ($q\bar{q}$ + *excited glue*)
  - hybrid baryons (qqq + *excited glue*)
- such states not well understood
  - quark model fails
  - perturbative methods fail
- lack of understanding makes identification difficult!
- confront gluon field behavior
  - bags, strings, …
- clues to confinement





Constituent quark model

- much of our understanding of hadron formation comes from the *constituent quark model*
  - motivated by QCD
  - valence quarks interacting via Coulomb + linear potential
  - gluons: source of the potential, *dynamics ignored*



Quark model (continued)

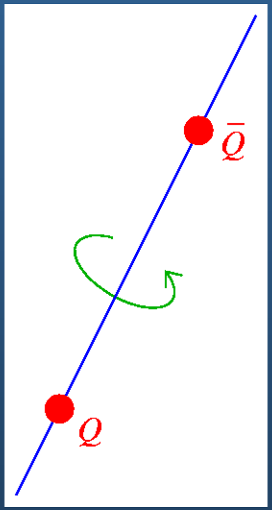
- *most* of observed low-lying hadron spectrum described reasonably well by quark model
  - agreement is amazing given the crudeness of the model
- mesons: only certain J^{PC} allowed:
  - $0^{+-}, 0^{--}, 1^{-+}, 2^{+-}, 3^{-+}, 4^{+-}, \ldots$ forbidden

$$P = (-1)^{L+1} \qquad L = 0,1,2,\ldots$$
$$C = (-1)^{L+S} \qquad S = 0,1$$

- experimental results now need input beyond the quark model
  - over-abundance of states
  - forbidden 1^{-+} states

Heavy-quark hybrid mesons

- more amenable to theoretical treatment than light-quark hybrids
- early work: Hasenfratz, Horgan, Kuti, Richard (1980), Michael, Griffiths, Rakow (1983)
- possible treatment like diatomic molecule (Born-Oppenheimer)
  - slow heavy quarks ←→ nuclei
  - fast gluon field ←→ electrons
    (and light quarks)



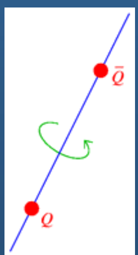
- gluons provide adiabatic potentials $V_{Q\bar{Q}}(r)$
  - gluons fully relativistic, interacting
  - potentials computed in lattice simulations
- nonrelativistic quark motion described in *leading order* by solving Schrodinger equation for each $V_{Q\bar{Q}}(r)$

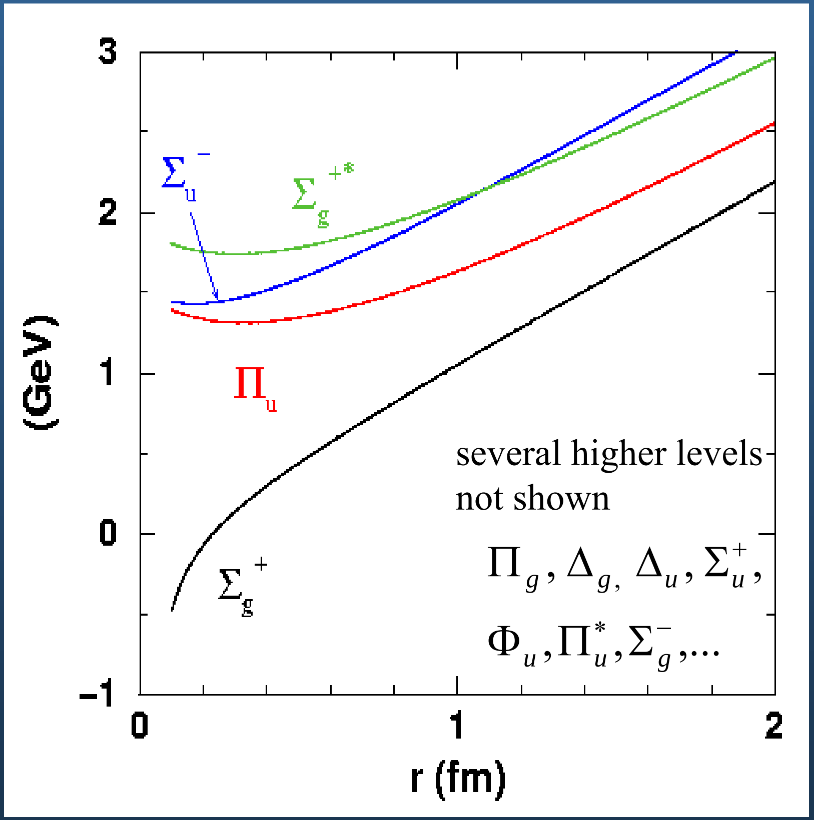
$$\left\{\frac{p^2}{2\mu}+V_{Q\bar{Q}}(r)\right\}\psi_{Q\bar{Q}}(r)=E\psi_{Q\bar{Q}}(r)$$

