Charm physics from lattice QCD

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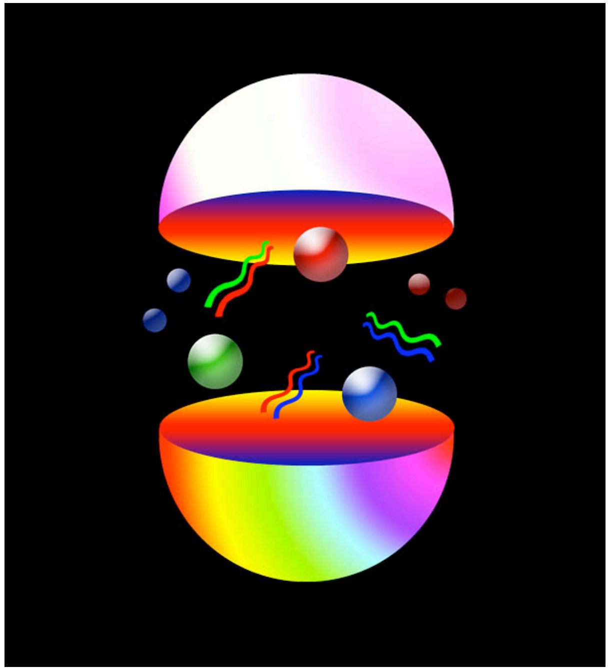
HPQCD collaboration

EINN 2007
Milos

QCD is key part of SM but quark confinement tricky

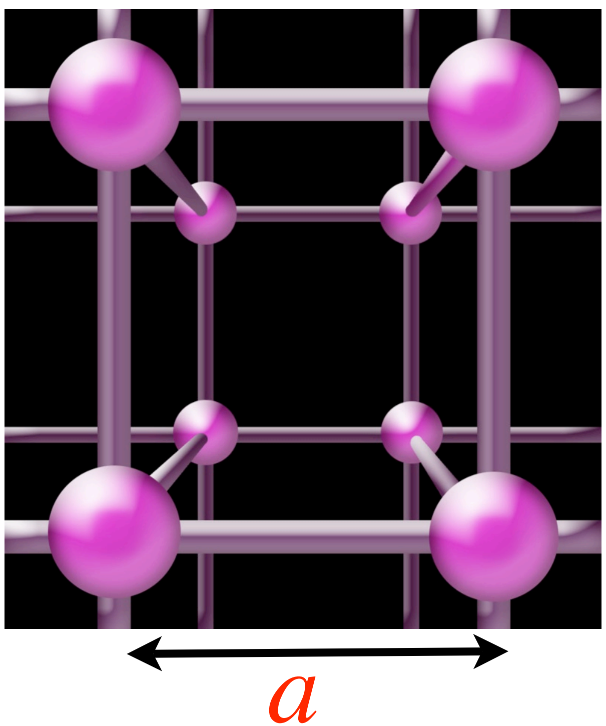


Lattice QCD enables calcn of QCD effects "from first principles". Done by numerical evaln of Path Integral in a 4-d vol. of space-time defined as a lattice

$$\int dA_\mu e^{-L_{QCD}} O(A_\mu)$$

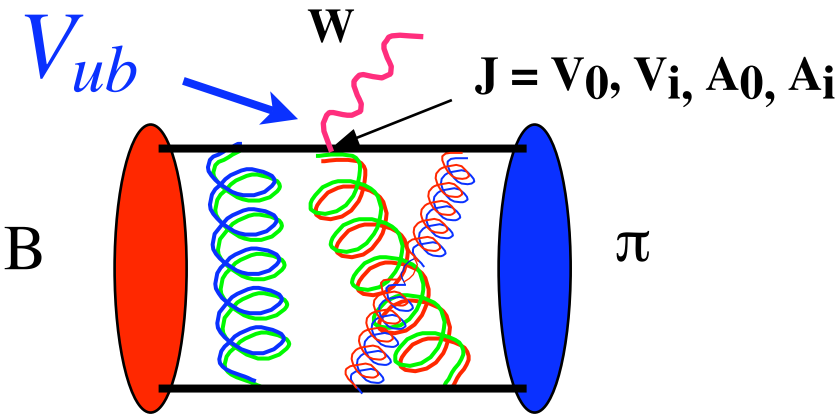
RECIPE

- Generate sets of gluon fields that contribute most to the PI
- Calculate averaged "hadron correlators" on these and fit to obtain masses and simple matrix elements
- Fix m_q and determine a to get physical results



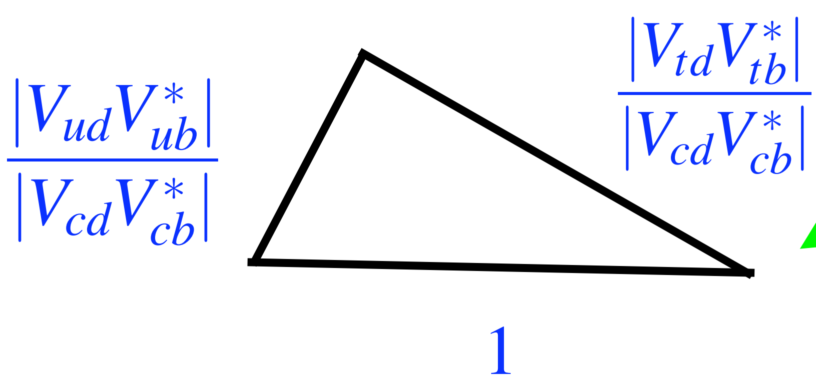
Where can lattice QCD have most immediate impact?

Precision calculations of electroweak decay rates for gold-plated hadrons ⟶ CKM physics



expt=(CKM)x(lattice calc.)

$$
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
\pi \to l\nu & K \to l\nu & B \to \pi l\nu \\
 & K \to \pi l\nu & \\
V_{cd} & V_{cs} & V_{cb} \\
D \to l\nu & D_s \to l\nu & B \to D l\nu \\
D \to \pi l\nu & D \to K l\nu & \\
V_{td} & V_{ts} & V_{tb} \\
\langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle &
\end{pmatrix}
$$



Unitarity triangle - test this!

I will concentrate on results relevant to this programme...

Why is lattice QCD so hard?

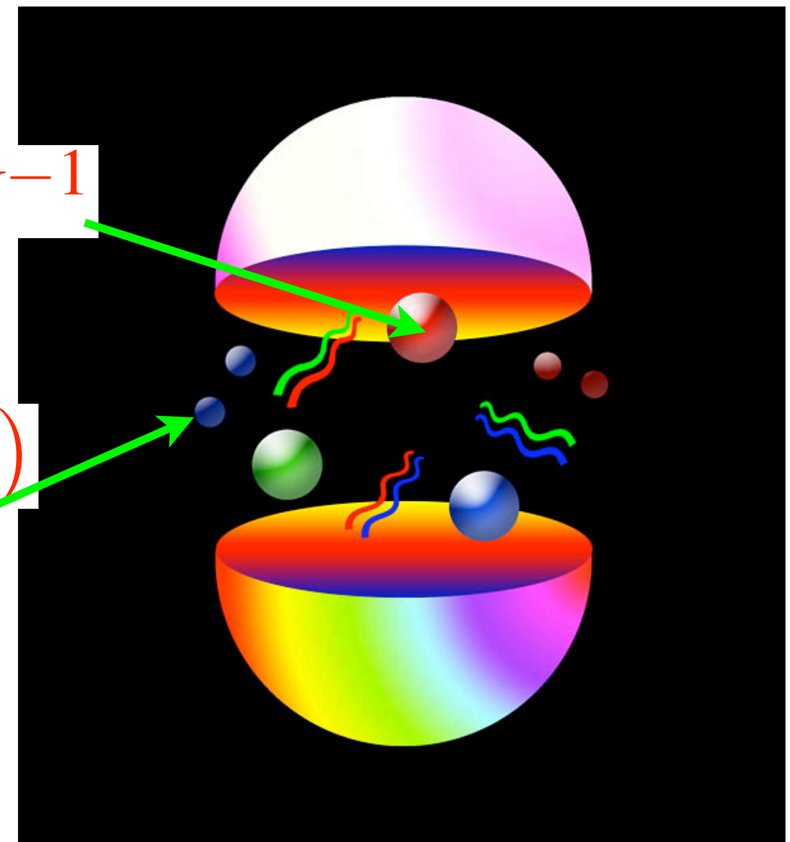
Handling light u,d, s quarks is a big headache

$$L_{q,QCD} = \overline{\psi}(\gamma \cdot D + m)\psi \equiv \overline{\psi} M \psi$$

Quarks must be 'integrated out' by inverting Dirac matrix M

valence quarks, calculate M^{-1}

sea quarks, include $det(M)$ in importance sampling gluons



Cost inc. as $m_q \rightarrow 0$

and also as $a \rightarrow 0$ $L \rightarrow \infty$

The story so far ....

