Probing the chiral regime with light dynamical fermions

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Outline

- Simulations with twisted mass fermions at maximal twist
  - dynamical quarks: $N_f = 2$ degenerate light flavours
  - physical masses: $m_\pi \sim 300$ MeV
  - continuum limit: $\mathcal{O}(a)$ improvement
  - large volumes: $L > 2$ fm
- Use of χ-PT :
  - quark mass dependence $\rightsquigarrow$ low-energy constants
  - finite volume corrections
- Determine fundamental parameters of QCD :

  $m_q, \ \langle \bar{q}q \rangle, \ldots$
- Outline:
  - setup
  - light-quarks sector
  - strange-quark sector
  - charm-quark sector
  - baryons
  - form factors

Simulations 3 lattice spacings

β	target a (fm)	$L^3 \cdot T$	target L (fm)	$a\mu$	N_{traj} $(\tau = 0.5)$	target m_{PS} (MeV)
4.05	~ 0.066	$32^3 \cdot 64$	2.2	0.0030	5200	~ 300
				0.0060	5600	~ 420
				0.0080	5300	~ 480
				0.0120	5000	~ 600
		$24^3 \cdot 48$	1.6	0.0060	3000×2	~ 420
		$20^3 \cdot 48$	1.3	0.0060	5300×2	~ 420
3.9	~ 0.086	$24^3 \cdot 48$	2.1	0.0040	10500	~ 300
				0.0064	5600	~ 380
				0.0085	5000	~ 440
				0.0100	5000	~ 480
				0.0150	5400	~ 590
		$32^3 \cdot 64$	2.8	0.0040	5000	~ 300
3.8	~ 0.100	$24^3 \cdot 48$	2.4	0.0060	4700×2	~ 360
				0.0080	3000×2	~ 410
				0.0110	2800×2	~ 480
				0.0165	2600×2	~ 580
		$20^3 \cdot 48$	2.0	0.0060	4000×2	~ 360

Correlators analysis techniques

- quark propagators : stochastic sources to include all spatial sources
- change the location of the time-slice source : reduce autocorrelations
- set of interpolating operators (π : $\bar{\psi}\gamma_5\tau_1\psi$, $\bar{\psi}\gamma_0\gamma_5\tau_1\psi$, $\bar{\psi}\gamma_0\tau_2\psi$) and smearing
- autocorrelation times $\tau_{\rm int}$:

β	target a (fm)	$a\mu$	$\tau_{\rm int}(P)$	source	$\tau_{\rm int}(am_{\rm PCAC})$	$\tau_{\rm int}(am_{\rm PS})$	$\tau_{\rm int}(af_{\rm PS})$
3.9	$\sim$ 0.086	0.0040	47(15)	random	23(05)	6(1)	7(1)
				cyclic	60(24)	7(1)	13(4)
		0.0085	13(3)	cyclic	66(27)	10(2)	11(2)

light-quarks sector

m_π f_π

LEC of χPT

$m_{u,d}$ $\langle\bar{q}q\rangle$

Charged pion: decay constant and χPT fits

Pseudo-scalar decay constant:

$$f_{\rm PS} = \frac{2\mu}{m_{\rm PS}^2} |\langle 0|P^1(0)|\pi^{\pm}\rangle|$$

- obtained from exact lattice Ward identity for maximally twisted mass fermions
- no need of renormalization factors : $Z_{\rm P} = 1/Z_\mu$

chiral perturbation theory (χPT)

- Use of continuum χPT to describe the dependence on :
  - the mass μ
  - finite spatial size L
- Simultaneous fit to $N_{\rm f} = 2$ χPT at NLO (Gasser, Leutwyler, 1987; Colangelo *et al.*, 2005)

$$m_{\rm PS}^2(L) = 2B_0\mu \left[1 + \frac{1}{2}\xi\tilde{g}_1(\lambda)\right]^2 \left[1 + \xi \ln(2B_0\mu/\Lambda_3^2)\right] ,$$

$$f_{\rm PS}(L) = f_0 \ [1 - 2\xi\tilde{g}_1(\lambda)] \left[1 - 2\xi \ln(2B_0\mu/\Lambda_4^2)\right]$$

where $\xi = 2B_0\mu/(4\pi f_0)^2$, $\lambda = \sqrt{2B_0\mu L^2}$, $f_0 = \sqrt{2}F_0$, $\tilde{g}_1(\lambda)$ is a known function

- fit parameters: B_0, f_0, Λ_3 and Λ_4
- extract low-energy constants: $\bar{l}_{3,4} \equiv \log(\Lambda_{3,4}^2/m_{\pi^\pm}^2)$
- $\mathcal{O}(a^2)$ effects appear at NNLO only

$\mathcal{O}(a^2)$ effects in χPT : $f_{\rm PS}$ and $m^2_{\rm PS}$

Power counting for lattice χPT:

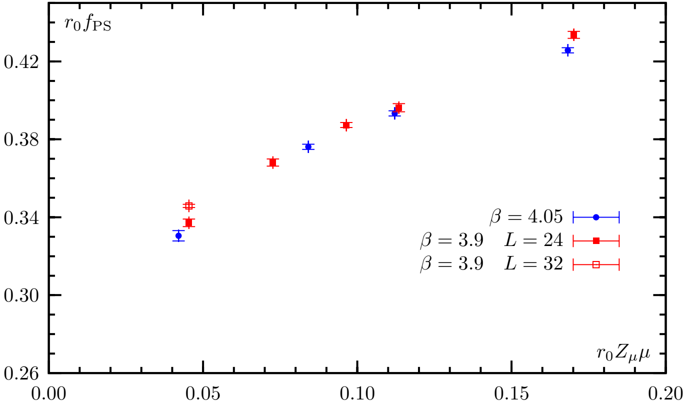
$$a \sim \mu \sim m_\pi^2 \sim p^2$$

$\mathcal{O}(a^2)$ effects appear at NNLO only

- $f_\pi|^{\rm Mtm} = f_\pi|^{\rm cont.} + \underset{\text{(NNLO)}}{\mathcal{O}(a^2)} + \underset{\text{(N}^3\text{LO)}}{\mathcal{O}(a^2 m_\pi^2)}$
- $m_\pi^2|^{\rm Mtm} = m_\pi^2|^{\rm cont.} + \underset{\text{(NNLO)}}{\mathcal{O}(a^2 m_\pi^2)} + \underset{\text{(N}^3\text{LO)}}{\mathcal{O}(a^4)}$

