

GDH sum rule on the neutron

Paolo Pedroni

INFN-Sezione di Pavia, Italy



gdh-Collaboration

The logo for the GDH Collaboration is shown. It consists of the letters 'g', 'd', and 'h' in a stylized, lowercase font. The 'g' is red, the 'd' is green, and the 'h' is blue. Above the 'd' and 'h' are two vertical arrows: a green one pointing up and a blue one pointing up. A red circle is drawn around the top of the blue arrow. To the right of this graphic, the word 'Collaboration' is written in a black, serif font.

The GDH collaboration

J. Ahrens¹⁰, S. Altieri^{16,17}, J.R. Annand⁷, G. Anton⁴, H.J. Arends¹⁰, K. Aulenbacher¹⁰, R. Beck¹⁰, M. Balckston³, C. Bradtke¹, A. Braghieri¹⁶, N.d'Hose⁶, H. Dutz², S. Goertz¹, P. Grabmayr¹⁸, K. Hansen⁹, J. Harmsen¹, S. Hasegawa¹⁴, T. Hasegawa¹², E. Heid¹⁰, K. Helbing⁴, H. Holvoet⁵, L. VanHoorebeke⁵, N. Horikawa¹⁵, T. Iwata¹⁴, P. Jennewein¹⁰, T. Kageya¹⁵, F. Klein³, R. Kondratiev¹³, J. Krimmer¹⁸, M. Lang¹⁰, B. Lannoy⁵, V. Lisin¹³, J.C. McGeorge⁷, A. Meier¹, D. Menze², W. Meyer¹, T. Michel⁴, J. Naumann⁵, A. Panzeri^{16,17}, P. Pedroni¹⁶, T. Pinelli^{16,17}, I. Preobrajenski^{10,13}, E. Radtke¹, E. Reichert¹¹, G. Reicherz¹, Ch. Rohlof², G. Rosner⁷, T. Rostomyan¹⁶, D. Ryckbosch⁵, B. Schoch², M. Schumacher⁸, B. Seitz⁸, T. Speckner⁴, M. Steigerwald¹⁰, N. Takabayashi¹⁴, G. Tamas¹⁰, A. Thomas⁹, R. van de Vyver⁴, A. Wakai¹⁴, W. Weihofen⁸, H. Weller³, F. Zapadtko⁸, G. Zeitler³

¹Institute of Experimental Physics, Ruhr-University, **Bochum**, Germany

²Physics Institute, University of **Bonn**, Germany

³Dept. of Physics-Duke University and TUNL, **Durham**, NC, USA

⁴Physics Institute, University of Erlangen-Nuernberg, **Erlangen**, Germany

⁵Nuclear Physics Laboratory, **Gent**, Belgium

⁶**CEA Saclay**, DSM/DAPNIA/SPhN, Gif-sur-Yvette, France

⁷Department of Physics & Astronomy, University of **Glasgow**, U.K.

⁸II. Physics Institute, University of **Goettingen**, Germany

⁹Department of Physics, University of **Lund**, Sweden

^{10,11} Institute of Nucl. Physics and Inst. of Physics, University of **Mainz**, Germany

¹² Faculty of Engineering, Miyazaki University, **Miyazaki**, Japan

¹³ INR, Academy of Science, **Moscow**, Russia

^{14,15} Department of Physics and CIRSE, Nagoya University, **Nagoya**, Japan

^{16,17} INFN Sezione di Pavia and Dept. of Nucl. Physics of the University, **Pavia**, Italy

¹⁸ Physics Institute, University of **Tuebingen**, Germany

SUMMARY

- Physics motivation
- Experimental set-up

- **Results**

$$\vec{\gamma} \vec{d} \rightarrow \left\{ \begin{array}{l} X \\ N\pi(\pi)N_{Spect} \\ pn \end{array} \right.$$

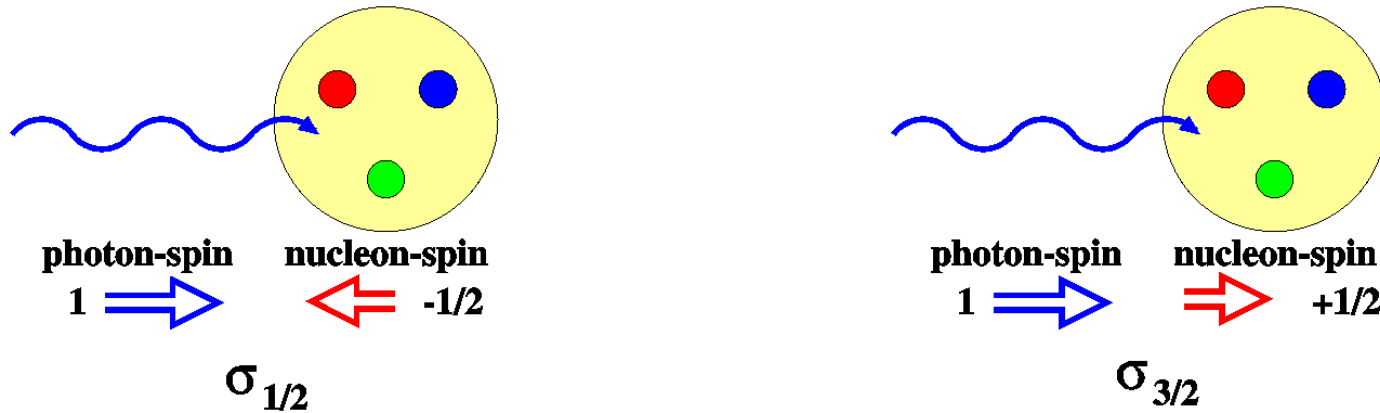
for $E_\gamma < 2 \text{ GeV}$ (GDH sum rule)

for $E_\gamma < 800 \text{ MeV}$

- Conclusions

Gerasimov-Drell-Hearn sum rule

- Proposed in 1966 (and never verified up to now...)
- Prediction on the absorption of circularly polarized photons by longitudinally polarized nucleons



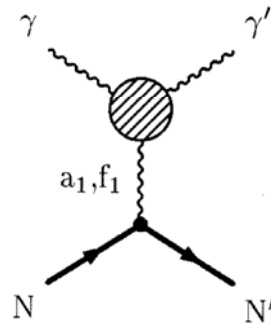
- Based on very general quantum mechanical principles (causality, optical theorem, gauge and Lorentz invariance)

Anomalous magnetic moment of the nucleon

$$I_{GDH} = \int_{m_\pi}^{\infty} \frac{\sigma_{3/2}(E_\gamma) - \sigma_{1/2}(E_\gamma)}{E_\gamma} dE_\gamma = \frac{2\pi^2 \alpha}{m^2} \kappa^2$$

Why could the GDH sum rule be violated ?

- The only "weak" hypothesis is the assumption that Compton scattering $\gamma N \rightarrow \gamma' N'$ becomes spin independent when $\nu \rightarrow \infty$
- A violation of this assumption can not be easily explained
- Possible hypotheses for violation:
 - ✓ Exchange of a_1 -like ($J=1+$) mesons between γ and N



- ✓ Non pointlike (constituent) quarks ?
- ✓ Photoproduction of gravitons ?

GDH sum rule:

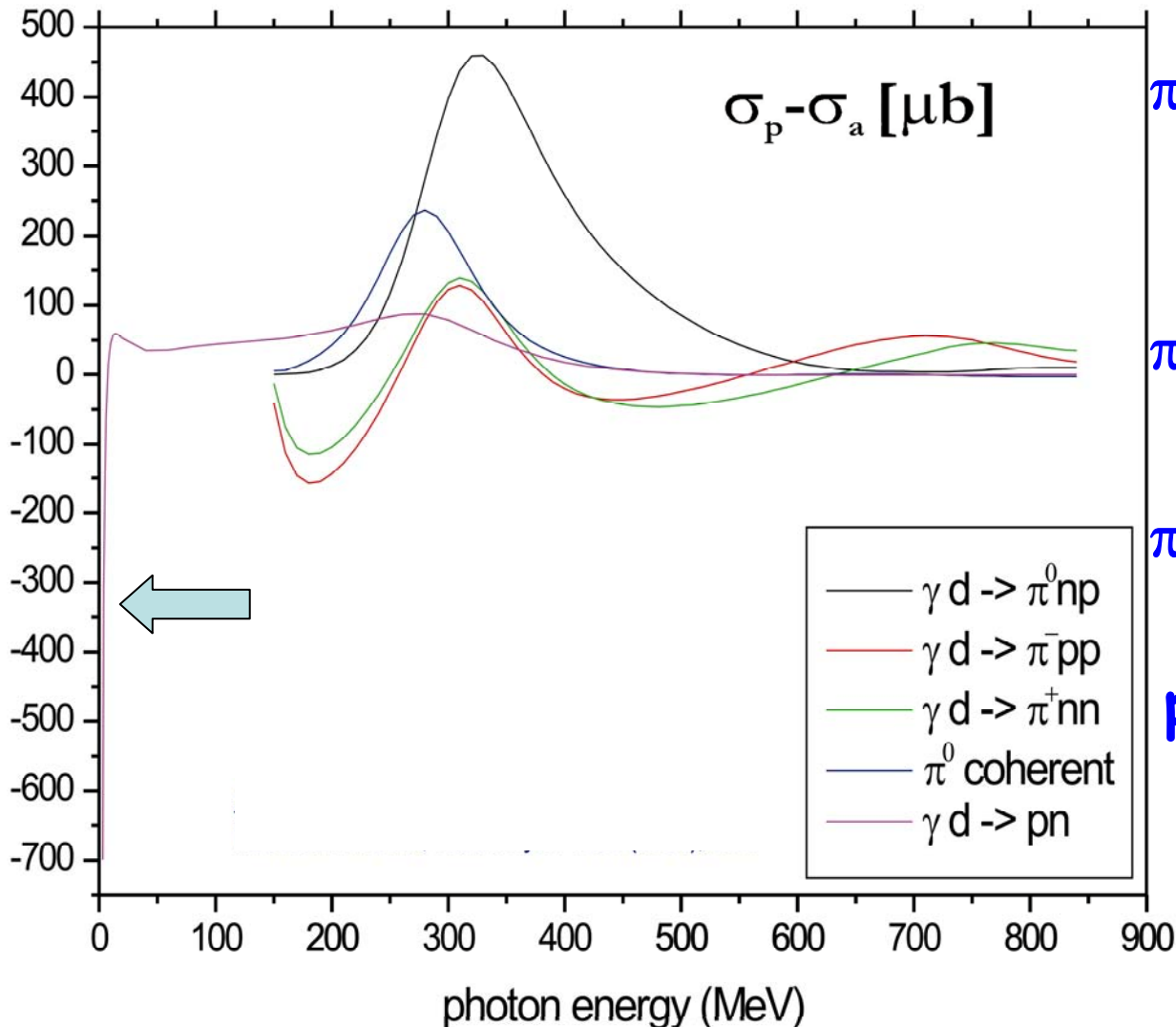
- ✓ Fundamental check of our knowledge of the γN interaction
- ✓ Important comparison for photoreaction models
- ✓ Helicity dependence of partial channels (pion photoproduction) is an essential tool for the study of the baryon resonances (interference terms between different electromagnetic multipoles)
- ✓ Valid for any system with $k \neq 0$ (^2H , ^3He). "Link" between nuclear and nucleon degrees of freedom

$$I_{GDH}^{deut} = \int_{2.2\text{MeV}}^{\infty} \frac{\sigma_p - \sigma_a}{E_\gamma} dE_\gamma = 0.6 \mu b \ll I_{GDH}^{proton} + I_{GDH}^{neutron} (\approx 430 \mu b)$$

↓
($\kappa_{deut} = -0.14 \text{ n.m.}$)

AFS model

Arenhoevel, Fix, Schwamb, PRL 93, 202301 (04)



πNN πN from MAID
+nuclear effects

$\pi\pi NN$ EPJA 25,114 (05)

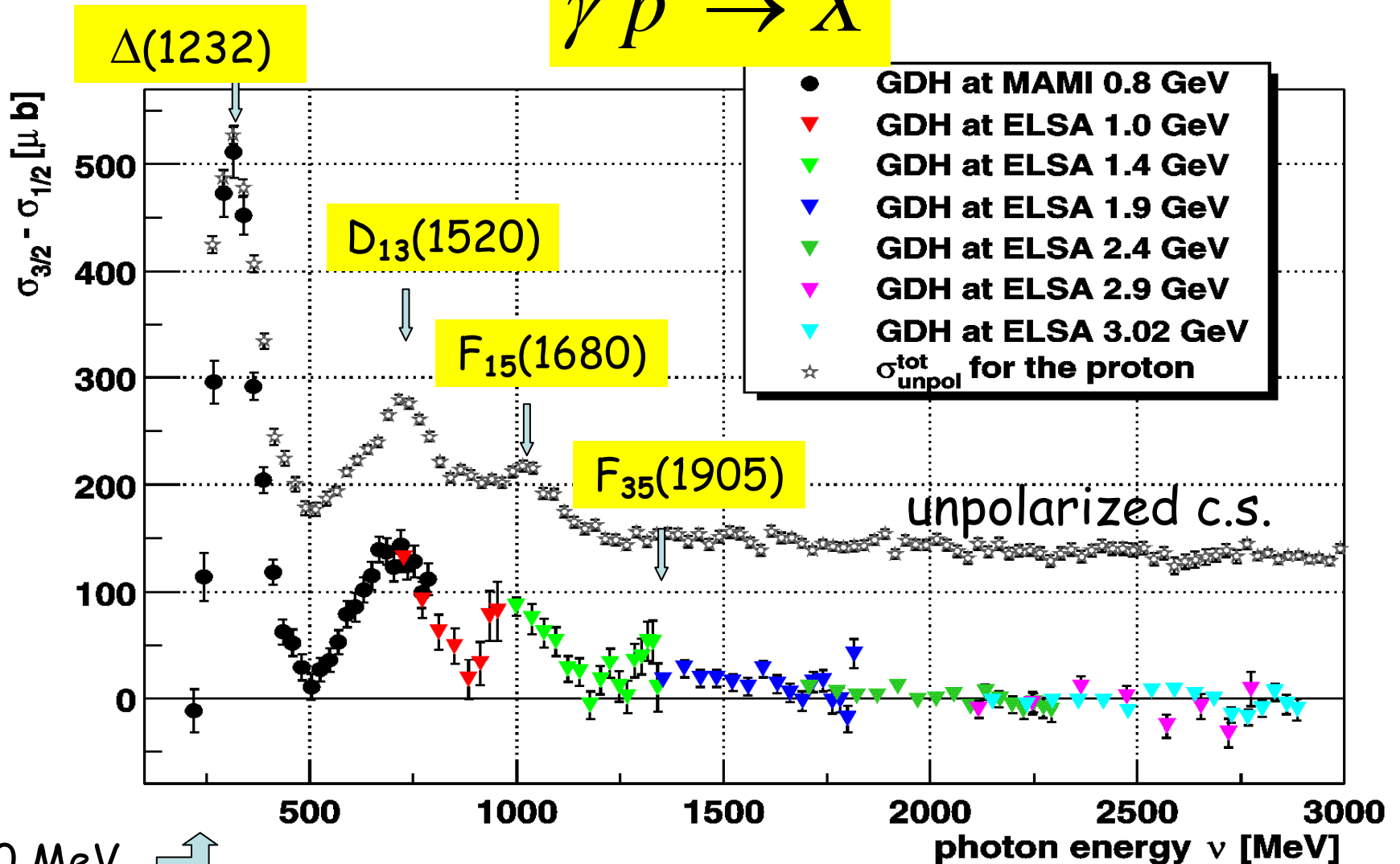
$\pi^0 d$ PLB 407,1 (97)

pn NPA 690,682 (01)

$$\left[I_{GDH}^{deut} \right]_{AFS} = 27 \mu\text{b}$$

Experimental status

$$\vec{\gamma} \vec{p} \rightarrow X$$



- GDH at MAMI 0.8 GeV
- ▼ GDH at ELSA 1.0 GeV
- ▼ GDH at ELSA 1.4 GeV
- ▼ GDH at ELSA 1.9 GeV
- ▼ GDH at ELSA 2.4 GeV
- ▼ GDH at ELSA 2.9 GeV
- ▼ GDH at ELSA 3.02 GeV
- ☆ $\sigma_{\text{unpol}}^{\text{tot}}$ for the proton

MAMI data: J. Ahrens et al., Phys. Rev. Lett. 87 (2001) 022003
 ELSA data: H. Dutz et al., Phys. Rev. Lett 91 (2003) 192001
 H. Dutz et al., Phys. Rev. Lett 93 (2004) 032003

GDH sum rule on the proton

E_γ (GeV)		I_{GDH} (mb)
0.14-0.20 *	MAID03	-29
	SAID04	-28
0.20-2.90	Measured (Mainz+Bonn)	$254 \pm 5 \pm 12$
> 2.90 (Regge approach)	Simula et al.	~ -13
	Bianchi-Thomas	~ -14
Total		~ 211
GDH sum rule		205

* Low energy theorems in the $N\pi$ threshold region (multipole analyses not very wrong ...)