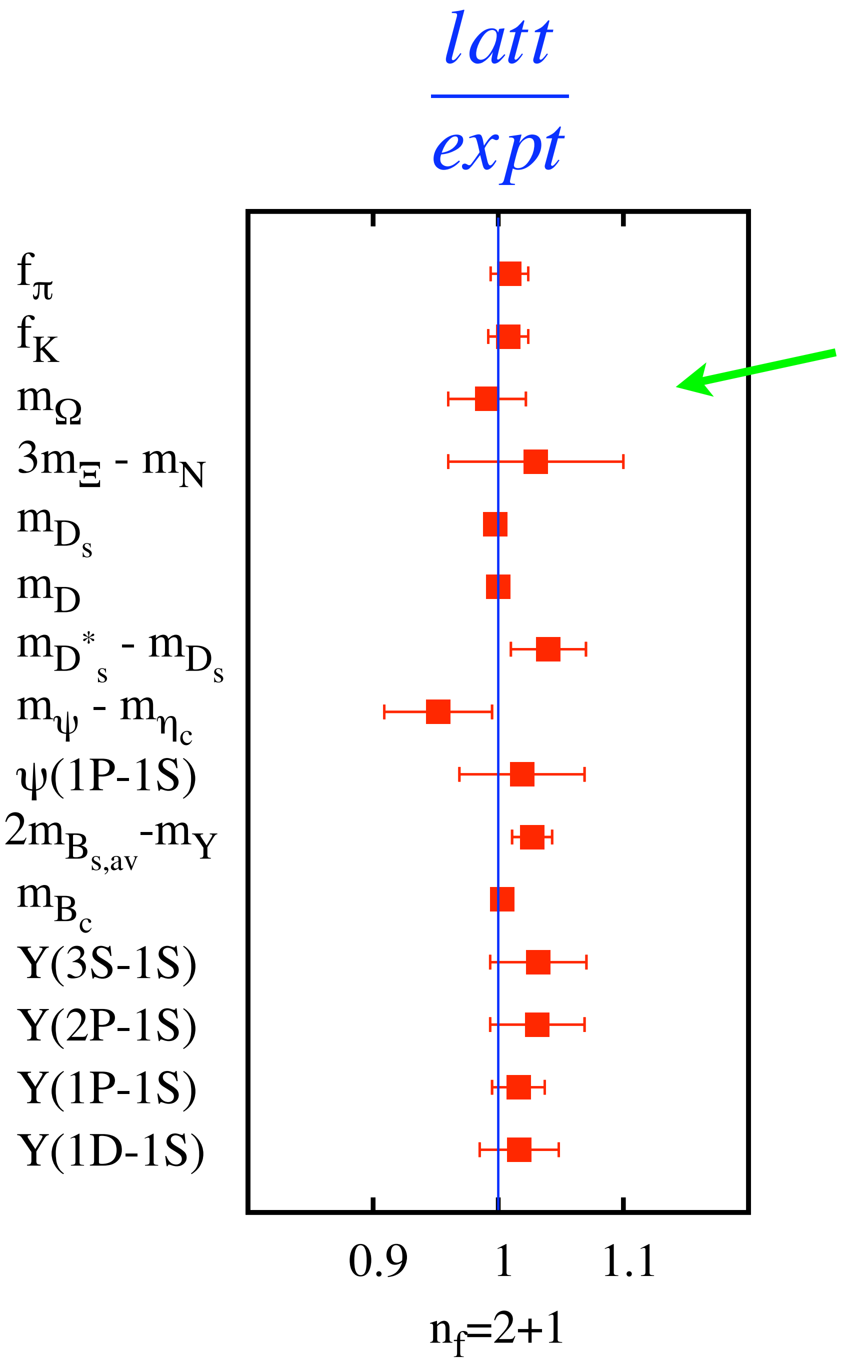
Early days (before 2000) - u, d, s sea quarks omitted or inc. with u/d masses 10-20x too big.
Systematic errors 10-20 % and theory not self-consistent

Now (since 2000) - possible to inc. u/d sea quarks with masses only 3-5x too large and extrapolate to real world.
Improved staggered quark formalism first to do this since numerically very fast. Uses $(det(M))^{0.25}$ to reduce 4 "tastes" to 1 flavor.

2007 - improved staggered calculations have matured, many values of a and $m_{u/d}$ from MILC collaborn.
Results using other formalisms now appearing

Future - looks good. Lots of analysis to be done ....

2007 results



Essential to check how lattice QCD is doing vs well-known gold-plated experimental quantities

Update of 2003 results (MILC/ FNAL/HPQCD) using improved staggered sea quarks.

Fix QCD params from:

$\Upsilon(2S-1S), m_\pi, m_K, m_{\eta_c}, m_\Upsilon$

Quenched Approx. is dead!

New (HPQCD): Highly improved staggered quarks (HISQ) - improves disc. errors further over asqtad. Allows use for c quarks.

Heavy (b,c) valence quarks in lattice QCD

Discretise Dirac equation onto lattice → syst. errors

$$L_{q,QCD} = \overline{\psi}(\gamma \cdot D + m)\psi \equiv \overline{\psi} M \psi$$

For light quarks, discretisation errors set by $\Lambda_{QCD} a$

For $a \approx 0.1 fm \quad (\Lambda_{QCD} a)^2 \approx 0.06$

Multiple values of a allows fit and extrapolation to $a = 0$

For b, c quarks discretisation errors set by $m_Q a$

$$a \approx 0.1 fm \quad m_c a \approx 0.6, m_b a \approx 2.5$$

For b quarks, use the fact that $m_b a$ is not a dynamical scale to write down an effective theory in which it is removed.

Possibilities: HQET, NRQCD, FNAL heavy quarks

← handles Υ *and* B

Now disc. errors set by e.g (mom. in bound state)a

For c quarks, have a choice of effective theory or beating down disc. errors in relativistic theory

FNAL/MILC use FNAL heavy (clover) quarks

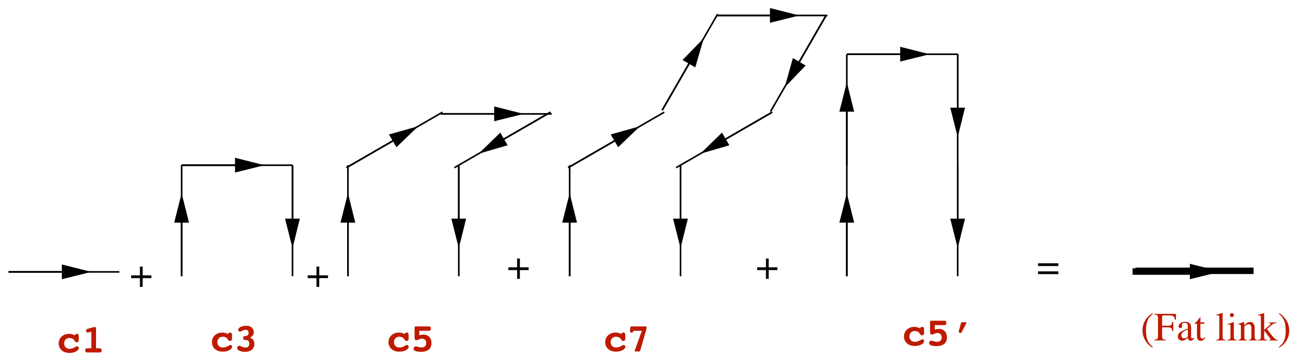
NEW - HPQCD use Highly Improved Staggered Quarks as a relativistic method

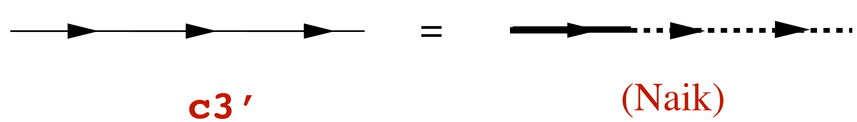
How good a disc. needed for precise charm results?