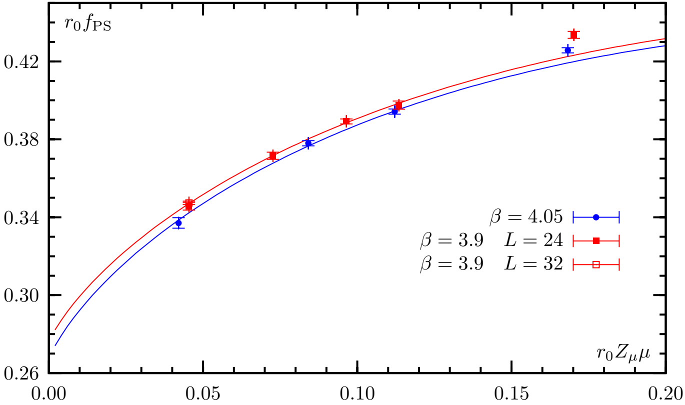
Consistent use of continuum χPT at NLO

χPT fits: $f_{\rm PS}$ vs. $Z_\mu \mu$ — $\beta = 4.05$ and 3.9



we use at $\beta = 4.05$: $r_0/a = 6.61(3)$
$\beta = 3.9$: $r_0/a = 5.22(2)$

χPT fits: f_{PS} vs. $Z_\mu \mu$ $\beta = 4.05$ and 3.9



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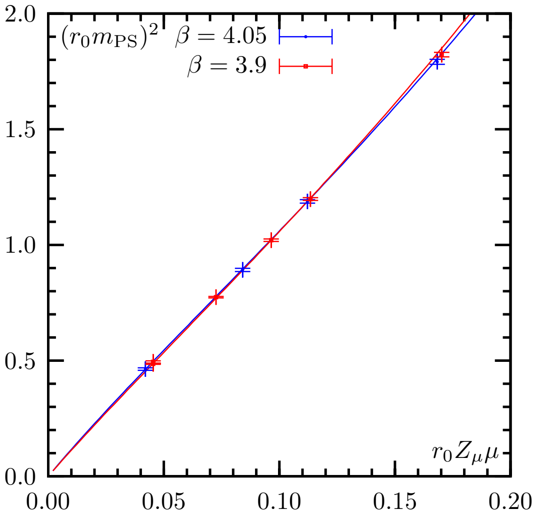
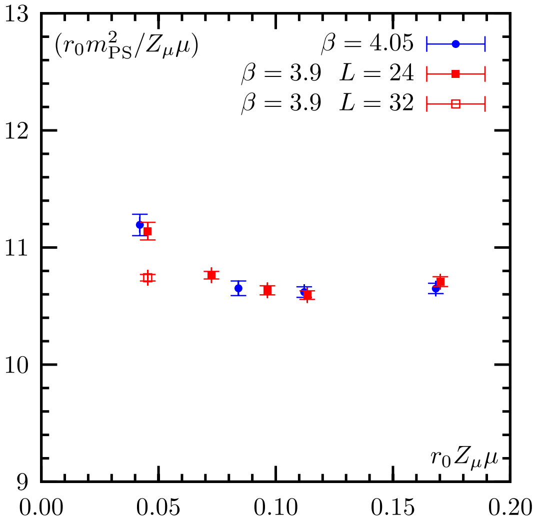
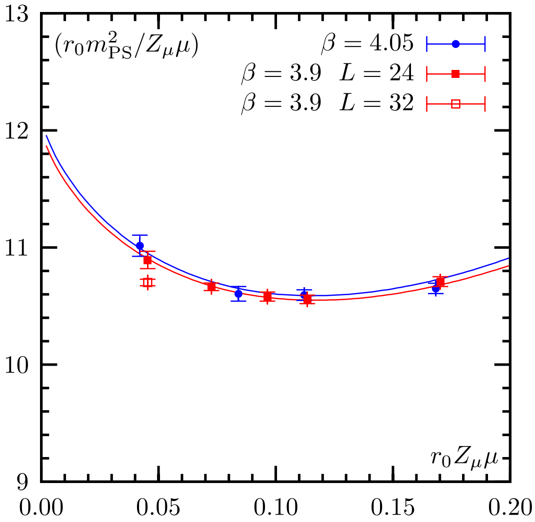


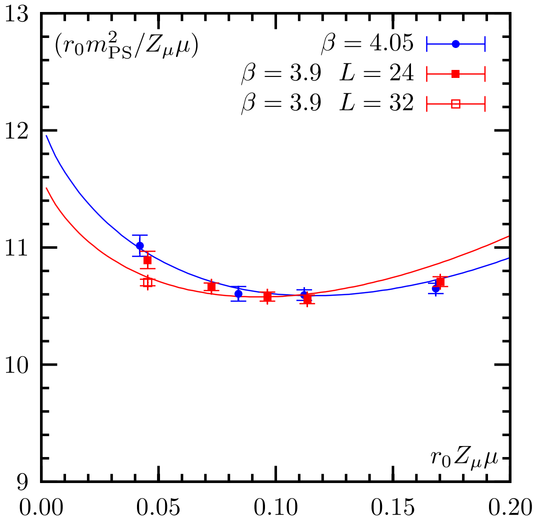
χPT fits: $m_{\rm PS}^2/(Z_\mu \mu)$ vs. $Z_\mu \mu$ $\beta = 4.05$ and 3.9



χPT fits: $m^2_{\rm PS}/(Z_\mu\mu)$ vs. $Z_\mu\mu$ $\beta = 4.05$ and 3.9



χPT fits: $m_{PS}^2/(Z_\mu\mu)$ vs. $Z_\mu\mu$ — $\beta = 4.05$ and 3.9



χPT fits: results $\beta = 3.9$ and 4.05

β	parameter	This work	[hep-lat/0701012)
4.05	$2aB_0$	3.88(7)	-
	af_0	0.0404(7)	-
	$\chi^2_{\rm red}$	0.8	-
	$\bar{l}_3$	3.70(16)	-
	$\bar{l}_4$	4.65(5)	-
3.9	$2aB_0$	4.84(3)	4.99(6)
	af_0	0.0528(4)	0.0534(6)
	$\chi^2_{\rm red}$	1.3	0.15
	$\bar{l}_3$	3.37(8)	3.65(12)
	$\bar{l}_4$	4.60(3)	4.52(6)

- The "physical point" $a\mu_\pi$ is determined by requiring $m_{\rm PS}/f_{\rm PS} = 135/130.7 = 1.033 \rightsquigarrow$ *e.g* at $\beta = 3.9$ we get : $a\mu_\pi = 0.00073(2)$
- Taking $f_\pi = 130.7$ MeV, we obtain : $a = 0.0858(5)$ fm
- Using $r_0/a = 5.22(2)$ we get : $r_0 = 0.448(3)$ fm