- conventional mesons from Σ_g^+; hybrids from $\Pi_u, \Sigma_u^-, \ldots$

Excitations of static quark potential

- gluon field in presence of static quark-antiquark pair can be *excited*
- classification of states: (notation from molecular physics)
  - magnitude of glue spin projected onto molecular axis
    $\Lambda = 0,1,2,\ldots$
    $= \Sigma, \Pi, \Delta, \ldots$
  - charge conjugation + parity about midpoint
    $\eta = g$ (even)
    $= u$ (odd)
  - chirality (reflections in plane containing axis) Σ^+, Σ^-
    $\Pi, \Delta, \ldots$ doubly degenerate
    (Λ doubling)





Juge, Kuti, Morningstar, PRL 90, 161601 (2003)

Leading Born-Oppenheimer

- replace covariant derivative $\vec{D}^2$ by $\vec{\nabla}^2$ → neglects retardation
- neglect quark spin effects
- solve radial Schrodinger equation

$$\frac{-1}{2\mu}\frac{d^2u(r)}{dr^2}+\left\{\frac{\langle L_{q\bar{q}}^2\rangle}{2\mu r^2}+V_{q\bar{q}}(r)\right\}u(r)=Eu(r)$$

- angular momentum

$$\vec{J}=\vec{L}+\vec{S} \qquad \vec{S}=\vec{s}_q+\vec{s}_{\bar{q}} \qquad \vec{L}=\vec{L}_{q\bar{q}}+\vec{J}_g$$

- in LBO, L and S are good quantum numbers
- centrifugal term

$$\langle \vec{L}_{q\bar{q}}^2\rangle = L(L+1)-2\Lambda^2+\langle \vec{J}_g^2\rangle$$

$$\langle \vec{J}_g^2\rangle = 0 \quad (\Sigma_g^+)$$
$$= 2 \quad (\Pi_u, \Sigma_u^-)$$