$m_c a \approx 0.4,\ (m_c a)^2 \approx 0.2,\ \alpha_s (m_c a)^2 \approx 0.06,\ (m_c a)^4 \approx 0.04$

Remove *all* these errors by improved disc. and check this

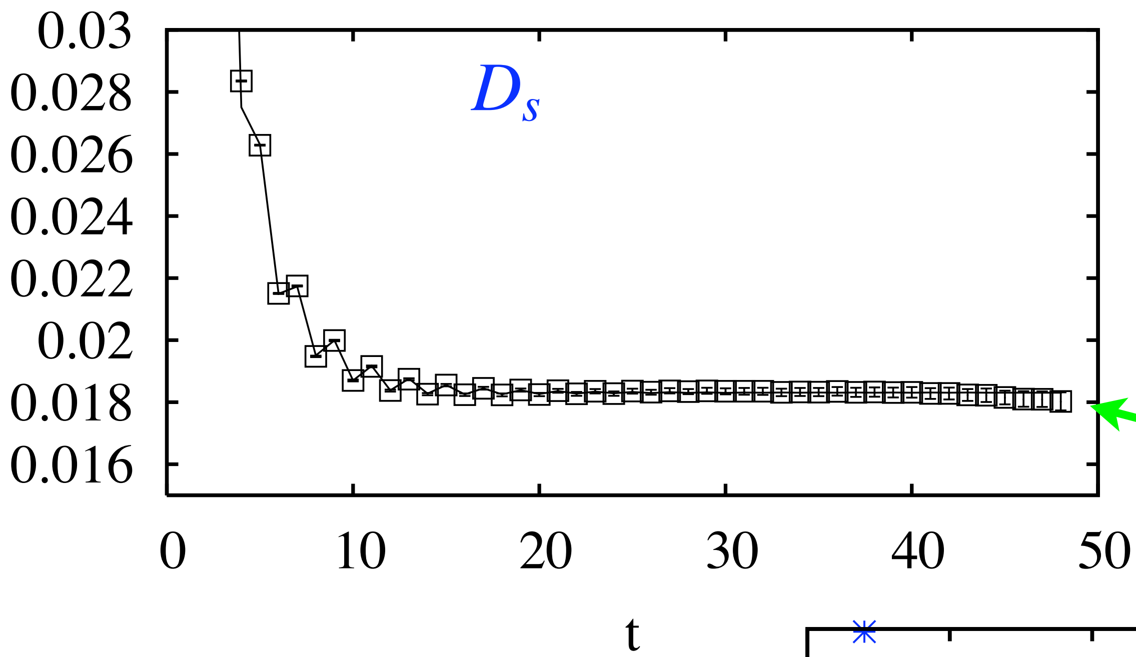
Improved staggered quarks remove all tree-level a^2 errors





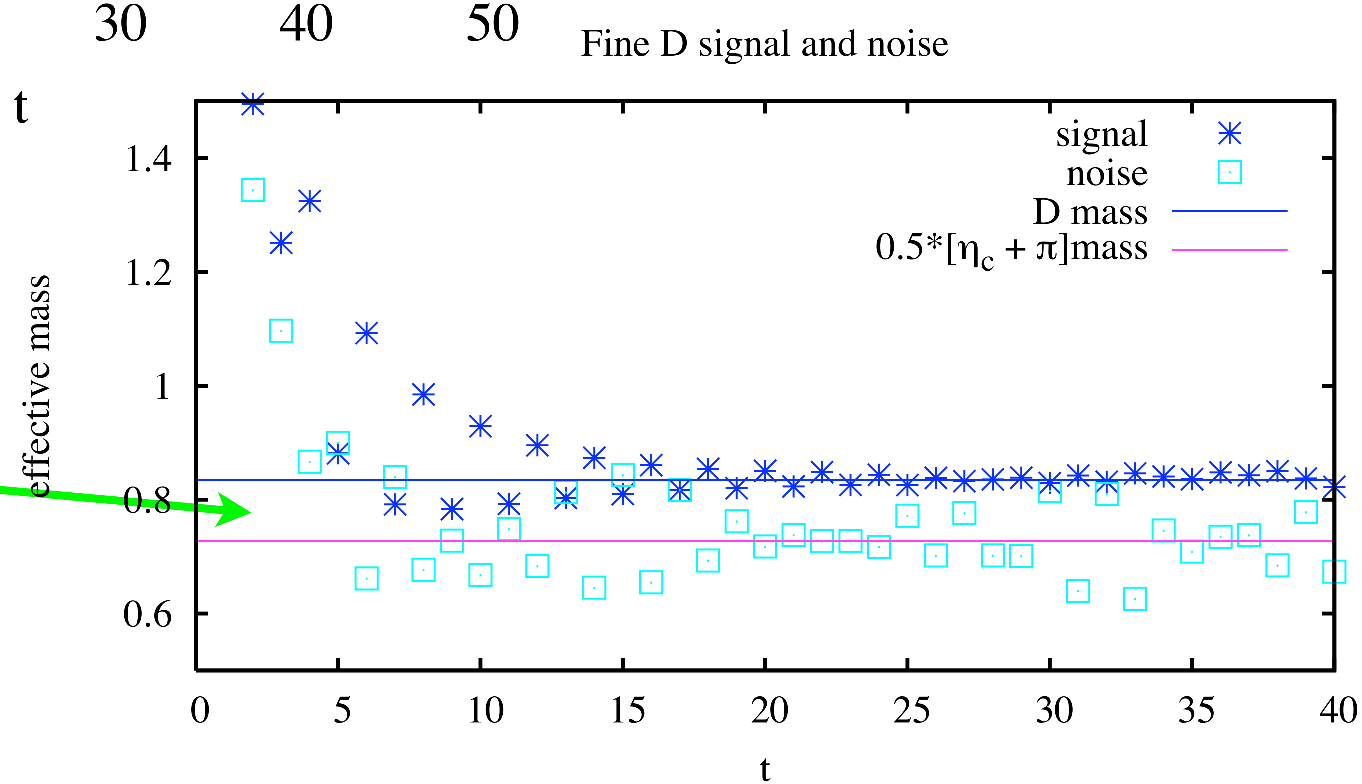
Highly Improved Staggered Quarks (HISQ) reduce further ‘taste-changing’ errors in staggered formalism → very accurate

Excellent statistical accuracy from random wall sources (as used by MILC for light mesons)



$$\sum a_i e^{-M_i t} + (-1)^t a_{ip} e^{-M_{ip} t}$$
$$+ (t \to T - t)$$

correlator/(fit ground state)



Can even study the noise in the correlator and see what mass it has ...