GDH sum rule: predictions (2005)

Proton	$I_{GDH} (\mu b)$	Neutron	$I_{GDH} (\mu b)$
$\gamma p \rightarrow N\pi$	172	$\gamma n \rightarrow N\pi$	133
$\gamma p \rightarrow N\pi\pi$	94	$\gamma n \rightarrow N\pi\pi$	82
$\gamma p \rightarrow N\eta$	-8	$\gamma n \rightarrow N\eta$	-6
$\gamma p \rightarrow K\Lambda (\Sigma)$	-4	$\gamma n \rightarrow K\Lambda (\Sigma)$	2
$\gamma p \rightarrow N\rho(\omega)$	0	$\gamma n \rightarrow N\rho(\omega)$	2
Regge contrib. ($E_\gamma > 2 \text{ Gev}$)	~ -15	Regge contrib. ($E_\gamma > 2 \text{ Gev}$)	~ 20
TOTAL	~ 240	TOTAL	~ 230
GDH	205	GDH	233

$N\pi$: SAID $K\Lambda(\Sigma)$: Sumowidagdo et al., PRC 65,0321002 (02)

$N\eta$: MAID $N\pi\pi$: Fix, Arenhoevel EPJA 25, 114 (2005)

$N\rho$: Zhao et al., PRC 65, 032201 (03) Regge : Bianchi-Thomas , PLB 450,439(99)

GDH sum rule on the neutron

- **No Free neutron target available**
- Model dependent results from nuclear targets
- Experimental goal: have a "small" and "reliable" model dependence
- Two different (and complementary) targets
 - =) **Deuterium** (deuterated butanol / ${}^6\text{LiD}$) 
 - =) ${}^3\text{He}$ (high pressure gas target - under development)

- Measurement of partial channels like

$$\vec{\gamma} \vec{d} \rightarrow pn$$

$$\vec{\gamma} \vec{d} \rightarrow p\pi^- p_s (n\pi^+ n_s) (p\pi^0 n_s)$$

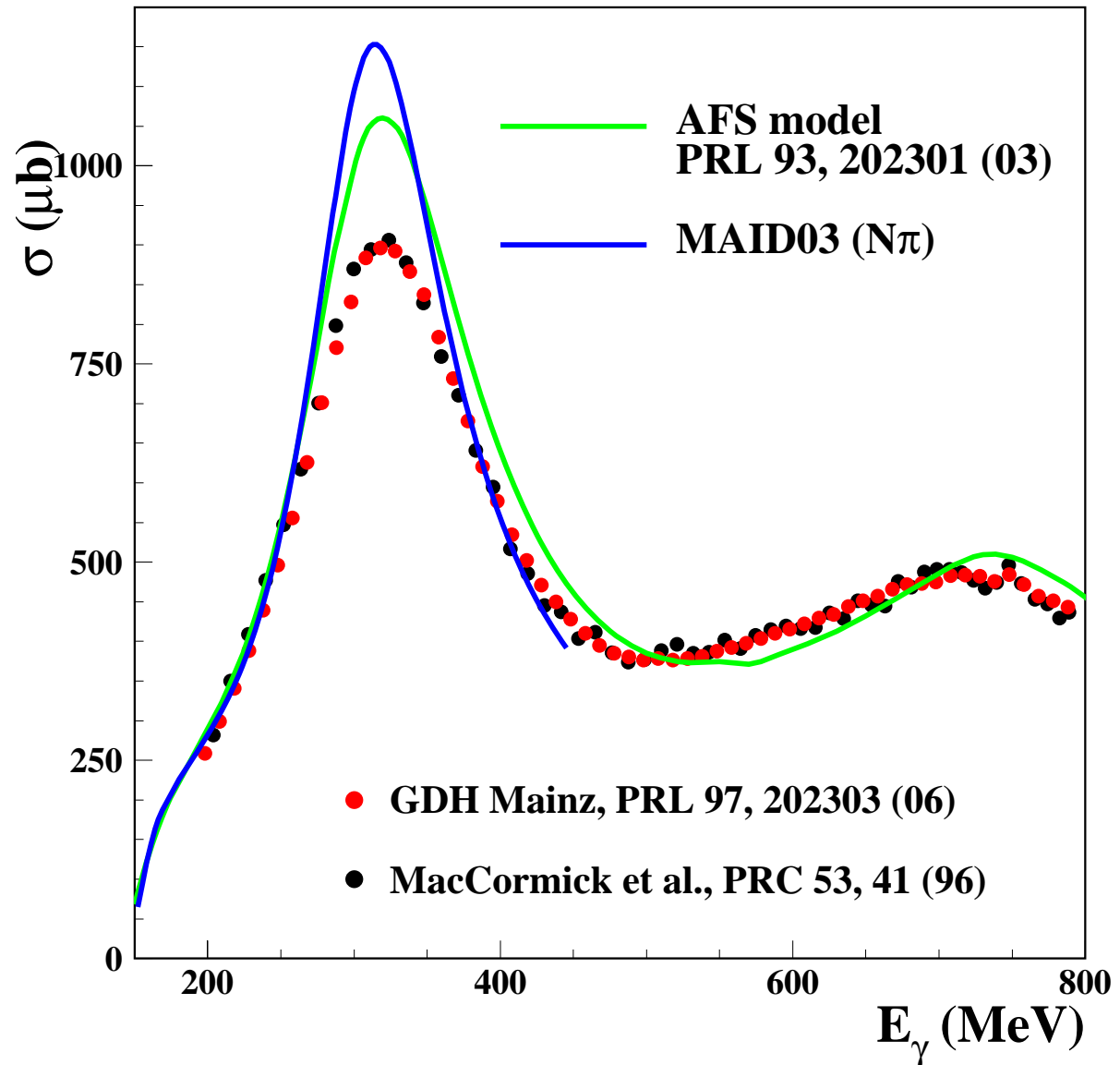
Experimental set-up

- **Tagged photon beam** Mainz: $m_\pi \leq E_\gamma \leq 800\text{MeV}$
Bonn: $0.6\text{ GeV} \leq E_\gamma \leq 2.9\text{ GeV}$
- **Circularly polarized photons**
from bremsstrahlung of linearly polarized electrons
- **Longitudinally polarized protons and neutrons**
Frozen spin (deut.)butanol/ ^6LiD target (Bonn, Bochum, Nagoya)
- **Large acceptance hadron detector**
 - Mainz: DAPHNE detector (Pavia, Saclay) + forward angle detectors (Pavia, Mainz)
 - Bonn: GDH Detector (Erlangen, Tuebingen, Gent)

Total inclusive cross section on the deuteron

$$\gamma d \rightarrow X$$

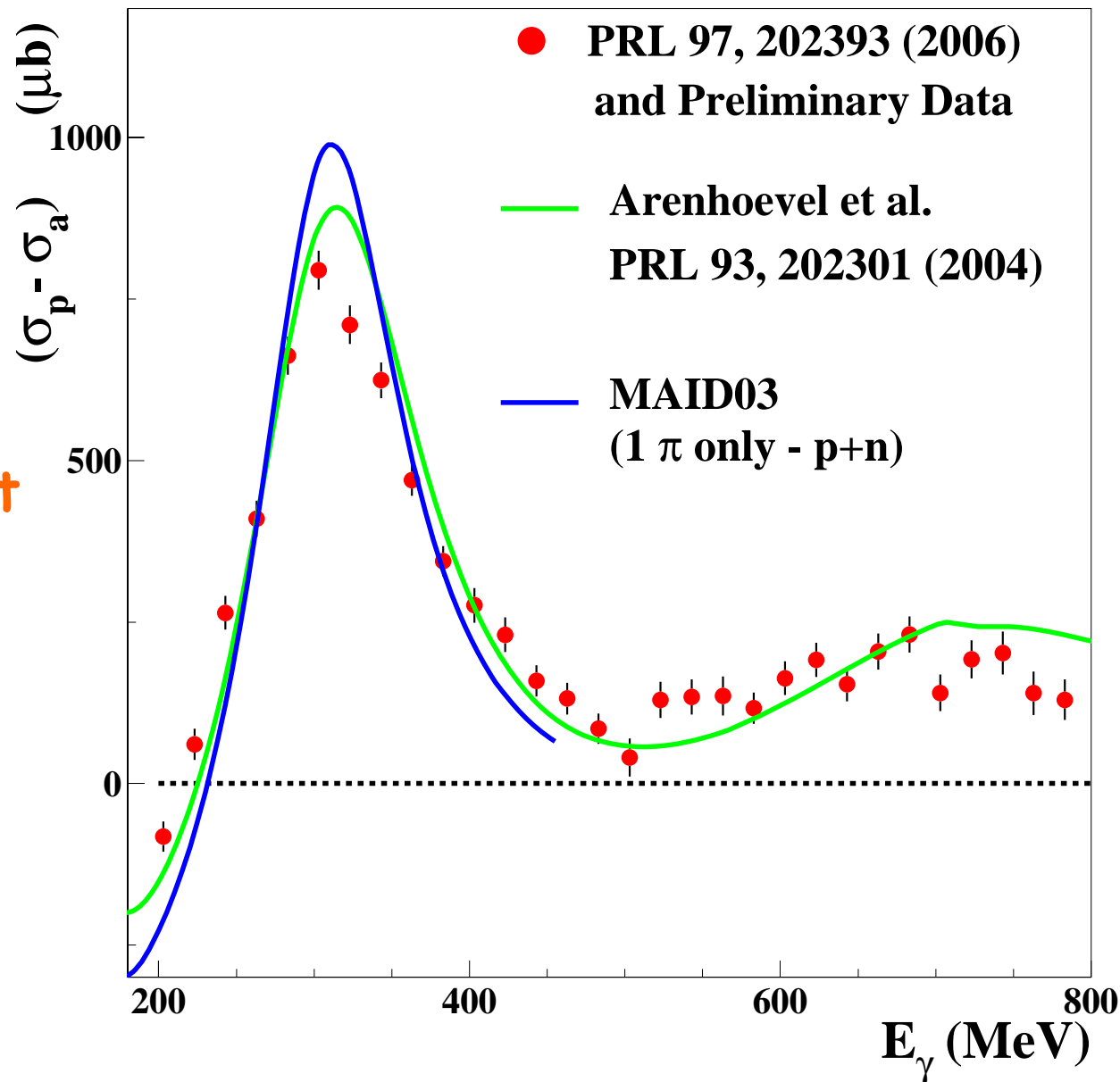
unpolarized
data



Total inclusive cross section

$$\vec{\gamma} \vec{d} \rightarrow X$$

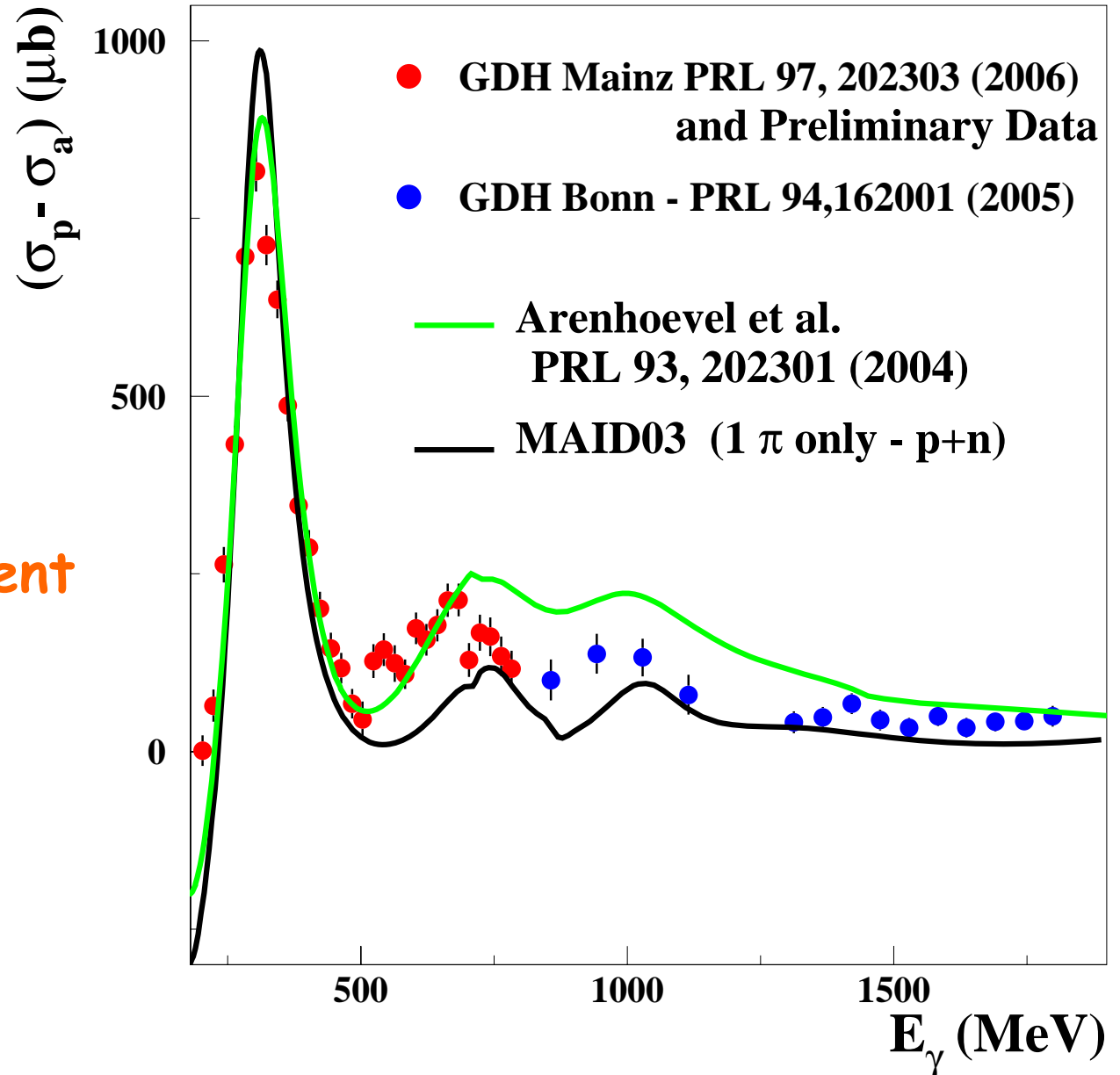
helicity dependent
data
(full statistics)



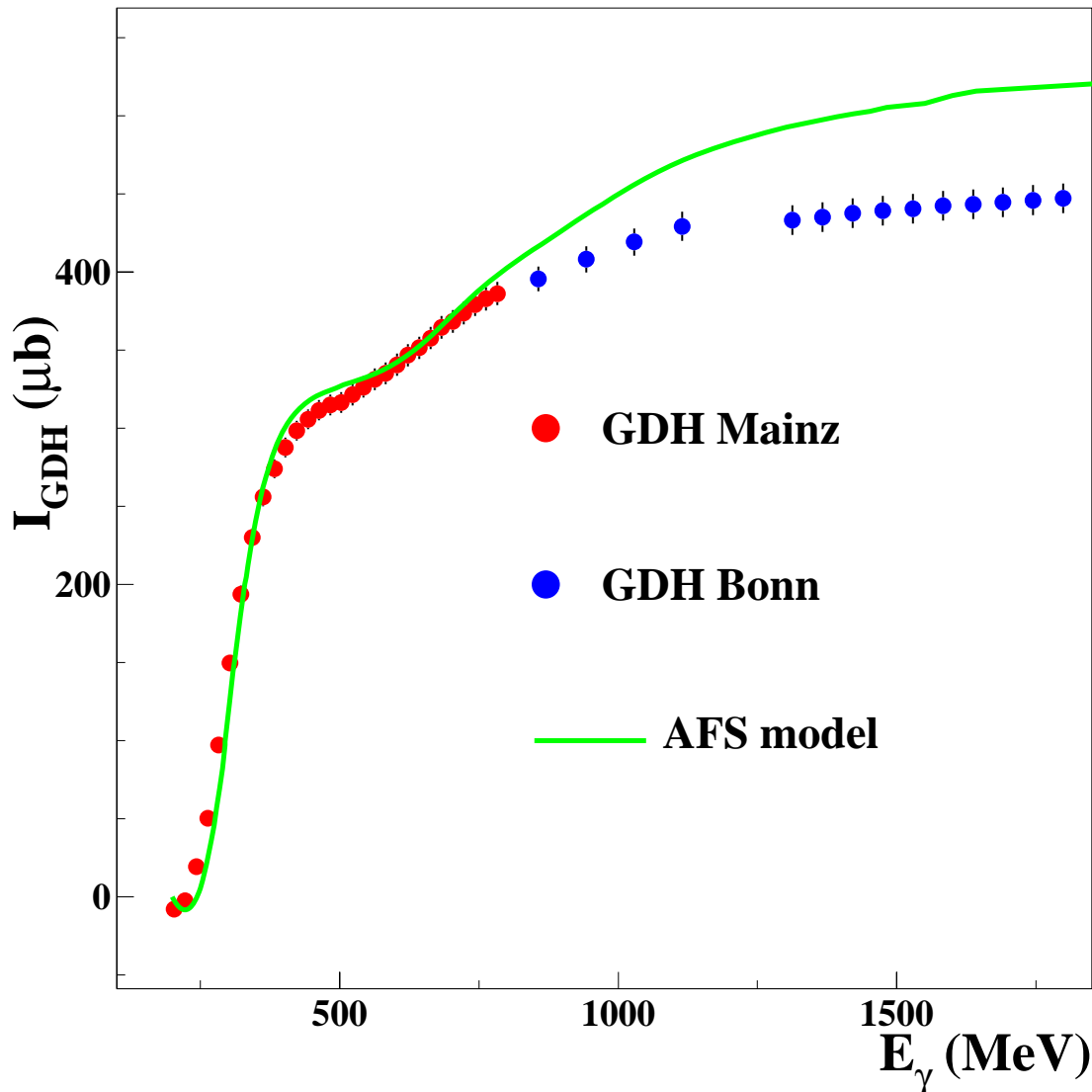
Total inclusive cross section on the deuteron