Combined fits $\beta = 3.9$ and 4.05

$$m^2_{\rm PS} = 2B_0\mu \left[1 + \xi \ln(2B_0\mu/\Lambda_3^2)\right]$$

$$f_{\rm PS} = f_0 \left[1 - 2\xi \ln(2B_0\mu/\Lambda_4^2)\right]$$

where $\xi = 2B_0\mu/(4\pi f_0)^2$, $f_0 = \sqrt{2}F_0$,

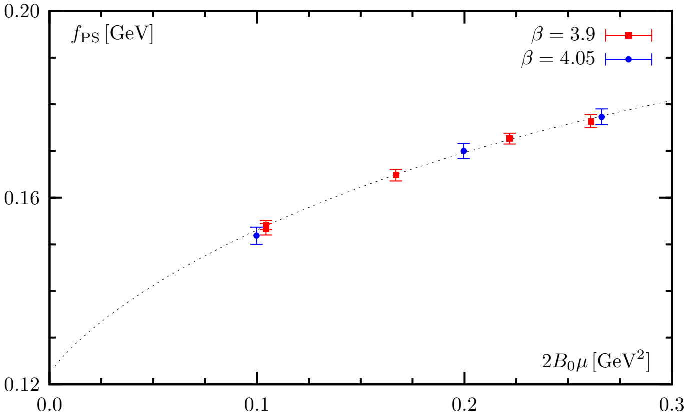
- 6 fit parameters: $(aB_0)|_{\beta=3.9}$, $(aB_0)|_{\beta=4.05}$, $(af_0)|_{\beta=3.9}$, $(af_0)|_{\beta=4.05}$, Λ_3/f_0 and Λ_4/f_0
- 16 data points ; higher masses ($m_{\rm PS} \sim 600$ MeV) not included in the fit
- Precise results for the LEC

β	3.9	4.05	combined	
$\chi^2_{\rm red}$	1.3	0.8	1.2	
a (fm)	0.0858(5)	0.0657(11)	0.0855(5)(3)	0.0666(6)(9)
$\bar{l}_3$	3.37(8)	3.70(16)	3.44(8)(26)(6)	
$\bar{l}_4$	4.60(3)	4.65(5)	4.61(4)(3)(7)	

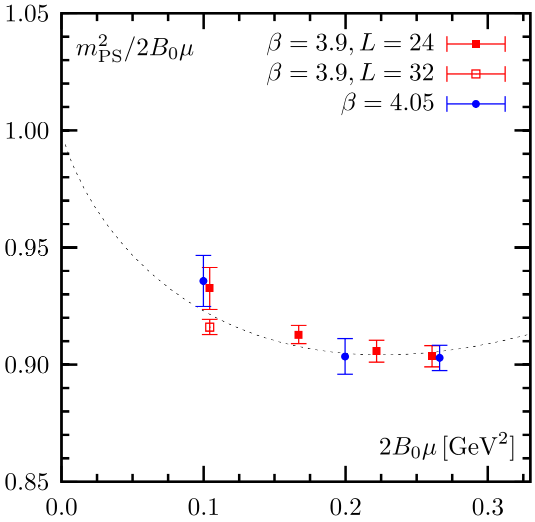
Consistent with independent measurements:

- $a|_{\beta=3.9}/a|_{\beta=4.05} = 1.284(14)(18)$ $\qquad (r_0/a)|_{\beta=4.05}/(r_0/a)|_{\beta=3.9} = 1.266(8)$
- fit : $Z_\mu|_{\beta=3.9}/Z_\mu|_{\beta=4.05} = 1.007(17)$ $\qquad$ RI-MOM : $Z_\mu|_{\beta=3.9}/Z_\mu|_{\beta=4.05} = 1.05(3)(7)$

Combined fits: $f_{\rm PS}$ vs. $2B_0\mu$ $\beta = 4.05$ and 3.9



Combined fits: f_{PS} vs. $2B_0\mu$ $\beta = 4.05$ and 3.9



Continuum extrapolation: f_{PS}

Strategy:

- Bring volumes to a reference volume: $L_{\mathrm{ref}} = 2.2$ fm
- Interpolate data points to some reference pion masses:

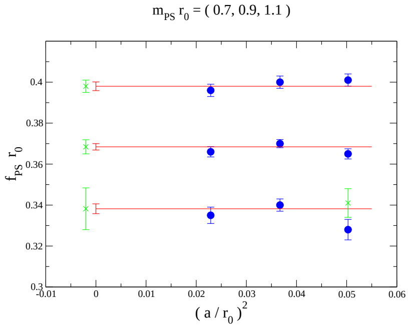
  $$m_{\mathrm{PS}} r_0 = (0.7, 0.8, 0.9, 1.0, 1.1, 1.25)$$

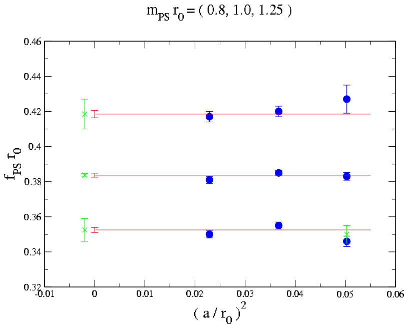
- Estimate continuum limit by extrapolating at fixed volume:

  Weighted average of data at $\beta = 4.05$ and 3.9

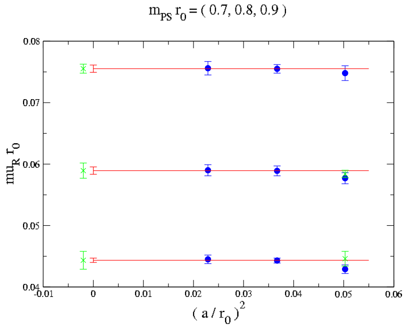
  Use the coarse lattice ($\beta = 3.8$) to include a systematic error

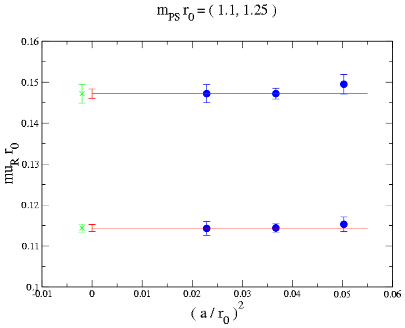
Continuum extrapolation: f_{PS}





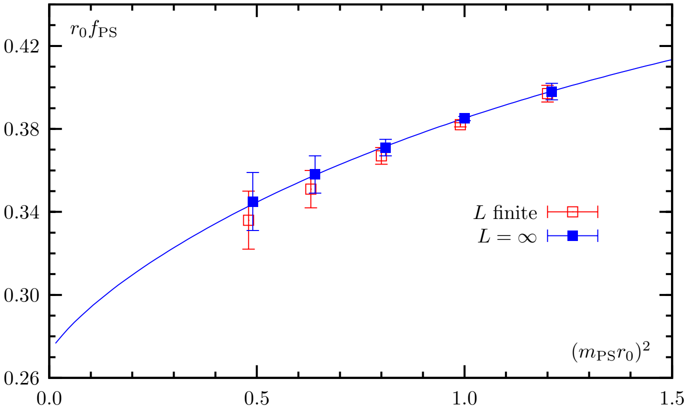
Continuum extrapolation: μ_R





Chiral fits in the continuum

PRELIMINARY



β	3.9	4.05	combined	continuum
r_0 (fm)	0.448(3)	0.434(7)	0.443(5)[9]	0.441(14)
$\bar{l}_4$	4.60(3)	4.65(5)	4.61(4)(3)(7)	4.79(14)

low-energy constants (LEC)

