Electromagnetic and spin polarisabilities from lattice QCD

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Outline

- Hadron polarisabilities
- Lattice techniques
- Chiral perturbation theory
  - QCD results from unphysical hadrons

Hadron polarisabilities

- Hadron polarisabilities describe the deformation of a particle in an external (EM) field
- Quadratic energy shifts from effective Hamiltonian:

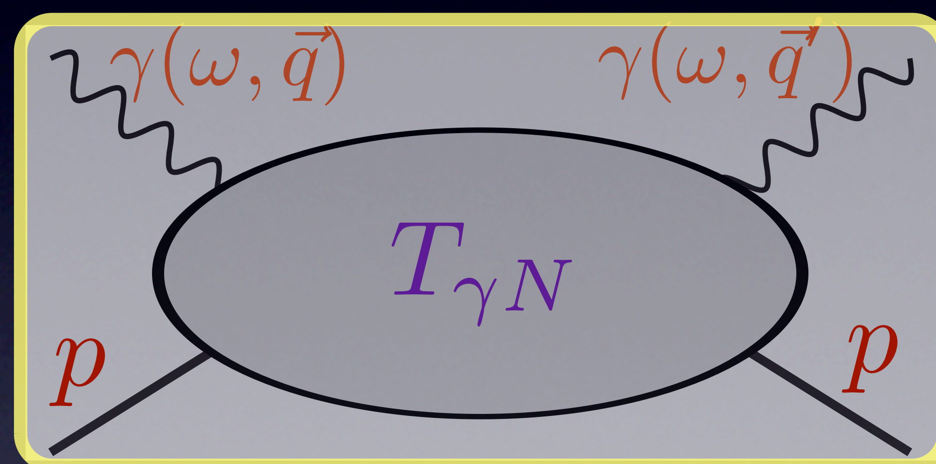
$$\mathcal{H} = \mathcal{H}_0 - \vec{\mu} \cdot \vec{B} - 2\pi\alpha|\vec{E}|^2 - 2\pi\beta|\vec{H}|^2 - 2\pi\gamma_1\vec{\sigma} \cdot \vec{E} \times \dot{\vec{E}} + \dots$$

Magnetic moment Electric pol Magnetic pol First spin pol

- Electric and magnetic polarisabilities: ability to align with or against the applied field
- Spin and higher order polarisabilities are less intuitive: more detailed view of EM structure

Compton scattering

- p, d: Experimentally measured in the low frequency limit of real Compton scattering



- Thomson limit and Low–Gell-Mann–Goldberger LET determined by Born terms (charge and magnetic moment)

$$T_{\gamma N} = f(\underbrace{\omega, \vec{q}, \vec{q}', \vec{\epsilon}, \vec{\epsilon}', \vec{\sigma}}_{\text{Kinematics}}; Z, \mu, \underbrace{\alpha, \beta}, \underbrace{\gamma_{1,\ldots,4}}) + \mathcal{O}(\omega^4)$$

Kinematics

- Next order given in terms EM and spin polarisabilities

Experiment

- MAMI, Saskatoon, JLab, OOPS, ELSA, HIγS
- EM and 2 combinations of spin polarisabilities are measurable for the proton but *difficult* experiments
- Neutron accessed via (quasi-)elastic Compton scattering on the deuteron - *even more difficult*

$$\alpha_p = 12.0(6), \quad \beta_p = 1.9(6), \quad \alpha_n = 13(2), \quad \beta_n = 3(2) \quad 10^{-4}\,\mathrm{fm}^3$$

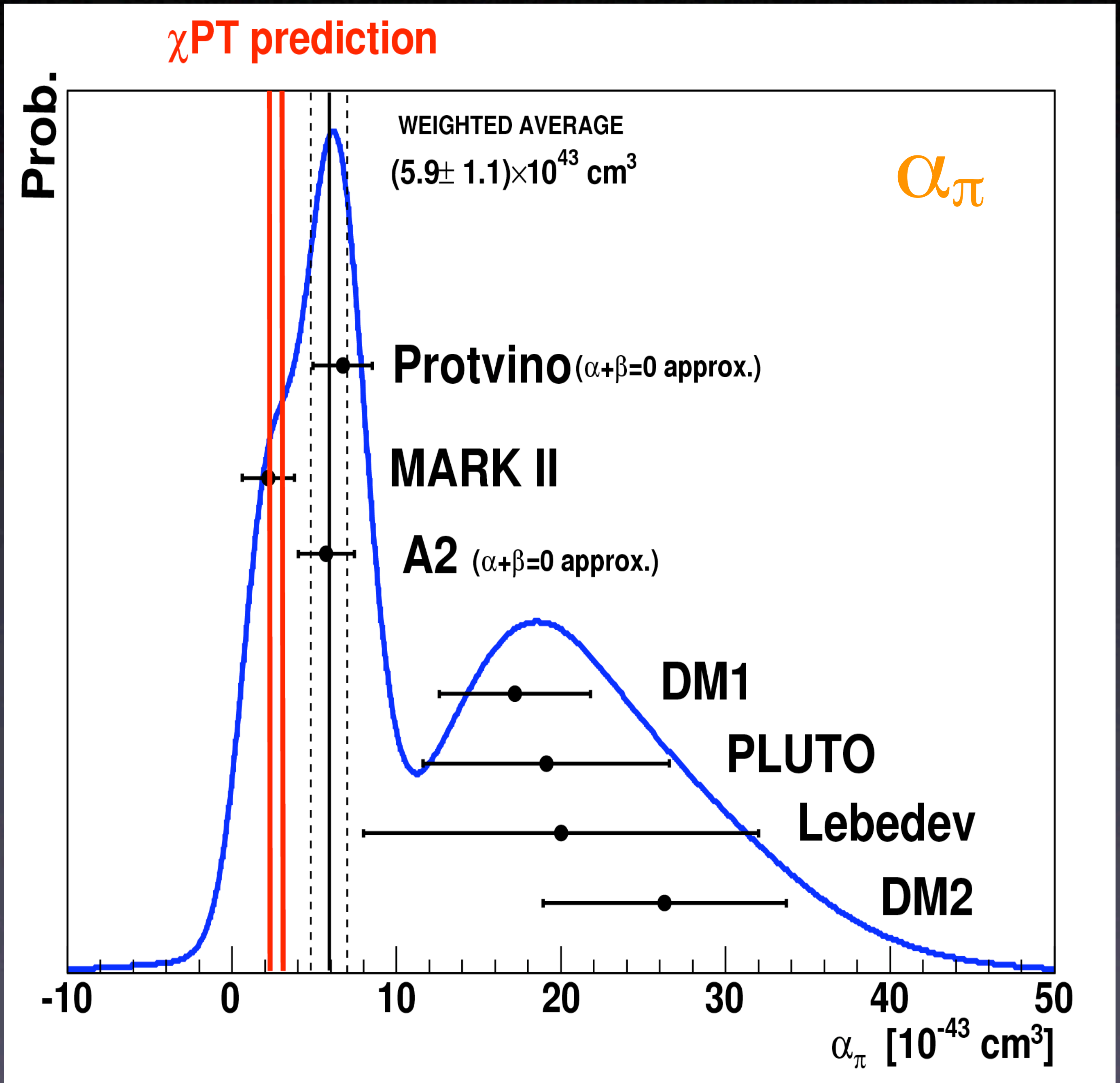
$$\gamma_\pi^{(p)} = -39(2), \quad \gamma_0^{(p)} = -1.0(1), \quad \gamma_\pi^{(n)} = 59(4), \quad 10^{-4}\,\mathrm{fm}^4$$

[de Jaeger & Hyde-Wright 05]

- Sign and small size of polarisabilities indicates tightly bound diamagnetic system - hard to deform
  - Spin polarisabilities - HIγS: ~10% errors by 2010

Pion polarisabilites

- A number of different measurements: most involve nuclear targets
- 3σ disagreement with two-loop χPT
- Big improvements in future: COMPASS, JLab@12 GeV