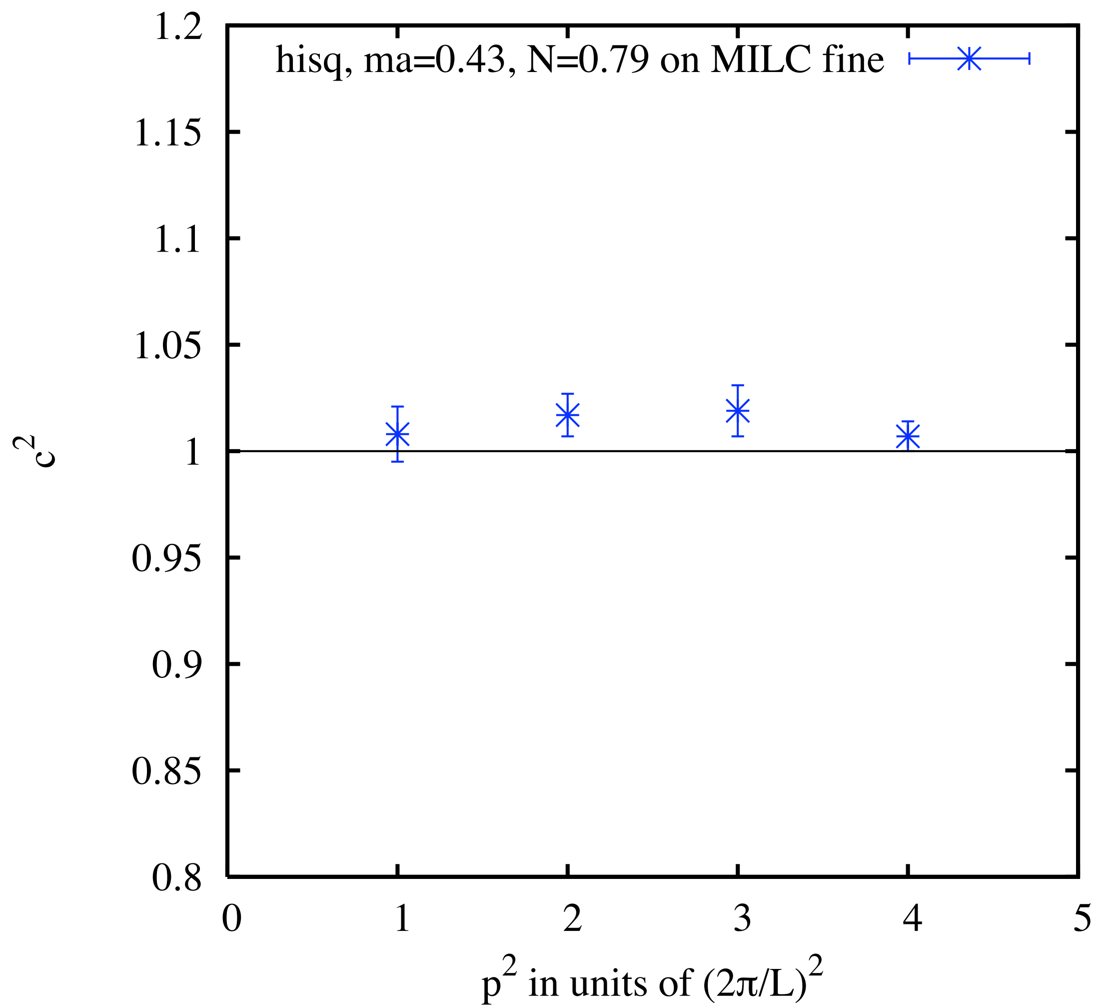
Important tests e.g. using charmonium

$$c^2 = \frac{E^2 - m^2}{p^2}$$

is sensitive test of a^2 errors.

We adjust Naik coeff. so

$$c^2 = 1$$

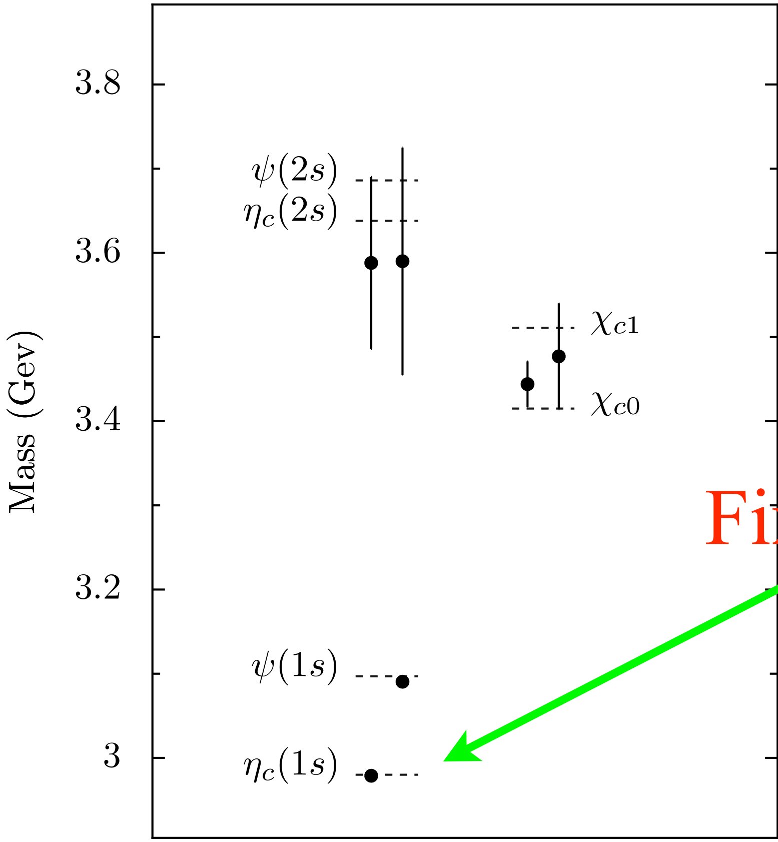


Removes errors at $\alpha_s (m_c a)^2, (m_c a)^4$

Follana et al, hep-lat/0610092

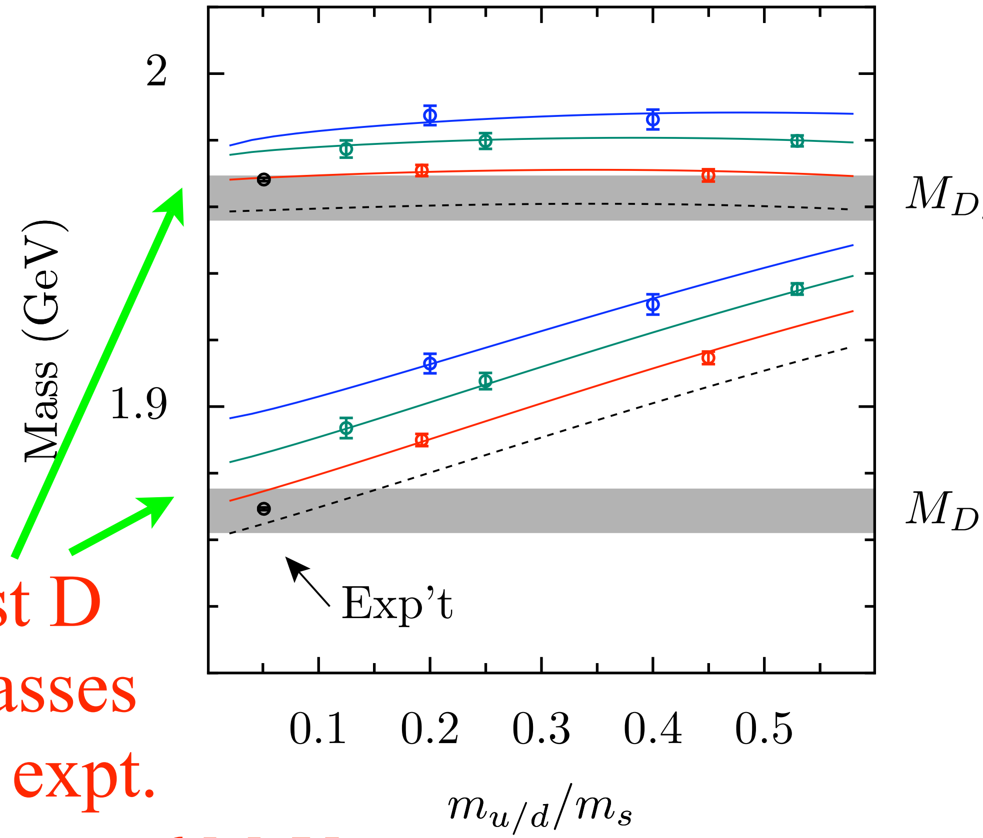
Compare charmonium and D spectrum

using MILC configs with sea quarks



Fix m_c

test D masses vs expt.