Accurate determinations of $\bar{l}_{3,4} \equiv \log(\Lambda^2_{3,4}/m^2_{\pi^\pm})$

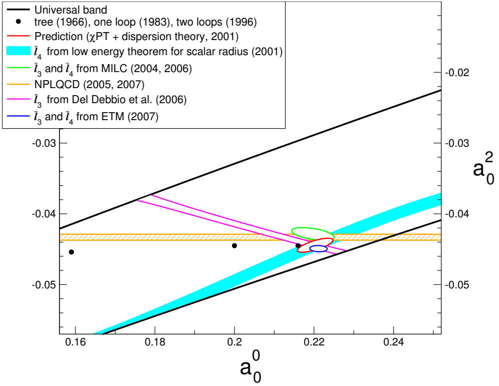
$$\bar{l}_3 = 3.44(8)(26)(6)$$
$$\bar{l}_4 = 4.61(4)(3)(7)$$

Other estimates (Leutwyler, hep-ph/0612112 ; lattice 2007)

- $\bar{l}_3$:
  - $\bar{l}_3 = 2.9 \pm 2.4$ from the mass spectrum of the pseudoscalar octet
  - $\bar{l}_3 = 0.8 \pm 2.3$ from MILC
  - $\bar{l}_3 = 3.0 \pm 0.5$ from CERN
  - $\bar{l}_3 = 3.49 \pm 0.12$ from QCDSF
  - $\bar{l}_3 = 2.9 \pm 0.5$ from JLQCD
  - $\bar{l}_3 = 3.13 \pm 0.33$ from RBC/UKQCD
- $\bar{l}_4$:
  - $\bar{l}_4 = 4.3 \pm 0.9$ from f_K/f_π
  - $\bar{l}_4 = 4.4 \pm 0.2$ from the radius of the scalar pion form factor
  - $\bar{l}_4 = 4.0 \pm 0.6$ from MILC
  - $\bar{l}_4 = 4.69 \pm 0.14$ from QCDSF
  - $\bar{l}_4 = 4.3 \pm 0.6$ from JLQCD
  - $\bar{l}_4 = 4.42 \pm 0.14$ from RBC/UKQCD

$\pi\pi$ scattering

S-wave scattering lengths a_0^0 and a_0^2 (Leutwyler, 2007)



- radius of the scalar pion form factor :
  - This work: $\langle r^2 \rangle = 0.637 \pm 0.026\ \mathrm{fm}^2$ (statistical)
  - Colangelo *et. al*, 2001 : $\langle r^2 \rangle = 0.61 \pm 0.04\ \mathrm{fm}^2$

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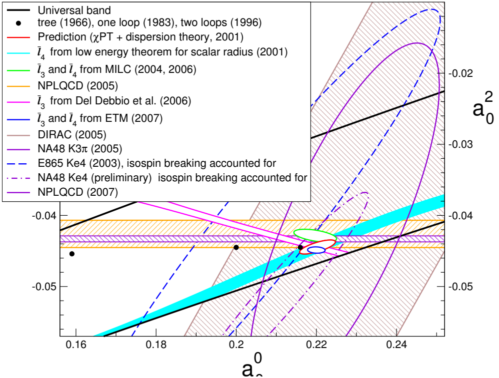

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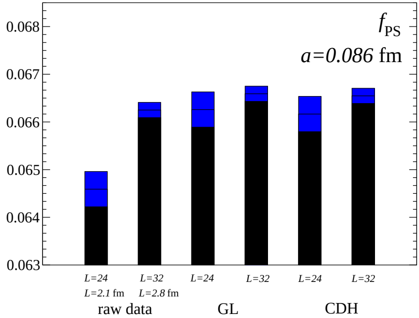
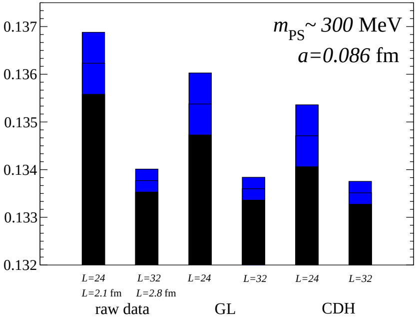
FSE : $f_{\rm PS}$ and $m_{\rm PS}$ $a\mu = 0.0040, \quad \beta = 3.9$

Data shows an exponential behaviour as a function of $m_{\rm PS}L$

Study of finite size corrections:





Renormalization and quark masses

- renormalization constants of bilinear quark operators : Z_P

  non-perturbatively (RI-MOM)

- u-d quark mass:

β	3.9	4.05	combined	
$a\mu_\pi$	0.000732(15)	0.000537(12)	0.000719(13)	0.000568(13)
$Z_P[\overline{\mathrm{MS}}, 2\ \mathrm{GeV}]$	0.46(1)(2)	0.44(1)(2)	-	-
$m_{u,d}[\overline{\mathrm{MS}}, 2\ \mathrm{GeV}]$ (MeV)	3.66(11)(16)	3.67(13)(24)	3.61(10)(16)	3.82(13)(24)

From PQ analysis: $\beta = 3.9$

$$m_{u,d}[\overline{\mathrm{MS}}, 2\ \mathrm{GeV}] = 4.01(12)(37)\ \mathrm{MeV}$$

Chiral condensate $\langle\bar{q}q\rangle$

From the χPT fits we extract:

$$\langle\bar{q}q\rangle = -Z_P F_0^2 B_0$$

- $\beta = 3.90:\quad (-\langle\bar{q}q\rangle)^{1/3} = 266(3)(5)\,\mathrm{MeV}$
- $\beta = 4.05:\quad (-\langle\bar{q}q\rangle)^{1/3} = 266(6)(6)\,\mathrm{MeV}$
- Comparison: ϵ regime gives at $\beta = 3.90$

  $(-\langle\bar{q}q\rangle)^{1/3} = 262(12)(4)\,\mathrm{MeV}$

strange-quark sector : partially quenched

$$m_K \qquad f_K$$

$$m_s \qquad |V_{us}|$$

strange-quark sector : setup and strategy

- Setup :
  - quark masses (partially quenched)