A. Guskov (COMPASS) ICHEP'06

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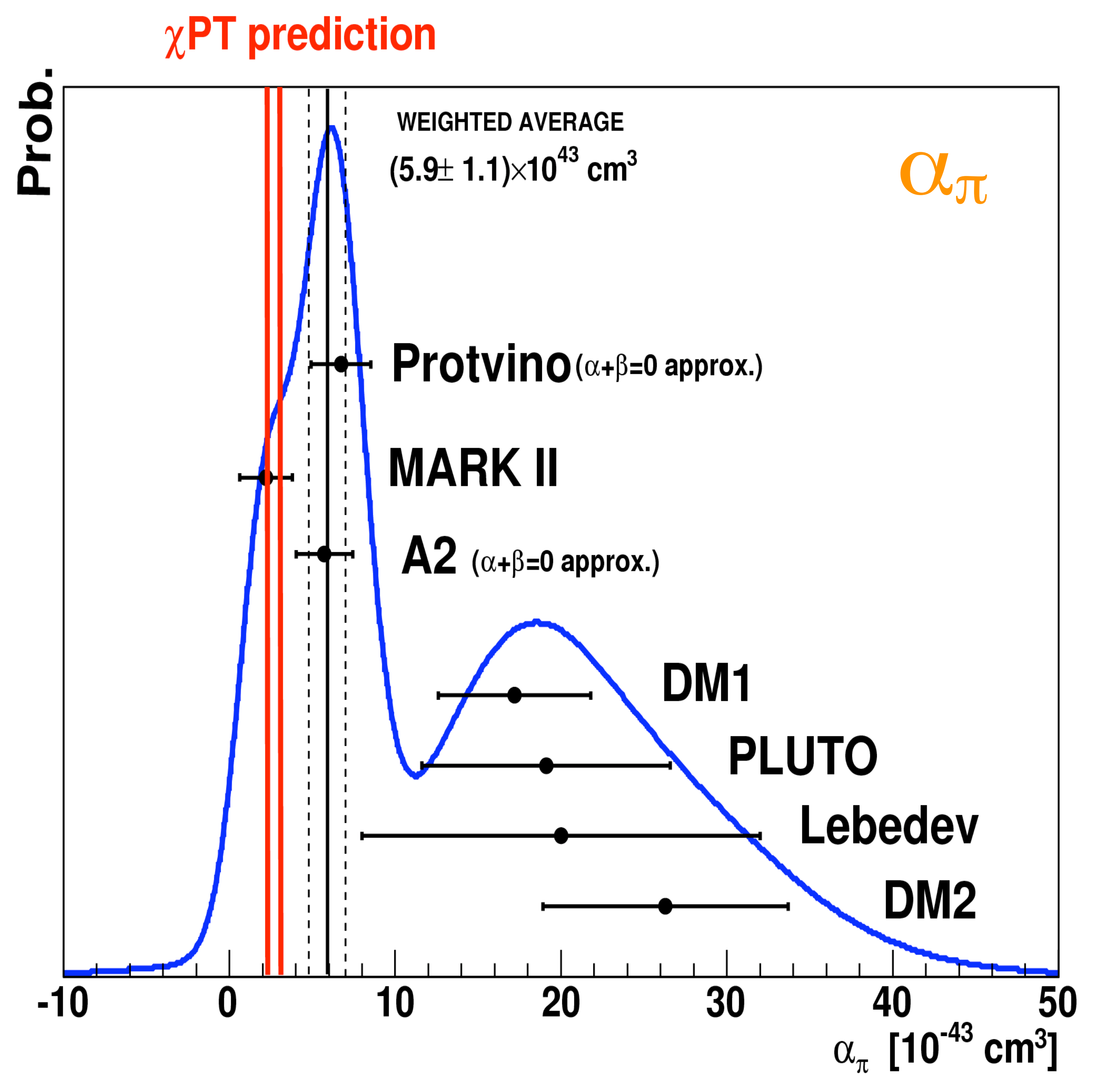
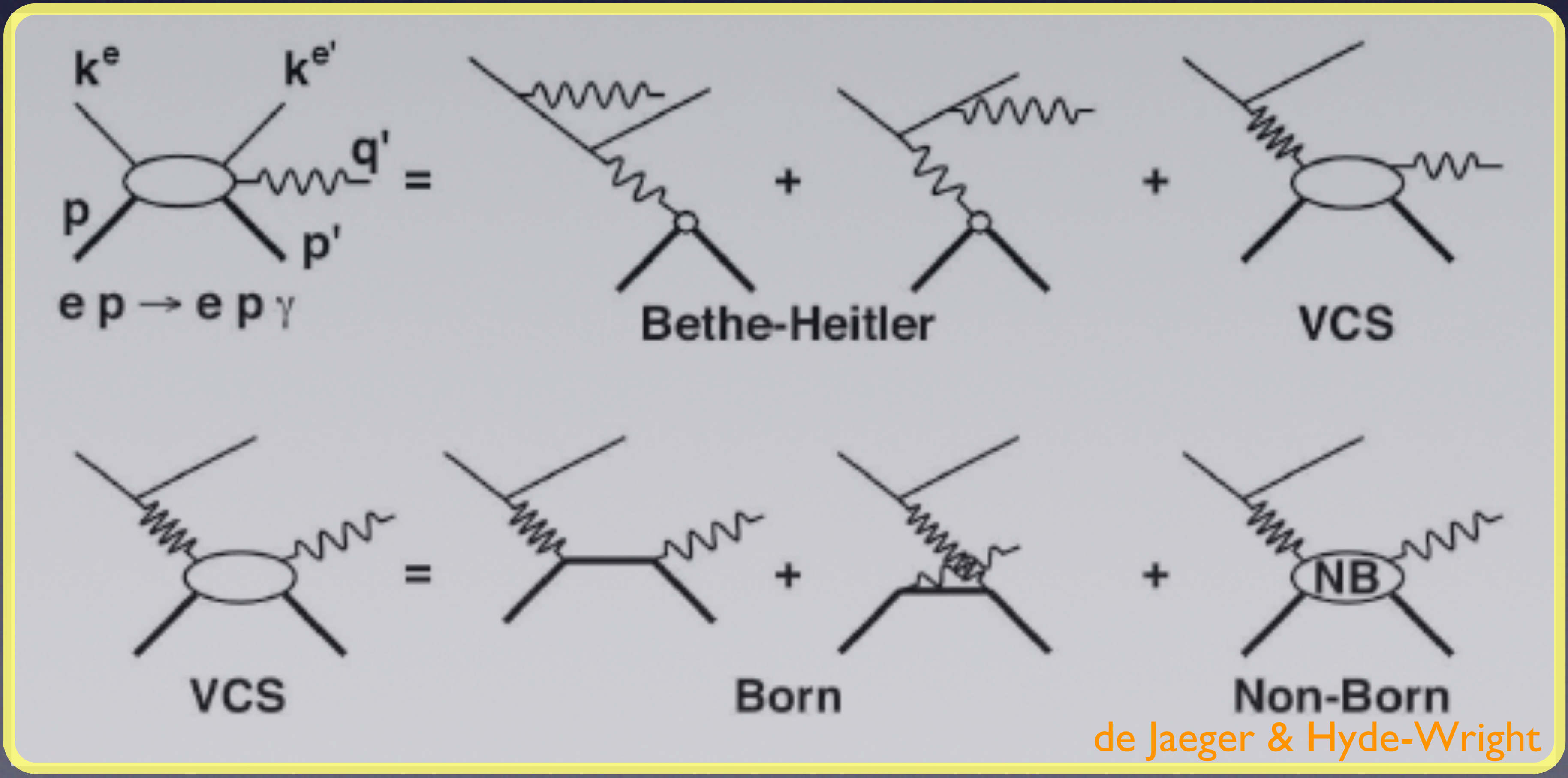


See talk on Friday: Daniele Panzieri

A. Guskov (COMPASS) ICHEP'06

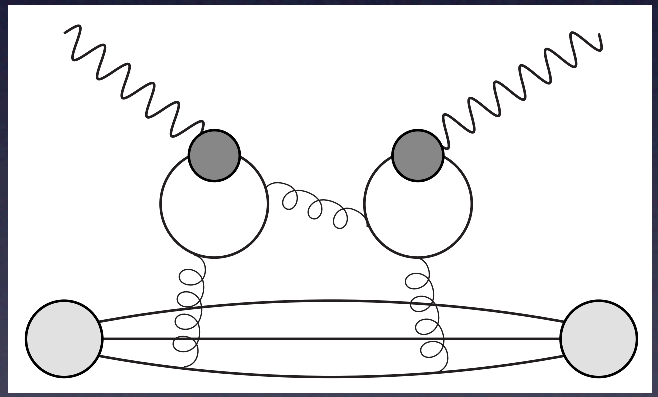
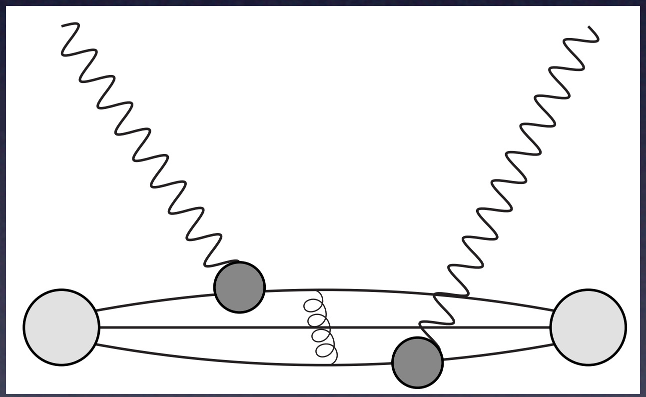
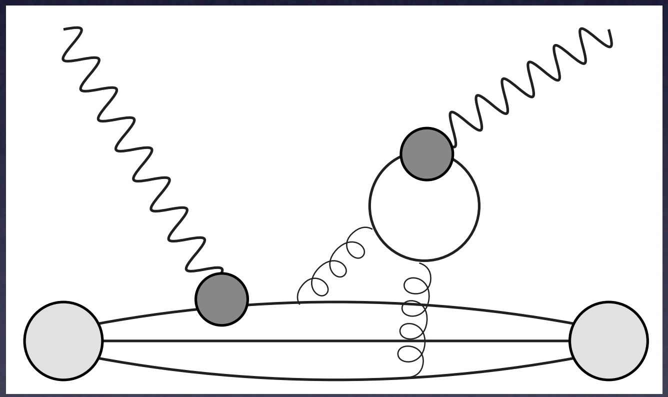
Further polarisabilities

- Higher orders in the frequency expansion gives higher order polarisabilities [Holstein *et al.* '99]
- Virtual and doubly virtual Compton scattering leads to generalised polarisabilities [Guichon, Liu & Thomas '95]



Lattice approaches

1. Four point correlators: $\langle 0|\chi(x_1)J^\mu(y_1)J^\nu(y_2)\overline{\chi}(x_2)|0\rangle$
   - Analogous to experimental measurement
   - Many disconnected contractions and doubly extended propagators eg:

   - Extraction of magnetic/spin pols requires momentum extrapolation
   - Pols: used by Engelhardt with some success