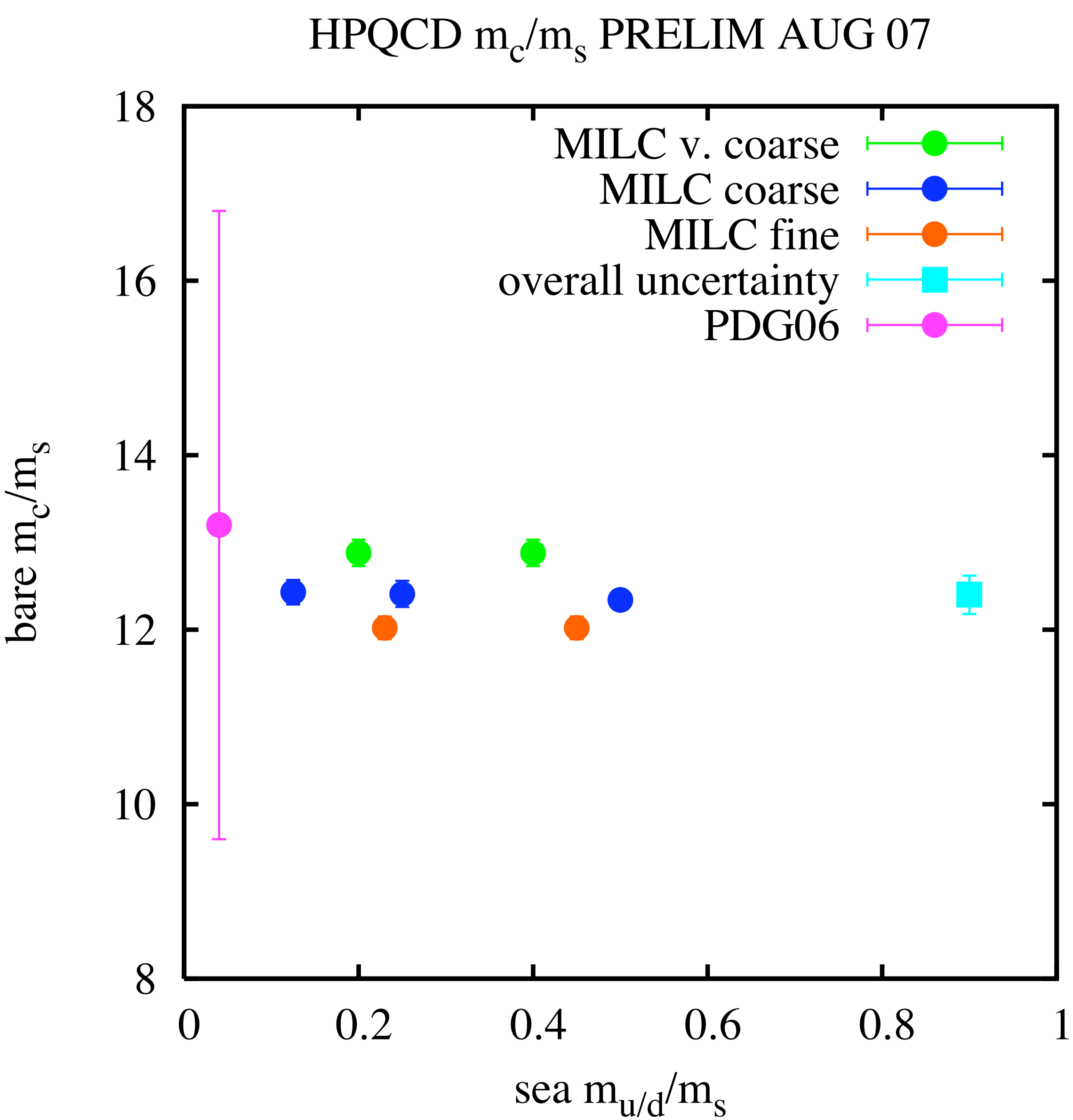


lattice errors 6 MeV

Another key test of disc. errors since charmonium and D have different dynamics → more tests of lattice QCD possible. Same action as for u,d,s.

E.Follana et al, 0706.1726[hep-lat]

Will lead to a very accurate value for m_c



Since using same formalism for s and c can get accurate ratio of bare lattice quark masses.

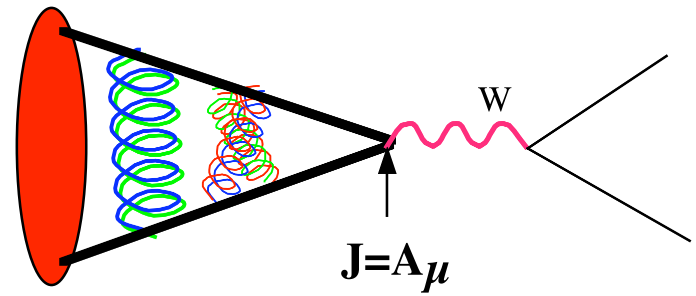
For accurate $\overline{MS}$ ratio need a 1-loop lattice perturbative calcn.

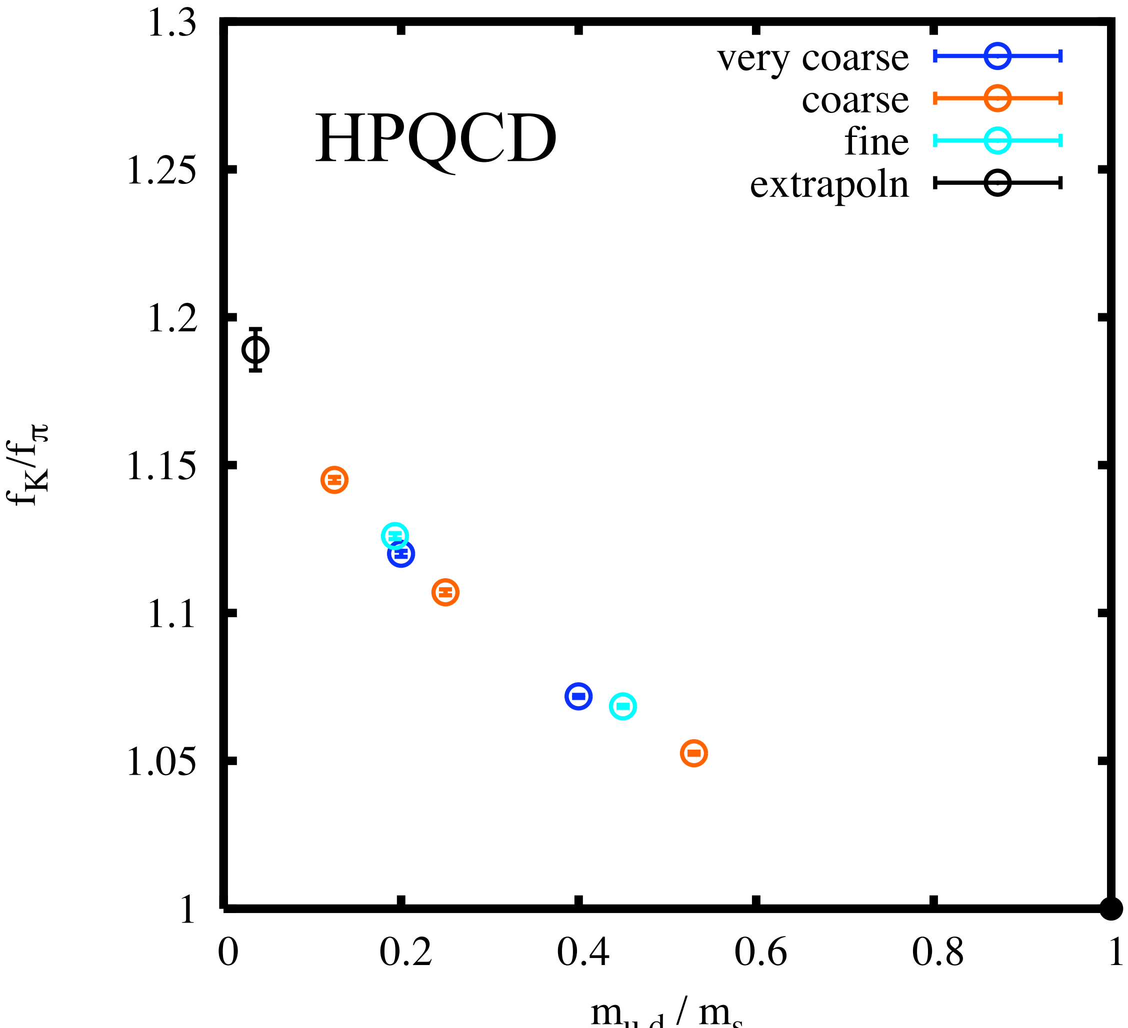
Lattice nonperturbative methods also underway.

Leptonic decay rate gives CKM element or test vs expt.

π leptonic decay

$$Br(\pi \to \mu\nu) \propto V_{ud}^2 f_\pi^2$$





$$\frac{\Gamma(K \to \mu\nu)}{\Gamma(\pi \to \mu\nu)} \to \frac{V_{us}^2 f_K^2}{V_{ud}^2 f_\pi^2}$$