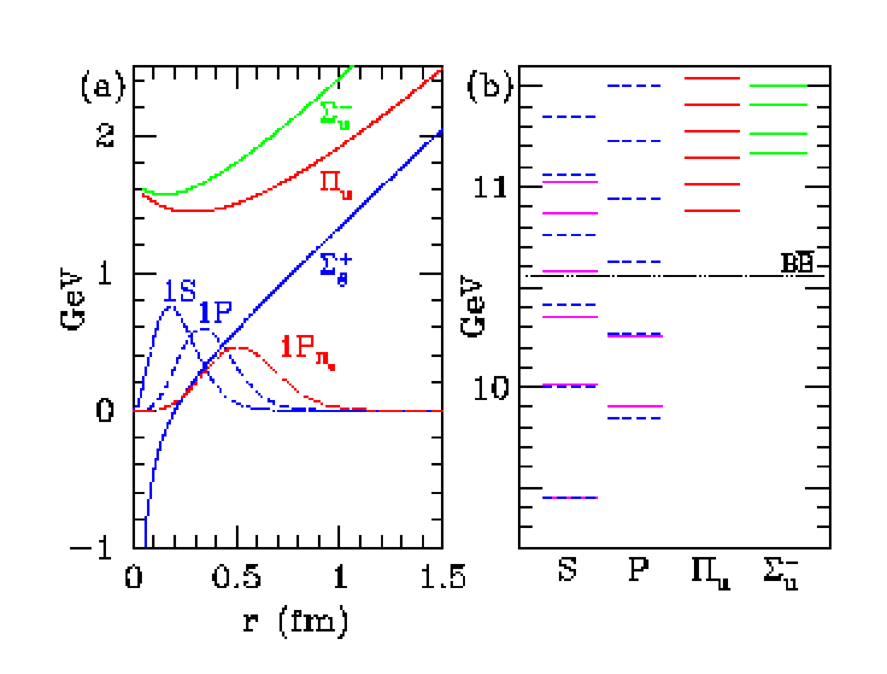
- J^{PC} eigenstates → Wigner rotations

$$|LSJM;\Lambda\eta\rangle+\varepsilon|LSJM;-\Lambda\eta\rangle$$

  - η is CP, $\varepsilon=\pm1$ for $\Lambda\ge1$, $\varepsilon=\pm1$ for $\Sigma^\pm$
- LBO allowed J^{PC} → $P=\varepsilon(-1)^{L+\Lambda+1}$, $C=\eta\varepsilon(-1)^{L+S+\Lambda}$

Leading Born-Oppenheimer spectrum

- results obtained (in absence of light quark loops)
- good agreement with experiment below $B\bar{B}$ threshold
- plethora of hybrid states predicted (caution! quark loops)
- but is a Born-Oppenheimer treatment valid?



LBO degeneracies:

$\Sigma_g^+(S):\quad 0^{-+}, 1^{--}$

$\Sigma_g^+(P):\quad 0^{++}, 1^{++}, 2^{++}, 1^{+-}$

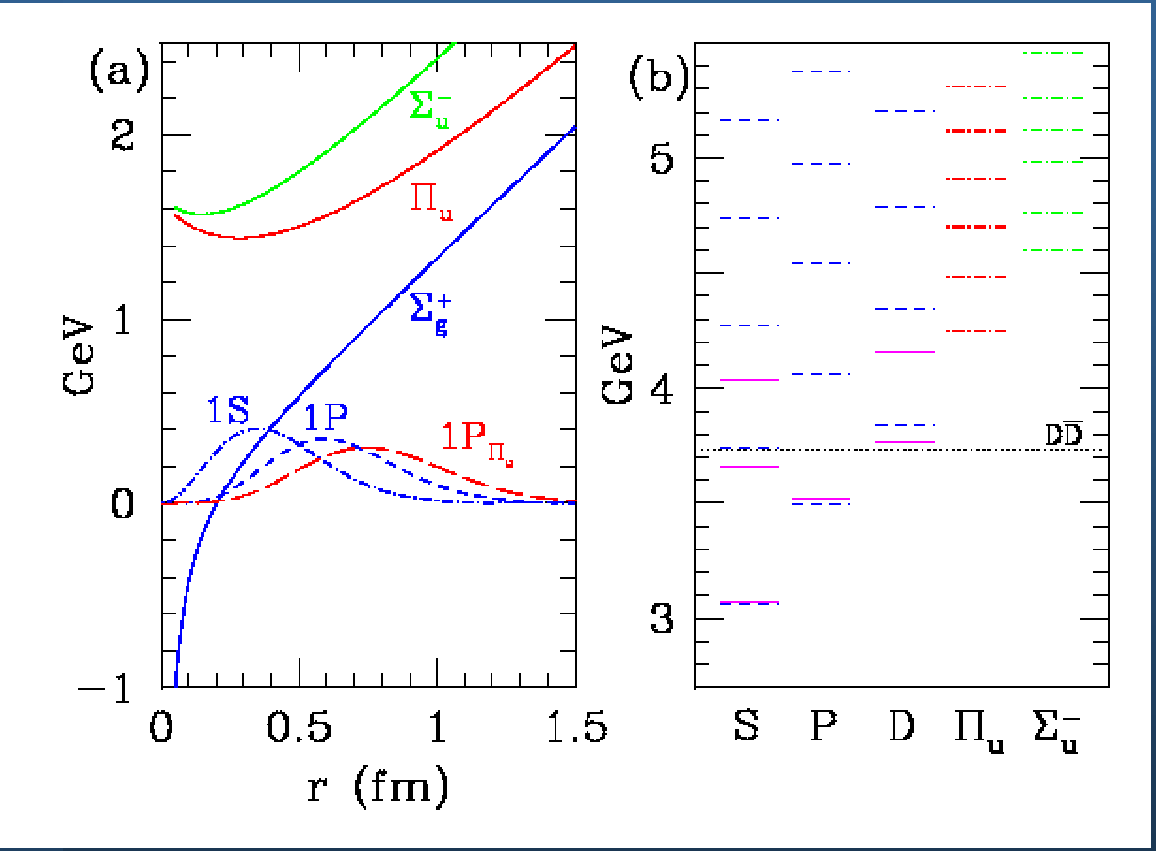
$\Pi_u(P):\quad 0^{-+}, 0^{+-}, 1^{++}, 1^{--},$