Lattice approaches

2. Energy shifts in two point correlators in external U(1) field
   - *QCD*: external field must be known during gauge field generation
     - costly but multipurpose
   - *Quenched QCD*: external field can be added after gauge configurations are generated
     - unrelated to physical poalarisabilities
   - *Partially quenched QCD*: physical results from unphysical hadrons

External field method

- Quenched external fields simple to apply:

$$U_\mu^a(x) \to U_\mu^a(x) \cdot U_\mu^{\rm ext}(x) \qquad U_\mu^{(a)}(x) = e^{i\,a\,g\,A_\mu^{(a)}(x)}$$

  - E.g.: magnetic field $\vec{B} = (0, 0, B)$

Quantised for single-valuedness

$$U_0^{\rm ext} = U_2^{\rm ext} = U_3^{\rm ext} = 1,\ U_1(x) = e^{ieBx_2}$$

  - Look for shift in energy quadratic in |B|

$$C_{\uparrow\uparrow}(\tau, B) = \sum_{\vec{x}} \langle 0|\chi_\uparrow(\vec{x}, t)\overline{\chi}_\uparrow(0)|0\rangle$$

$$= \exp\left[-(M - \mu|B| + 2\pi\beta|B|^2)\tau\right] + \mathcal{O}(|B|^3)$$

Magnetic polarisability (β)

Magnetic moment (μ)

Field constraints

- Field values are restricted by a number of constraints
  - Perturbative in EFT: $|eB|, \ |eE| < m_\pi^2$
  - Periodicity of box: e.g. magnetic field

$$U_\mu(x + L\hat{\nu}) = U_\mu(x)$$

$$a^2|eB| = \frac{2\pi n}{L}, \quad n \in Z$$

  - Landau levels well represented
- *Caveat: existing calculations do not satisfy these conditions!*

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Periodicity up to gauge transformation

  - Landau levels well represented
- *Caveat: existing calculations do not satisfy these conditions!*

External field method

- Can study more than energy shifts - hadronic correlator analysis ≡ effective field theory
  - matching behaviour of QCD correlator to EFT correlator (not just in ε-regime)
- E.g.: proton in constant electric field (T=∞)

$$C_{ss'}(\tau;E) = \sum_{\vec{x}} \langle 0|\chi_s(\vec{x},t)\overline{\chi}_{s'}(0)|0\rangle_E$$
$$= \delta_{s,s'} \exp\left[-(M + 2\pi\alpha|E|^2)\tau - \frac{q^2|E|^2}{6M}\tau^3\right] + \ldots$$

Acceleration of proton at large times

Electric polarisability

- Valid for $L^{-1} < m_\pi$, $|eE| < m_\pi^2$

External field method

- All meson/baryon polarisabilities can be calculated
  - utilise all information in hadron correlators including spin-flip matrix elements
  - spin polarisabilities require space/time varying U(1) fields: E.g. γ_{E1E1}

$$U_\mu^{\text{ext}} = e^{iaeA_\mu(x)},\ A_\mu(x) = \left(-\frac{a_6 t^2}{2a}, \frac{-ib_6 t}{2}, 0, 0\right)$$

$$\frac{C_{\uparrow\uparrow}(\vec{p},\tau;A)}{C_{\downarrow\downarrow}(\vec{p},\tau;A)} = \exp\left[\frac{2\pi}{a} a_6\, b_6\, \gamma_{E_1E_1}\, \tau\right] + \ldots$$

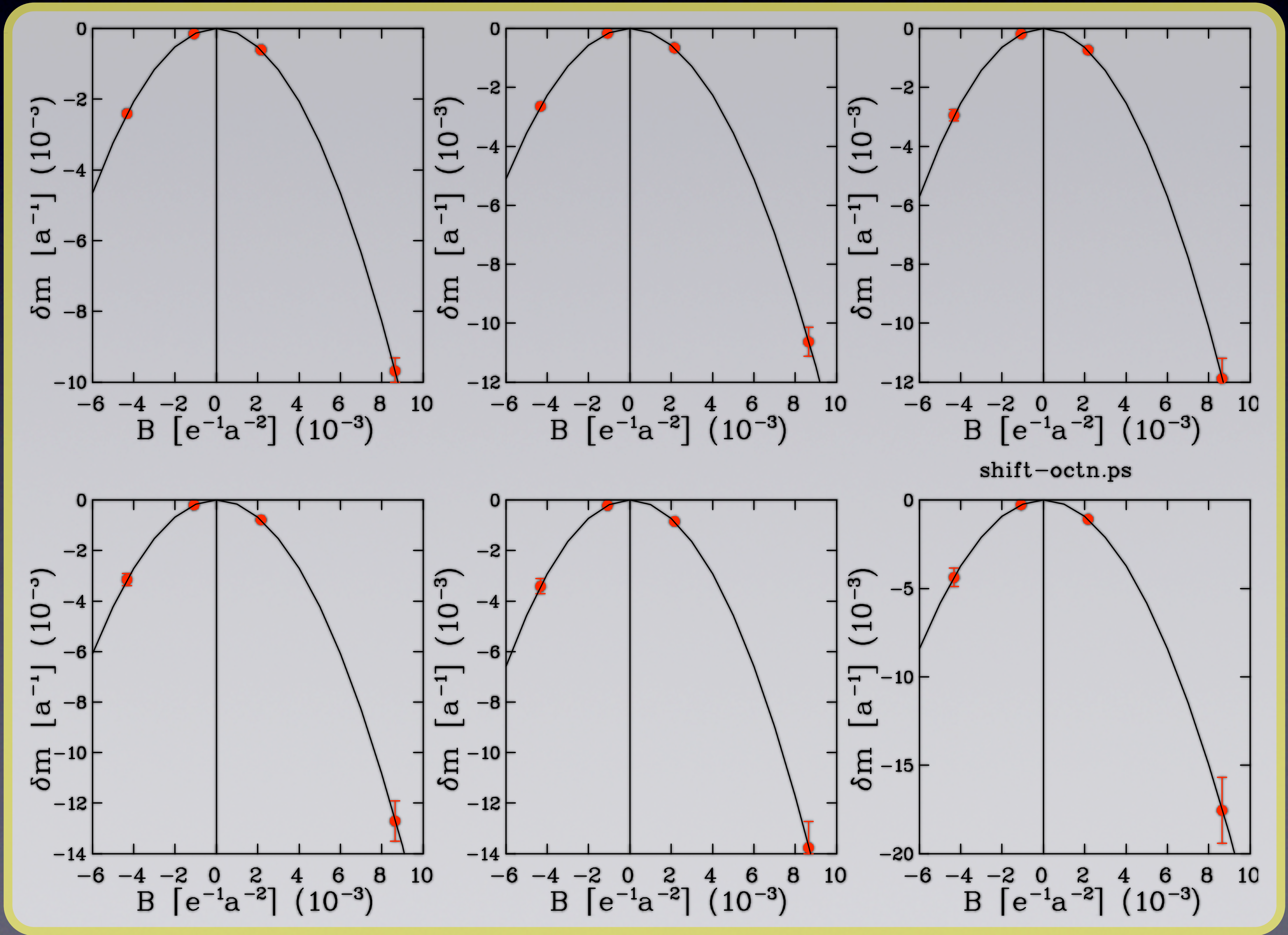
Quenched polarisabilities

- External field calculations of magnetic moments and EM polarisabilities have a long history
  - Martinelli *et al.*, Bernard *et al.*: μ for n, p, Δ [83]
  - Fiebig *et al.*: α for neutron [89]
  - Christensen *et al.*: α for uncharged particles [05]
  - Lee *et al.*: μ, β for many hadrons [05]
  - Shintani *et al.*: α for neutron [06]

Quenched magnetic polarisabilities

[Lee *et al.*, hep-lat/0509065]

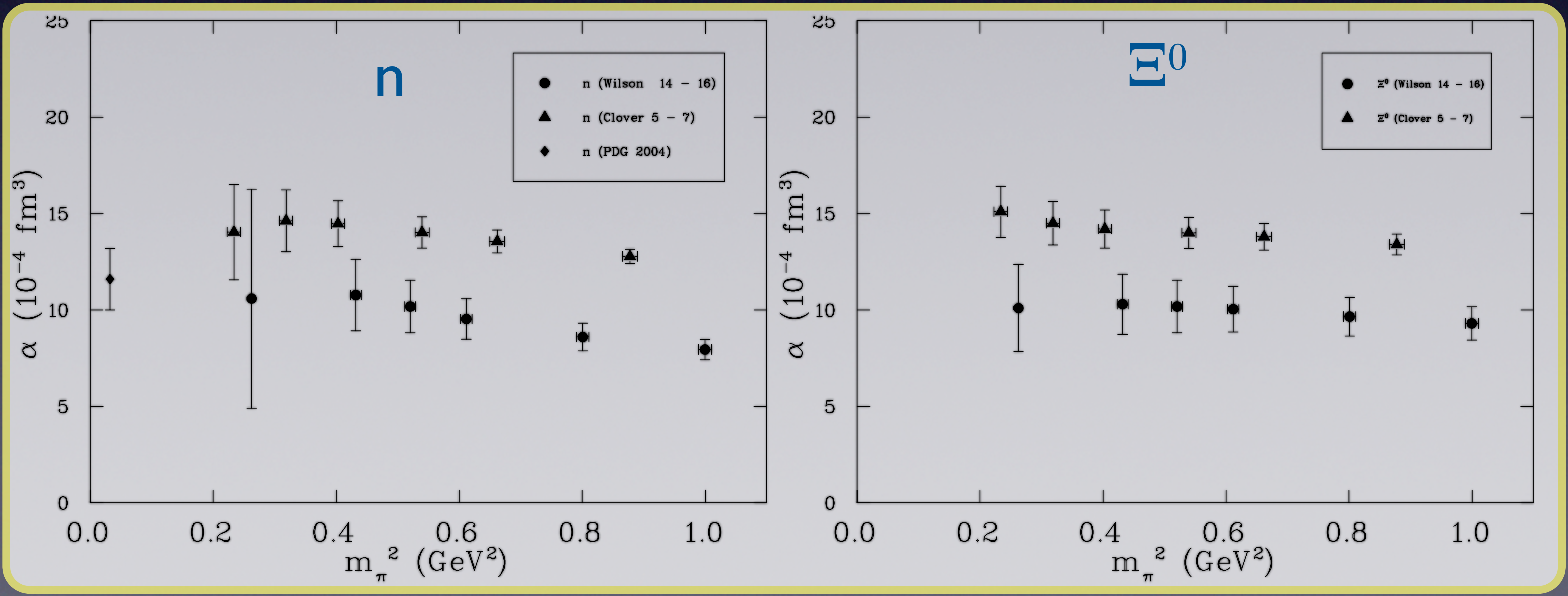
- Use four (non-quantised)field values



Quenched electric polarisabilities

[Christensen *et al.*, hep-lat/0408024]