$$\vec{\gamma} \vec{d} \rightarrow X$$

helicity dependent
data



GDH sum rule on the deuteron



For 0.5-2.0 GeV

(nuclear effects are expected to be "small")

$$I_{GDH}^{Exp-Deuteron} \cong 2 \cdot I_{GDH}^{Exp-proton}$$

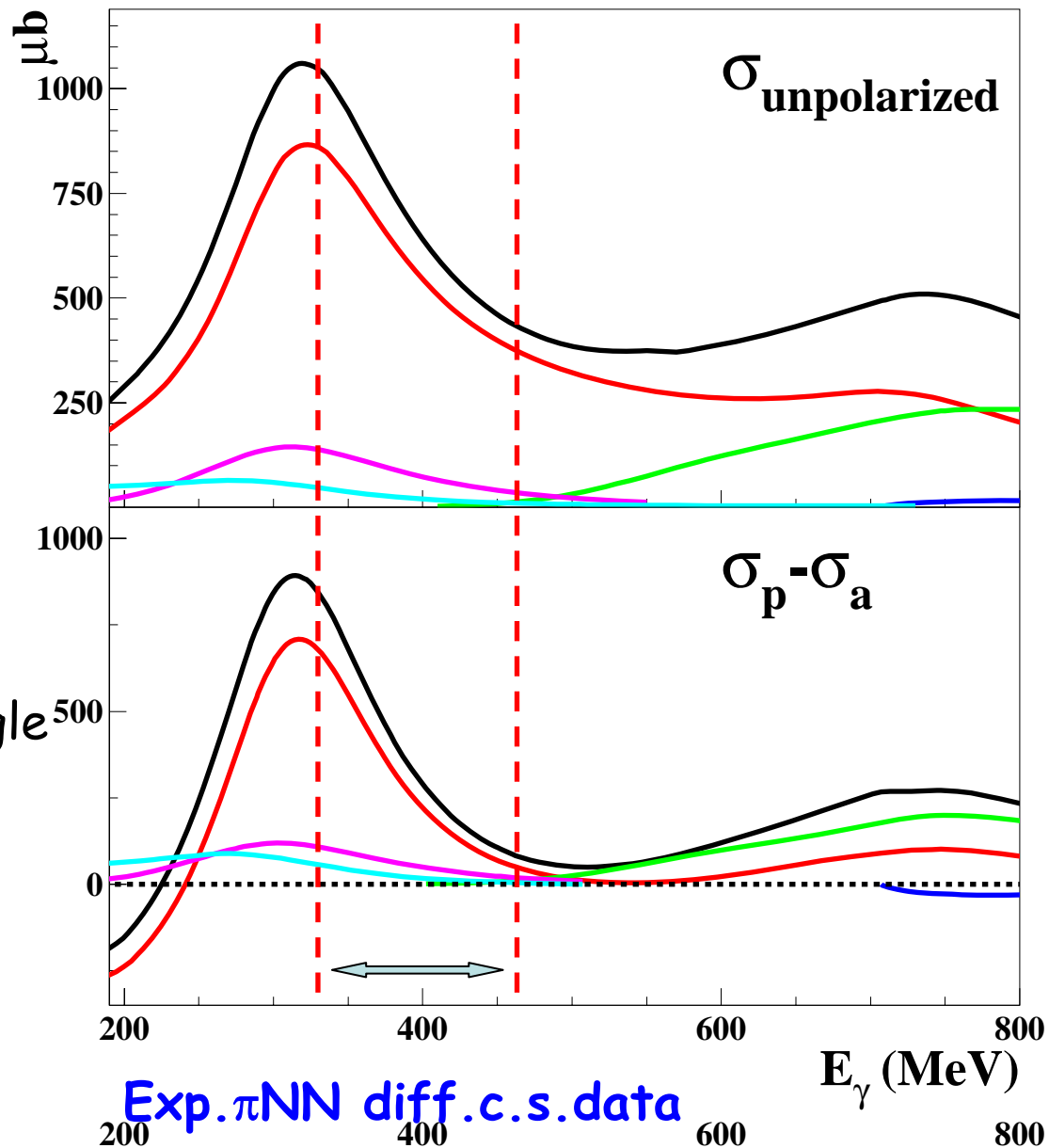
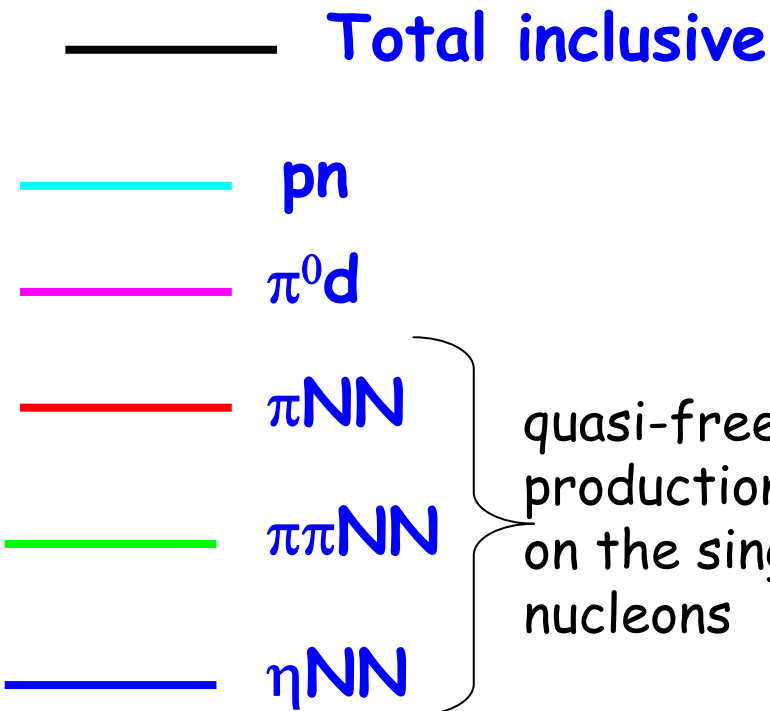


$$I_{GDH}^{neutron} \approx I_{GDH}^{proton}$$

Running GDH integral for the deuteron

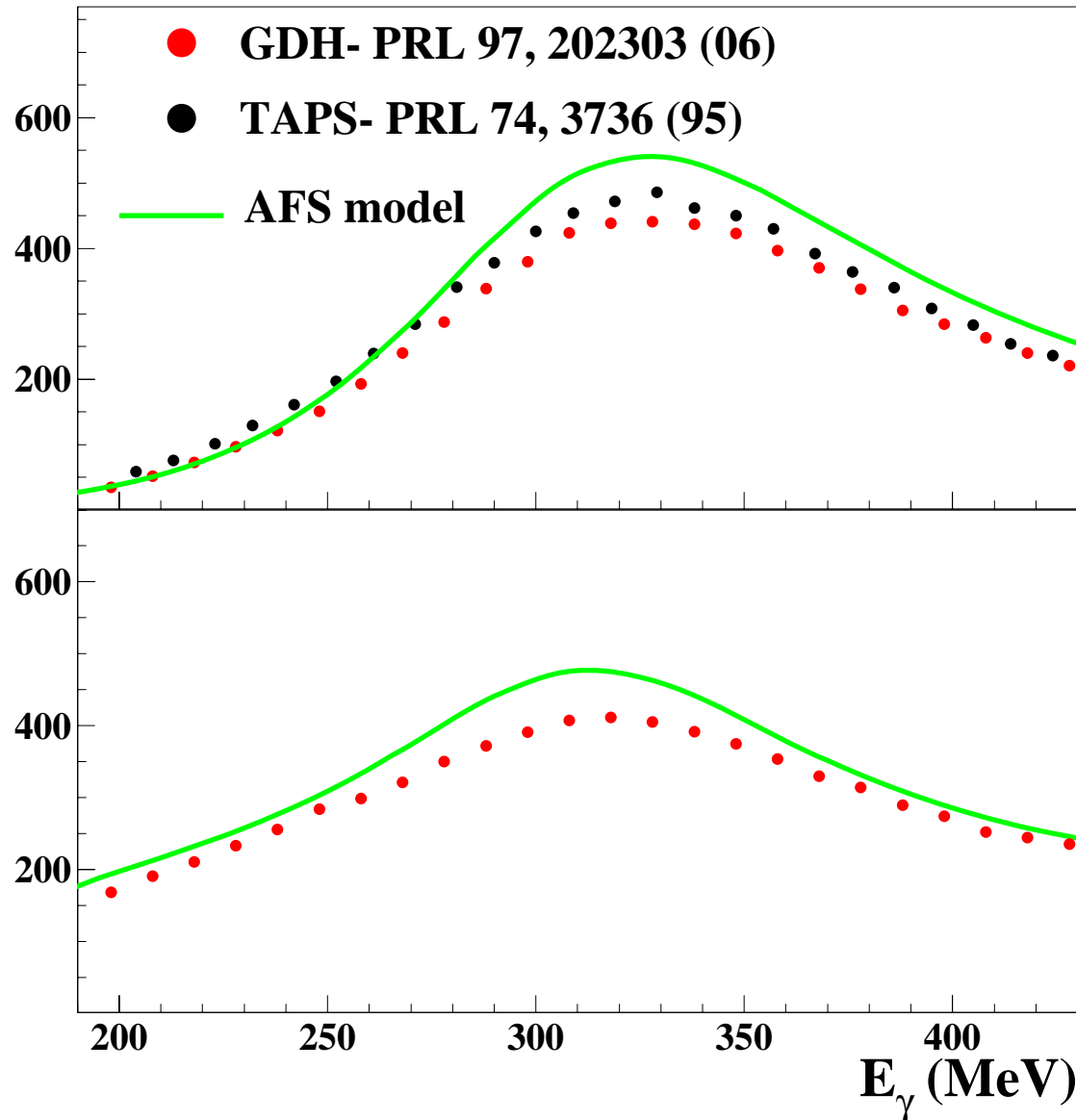
Study of Partial channels

AFS model



Unpolarized cross sections

σ (μb)

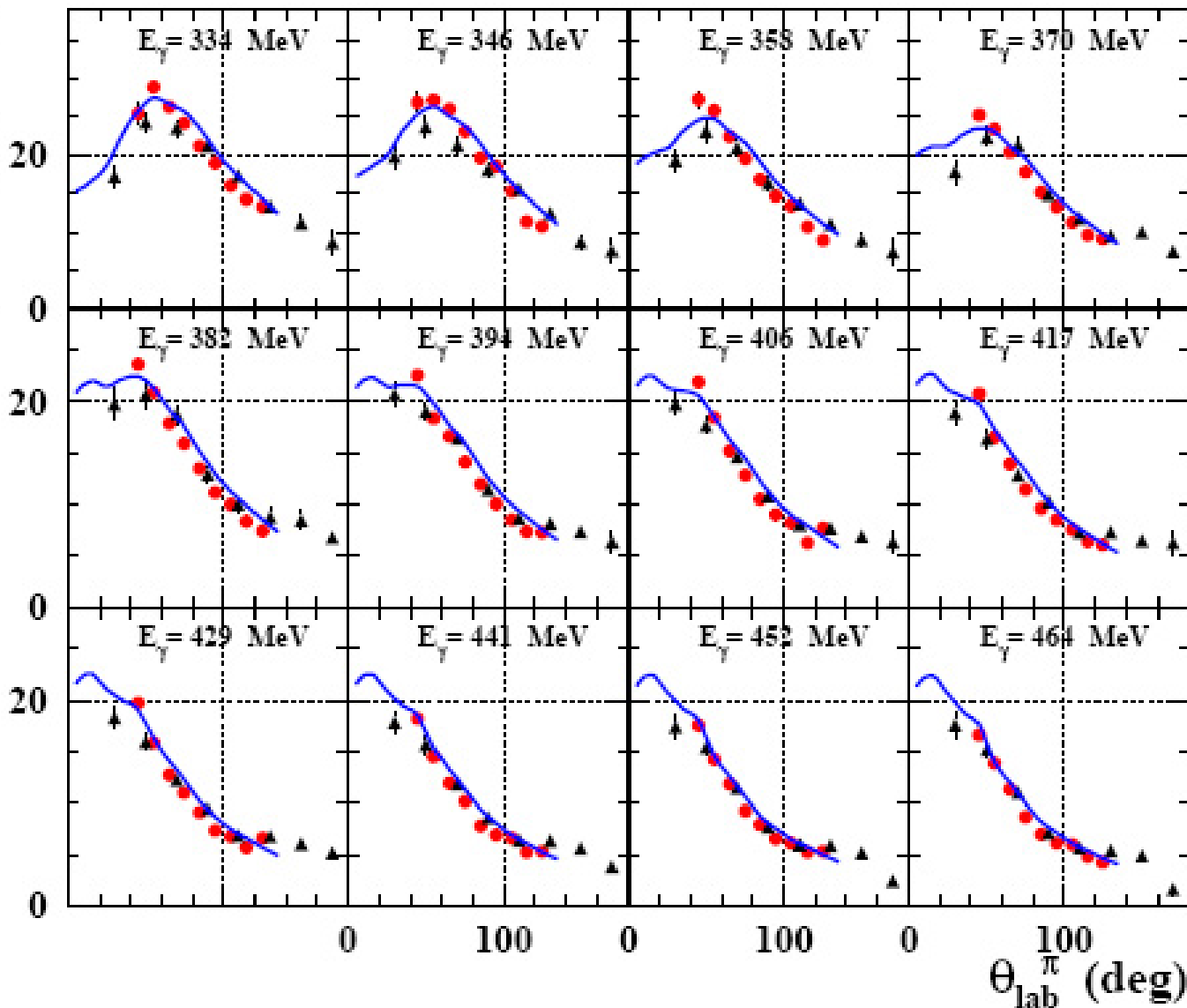


$\pi^0 X(pn, d)$

$\pi^\pm NN$

$$\gamma d \rightarrow p \pi^- p_s$$

$(d\sigma/d\Omega)$ ($\mu\text{b/sr}$)



● GDH Preliminary

— AFS model

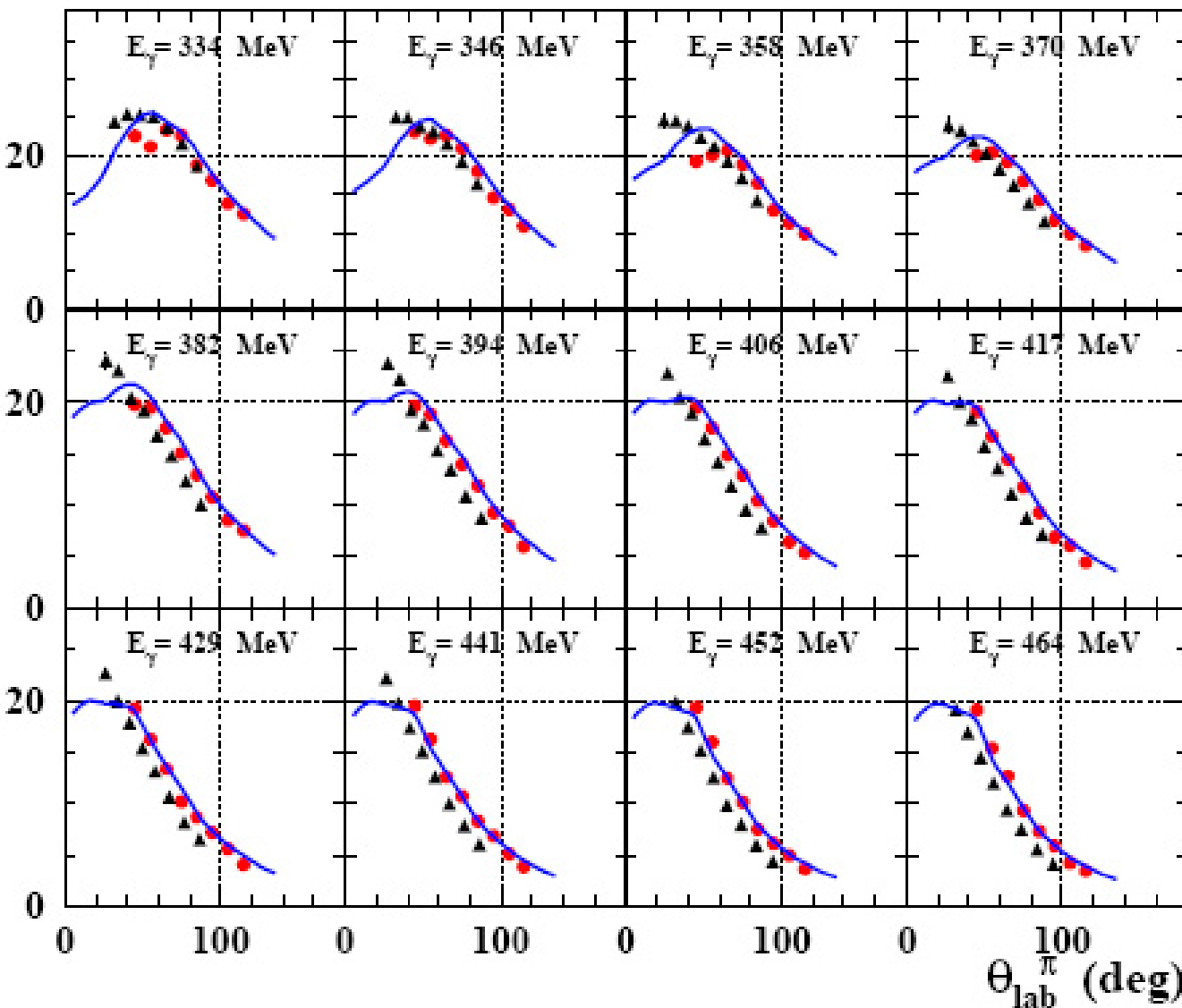
▲ Benz et al.,
NPB 65, 158 (73)