    $\mu_{\rm sea} = \mu_S$ and $\mu_{\rm val} = \{\mu_1, \mu_2\}$

    - light : μ_S and $\mu_1 \in [1/6\,;\,2/3]\, m_s$
    - strange : $\mu_{1,2} \sim m_s$ (and $\mu_2 \geq \mu_1 = \mu_S$)
  - lattice spacing : $\beta = 3.9$ $a \sim 0.09$ fm
  - volume : $L \sim 2.1$ fm and $m_{\rm PS} L \geq 3.2$
  - statistics : 240 confs for each μ_S
  - stochastic all to all propagators
- Strategy:
  - extrapolation to $m_{u,d}$ and interpolation to m_s
  - experimental inputs:
    - light : $a\mu_{u,d}$ from $(m_\pi/f_\pi)^{\rm exp.}$
    - a from $(f_\pi)^{\rm exp.}$
    - strange : $a\mu_s$ from $(m_K)^{\rm exp.}$

strange-quark sector : mass dependence

chiral perturbation theory (χPT)

- Use of continuum PQχPT to describe the dependence on :
  - the mass μ — NLO (S. Sharpe)
  - finite spatial size L — 1-loop (D. Becirevic and G. Villadoro)
- fit to $N_f = 2$ PQχPT at NLO + "local" NNLO

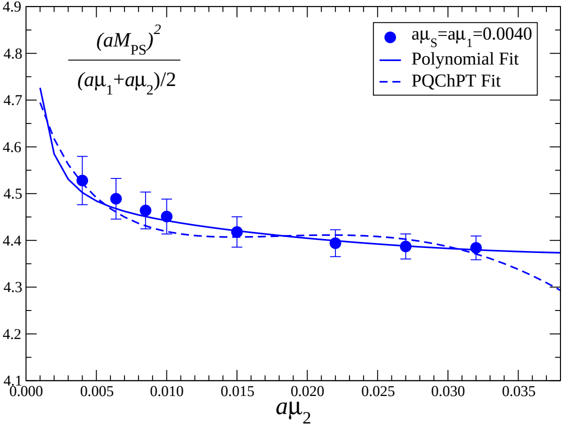
$$m^2_{\rm PS}(\mu_S, \mu_1, \mu_2) = B_0(\mu_1 + \mu 2) \left[1 + \frac{\xi_1(\xi_S - \xi_1)\ln(2\xi_1)}{\xi_2 - \xi_1} - \frac{\xi_2(\xi_S - \xi_2)\ln(2\xi_2)}{\xi_2 - \xi_1} + a_V\xi_{12} + a_S\xi_S + \right.$$
$$\left. + a_{VV}\xi_{12}^2 + a_{SS}\xi_S^2 + a_{VS}\xi_{12}\xi_S + a_{VD}\xi_{D12}^2 \right]$$

$$f_{\rm PS}(\mu_S, \mu_1, \mu_2) = f_0 \left[1 - \xi_{1S}\ln(2\xi_{1S}) - \xi_{2S}\ln(2\xi_{2S}) + \frac{\xi_1\xi_2 - \xi_S\xi_{12}}{2(\xi_2 - \xi_1)} \ln\left(\frac{\xi_1}{\xi 2}\right) + (b_V + 1/2)\xi_{12} + \right.$$
$$\left. + (b_S - 1/2)\xi_S + b_{VV}\xi_{12}^2 + b_{SS}\xi_S^2 + b_{VS}\xi_{12}\xi_S + b_{VD}\xi_{D12}^2 \right]$$

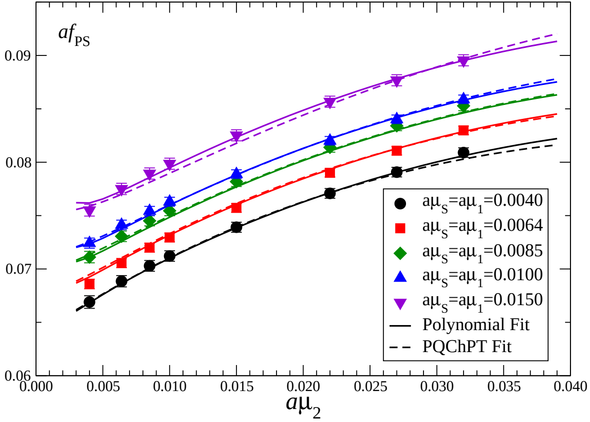
where $\xi_{ij} = B_0(\mu_i + \mu_j)/(4\pi f_0)^2$, $\xi_i = \xi_{ii}$, $\xi_{Dij} = B_0(\mu_i - \mu_j)/(4\pi f_0)^2$, $f_0 = \sqrt{2}F_0$

- We also consider polynomial fit functions
- 14 (combined) fit parameters: $B_0, f_0, a_V, a_S, a_{VV}, a_{SS}, a_{VS}, a_{VD}, b_V, \ldots$
- 300 data points: 150 combinations of quark masses (if $\mu_2 \geq \mu_1 = \mu_S$: 30 comb.)

strange-quark sector : $m^2_{\rm PS}/\mu$ vs. μ $\beta = 3.9$



strange-quark sector : $f_{\rm PS}$ vs. μ $\beta = 3.9$



Renormalization, m_s and f_K $\beta = 3.9$

Results:

- Central values: average of polynomial and PQχPT fits with FS corrections and $\mu_2 \geq \mu_1 = \mu_S$
- Systematic error: spread between polynomial/PQχPT and FSE
- As before: renormalization constants of bilinear quark operators : Z_P non-perturbatively (RI-MOM)
- Strange quark mass :

$$m_s[\overline{\mathrm{MS}}, 2\ \mathrm{GeV}] = 109(3)(8)\ \mathrm{MeV} \qquad m_s/m_{u,d} = 27.3(2)(9)$$

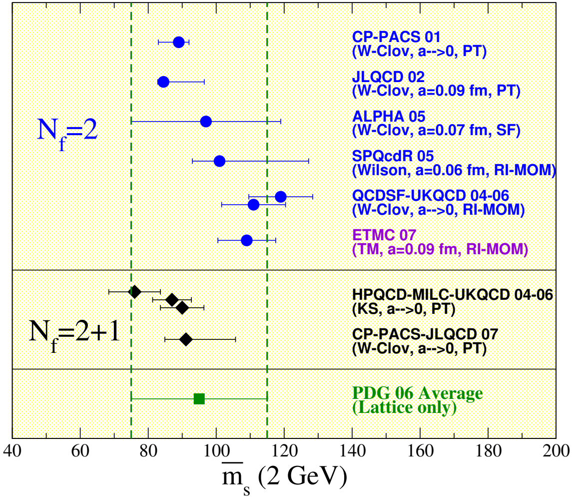
- decay constant :