$1^{+-}, 1^{-+}, 2^{+-}, 2^{-+}$

Juge, Kuti, Morningstar, Phys Rev Lett **82**, 4400 (1999)

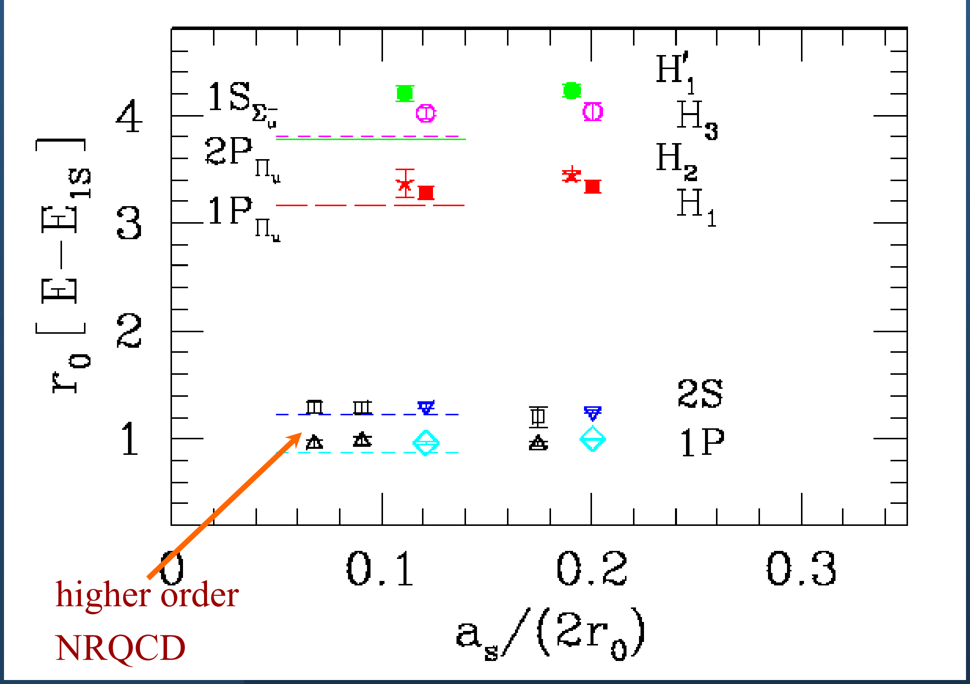
Charmonium LBO

- same calculation, but for charmonium



Testing LBO

- test LBO by comparison of spectrum with NRQCD simulations
  - include retardation effects, but no quark spin, no $\vec{p}^{\,4}$, no light quarks
  - allow possible mixings between adiabatic potentials
- dramatic evidence of validity of LBO
  - level splittings agree to 10% for 2 conventional mesons, 4 hybrids