Preliminary

$$\gamma d \rightarrow n \pi^+ n_s$$

quasi-free reaction on the proton

$(d\sigma/d\Omega)$ ($\mu\text{b/sr}$)



● GDH Preliminary

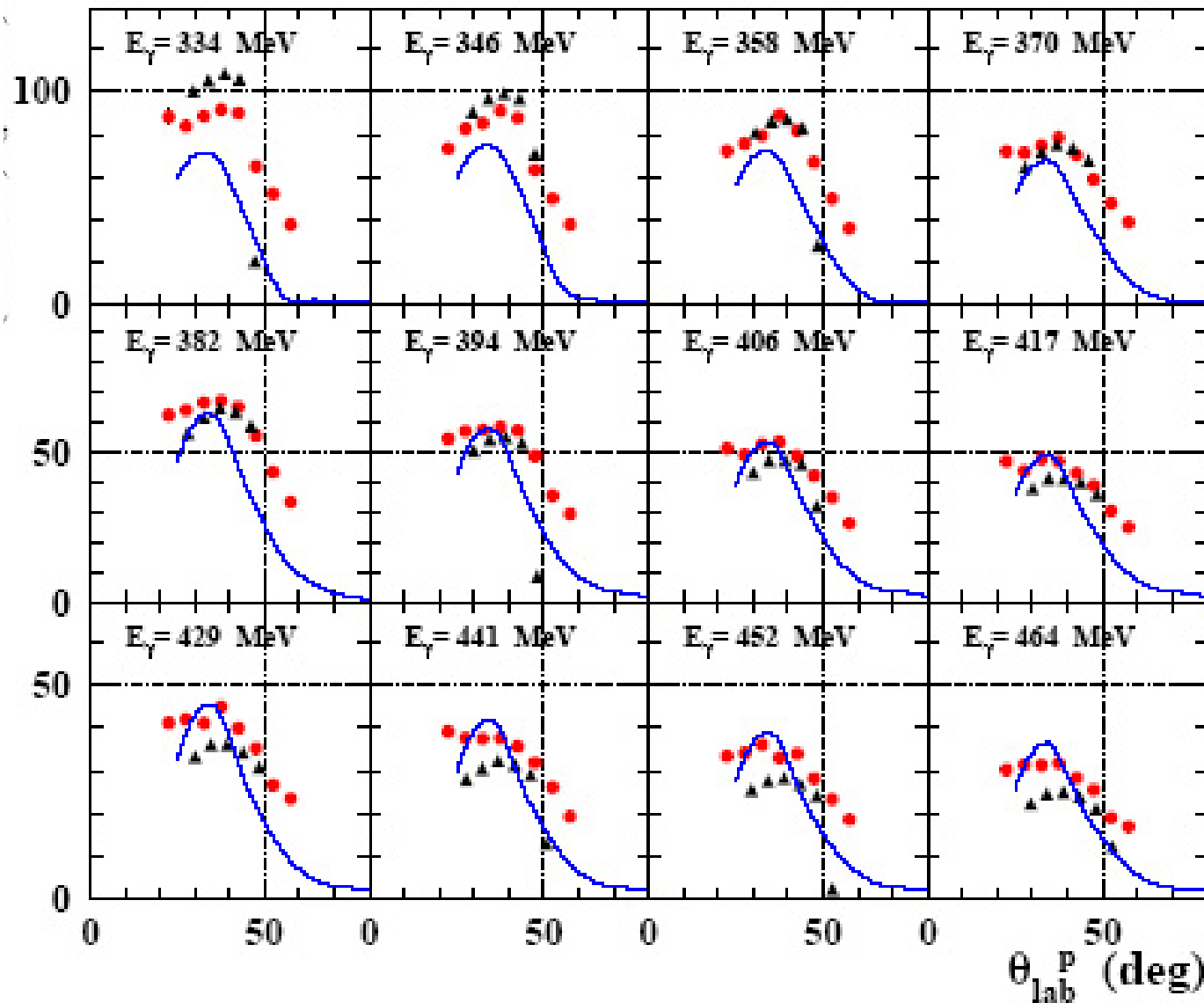
— AFS model

▲ GDH $\gamma p \rightarrow n \pi^+$
Ahrens et al.,
EPJA 26, 135 (05)

Preliminary

$$\gamma d \rightarrow p \pi^0 n_s$$

$(d\sigma/d\Omega)$ ($\mu\text{b/sr}$)



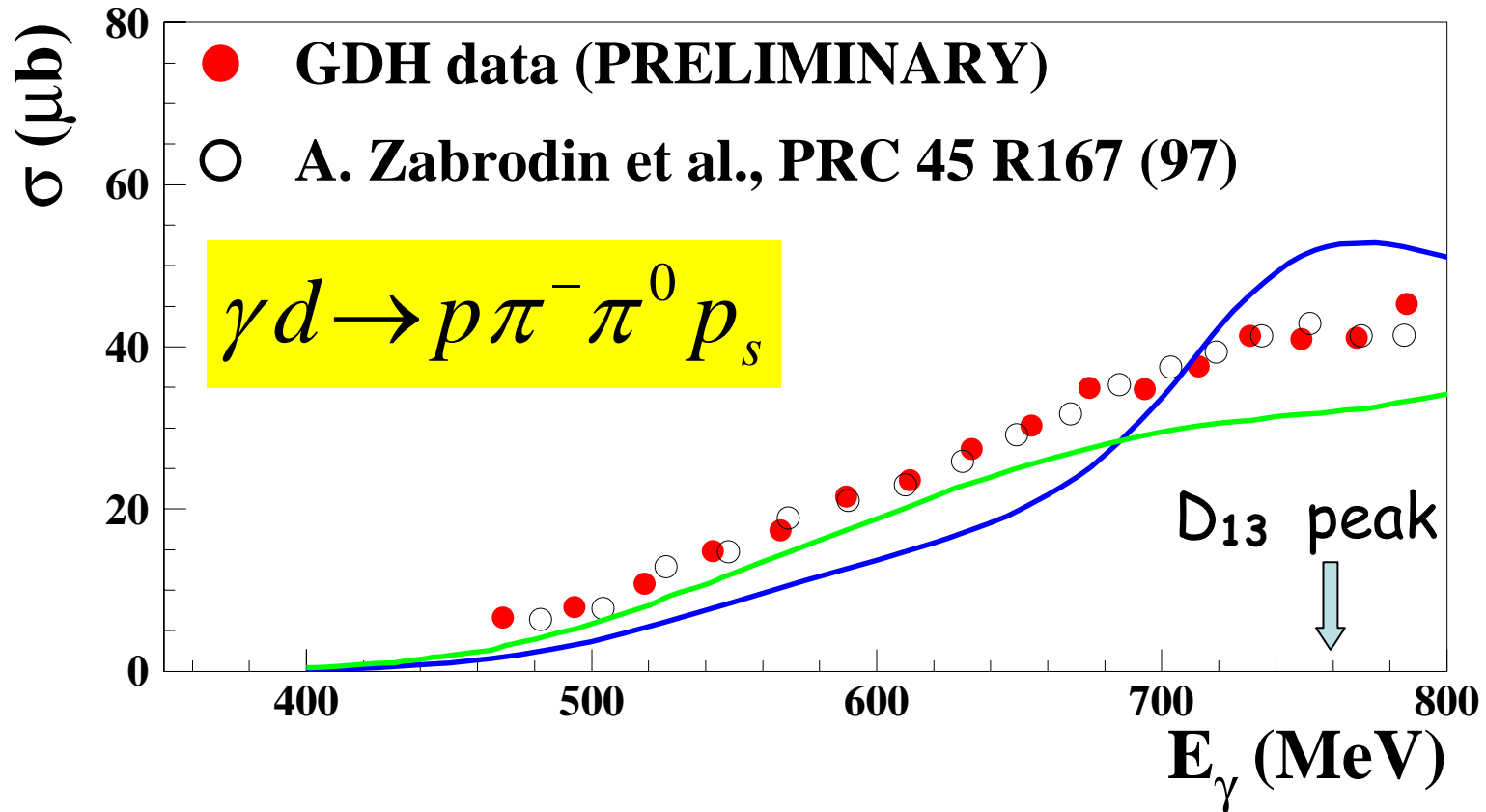
● GDH Preliminary

— AFS model

▲ GDH $\gamma p \rightarrow p \pi^0$
Ahrens et al.,
EPJA 26, 135 (05)

Preliminary

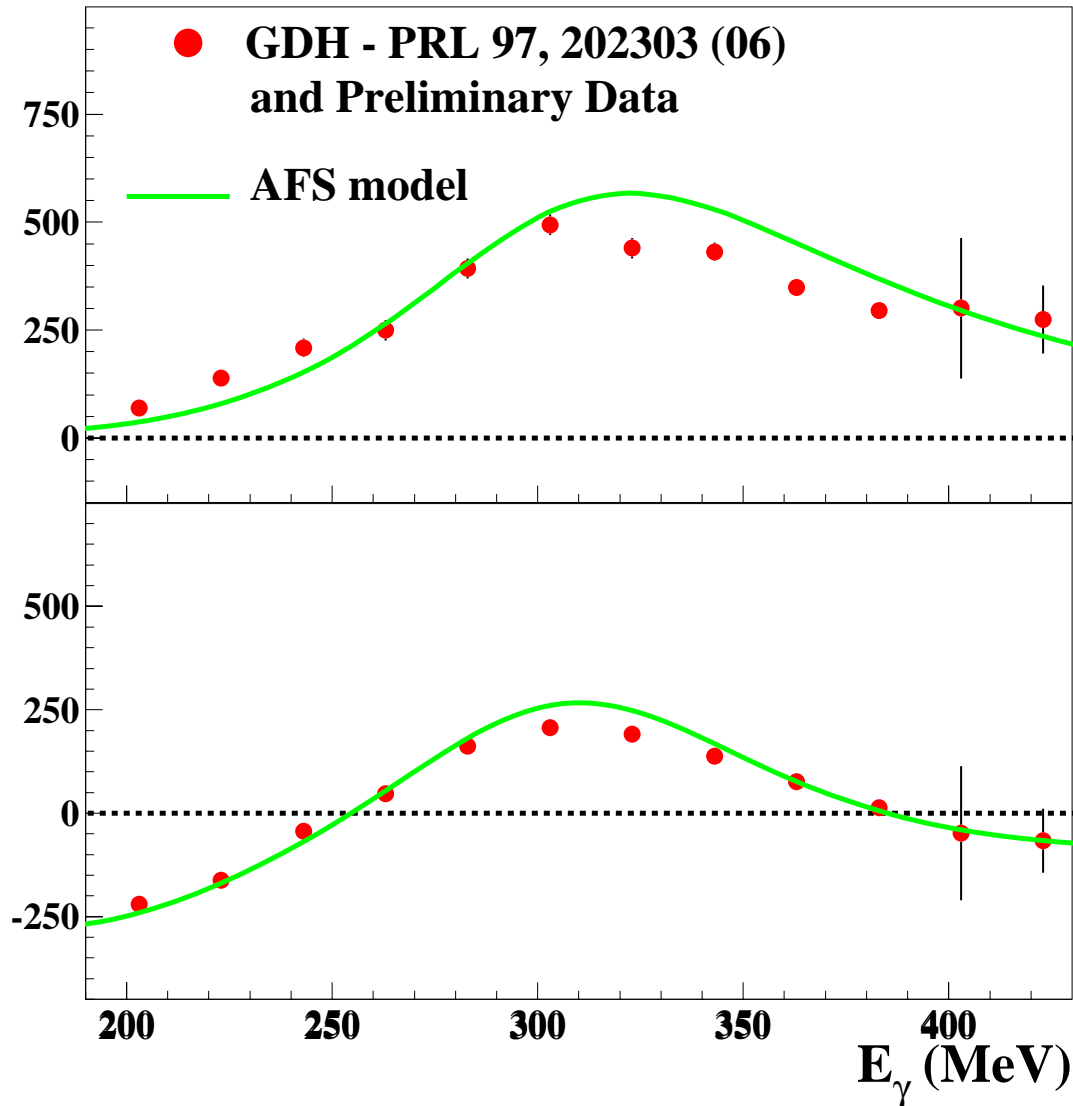
Double pion photoproduction



- Fix-Arenhovel model, EPJA 25, 114 (05) dominant mechanism $\gamma n \rightarrow D_{13}(1520) \rightarrow p \pi^- \pi^0$
- Valencia model, NPA 600, 413 (96) dominant mechanism $\gamma n \rightarrow \Delta^+ \pi^- \rightarrow p \pi^- \pi^0$

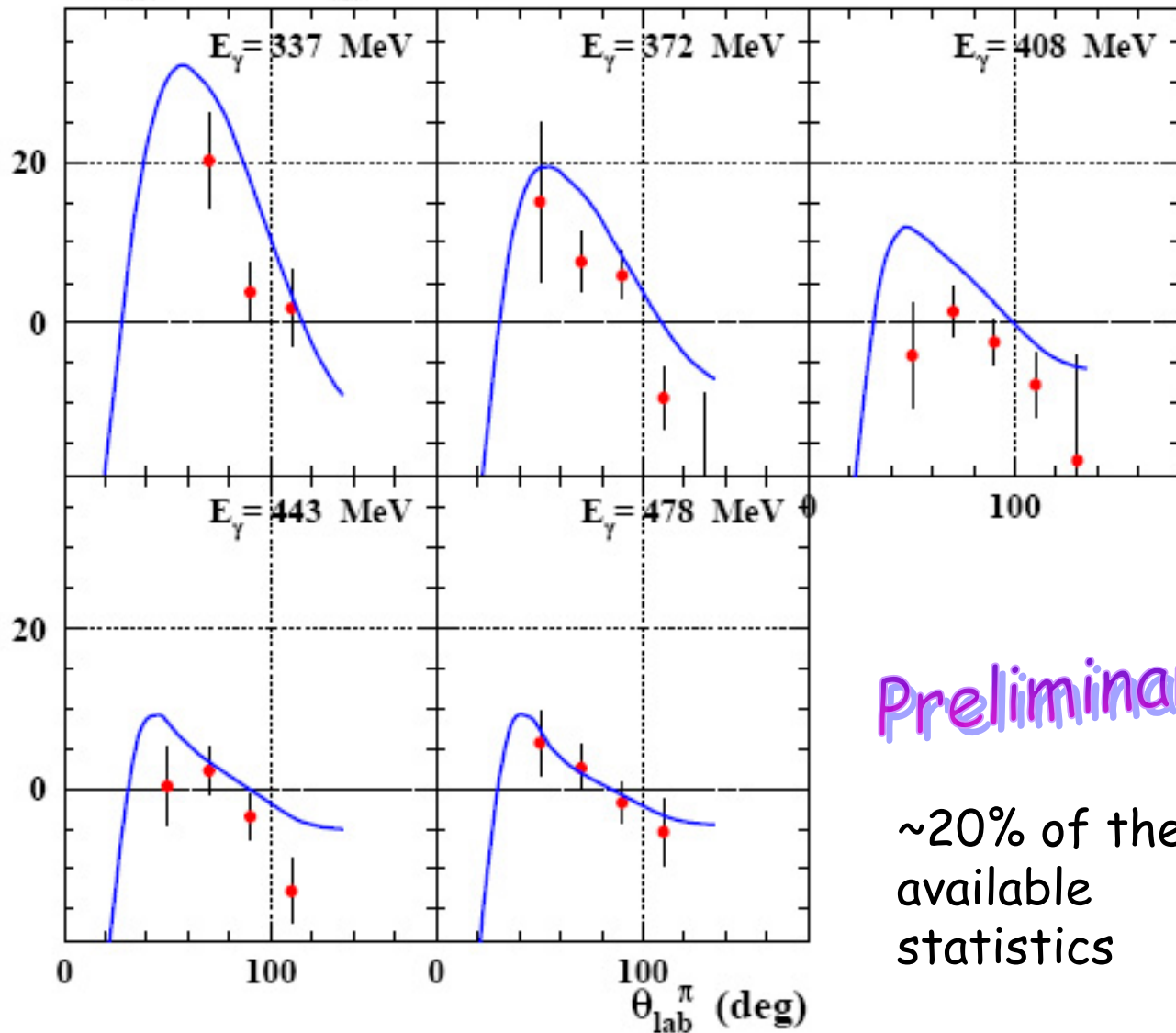
Helicity dependent cross sections

$$\sigma_p - \sigma_a (\mu b)$$



$$\vec{\gamma} d \rightarrow p \pi^- p_s$$

$d\sigma_p/d\Omega - d\sigma_a/d\Omega$ ($\mu\text{b}/\text{sr}$)