$$f_K = 158.8 \pm 1.3 \pm 2.4\ \mathrm{MeV} \qquad f_K/f_\pi = 1.214(10)(18)$$

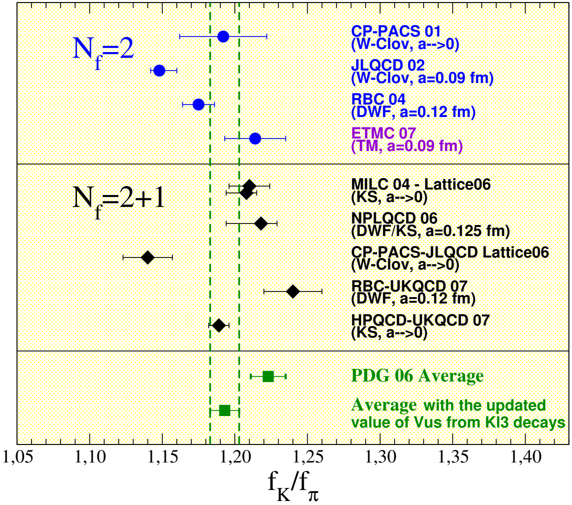
- $|V_{us}|$:

$$|V_{us}|/|V_{ud}| = 0.2275(6)(39) \qquad |V_{us}| = 0.2215(5)(38)$$

strange-quark sector : m_s comparison of results



strange-quark sector : f_K/f_π comparison of results



charm-quark sector partially quenched

m_D f_D

m_{D_s} f_{D_s}

m_c

charm-quark sector : setup and strategy

- Setup :
  - quark masses (partially quenched)

    $$\mu_{\rm sea} = \mu_S \quad \text{and} \quad \mu_{\rm val} = \{\mu_1, \mu_2\}$$

    - light : μ_S and $\mu_1 \in [1/6\,;\ 2/3]\, m_s$
    - strange : $\mu_{1,2} \sim m_s$
    - charm : $\mu_2 \sim m_c$
  - lattice spacings : $\beta = 3.9$ and 4.05 — $a = (0.07, 0.09)$ fm
  - volume : $L \sim 2.1$ fm and $m_{\rm PS} L \geq 3.2$
  - statistics : 240 confs. at $\beta = 3.9$ (each 20 traj. $\tau = 0.5$)

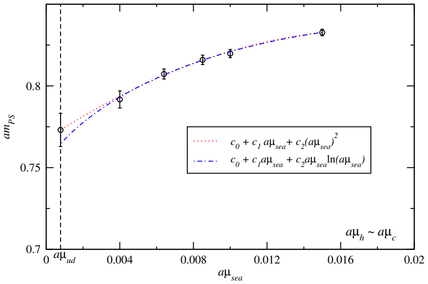
    130 confs. at $\beta = 4.05$ (each 20 traj. $\tau = 1.0$)
  - stochastic all to all propagators

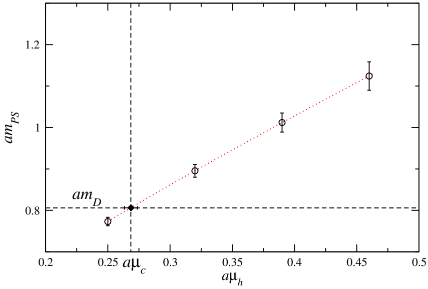
charm-quark sector : setup and strategy

- Strategy:
  - extrapolation to $m_{u,d}$ and interpolations to m_s and m_c
  - experimental inputs:
    - light : $a\mu_{u,d}$ from $(m_\pi/f_\pi)^{\mathrm{exp.}}$
    - a from $(f_\pi)^{\mathrm{exp.}}$
    - strange : $a\mu_s$ from $(m_K)^{\mathrm{exp.}}$
    - charm : $a\mu_c$ from $(m_D)^{\mathrm{exp.}}$

charm-quark sector : $m_D \rightsquigarrow m_c$ $\beta = 3.9$

Illustration of light and heavy quark-mass dependences: $m_{\rm PS}(\mu_{\rm sea}, \mu_{\rm val}^{\rm light}, \mu_{\rm val}^{\rm heavy})$





$$\mu_{\rm val}^{\rm light} = \mu_{\rm sea}$$
$$\mu_{\rm val}^{\rm heavy} \sim \mu_c$$

$$am_{\rm PS}(\mu_{\rm val}^{\rm heavy}) = A + Ba\mu_{\rm val}^{\rm heavy} + C/a\mu_{\rm val}^{\rm heavy}$$

Renormalization, m_c — $\beta = 3.9$ and 4.05

Results: (PRELIMINARY)

- As before: renormalization constants of bilinear quark operators : Z_P non-perturbatively (RI-MOM): *preliminary* for $\beta = 4.05$
- Central values: polynomial fit
- Systematic error: Z_P, a, spread between polynomial/χ-log fits
- Charm quark mass :

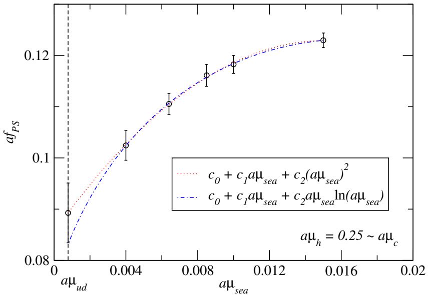
$$\beta = 3.9 \;:\quad m_c[\overline{\mathrm{MS}}, 2\ \mathrm{GeV}] = 1.481(21)(63)(8)(^{+21}_{-27})\ \mathrm{GeV}$$

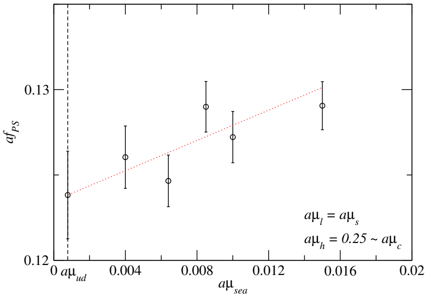
$$\beta = 4.05 \;:\quad m_c[\overline{\mathrm{MS}}, 2\ \mathrm{GeV}] = 1.475(43)(62)(16)(^{+0}_{-9})\ \mathrm{GeV}$$

charm-quark sector : f_D $\beta = 3.9$

Illustration of light valence and sea quark-mass dependences:

$f_{\rm PS}(\mu_{\rm sea}, \mu_{\rm val}^{\rm light}, \mu_{\rm val}^{\rm heavy})$



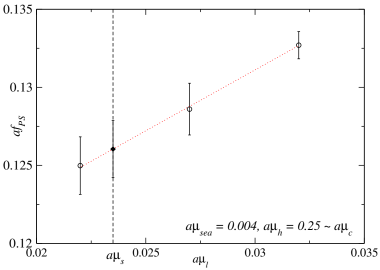


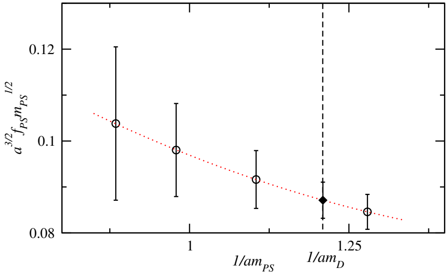
$\mu_{\rm val}^{\rm light} = \mu_{\rm sea}$