Using HISQ for valence u/d,s on MILC gives

$$V_{us} = 0.2262(14)$$

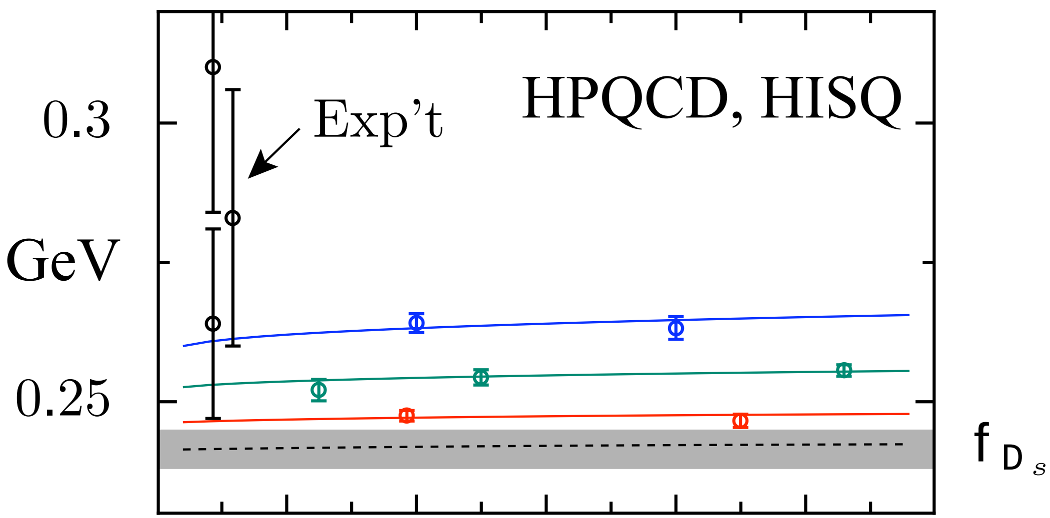
competitive with PDG from Kl3

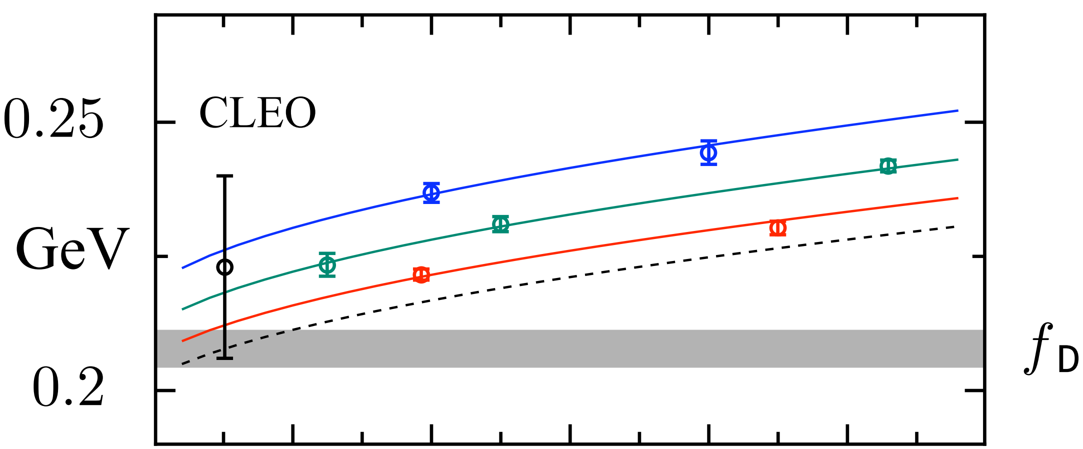
E.Follana et al, 0706.1726[hep-lat]

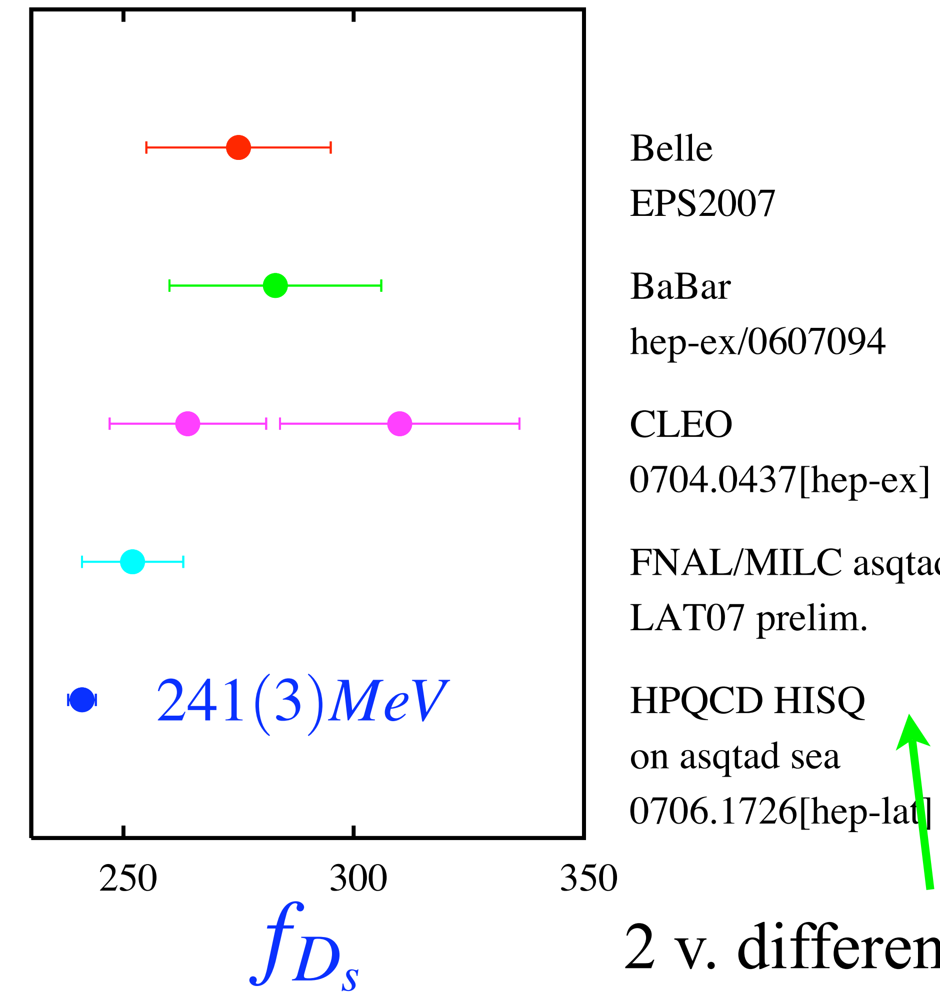
2007 New Results for D, D_s decay constants, for comparison to experiment

$D_x \to l\nu$

Lattice inc u,d,s sea vs expt





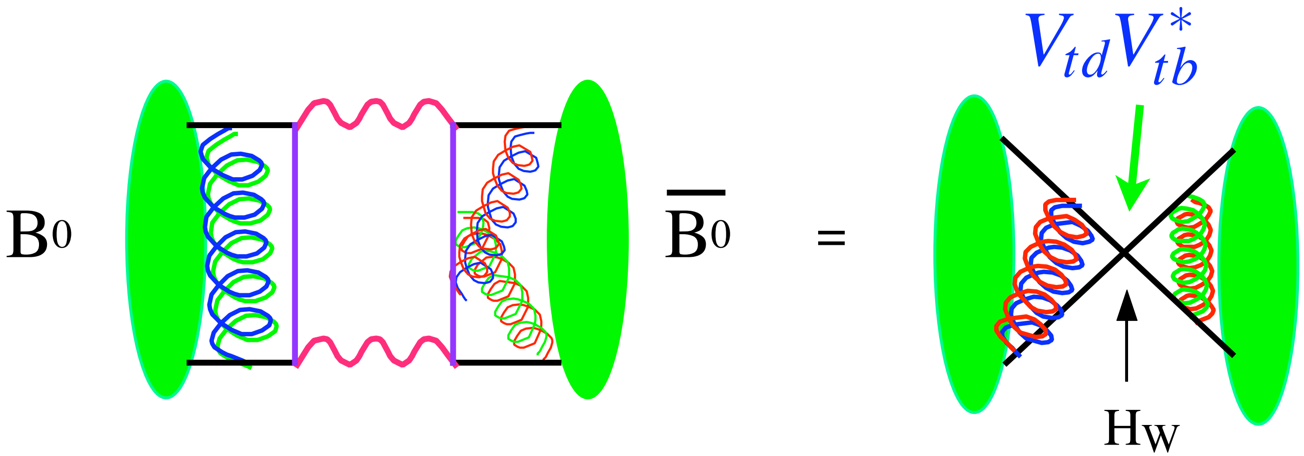


2 v. different lattice calcs

Next two years: more lattice results and improved semileptonic form factors also ...

- expt uses Vcs = Vud
- error will improve to few % in 2 yrs
- not all em corrns calculated ..

b physics - B^0 mixing and CKM constraint



Parameterise with $f_B^2 B_B$ where f_B is decay constant.

$$\Delta M_x = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tx}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_x} f_{B_x}^2 \hat{B}_{B_x}$$

Take exptl ratio from oscillation rates for B_s and B_d

$$\left|\frac{V_{td}}{V_{ts}}\right| = \xi \sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}}\ , \quad \xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_B \sqrt{B_B}}$$