• GDH Preliminary

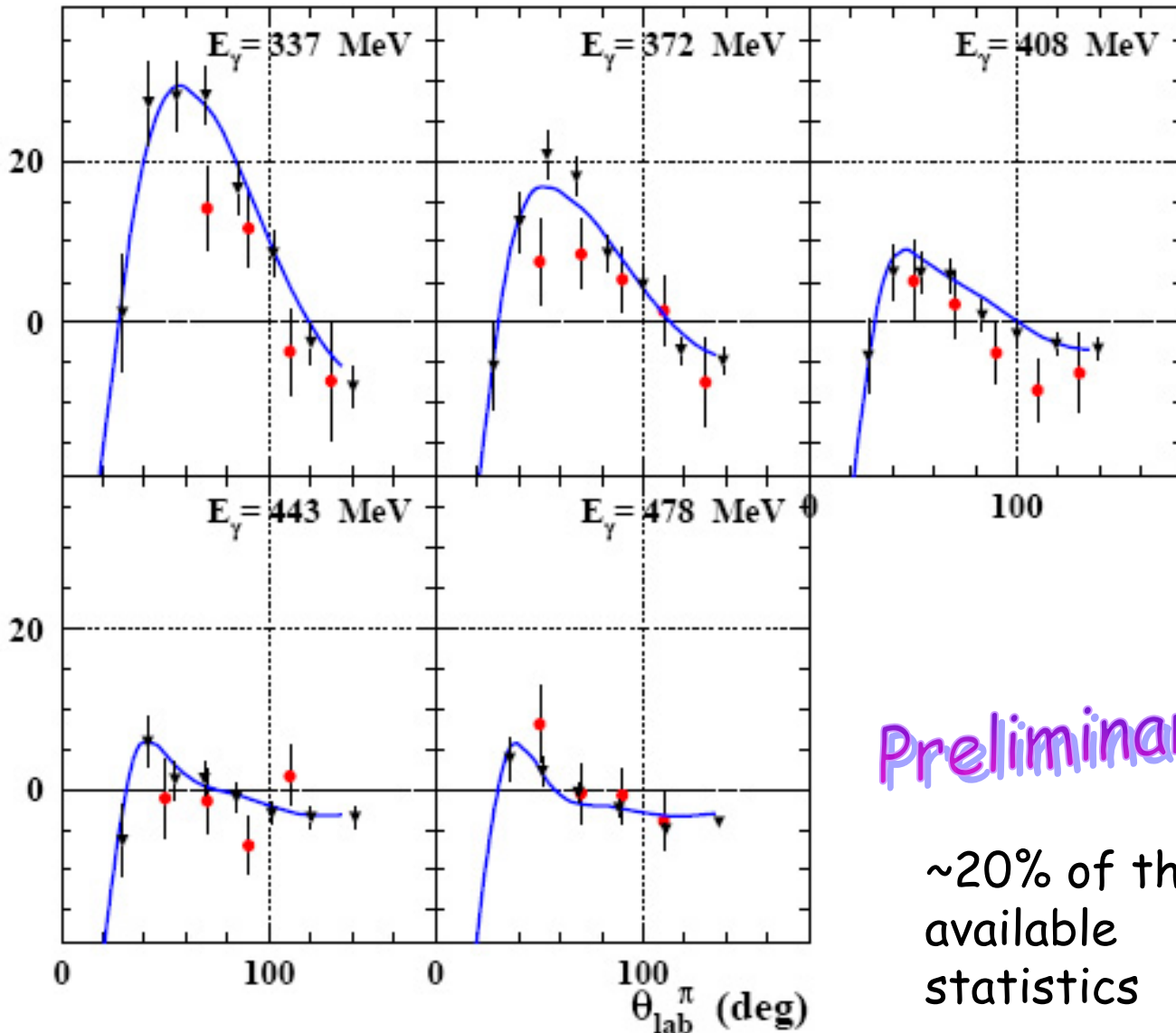
— AFS model

Preliminary

~20% of the
available
statistics

$$\vec{\gamma} \vec{d} \rightarrow n \pi^+ n_s$$

$d\sigma_p/d\Omega - d\sigma_a/d\Omega$ ($\mu\text{b}/\text{sr}$)



● GDH Preliminary

— AFS model

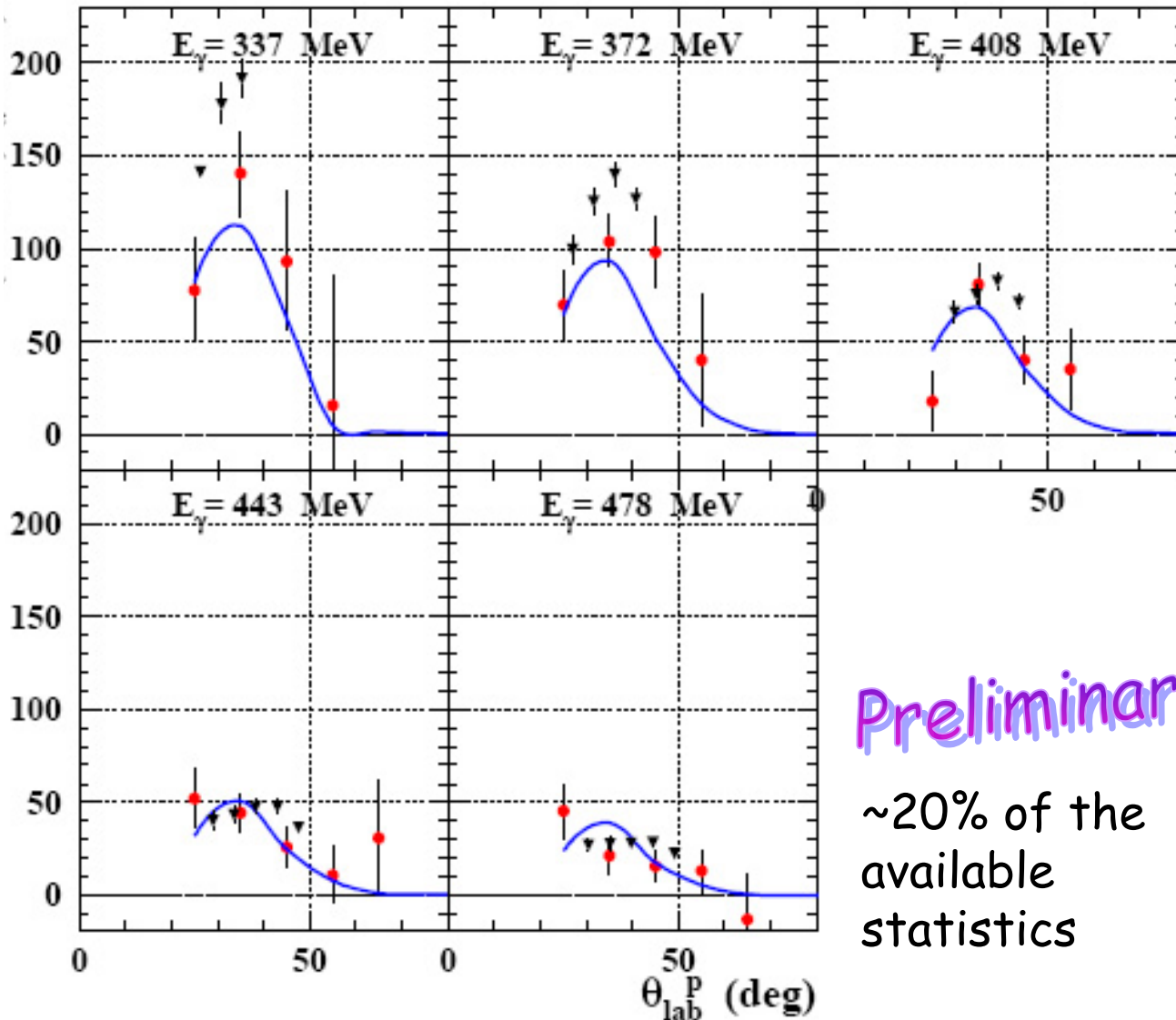
▲ GDH $\gamma p \rightarrow n \pi^+$
Ahrens et al.,
EPJA 26, 135 (05)

Preliminary

~20% of the
available
statistics

$$\vec{\gamma} \vec{d} \rightarrow p \pi^0 p_s$$

$d\sigma_p/d\Omega - d\sigma_a/d\Omega$ ($\mu\text{b/sr}$)



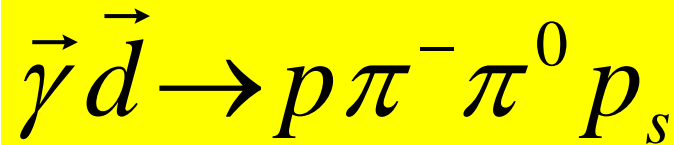
● GDH Preliminary

— AFS model

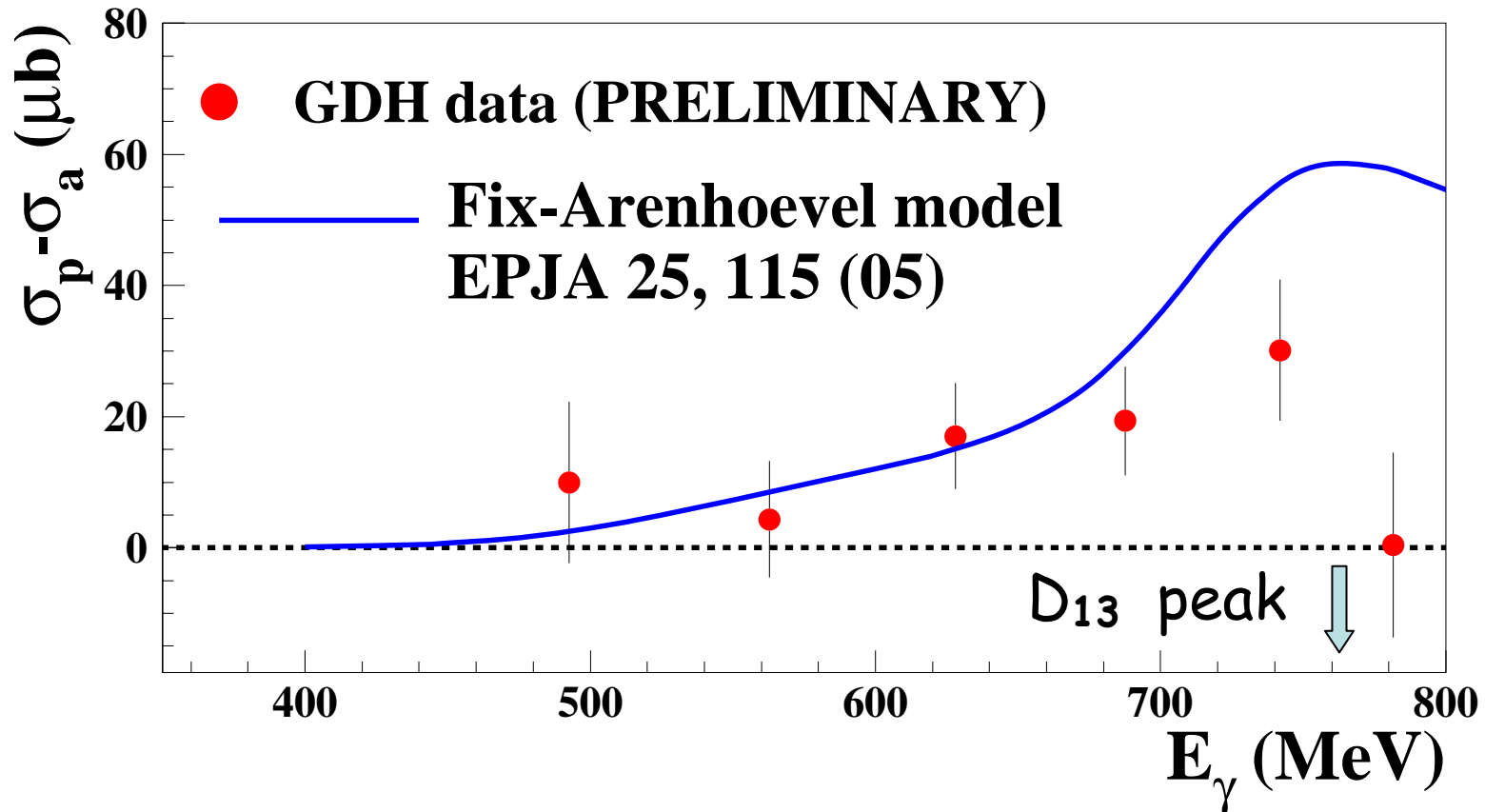
▲ GDH $\gamma p \rightarrow p \pi^0$
Ahrens et al.,
EPJA 26, 135 (05)

Preliminary

~20% of the
available
statistics



quasi-free reaction on the neutron



Effects due to the intermediate excitation of the $D_{13}(1520)$ resonance are much smaller than the AFS model predictions

$$\vec{\gamma} \vec{p} \rightarrow n \pi^+ \pi^0$$

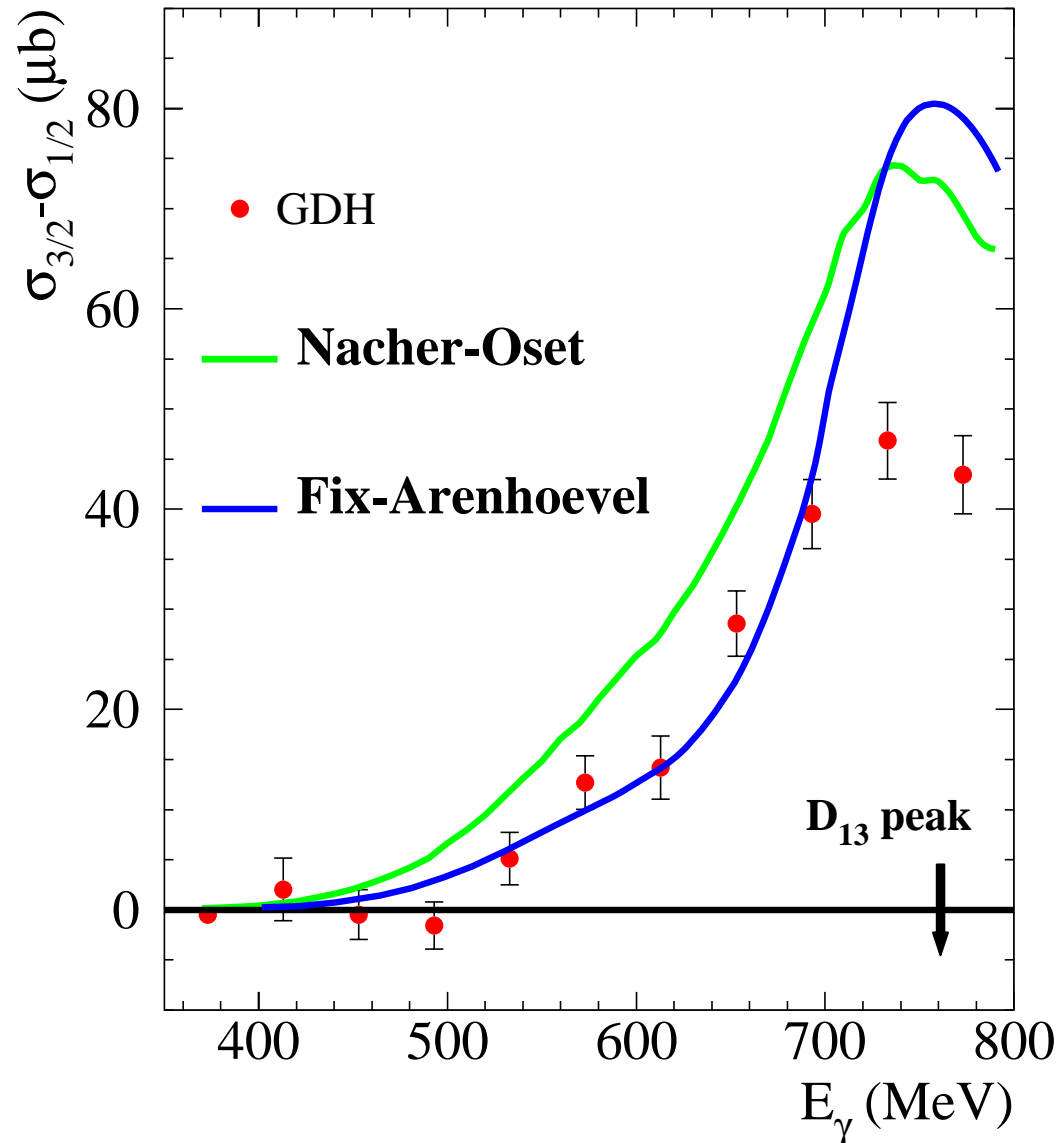
■ GDH data

PLB 551, 49 (2003)

➤ D_{13} plays a significant role ($\gamma p \rightarrow D_{13} \rightarrow N \rho$)