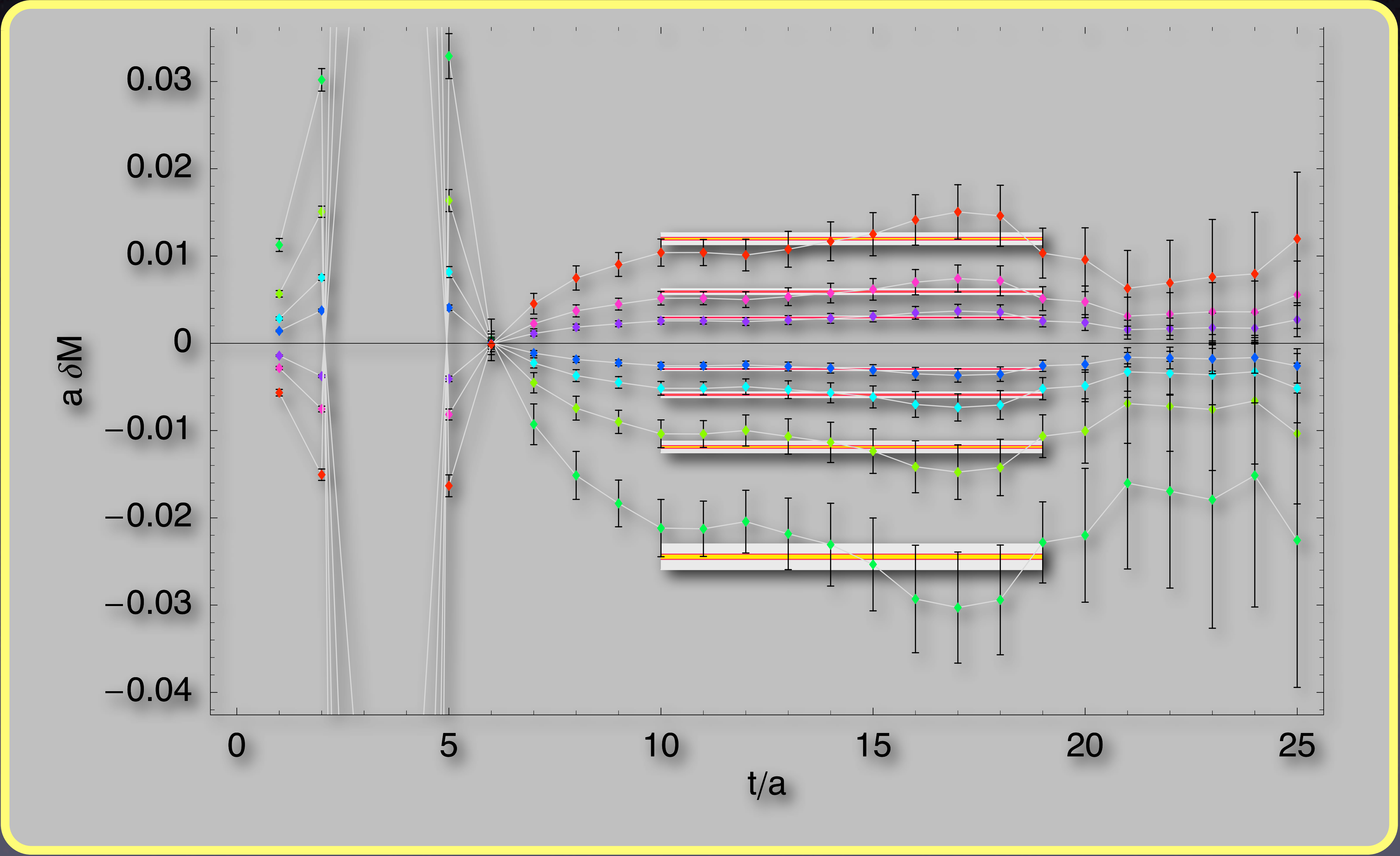
- Also do calculations with four field values (pos/neg)
- Neutral particles n, Σ^0, Ξ^0, Δ^0 Σ^{*0}, Ξ^{*0}, π^0, K^0, ρ^0, K^{*0}



Quenched magnetic moments

[WD, Tiburzi, Walker-Loud]