$\mu_{\rm val}^{\rm heavy} \sim \mu_c$

charm-quark sector : f_D $\beta = 3.9$

Illustration of strange and charm quark-mass dependence: $f_{\rm PS}(\mu_{\rm sea}, \mu^{\rm l,s}_{\rm val}, \mu^{\rm heavy}_{\rm val})$





$$a^{3/2} f_{\rm PS}\sqrt{m_{\rm PS}} = A + B/am_{\rm PS} + C/(am_{\rm PS})^2$$

f_D and f_{D_s} — $\beta = 3.9$ and 4.05

Results (Preliminary)

- Central values: polynomial fits
- Systematic error: a, spread between polynomial/χ-log fit
- Decay constant :

$$\beta = 3.9 \ : \quad f_D = 205(13)(3)(17)\ \mathrm{MeV} \qquad f_{D_s} = 287(4)(3)(3)\ \mathrm{MeV}$$

$$\beta = 4.05 \ : \quad f_D = 230(28)(6)(6)\ \mathrm{MeV} \qquad f_{D_s} = 270(4)(5)(7)\ \mathrm{MeV}$$

- m_{D_s}/m_D

$$\beta = 3.9 \ : \quad m_{D_s}/m_D = 1.072(13)(4)(8)$$

$$\beta = 4.05 \ : \quad m_{D_s}/m_D = 1.047(29)(2)(2)$$

baryon sector

$$m_N$$

$$m_\Delta$$

baryon sector : setup

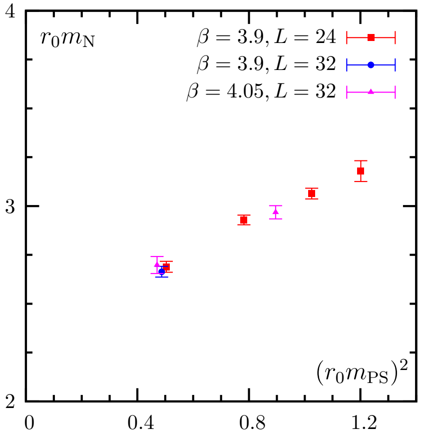
- Setup :
  - quark masses

    $$\mu_{\rm sea} = \mu_{\rm val}$$

    - light sector: $m_{\rm PS} \in [300, 500]$ MeV
  - lattice spacings : $\beta = 3.9$ and 4.05 $\quad a = (0.07, 0.09)$ fm
  - volumes : $L \sim 2.1$ fm and $m_{\rm PS} L \geq 3.2$

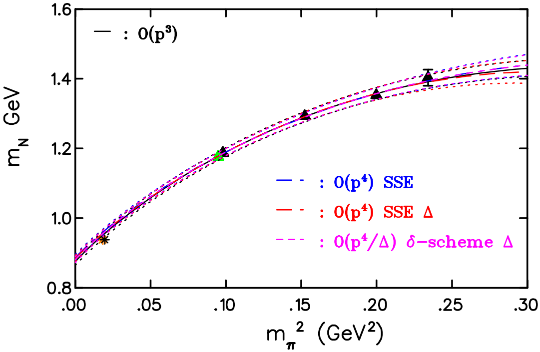
    $L \sim 2.8$ fm and $m_{\rm PS} L \geq 3.7$
  - point-like sources randomly located

baryon sector : nucleon $\beta = 3.9$ and 4.05



- cut-off effects appear to be small
- finite volume effects for smallest mass value at $\beta = 3.9$ negligible

baryon sector : nucleon χ-fits $\beta = 3.9$



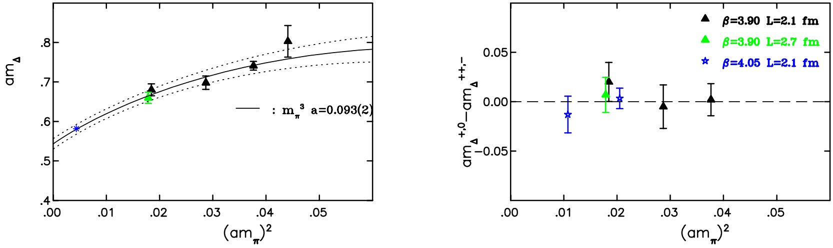
- χ-fit : fitting m_0 and c_1 (1-loop: $\mathcal{O}(p^3)$)

$$m_{\rm N} = m_0 + c_1 m_{\rm PS}^2 - \frac{3g_A^2}{32\pi f_{\rm PS}^2} m_{\rm PS}^3$$

- if $m_{\rm N}$ is used to set the scale for $\beta = 3.9$:
  $a(\beta = 3.9) = 0.0873(10)(5)$ fm

baryon sector : Δ χ-fits and splitting $\beta = 3.9$

(PRELIMINARY)



- χ-fit : fitting m_0 and c_1 (1-loop: $\mathcal{O}(p^3)$)

$$m_\Delta = m_0 - 4c_1 m_{\rm PS}^2 - \frac{3}{32\pi f_{\rm PS}^2}\frac{25H_A^2}{81} m_{\rm PS}^3$$

- $m_{\Delta^+} - m_{\Delta^{++}}$ mass splitting is consistent with zero

WME

Form factors

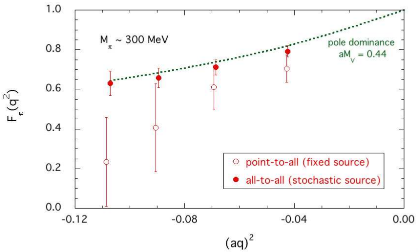
WME setup

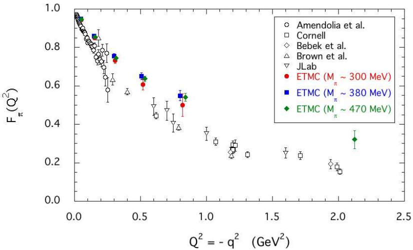
- Setup :
  - quark masses (partially quenched)

    $\mu_{\rm sea} = \mu_S$ and $\mu_{\rm val} = \{\mu_1, \mu_2, \mu_3\}$
    - light : μ_S and $\mu_{1,2,3} \in [1/6\,;\, 2/3]\, m_s$
    - strange : $\mu_{2,3} \sim m_s$
    - charm : $\mu_{2,3} \sim m_c$
  - lattice spacings : $\beta = 3.9$ $a = 0.09$ fm
  - volume : $L \sim 2.1$ fm and $m_{\rm PS} L \geq 3.2$
  - statistics : 240 confs. (each 20 traj. $\tau = 0.5$)
  - stochastic all to all propagators
  - twisted boundary conditions
- Study : pion form factor, $K_{\ell 3}$, Isgur-Wise, $B \to \pi$

pion form factor $\beta = 3.9$

(PRELIMINARY)