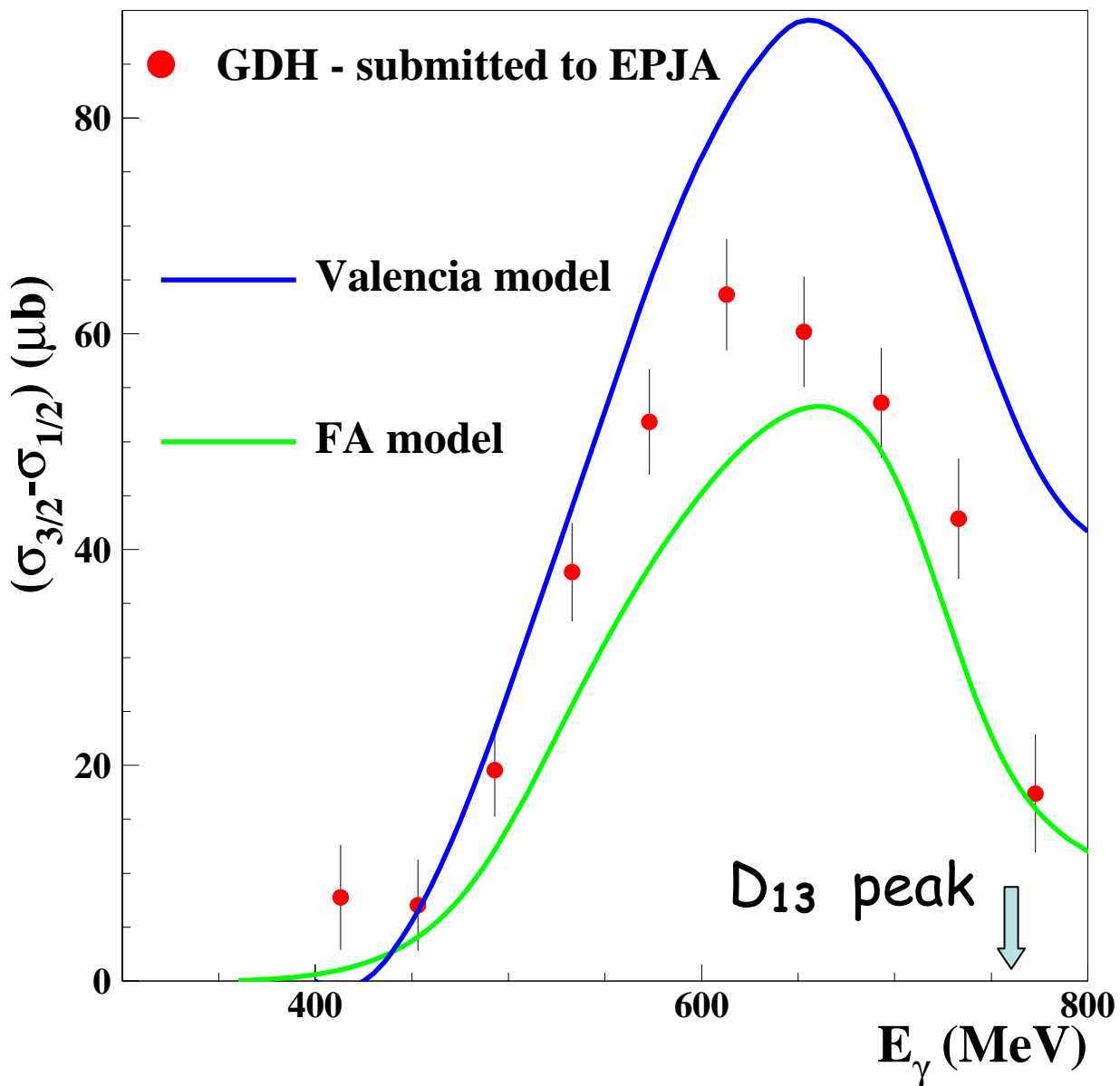
➤ Nacher-Oset model:
NPA 695, 295 (01)

➤ Fix-Arenhoevel model:
EPJA 25, 114 (05)



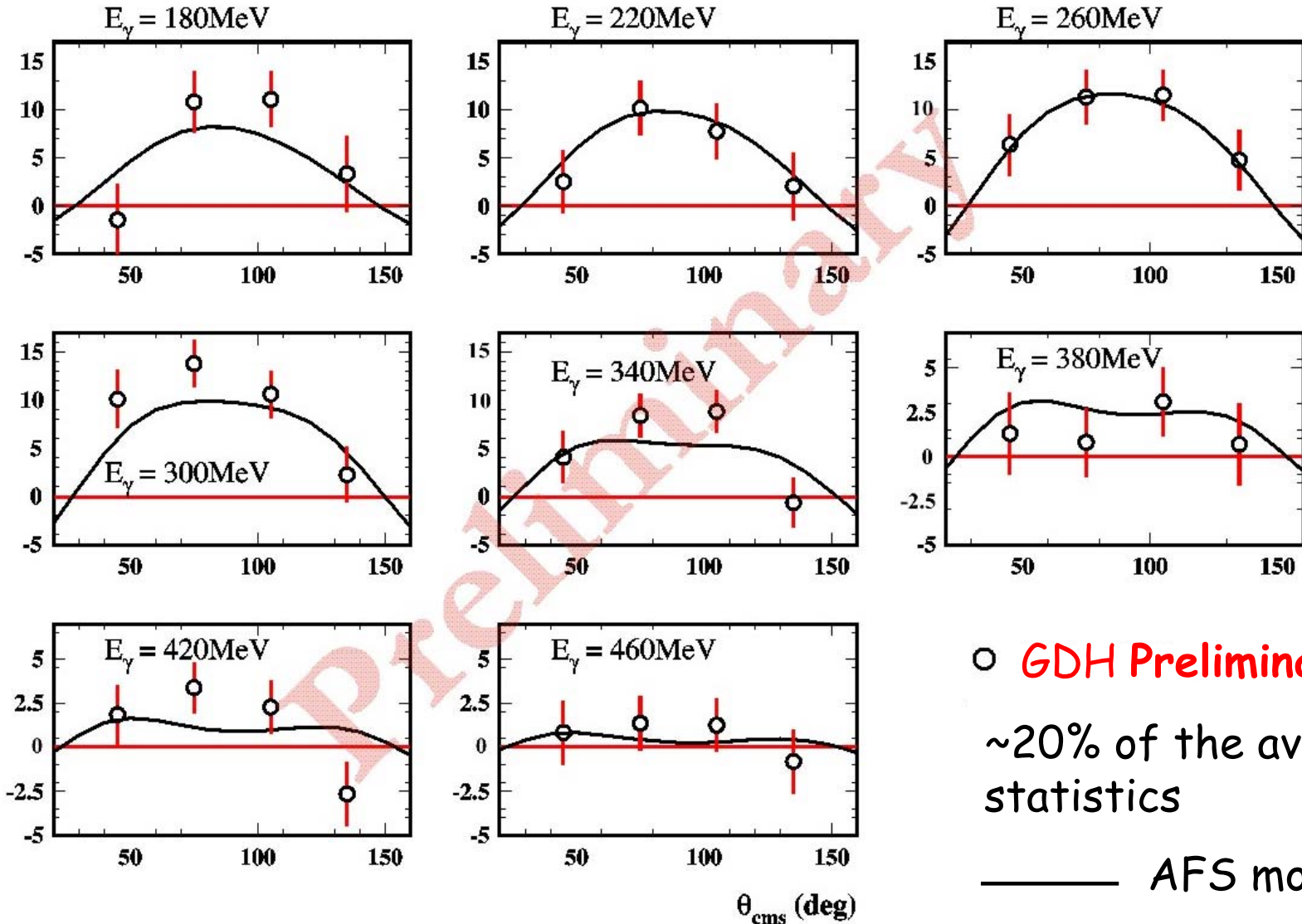
$$\vec{\gamma} \vec{p} \rightarrow p \pi^+ \pi^-$$

- D_{13} plays a small role
- Most important mechanism $\gamma p \rightarrow \Delta \pi$
- S -wave $\pi \Rightarrow \sigma_{3/2}$ is dominant



$$\vec{\gamma} d \rightarrow pn$$

$d\sigma_p/d\Omega - d\sigma_a/d\Omega$ ($\mu\text{b/sr}$)



○ GDH Preliminary

~20% of the available statistics

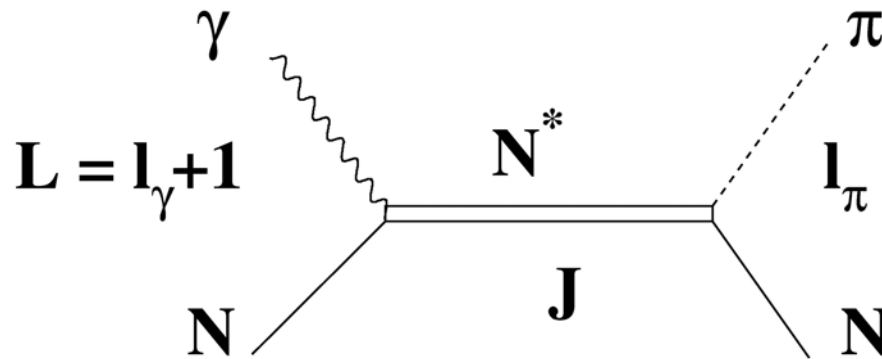
— AFS model

Conclusions

- After a long “hunt” for the GDH sum rule, we are almost there ...
 -) **proton** : sum rule ~ verified
 -) **neutron**: first data available on the deuteron (nuclear corrections !)
- Helicity dependent observables are a powerful (and essential) tool for a precise measurement of the baryon resonance properties
- At Mainz: CB@MAMI collaboration will improve/extend up to 1.5 GeV the $N\pi(\pi)(N\eta)$ GDH data (two proposals rated “A” by the PAC)
- **The game has just started**

Riserva

Connection between resonances and multipoles



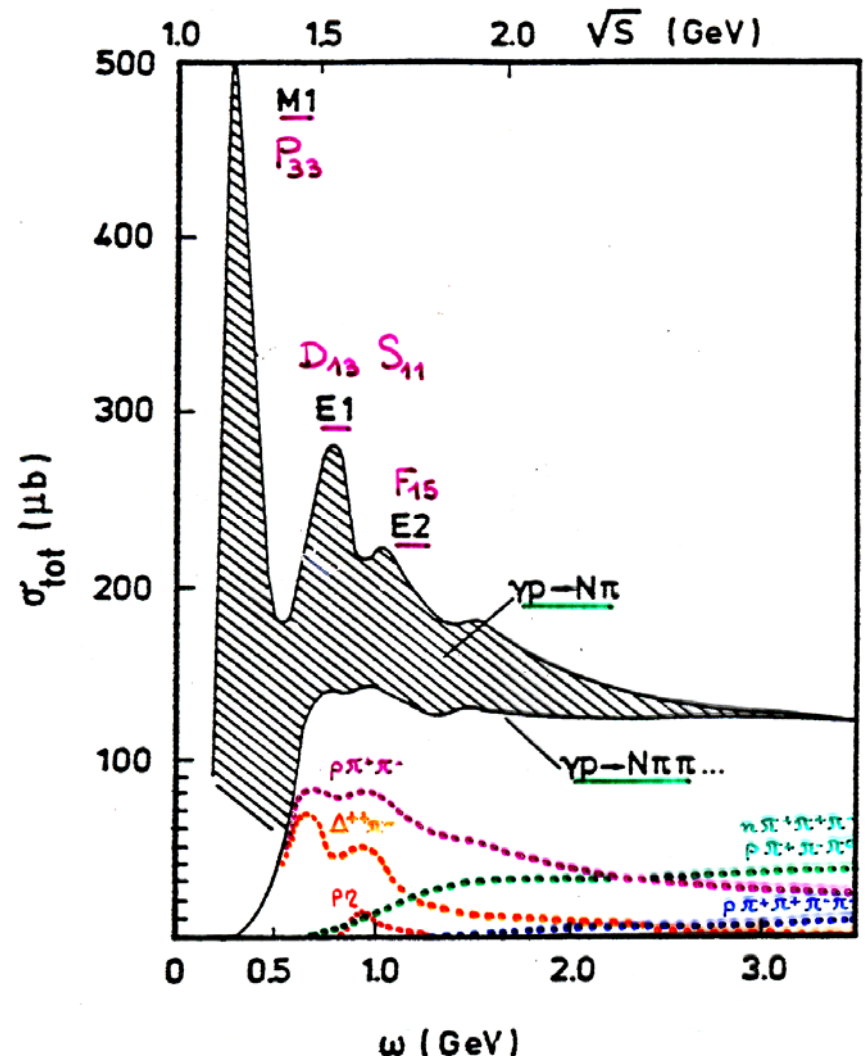
Photon L	Photon Multipole	J	P	Pion l_π	Pion Multipole	Resonance
1	E1	1/2	-	0	E_{0+}	S_{11}
		3/2	-	2	E_{2-}	D_{13}
	M1	1/2	+	1	M_{1-}	P_{11}
		3/2	+	1	M_{1+}	P_{33}
2	E2	3/2	+	1	E_{1+}	P_{33}
		5/2	+	3	E_{3-}	F_{15}
	M2	3/2	-	2	M_{2-}	D_{13}
		5/2	-	2	M_{2+}	D_{15}

➤ **Unpolarized cross section**

$$\sigma = |E_{0+}|^2 + |M_{1-}|^2 + 6|E_{1+}|^2 + 2|M_{1+}|^2 + 6|M_{2-}|^2 + 2|E_{2-}|^2 + \dots$$

Only the (few) most relevant multipoles can be accessed

Total photoabsorption cross section on the proton



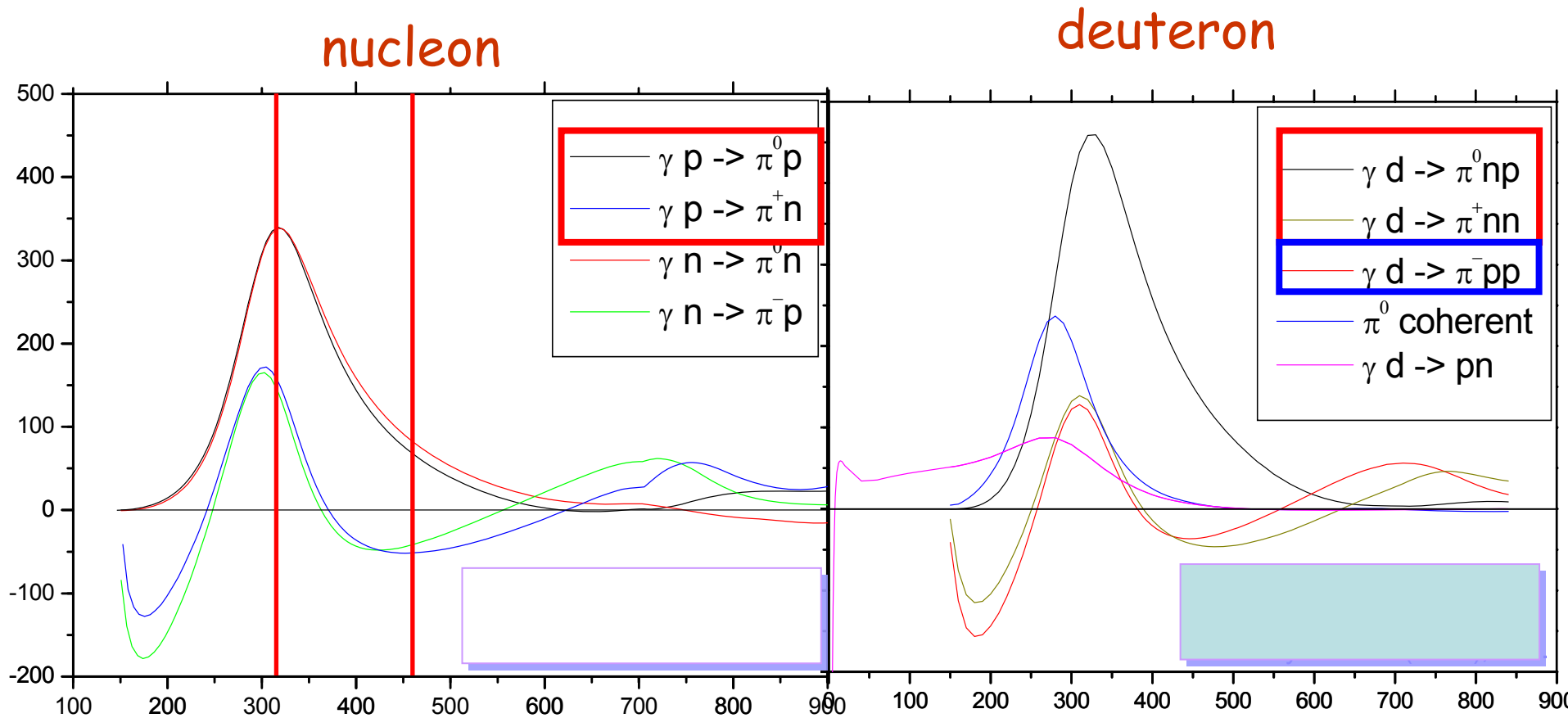
➤ Helicity dependent cross section ($\sigma_{3/2} - \sigma_{1/2}$)

$$(\sigma_{3/2} - \sigma_{1/2}) = \boxed{-} |E_{0+}|^2 \boxed{-} |M_{1-}|^2 \boxed{-} 3 |E_{1+}|^2 + |M_{1+}|^2 \boxed{-6E_{1+}^* M_{1+}} + \\ \boxed{-} 3 |M_{2-}|^2 + |E_{2-}|^2 \boxed{+6E_{2-}^* M_{2-}} + \dots$$

Change of sign / Interference terms between multipoles

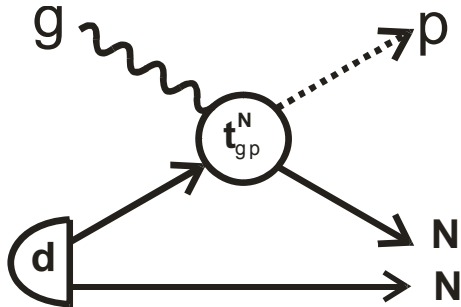
Single pion photoproduction

Helicity dependence of the photoabsorption reactions

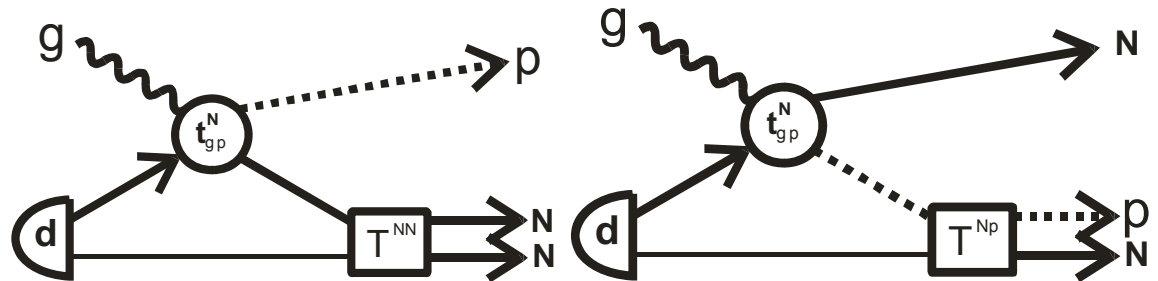


Deuteron Model

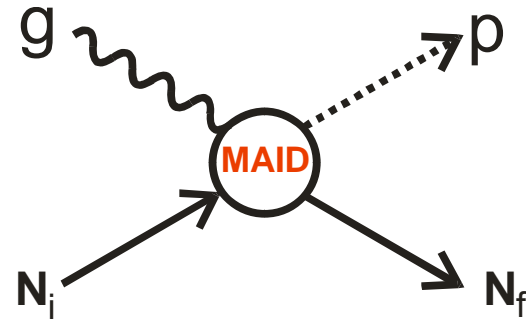
Impulse Approximation



+ Final State Interaction



- Elementary amplitude from **MAID**



- NN : Paris potential, partial waves up to $^{2s+1}L_j = {}^3D_3$
- pN : model of S. Nozawa *et al.*, partial waves up to $L_{2T2J} = D_{35}$