$$H_1, H_1' = 1^{--}, 0^{-+}, 1^{-+}, 2^{-+}$$

$$H_2 = 1^{++}, 0^{+-}, 1^{+-}, 2^{+-}$$

$$H_3 = 0^{++}, 1^{+-}$$

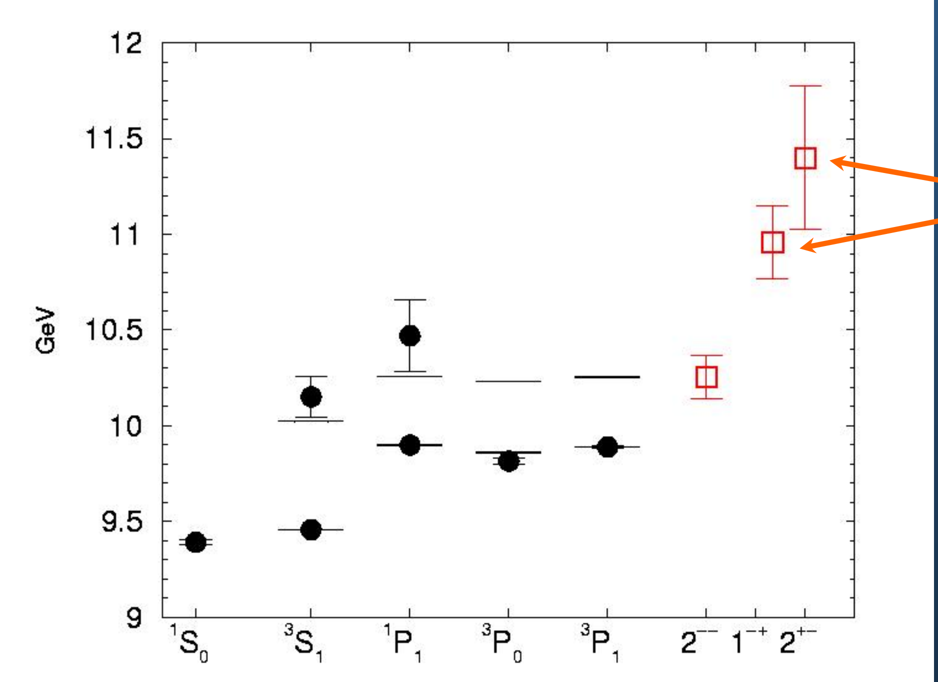
J^{PC}		Degeneracies	Operator
0^{-+}	S wave	1^{--}	$\tilde{\chi}^\dagger\ [\hat{\Delta}^{(2)}]^p\ \tilde{\psi}$
1^{+-}	P wave	$0^{++}, 1^{++}, 2^{++}$	$\tilde{\chi}^\dagger\ \hat{\Delta}\ \tilde{\psi}$
1^{--}	H_1 hybrid	$0^{-+}, 1^{-+}, 2^{-+}$	$\tilde{\chi}^\dagger\ \hat{B}\,[\hat{\Delta}^{(2)}]^p\ \tilde{\psi}$
1^{++}	H_2 hybrid	$0^{+-}, 1^{+-}, 2^{+-}$	$\tilde{\chi}^\dagger\ \hat{B}\times\hat{\Delta}\ \tilde{\psi}$
0^{++}	H_3 hybrid	1^{+-}	$\tilde{\chi}^\dagger\ \hat{B}\cdot\hat{\Delta}\ \tilde{\psi}$

lowest hybrid 1.49(2)(5) GeV above 1S

Bottomonium hybrids

- calculation of bottomonium hybrids in 2002 confirmed earlier results
  - quenched, several lattice spacings so $a \to 0$ limit taken
  - improved anisotropic gluon and fermion (clover) actions
  - good agreement with Born-Oppenheimer (but errors large)

Liao, Manke, PRD**65**, 074508 (2002)