- Use eight "weak" field values: spin difference



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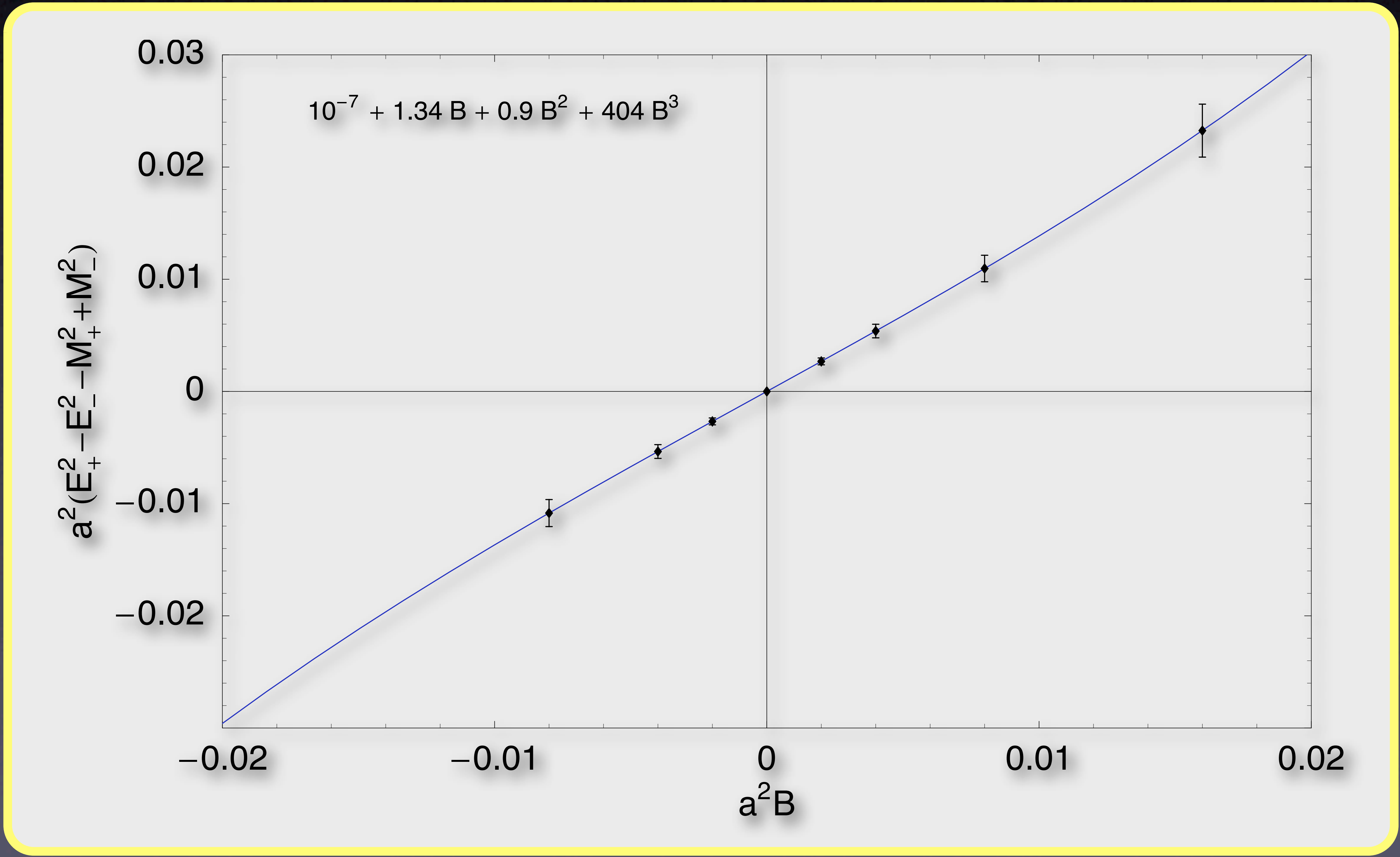
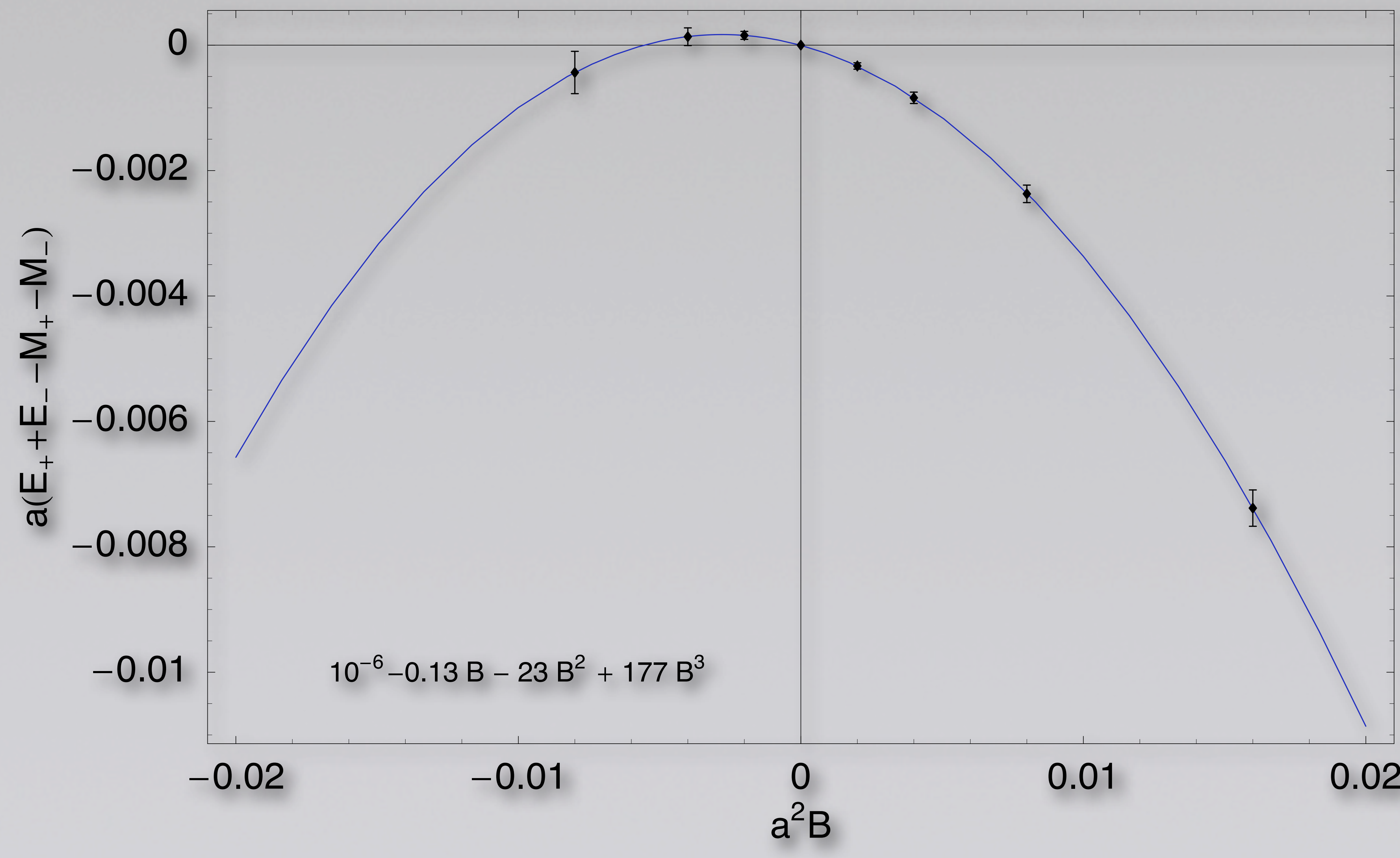
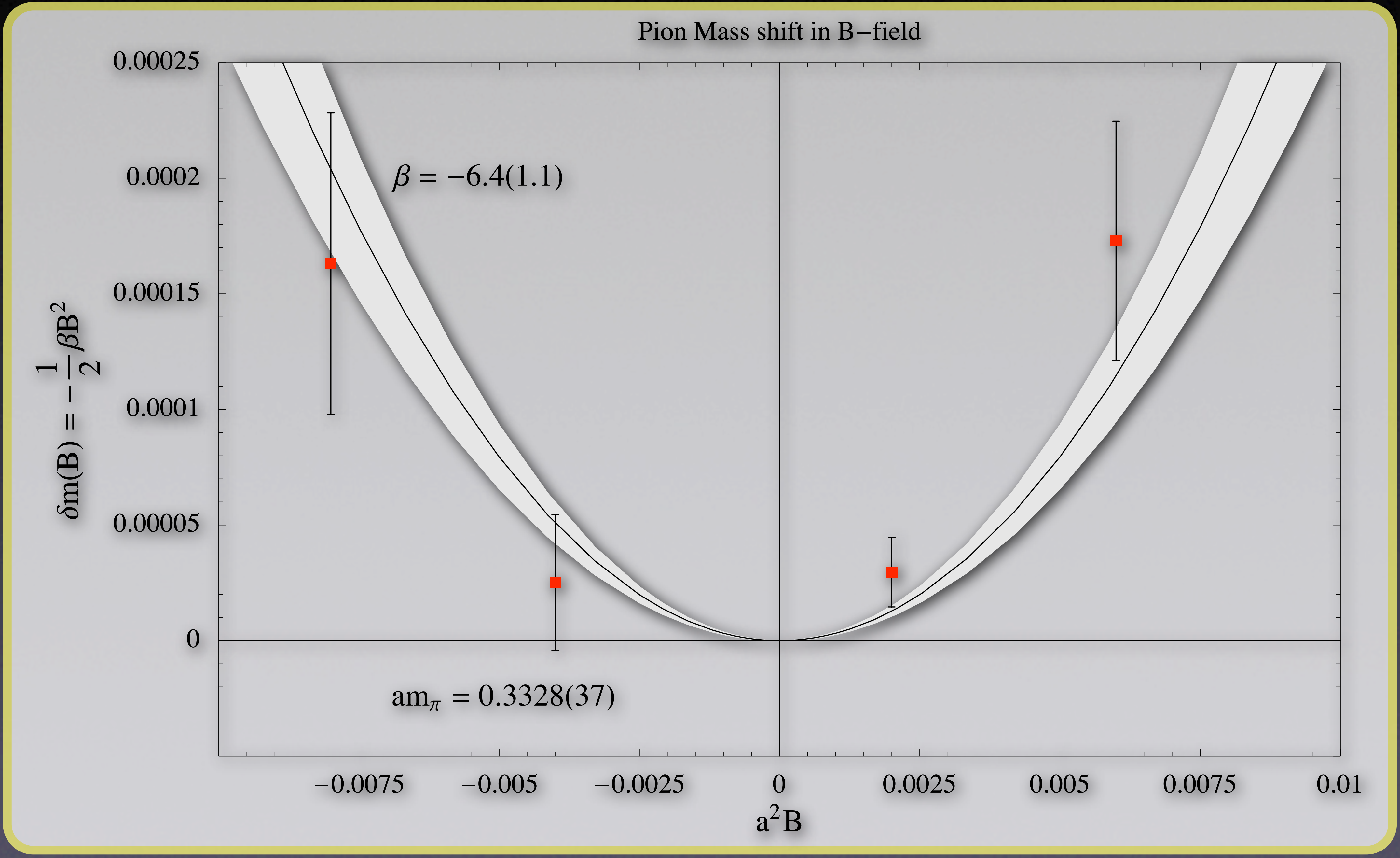