Total inclusive cross section

~~$$\sigma_{total} = \sum \text{partial channels}$$~~

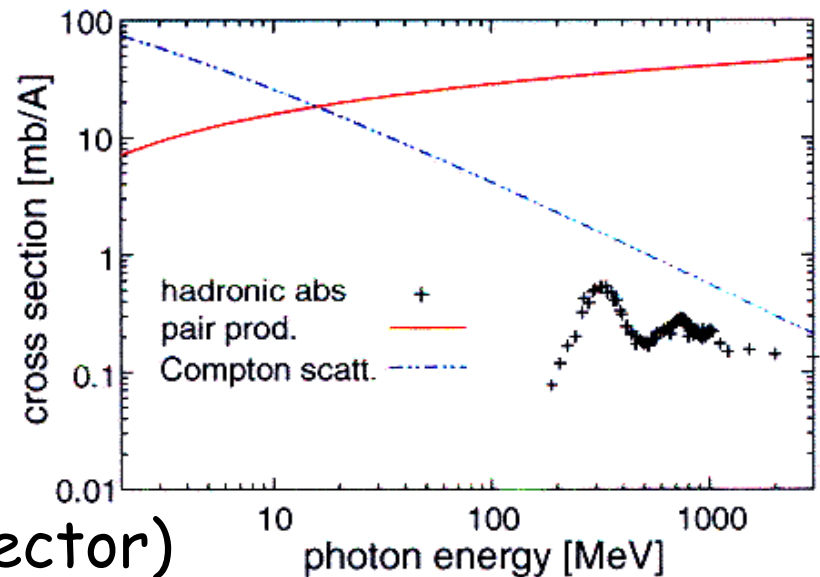
(not feasible)

$$\sigma_{total} = \sum \text{hadrons}$$

(inclusive method)

For each partial reaction channel, at least one reaction product has to be detected with (almost) complete acceptance (solid angle & efficiency)

- a) detector with a very high acceptance/particle detection efficiency (DAΦNE: 94% of 4π)
- b) Suppression of e.m. events (pair prod./Compton) at forward angles (Cerenkov detector)



Derivation of the GDH sum rule

- Forward ($\vartheta = 0$) Compton scattering amplitude (Lorentz and gauge invariance)

$$f(\nu) = f_1(\nu) \vec{\epsilon}_f \cdot \vec{\epsilon}_i + f_2(\nu) \vec{\sigma} \cdot (\vec{\epsilon}_f \times \vec{\epsilon}_i)$$

Spin independent

Spin dependent

($\vec{\epsilon}$ = photon polarization vector σ = nucleon spinor)

- Dispersion relation **without subtraction** for the spin dependent part of the amplitude

$$\text{Re } f_2(\nu) = \frac{2\nu}{\pi} \cdot \int_0^{\infty} \frac{\text{Im } f_2(\nu')}{(\nu'^2 - \nu^2)} d\nu'$$

$f_2 \rightarrow 0$ when $\nu \rightarrow \infty$

➤ Optical theorem

$$\text{Im } f_2(\nu) = \frac{\nu}{8\pi} [\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)]$$

➤ Low energy theorem

$$\lim_{\nu \rightarrow 0} f_2(\nu) = -\frac{\alpha}{2m^2} \kappa^2 \nu$$

$$\text{Re } f_2(\nu) = \frac{2\nu}{\pi} \int_0^{\infty} \frac{\text{Im } f_2(\nu')}{(\nu'^2 - \nu^2)} d\nu' \quad \xRightarrow{\nu \rightarrow 0} \quad \int_0^{\infty} \frac{\sigma_{3/2}(\nu') - \sigma_{1/2}(\nu')}{\nu'} d\nu' = \frac{2\pi^2 \alpha}{m^2} \kappa^2$$

Why could the GDH sum rule be violated ?

- The only "weak" hypothesis is: $f_2 \rightarrow 0$ when $\nu \rightarrow \infty$

- Violation:

$$\Rightarrow \text{Im } f_2(\nu) = (\sigma_{3/2} - \sigma_{1/2}) \rightarrow 0 \text{ when } \nu \rightarrow \infty$$

(from Regge theory $(\sigma_{3/2} - \sigma_{1/2}) \Rightarrow s^{-k}, k > 0$)

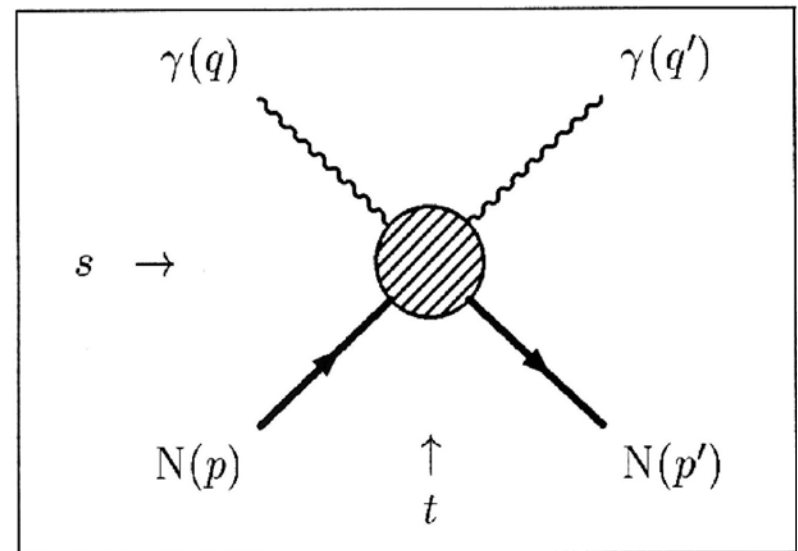
$$\Rightarrow \text{Re } f_2(\nu) \not\rightarrow 0 \text{ when } \nu \rightarrow \infty$$

Compton scattering amplitude is spin dependent
when $\nu \rightarrow \infty$

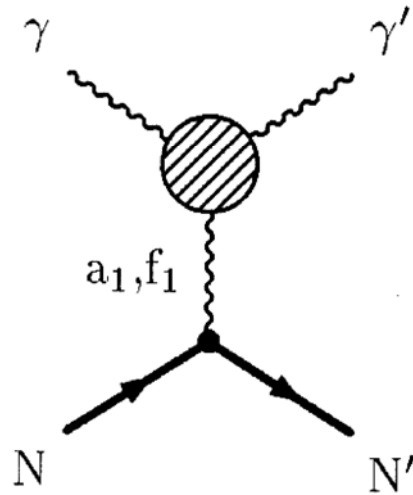


Scattering amplitude has a fixed $J=1+$ pole in the t channel

$$f_2(0) = f_2(\infty) + \int_0^{\infty} \frac{\text{Im } f_2(\nu')}{\nu'} d\nu$$

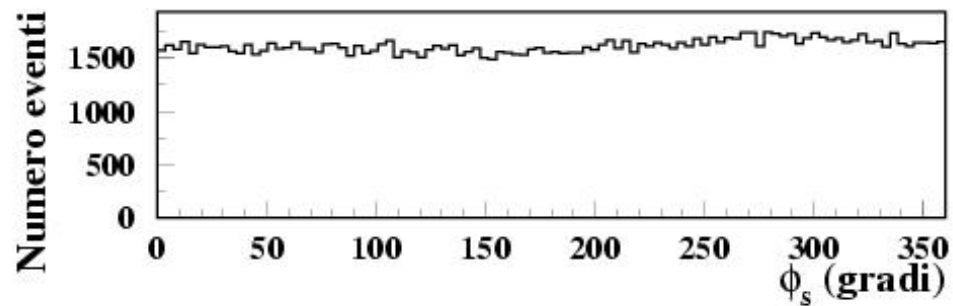
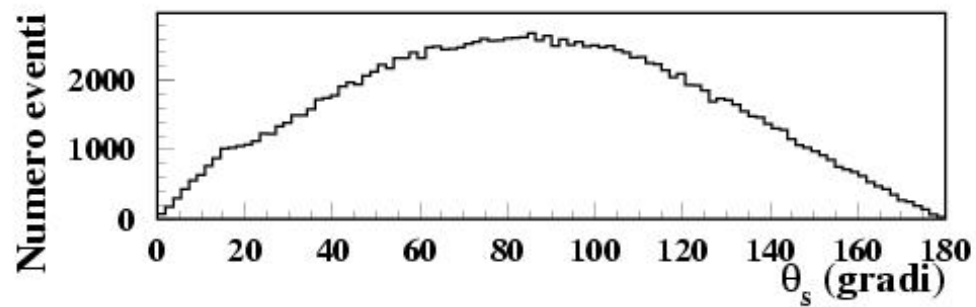
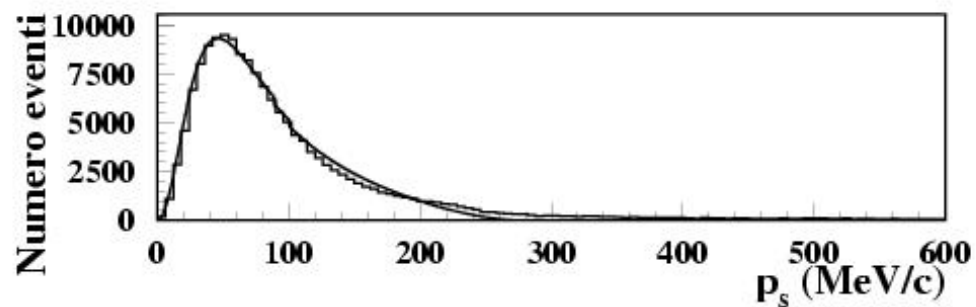


- Exchange of a_1 -like ($J=1+$) mesons between γ and N ?

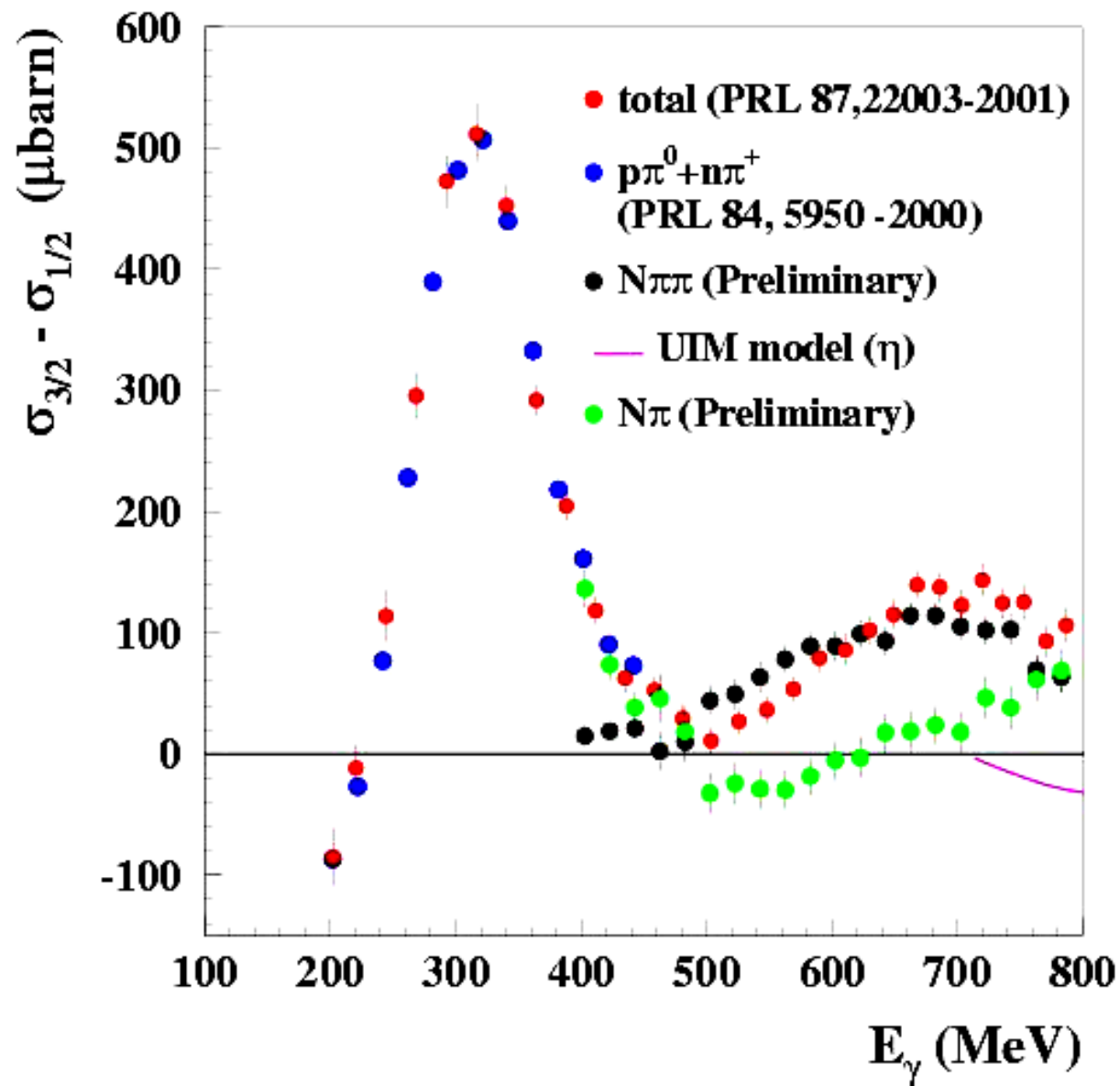


- Non pointlike (constituent) quarks ?
- photoproduction of graviton ?

$(d\sigma/d\Omega)$ ($\mu\text{b/sr}$)



Exclusive reactions



$$\vec{\gamma} \vec{p} \rightarrow X, N\pi(\pi), p\eta$$

$$\vec{\gamma} \vec{p} \rightarrow N \pi$$

Δ resonance region

■ GDH data

EPJA 31, 323 (04)

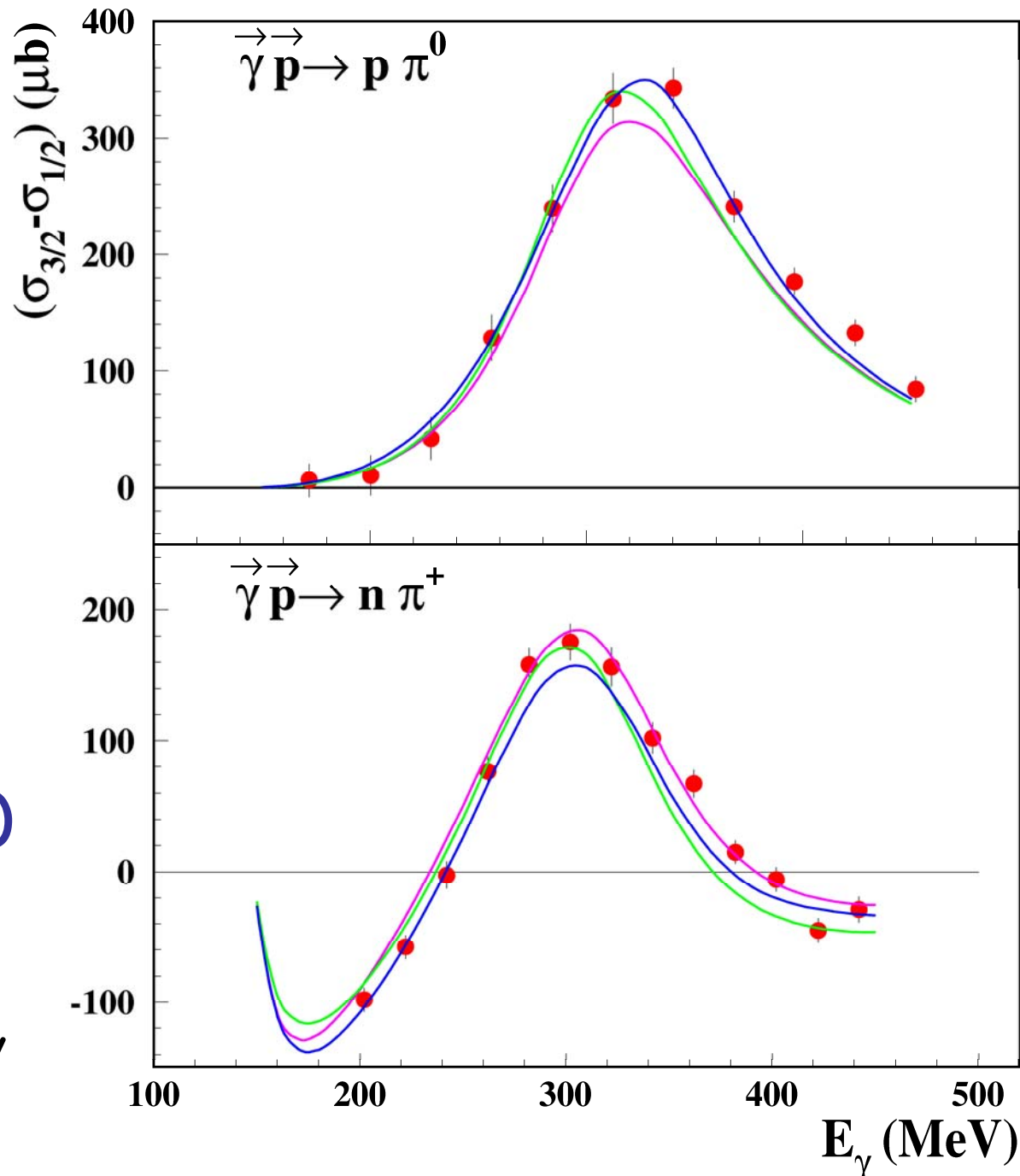
SAID (SM02k)