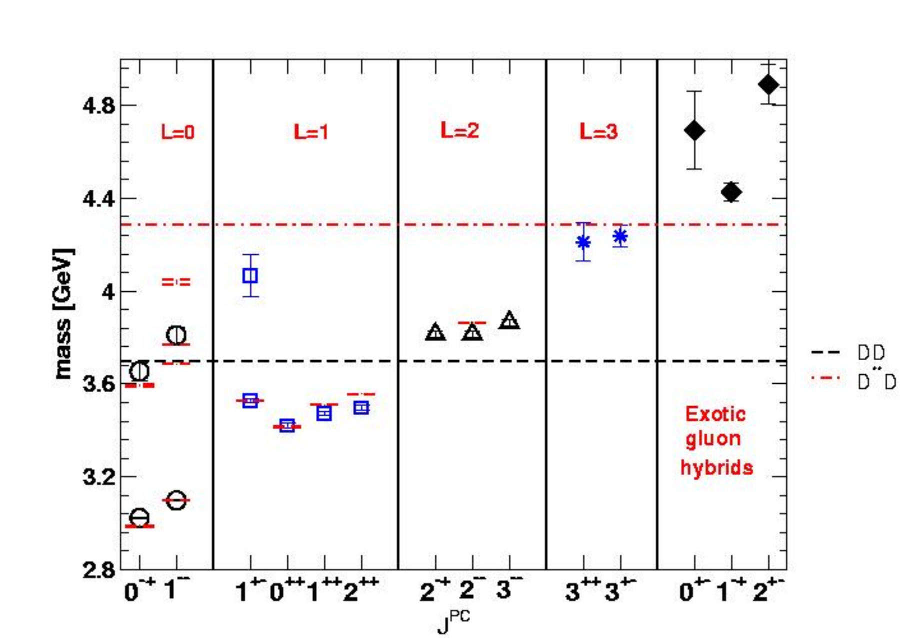


hybrids

Charmonium hybrids

- determination of some charmonium hybrids in 2002
  - quenched, several lattice spacings for continuum limit
  - improved, anisotropic gluon and fermion (clover) actions
  - results suggest significant (but not large) corrections from LBO

Liao, Manke, hep-lat/0210030



Light-quark hybrids

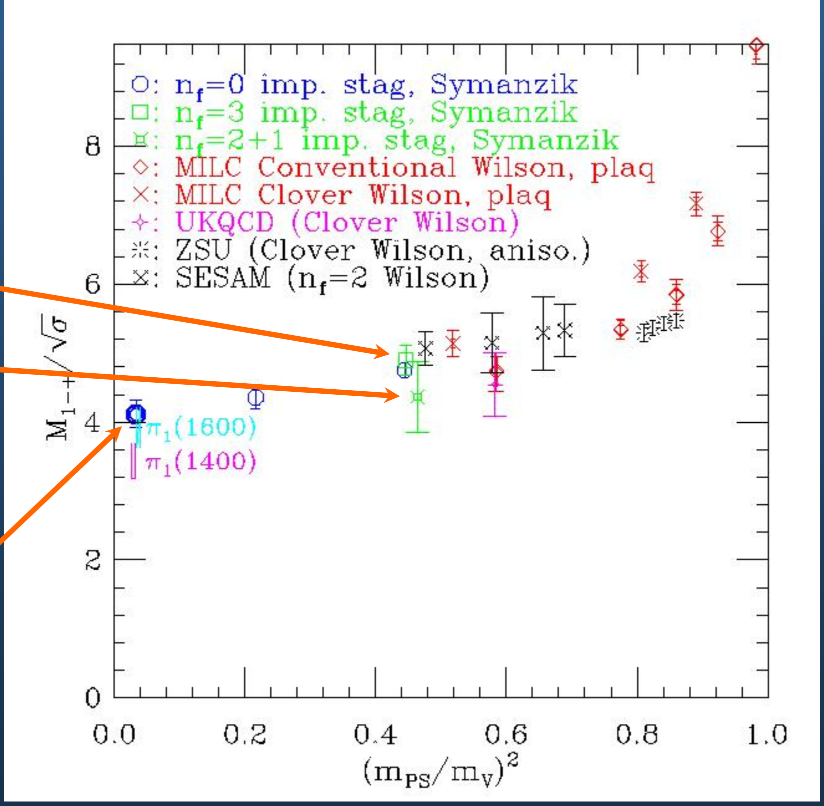
- determinations of exotic 1^{-+} hybrid meson from 2003 and earlier
  - improved staggered fermions (lighter quark masses)
  - quenched and unquenched, Wilson gluon action
  - $a \approx 0.09$ fm
  - lightest mass still above experiment

MILC, hep-lat/0301024

$N_f = 3, \quad m_u = m_d = m_s$
(around strange quark mass)