Quenched magnetic polarisabilities

- Spin average



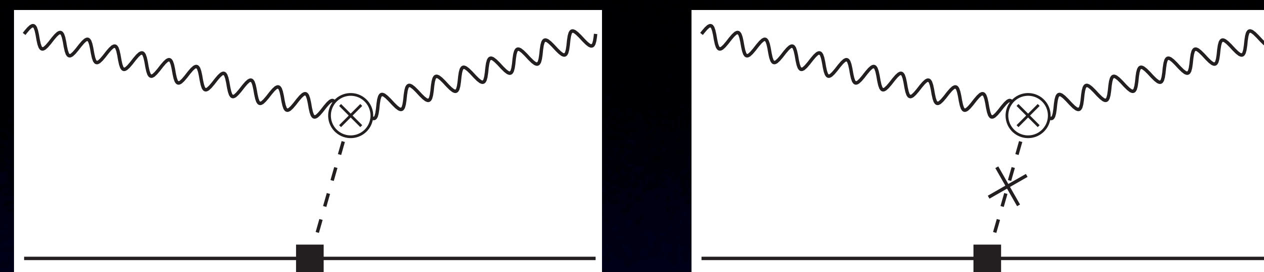
Quenched pion polarisabilities



Chiral perturbation theory

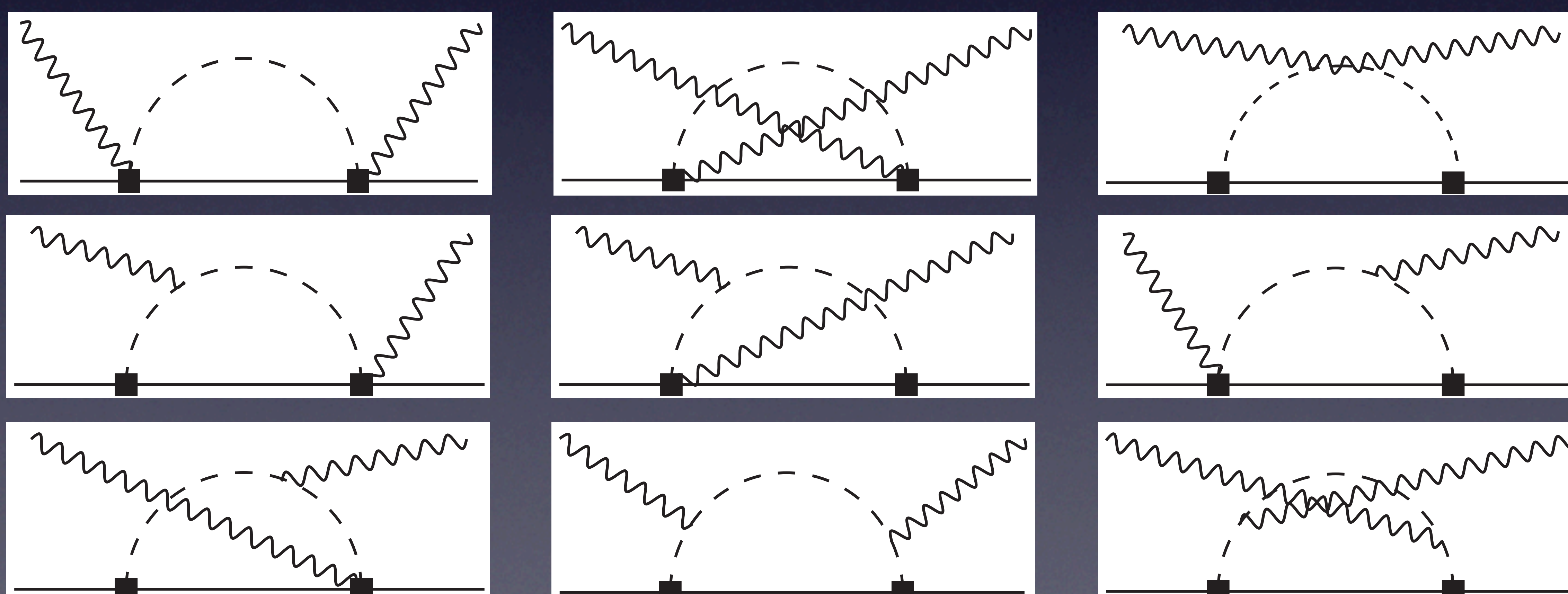
- Many studies of hadron polarisabilities in the context of chiral perturbation theory
  - pions and nucleons
- Extended to partially-quenched χPT at finite volume [WD, Tiburzi, Walker-Loud]
  - FV effects are significant
  - Post-multiplying unquenched gauge fields?

Nucleon polarisabilities



Anomalous $\pi^0 \rightarrow \gamma\gamma$

Δ-pole graphs



LOOPS

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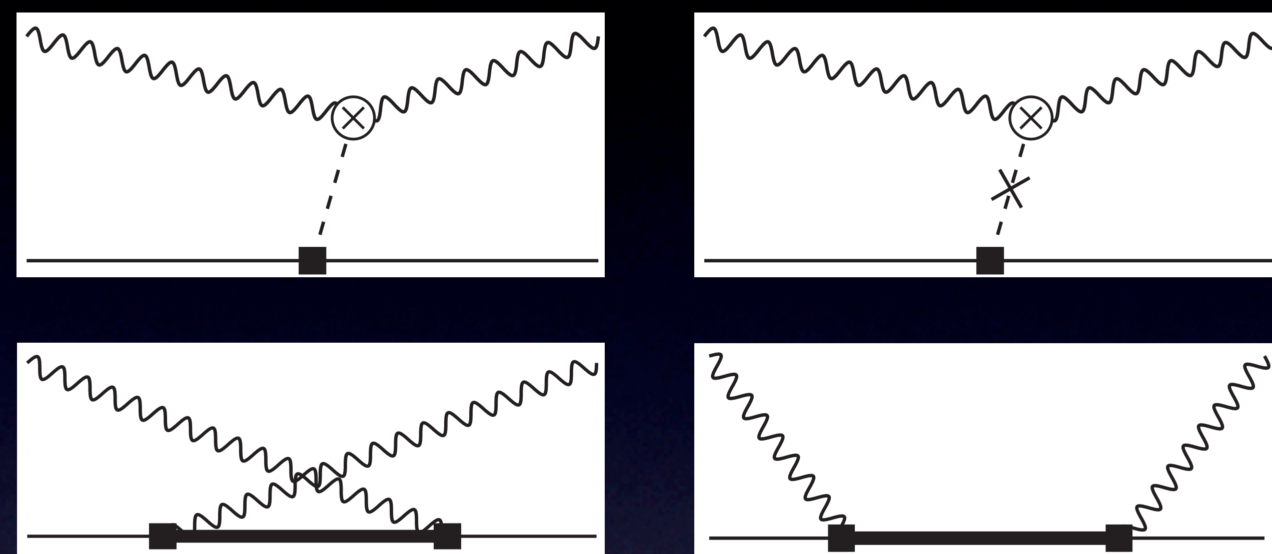

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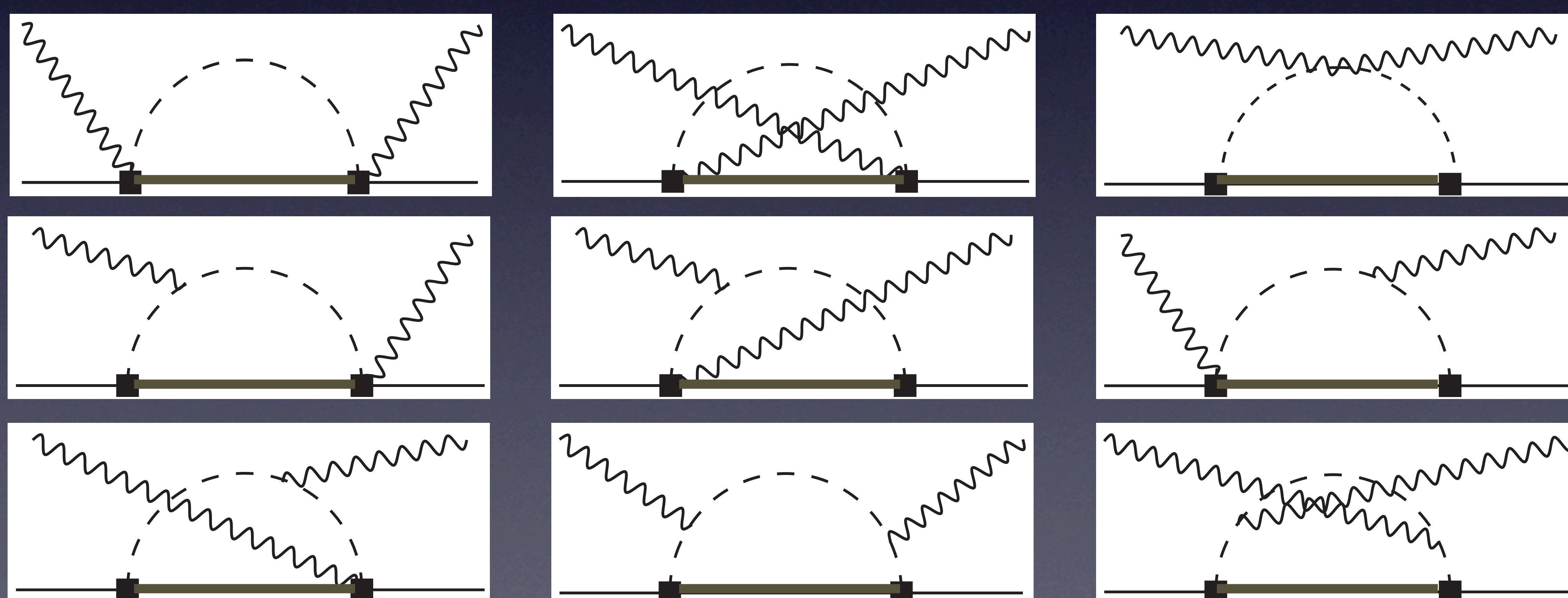

LOOPS

Infinite volume results

- Proton electric polarisability at NLO [SU(4|2)]

Involve axial couplings and quark charges

$$\alpha_p = \frac{e^2}{4\pi f^2}\left[\frac{5G_B}{192\pi}\frac{1}{m_\pi} + \frac{5G'_B}{192\pi}\frac{1}{m_{uj}} + \frac{G_T}{72\pi^2}F_\alpha(m_\pi) + \frac{G'_T}{72\pi^2}F_\alpha(m_{uj})\right]$$

Non-analytic function involving Δ isobar

$$F_\alpha(m) = \frac{9\Delta}{\Delta^2 - m^2} - \frac{\Delta^2 - 10m^2}{2(\Delta^2 - m^2)^{3/2}} \ln\left[\frac{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}}\right]$$

- Results for other polarisabilities similar but also have contributions from anomaly and Δ poles