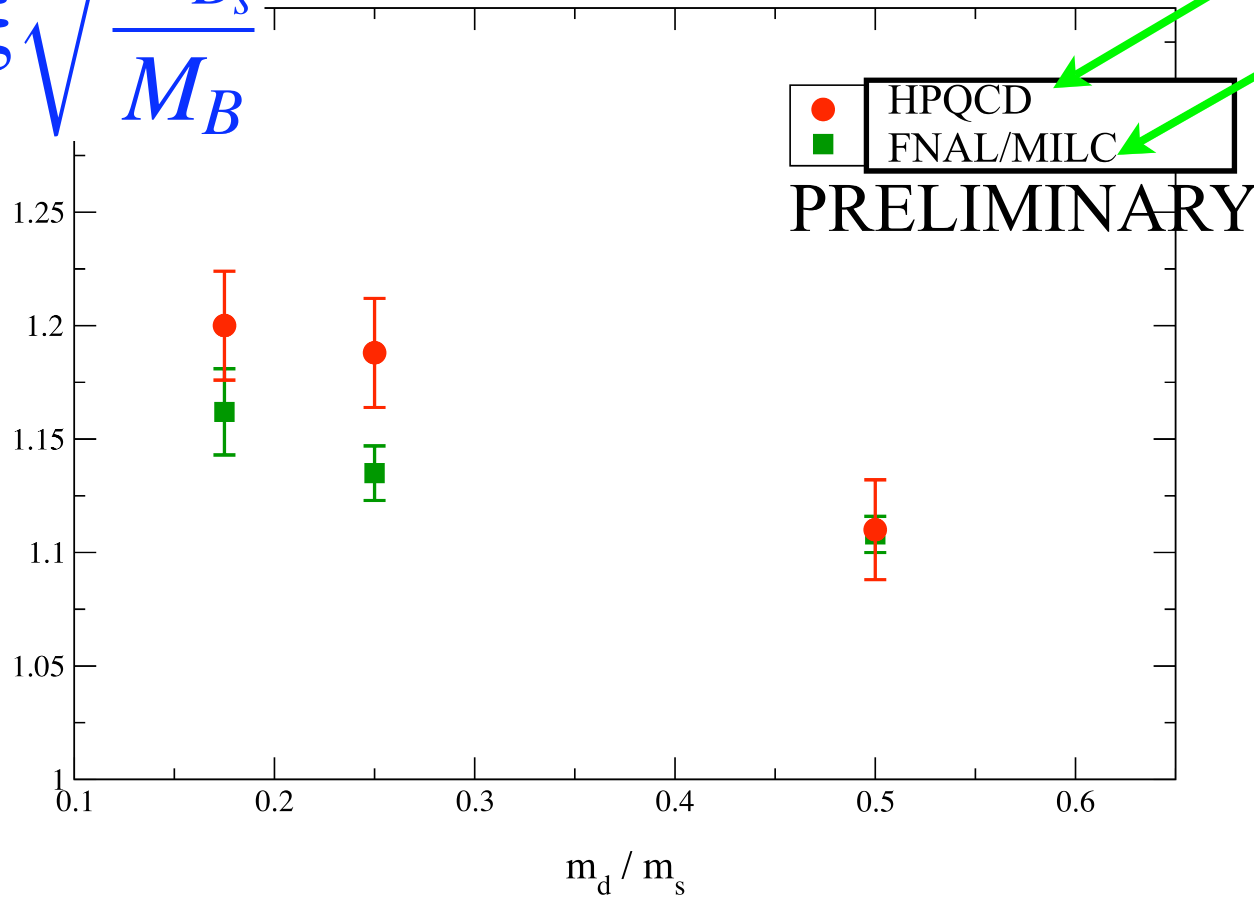
calculate in lattice QCD, renormln cancels

2007 New results for $\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}}$ inc. u, d, s sea quarks

$\xi\sqrt{\frac{M_{B_s}}{M_B}}$

NRQCD for b

FNAL for b



HPQCD
hep-lat/0610104

$f_{B_s}\sqrt{\hat{B}_{B_s}} = 0.281(21) GeV$

one value of a so far, main error is renormln of lattice 4-q operator

Can compare to easier lattice calc. of f_{B_s}/f_B

HPQCD, hep-lat/0507015 1.20(3) cf $f_{D_s}/f_D = 1.164(11)$

FNAL/MILC, Simone, LAT07 1.26(4)