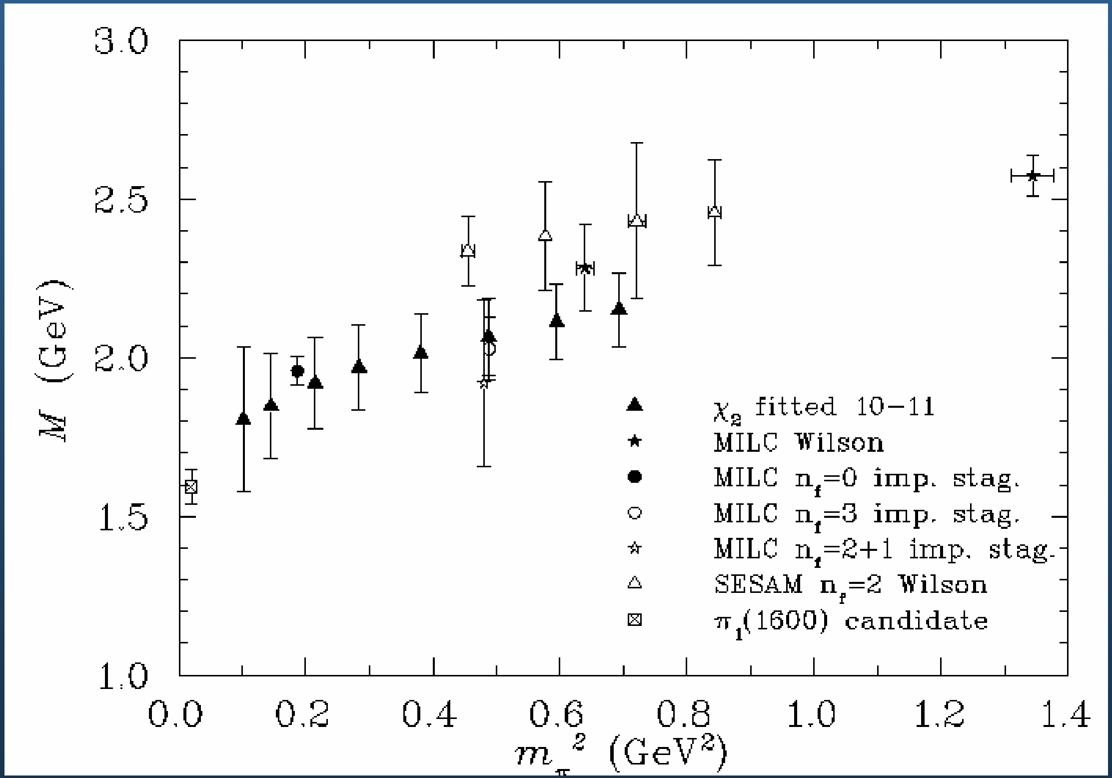
$m_u = m_d = 0.4 m_s$

quenched continuum limit



Recent light hybrid 1^{-+} meson study

- Hedditch et al., Phys.Rev.D**72**, 114507 (2005)
- quenched isotropic $20^3 \times 40$ lattice, FLIC fermions, improved gauge
- large errors, still not definitive



Light hybrid meson decay widths

- McNeile et al. (UKQCD) Phys.Rev. D**73**, 074506 (2006)
- N_f=2 dynamical clover fermions
- $m_\pi r_0 = 1.47$ and 1.29 (rather heavy)
- found hybrid 1^{-+} mass 2.2(2) GeV
- partial width to πb_1 of 400(120) MeV and to πf_1 of 90(6) MeV
- some evidence of coupling strength decrease as quark mass decreases

Charmonium exotics

- Dudek, Edwards, Mathur, Richards hep-lat/0611006
- quenched $12^3 \times 48$ anisotropic lattice $a = 0.1\text{fm}$, $a_s/a_t = 3$
- 1^{-+} mass around 4.2 GeV
  - lower than prior Manke study... but better effective mass
- signal for 0^{+-} and 2^{+-} exotics obtained (χ++, h+-, ψ--, η-+)