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Singular in chiral limit

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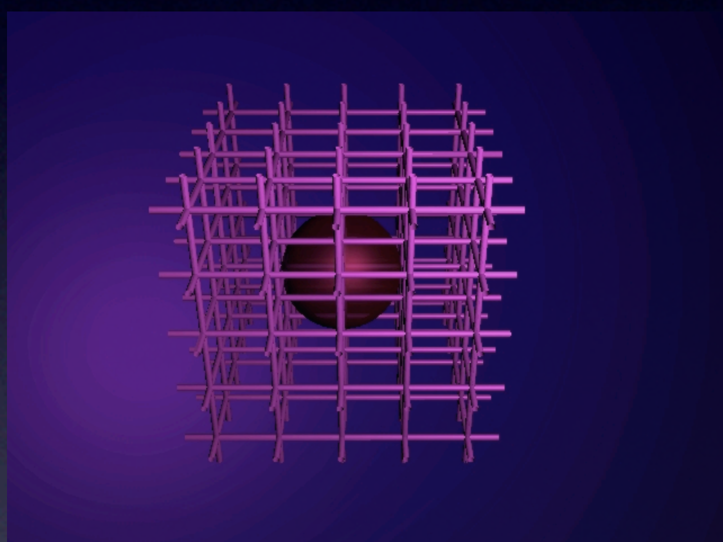
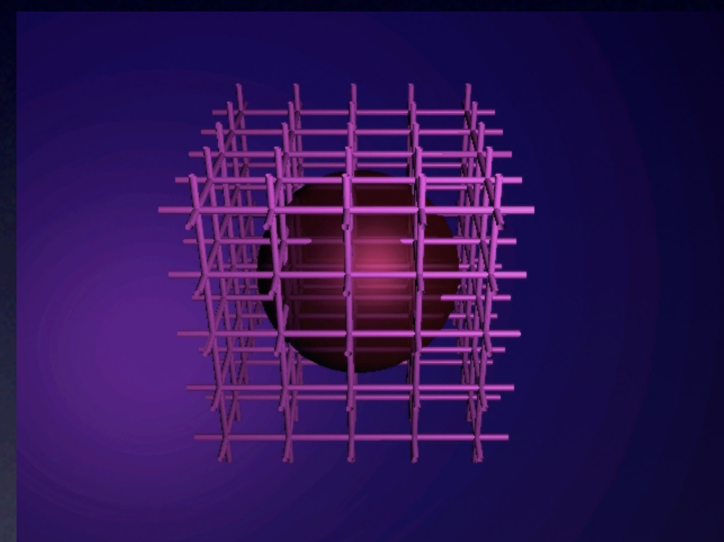
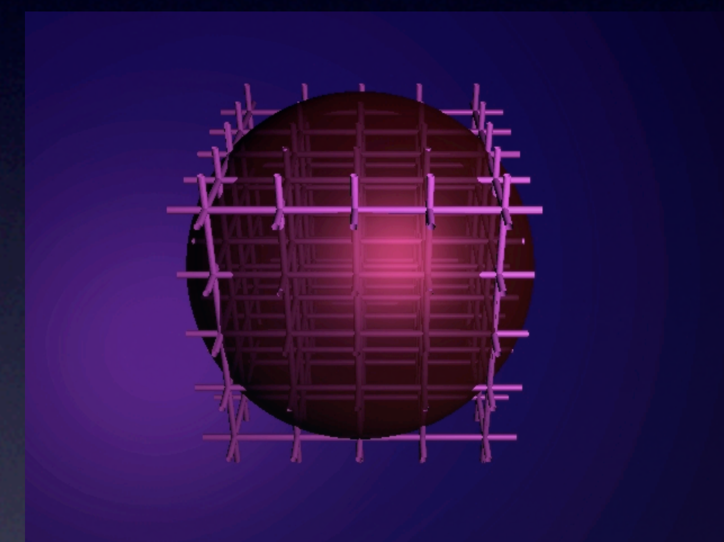
χPT at finite volume

- Volume dependence can be incorporated depending on pion mass and volume

		
$m_\pi L \gg 1$	$\mu_{\rm had} L \gg 1$	$\mu_{\rm had} L \lesssim 1$
p-regime	ε-regime (pion zero modes become non-perturbative)	"Out of luck"-regime

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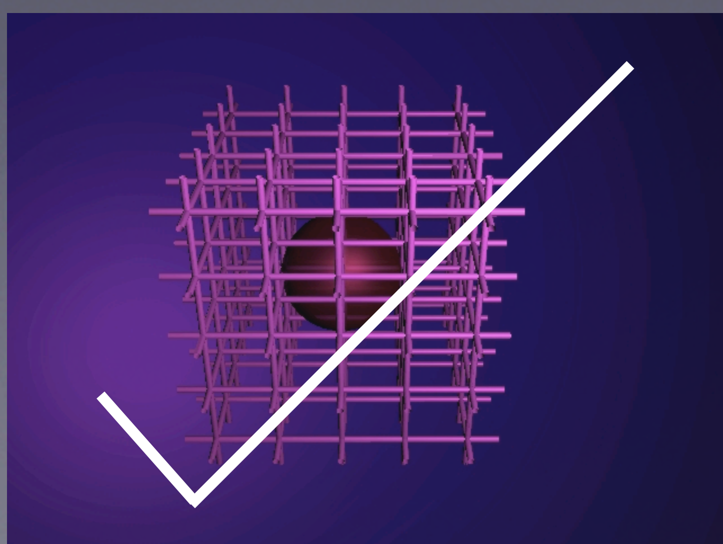	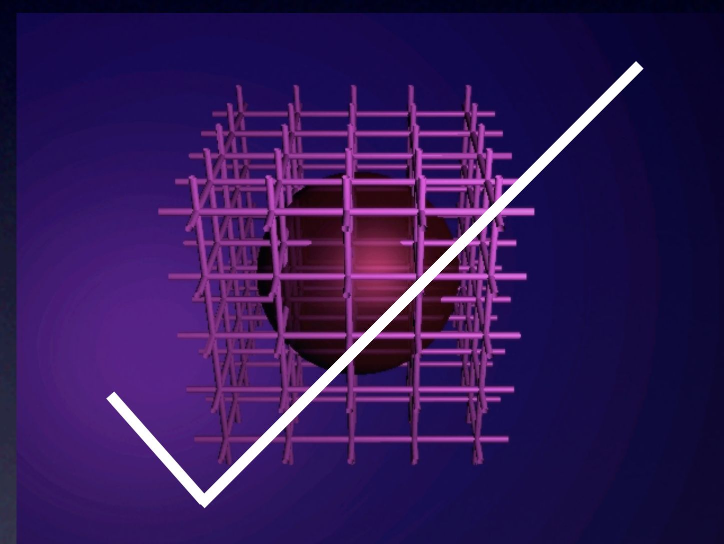	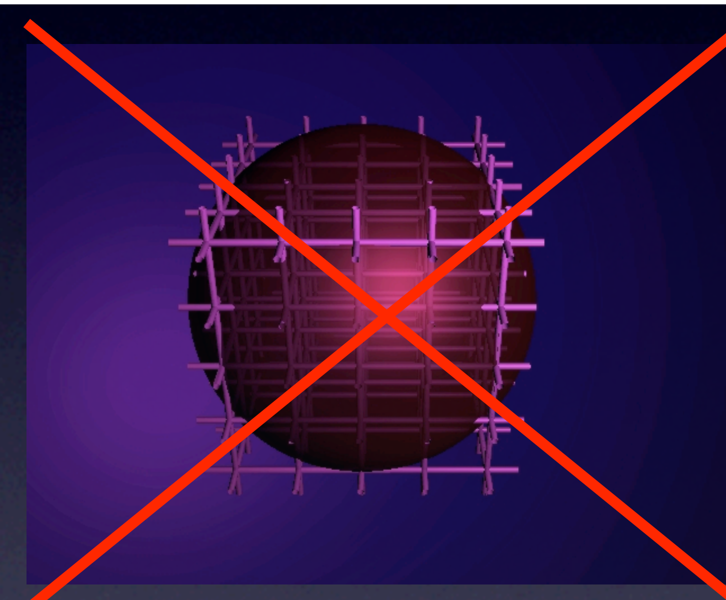
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Finite volume effects

- Polarisabilities are very sensitive to infrared scales
  - ➡ Expect large FV effects in lattice calculations
- Easily included in EFT for large volumes
  - Quantised momenta: $\vec{k} = \frac{2\pi}{L}\vec{n}$ for $n_i \in \mathbb{Z}$
  - Momentum integrals ⇒ mode sums

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_\pi^2 + i\epsilon} \rightarrow \int \frac{dk_0}{2\pi L^3} \sum_{\vec{k}} \frac{1}{k_0^2 - |\vec{k}|^2 - m_\pi^2 + i\epsilon}$$

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Poisson summation $\sum_{\vec{n}}\delta^{(3)}(\vec{y}-\vec{n}) = \sum_{\vec{m}} e^{2\pi i \vec{m}\cdot\vec{y}}$