HDT

MAID

As expected, no (big) discrepancies

(several precise measurements already performed)



$$\vec{\gamma} \vec{p} \rightarrow p \pi^0$$

$$\left(\frac{d\sigma}{d\Omega} \right)_{3/2} - \left(\frac{d\sigma}{d\Omega} \right)_{1/2} \quad (\mu b)$$

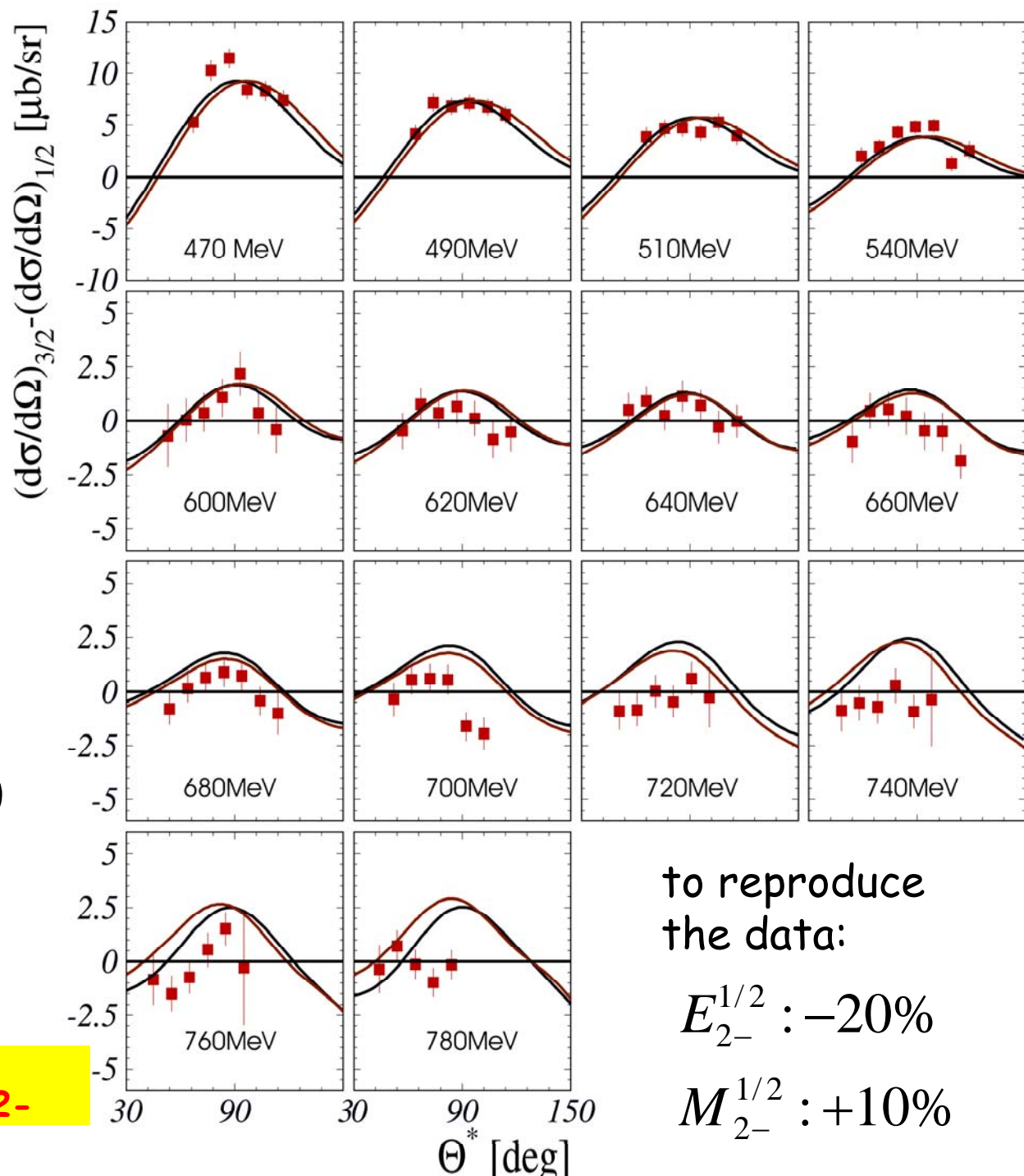
■ GDH data

PRL 88, 232002, 2002

— SAID (SM01)

— MAID2000

Sensitivity to $E_{2-}^* M_{2-}^*$



to reproduce
the data:

$$E_{2-}^{1/2} : -20\%$$

$$M_{2-}^{1/2} : +10\%$$

$$\vec{\gamma} \vec{p} \rightarrow n \pi^+$$

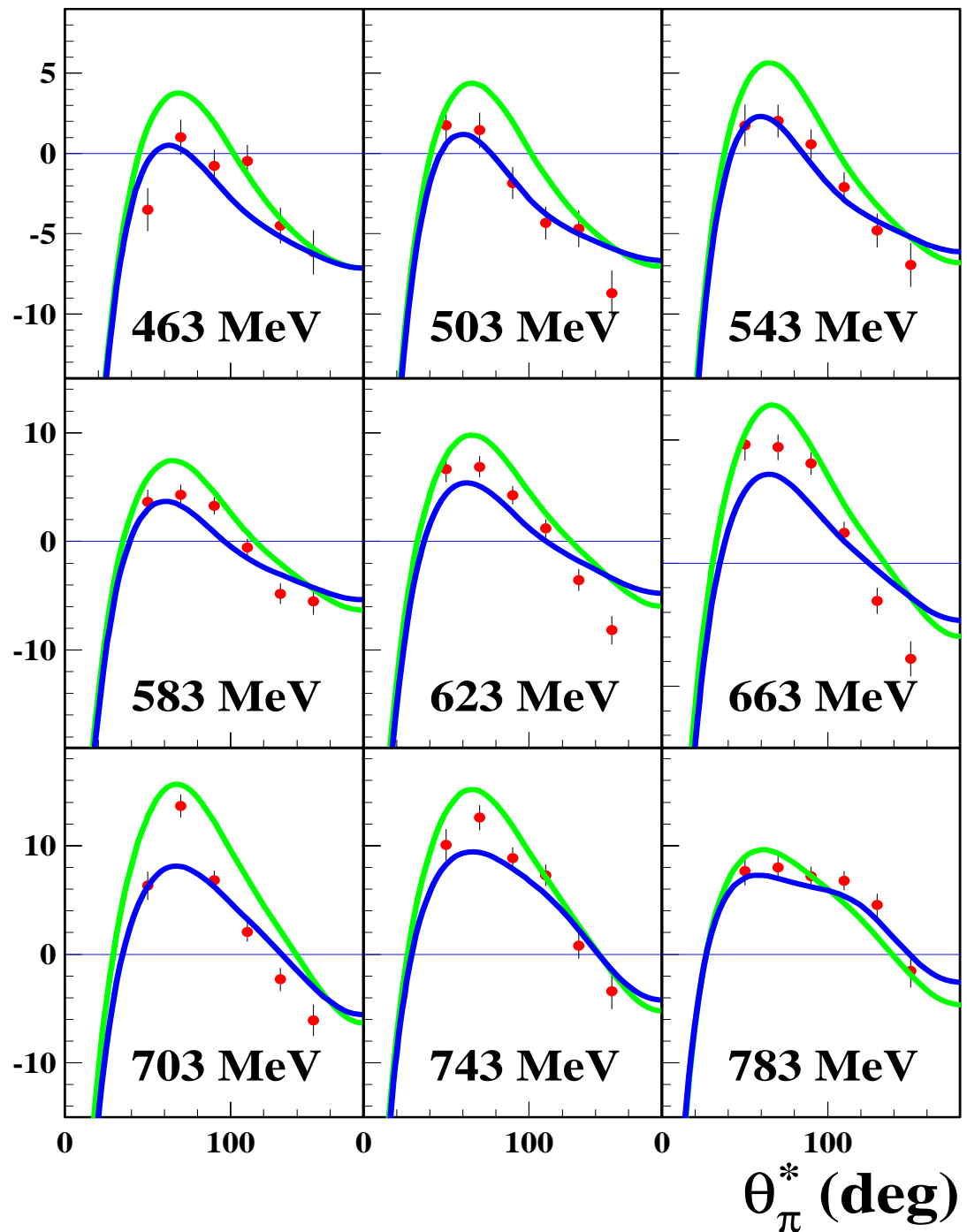
$$\left(\frac{d\sigma}{d\Omega} \right)_{3/2} - \left(\frac{d\sigma}{d\Omega} \right)_{1/2} \quad (\mu b)$$

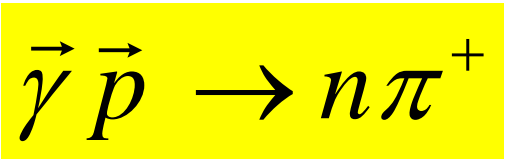
■ GDH data (submitted to PRL)

— SAID (FA04K)

— MAID2003

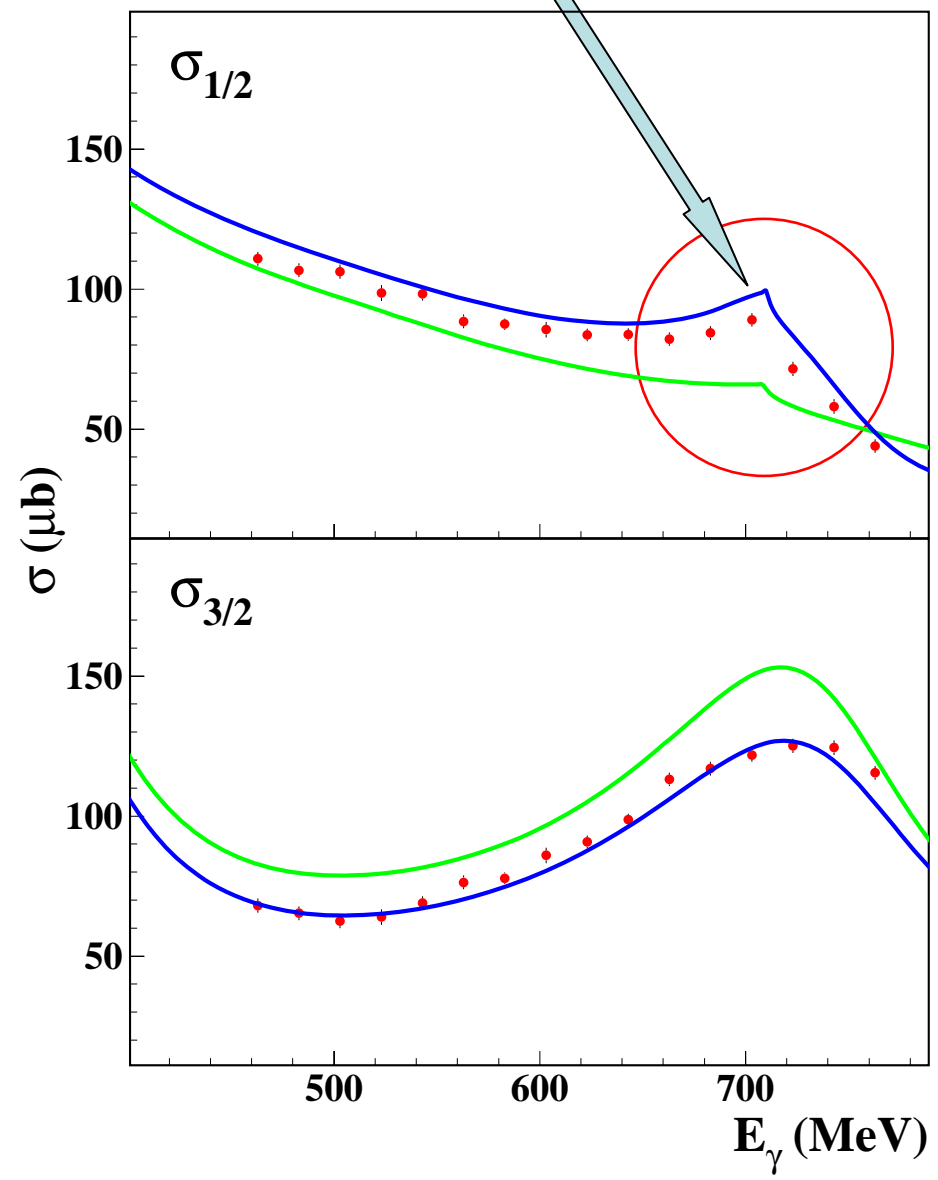
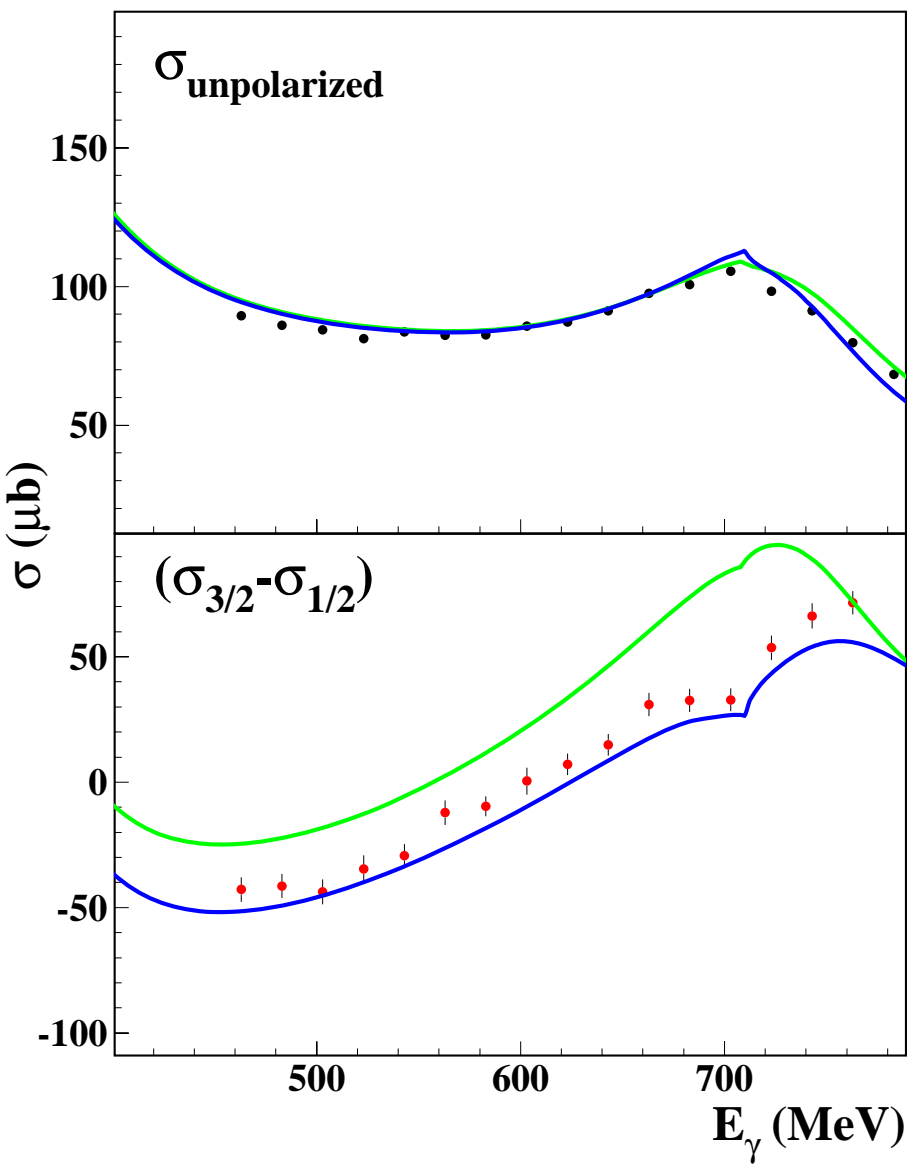
different contributions from E_{0+} , E_{2-} give now different model predictions





■ GDH data (PRC 2006)
— SAID (FA04K)
— MAID2003

Unitarity cusp (threshold of η production)



$$\vec{\gamma} \vec{p} \rightarrow p \pi^0 \pi^0$$

GDH data

PLB 624, 173 (05)

■ $\sigma_{3/2}$

■ $\sigma_{1/2}$

➤ D_{13} plays a significant role

➤ Which mechanisms do contribute to $\sigma_{1/2}$?
(P_{11} , ...)