- Statistic : 80 confs.

$$F^{\rm pole}(q^2) = 1/(1 - \frac{\langle r^2 \rangle}{6} q^2)$$

Conclusions

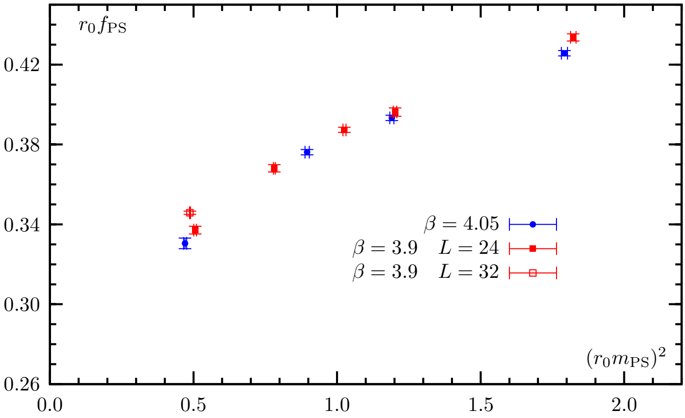
Summary:

- Perform a study of χPT at fixed lattice spacings and in the continuum
- Small discretization effects
- Good description of our pion data with continuum $N_f = 2$ NLO χPT
- Extraction of LEC, m_q and $\langle \bar{q}q \rangle$ with good statistical precision
- Study of FSE
- Preliminary results in the strange, charm, nucleon sectors and WME
- Setting the scale: pion, nucleon

On going:

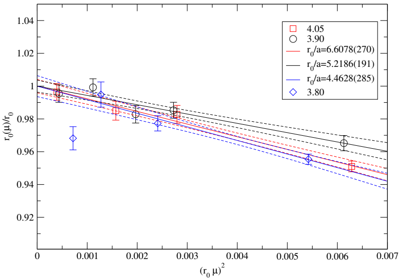
- χPT description of the data in the continuum
- Check of NNLO χPT in the continuum

Raw data: $f_{\rm PS}$ vs. $m^2_{\rm PS}$ $\beta = 4.05$ and 3.9



Setting the scale: r_0 vs. μ^2

- Sommer parameter r_0 : static inter-quark force



- HYP-smeared temporal links, APE smeared spatial links, correlator matrix
- statistical accuracy of less than 0.5%,
- compatible with μ^2 dependence

$\Rightarrow$ at $\mu \to 0$: $\beta = 4.05: \; r_0/a = 6.61(3)$ $\beta = 3.9: \; r_0/a = 5.22(2)$ $\beta = 3.8: \; r_0/a = 4.46(3)$

- setting the scale: use several quantities, e.g. m_π, f_π, m_K, m_{K^*}, f_K, m_N, ...

Finite Size Effects

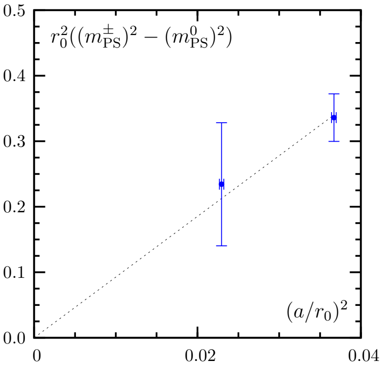
- comparison to NLO χPT [Gasser, Leutwyler, 1987, 1988] (GL)
  or resummed Lüscher formula [Colangelo, Dürr, Haefeli, 2005] (CDH)

	β	$m_{\mathrm{PS}} L$	meas (%)	GL (%)	CDH (%)
m_{PS}	3.9	3.3	$+1.8$	$+0.62$	$+1.13$
f_{PS}	3.9	3.3	-2.5	-2.48	-2.39
m_{PS}	4.05	3.0	$+6.2$	$+2.2$	$+6.1$
f_{PS}	4.05	3.0	-10.7	-8.8	-10.3
m_{PS}	4.05	3.5	$+1.1$	$+0.8$	$+1.5$
f_{PS}	4.05	3.5	-1.8	-3.4	-2.9

- as input for the parameters estimates from CDH were used
- CDH describes our data in general better than GL
  for the price of more parameters

Pion Mass Splitting

Flavour symmetry is broken at $\mathcal{O}(a^2)$ $\Rightarrow am^0_{\mathrm{PS}} \neq am^{\pm}_{\mathrm{PS}}$



- not easy to measure: disconnected contributions!
- $m^{\pm}_{\mathrm{PS}}$, m^0_{PS} mass splitting vanishes like a^2
- $am^0_{\mathrm{PS}} < am^{\pm}_{\mathrm{PS}}$ consistent with prediction from χPT for observed phase structure

at $\beta = 4.05$ splitting still as large as 16%, however ...

Pion Mass Splitting

Expect generically large a^2 artifacts all over the place?

- an analysis a la Symanzik shows that

$$(m_{\rm PS}^0)^2 = m_\pi^2 + a^2\zeta_\pi^2 + \mathcal{O}(a^2 m_\pi^2, a^4)\,, \qquad \zeta_\pi \equiv \langle\pi^0|\mathcal{L}_6|\pi^0\rangle|_{\rm cont}$$
$$(m_{\rm PS}^\pm)^2 = m_\pi^2 + \mathcal{O}(a^2 m_\pi^2, a^4)$$

- ζ_π has a dynamically large contribution:

$$a^2\zeta_\pi^2 \sim a^2|\hat{G}_\pi|^2\,, \qquad \hat{G}_\pi \equiv \langle 0|\hat{P}^3|\pi^0\rangle = \frac{f_\pi m_\pi^2}{2m_q} \sim (570\ {\rm MeV})^2$$

- $|\hat{G}_\pi|^2/\Lambda_{\rm QCD}^4 \sim 25$ $\rightarrow$ potentially large a^2 effects compared to their "natural" size $a^2\Lambda_{\rm QCD}^4$

- ζ_π enters only π^0-mass related quantities!

Pion Mass Splitting

Expect generically large a^2 artifacts all over the place?

- ζ_π enters only π^0-mass related quantities
  hence: no!

- indeed, we find:
  - splitting in the vector channel consistent with zero
  - Δ^{++}, Δ^{+} splitting consistent with zero
  - $f_{\mathrm{PS}}^{\pm}$ to f_{PS}^{0} difference small

- implication for Wilson and Wilson clover:
  ζ_π might contribute in many quantities