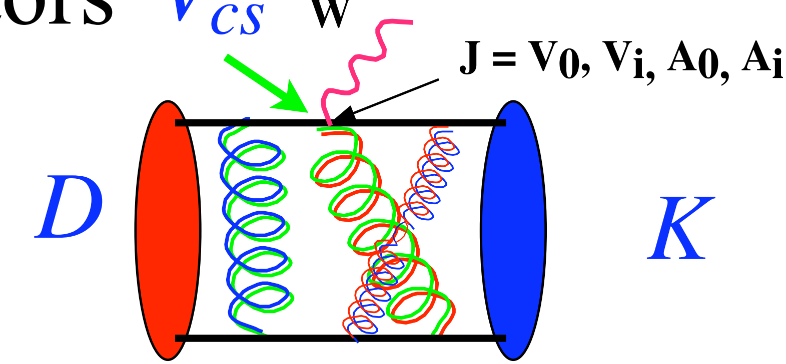
Gamiz, LAT07

Semileptonic decay rates/form factors V_{cs}

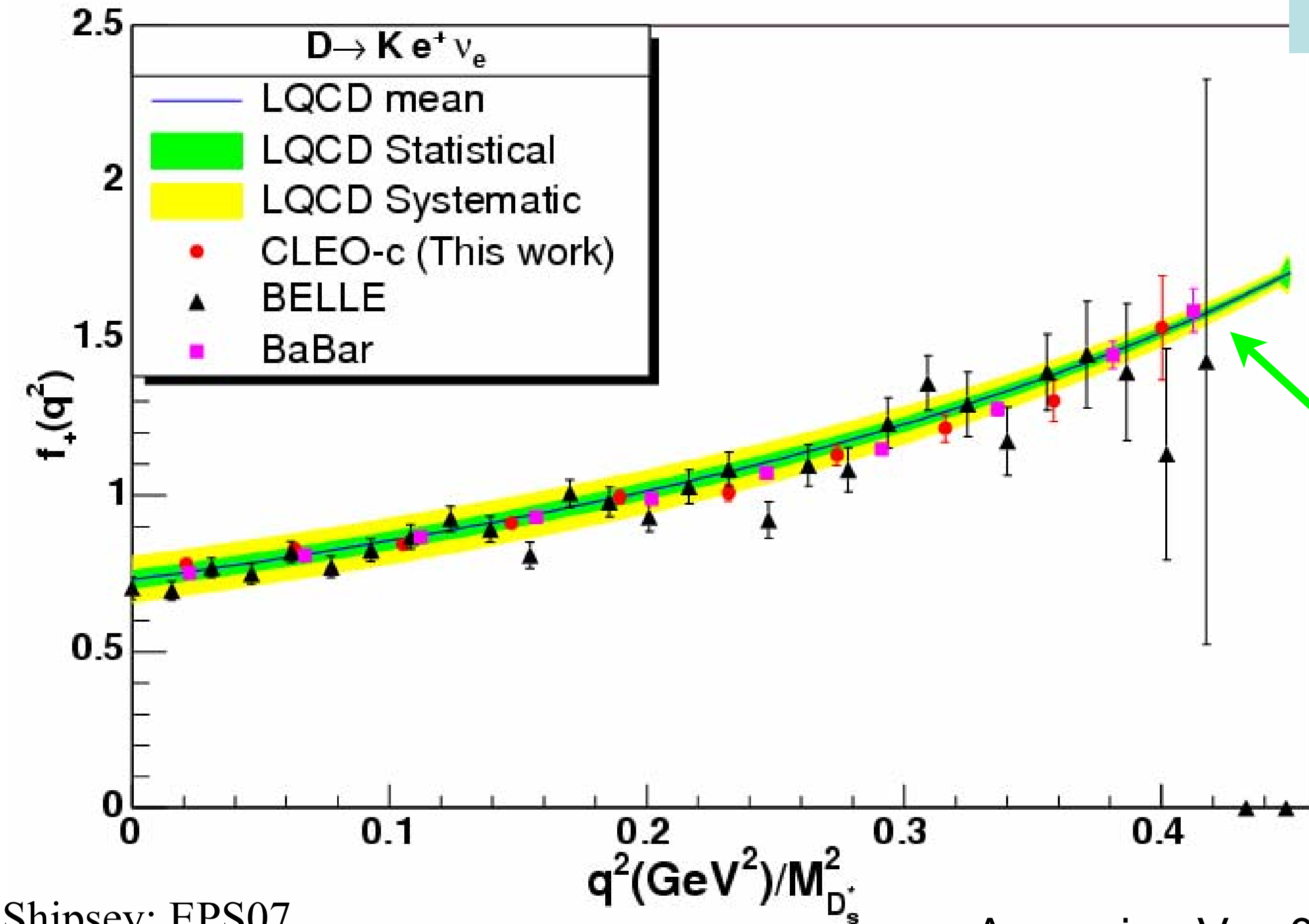
Much harder "3-pt" calculation

$$q^2 = m_D^2 + m_K^2 - 2m_D E_K$$

D rest frame



FNAL-MILC-HPQCD



$$\frac{d\Gamma}{dq^2} \propto |V_{cs}|^2 |f_+(q^2)|^2$$

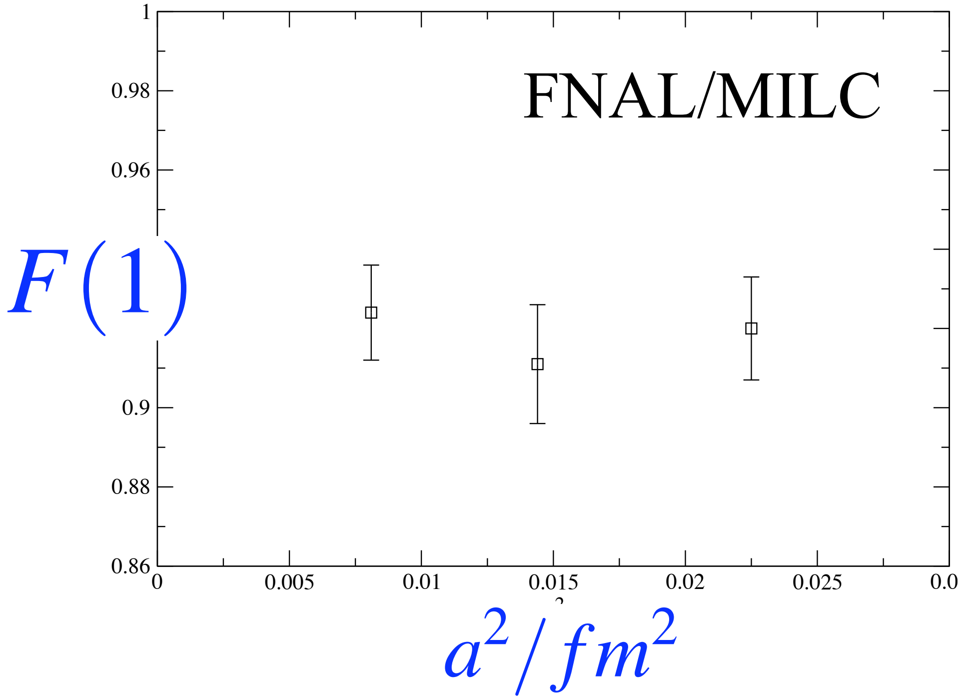
FNAL/MILC results compared to expt - theory error ~ 10%

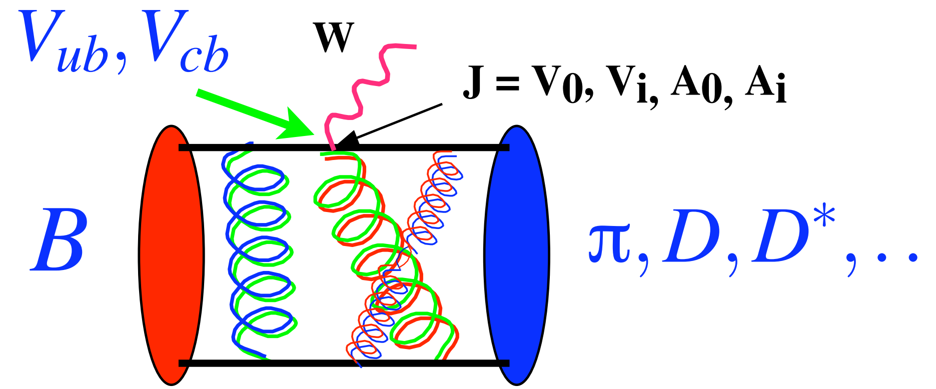
Better precision needed for good lattice test

Assuming V_{cs}=0.9745

Shipsey: EPS07,
FNAL/MILC: hep-ph/0408036

B excl. semileptonic decay and CKM constraint





$B \to D^* l \nu$ Laiho, LAT07

rate at zero recoil

$\propto |V_{cb} F(1)|^2$

FNAL/MILC with u, d, s sea quarks, 3 values of a

$$F(1) = 0.930(12)(19) \xrightarrow{\text{HFAG}} V_{cb} = 38.7(0.7)(0.9) \times 10^{-3}$$

stat syst expt latt

$B \to \pi l \nu$ Flynn+Nieves 0705.3553[hep-ph] combine lattice (HPQCD, FNAL/MILC) with LCSR, get

$$V_{ub} = 3.47(29) \times 10^{-3}$$

work underway to extend lattice results

Use lattice results with u, d, s sea quarks + experiment for constraints on unitarity triangle.

Lattice inputs (2+1 sea quarks):

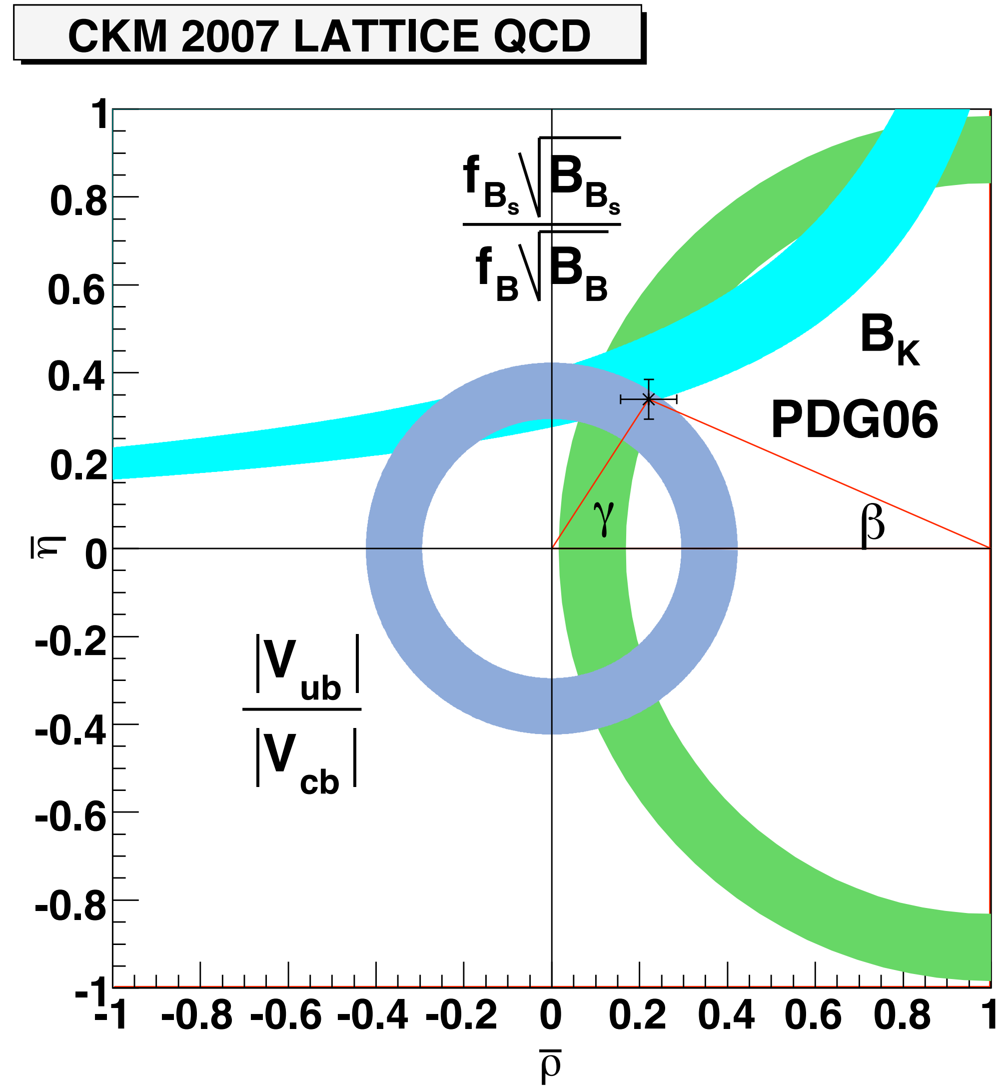
B_K

f_K/f_π , $f_+(K \to \pi l \nu)$

$F(B \to D^* l \nu)$

$f_+(B \to \pi l \nu)$

$$\frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}}$$



Errors on lattice results should halve over next two years

Improve checks in D physics

Conclusions

- Lattice calculations inc. sea quarks are in excellent shape.
- Precision continues to improve for the staggered quark formalism
- Significant new results this year in charm physics that now give accuracy similar to that for light hadrons
- More work needed to improve b physics further.

Future:

In next two years, lattice errors on CKM constraints should halve.

More checks will be done against gold-plated decays of D.

Precise values of m_c and m_b expected soon.