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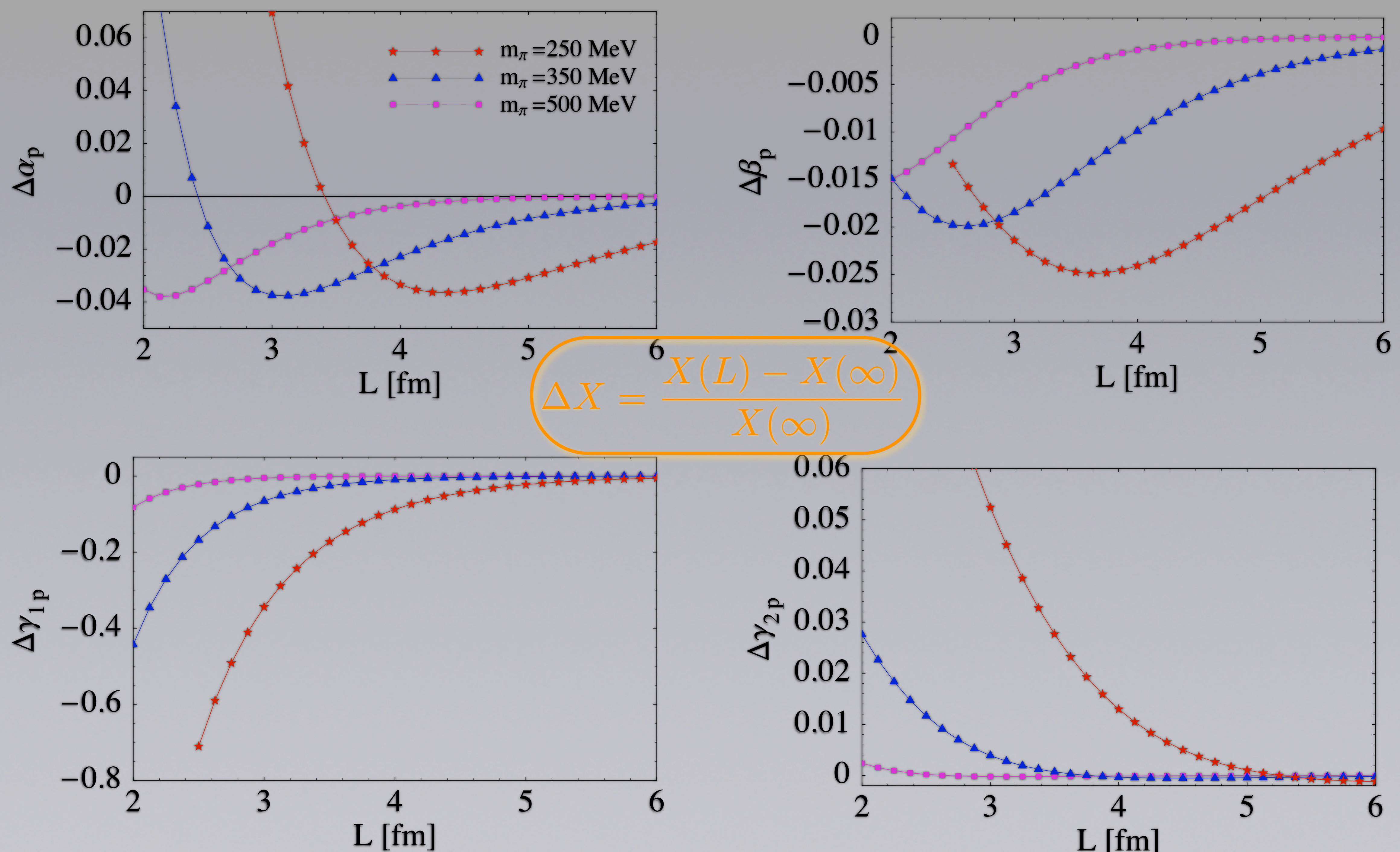
$$\xrightarrow{m_\pi L \to \infty} \sqrt{m_\pi/32\pi^3 L^3}\,\exp(-m_\pi L)$$

Volume Dependence: Proton



$$\Delta X = \frac{X(L) - X(\infty)}{X(\infty)}$$

Physical calculations

- Rigourous QCD results from unphysical calculations
- SU(3) electric charge matrix traceless:

$$\mathrm{diag}\{q_u, q_d, q_s\}$$

$$\downarrow$$

$$\mathrm{diag}\{q_u, q_d, q_s, q_j, q_l, q_r, q_u, q_d, q_s\}$$

- Turn sea charges off (unphysical hadrons)
  Still traceless: measure LECs
- Reconstruct *some* physical pols. using PQχPT:
  $\beta_p - \beta_n$, $\beta_{\Sigma+} - \beta_{\Sigma 0}$, $\beta_{\pi+}$, ...
- Errors are higher order in χPT

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Example: $\alpha_p - \alpha_n$

- $\alpha_p = \alpha_{loop}^{NLO} + \alpha_{loop}^{NNLO} + \alpha_{ct}^{NLLO} + \dots$
- Loop contributions are known functions of quark charges: easy to fix unphysical charges
- NNLO c.t.s come from tree level insertions of

$$\mathcal{L}_{ct} = \mathbf{E}^2 \left\{ a_{\alpha_E} \mathrm{str}[\overline{B} \mathcal{Q}^2 B] + b_{\alpha_E} \mathrm{str}[\overline{B} \mathcal{Q} B] \mathrm{str}[\mathcal{Q}] + c_{\alpha_E} \mathrm{str}[\overline{B} B] \mathrm{str}[\mathcal{Q}^2] \right\}$$

- Chargeless sea simulations insensitive to c_{α_E} but no contribution to $\alpha_p - \alpha_n$
- Errors at N^{3}LO: $\mathbf{E}^2 \mathrm{str}[\overline{B} m_q B] \mathrm{str}[\mathcal{Q}^2]$

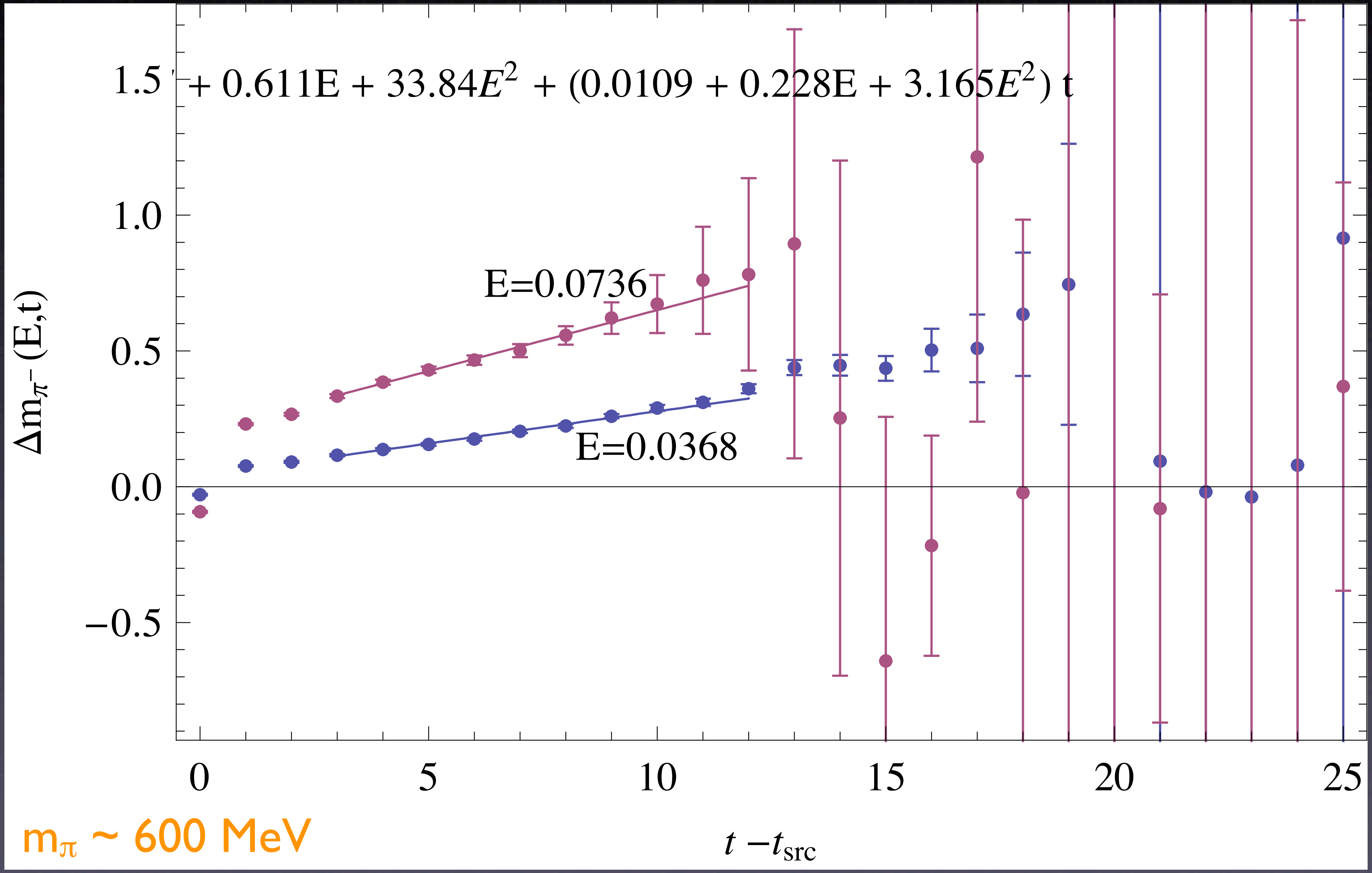
Physics from PQχPT

- Individual pion pols safe to N^{3}LO
    - also in SU(4|2)
- Given convergence of SU(6|3) chiral expansion
    - "Isovector" combinations of baryon pols.
        - Eg: β_p - β_n, $\beta_{\Sigma+}$ - $\beta_{\Sigma-}$
        - Errors are N$^{3(4)}$LO in χPT

Preliminary QCD calcs

- Computing combinations of QCD polarisabilities using USQCD resources
  - *Two other groups in USQCD also using BF methods*
- Clover on DWF lattices from RBC/UKQCD:
  16^3x32 (1.9 fm) m_π=400 MeV
  24^3x64 (2.7 fm) m_π=330, 420 MeV
- Tunings done and various 16^3x32 external fields run
- Investigate effects of non quantisation

π+ polarisabilities



Acceleration? slope linear in E, offset quadratic in E

Lattice polarisabilities

- All EM and spin polarisabilities can be measured with external fields
- Large volume effects and strong mass dependence require large volumes and small masses
- Higher order and generalised polarisabilities [(doubly)-virtual Compton scattering] are also measurable