Workshop on Hadron Physics on the Lattice

Conveners: C. Alexandrou and C. Michael

Summary

Outline

- Reaching the chiral regime - Highlights from Lattice 2007
- Workshop:

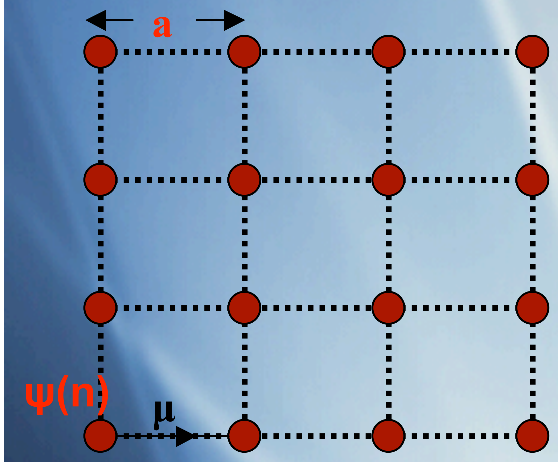
- Results using light dynamical fermions
  - Twisted mass - G. Herdozia and C.Urbach (ETMC)
  - Clover improved dynamical fermions: GPDs - G. Schierholz (QCDSF/UKQCD)
  - Overlap fermions: f_π, pion form factor - Sh. Hashimoto (JLQCD)
  - Hybrid approach: Form factors and GPDs - Th. Korzec, J. Negele (LHPC)
  - Staggered fermions: Charm Physics - C. Davies (HPQCD)
- Fundamental parameters of QCD - Quark masses - R. Sommer
- Harder Observables
  - π-π and NN scattering - A. Walker-Loud (NPLQCD)
  - Excited states - C. Morningstar
  - Electric and magnetic polarizabilities - Detmold
  - Neutron electric dipole moment - S. Simula
  - Nuclear Force - N. Ishii
  - Finite temperature and density - C. Allton
- Chiral extrapolation of lattice results - G. Colangelo

Lattice 2007: Highlights

• **Impressive progress in unquenched simulations using various fermions discretization schemes**

Lattice fermions

Basics:



$U_\mu(n)=e^{igaA_\mu(n)}$

$$S = S_G + S_F$$

$$S_F = \sum_{k,n} \bar{\psi}(k) D(k,n) \psi(n)$$

Difficult to simulate -->

90's Det[D]=1 : quenched

Integrate out

$$\langle O \rangle = \frac{\int \prod_n dU_n d\bar{\psi}_n d\psi_n O[U,\bar{\psi},\psi] e^{-S}}{Z} = \frac{\int \prod_n dU_n \det[D] O[U, D^{-1}] e^{-S_G}}{Z}$$

It is well known that naïve discretization of Dirac action

$$S_F = \int d^4x\, \bar{\psi}(x)\left[\gamma_\mu \partial_\mu + m\right]\psi(x) \quad \rightarrow \quad \partial_\mu \psi(x) = \frac{\psi(x+\mu) - \psi(x-\mu)}{2a}$$

leads to 16 species

A number of different discretization schemes have been developed to avoid doubling:

- **Wilson :** add a second derivative-type term --> breaks chiral symmetry for finite a
- **Staggered:** "distribute" 4-component spinor on 4 lattice sites --> still 4 times more species, take 4th root, non-locality

Nielsen-Ninomiya no go theorem: impossible to have doubler-free, chirally symmetric, local, translational invariant fermion lattice action ⟶ BUT…

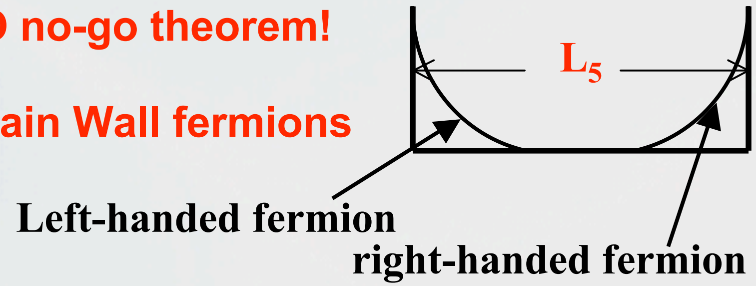
Chiral fermions

Instead of a D such that $\{\gamma_5, D\}=0$ of the no-go theorem, find a D such that

$\{\gamma_5, D\}=2a\, D\, \gamma_5 D$ ← **Ginsparg - Wilson relation**

Luescher 1998: realization of chiral symmetry but NO no-go theorem!

- **Kaplan: Construction of D in 5-dimensions → Domain Wall fermions**



- **Neuberger: Construction of D in 4-dimensions but with use of sign function → Overlap fermions**

$$D=\frac{1+\varepsilon(D_w)}{2}, \qquad \varepsilon(D_w)=\frac{D_w}{\sqrt{D_w^+ D_w}}$$

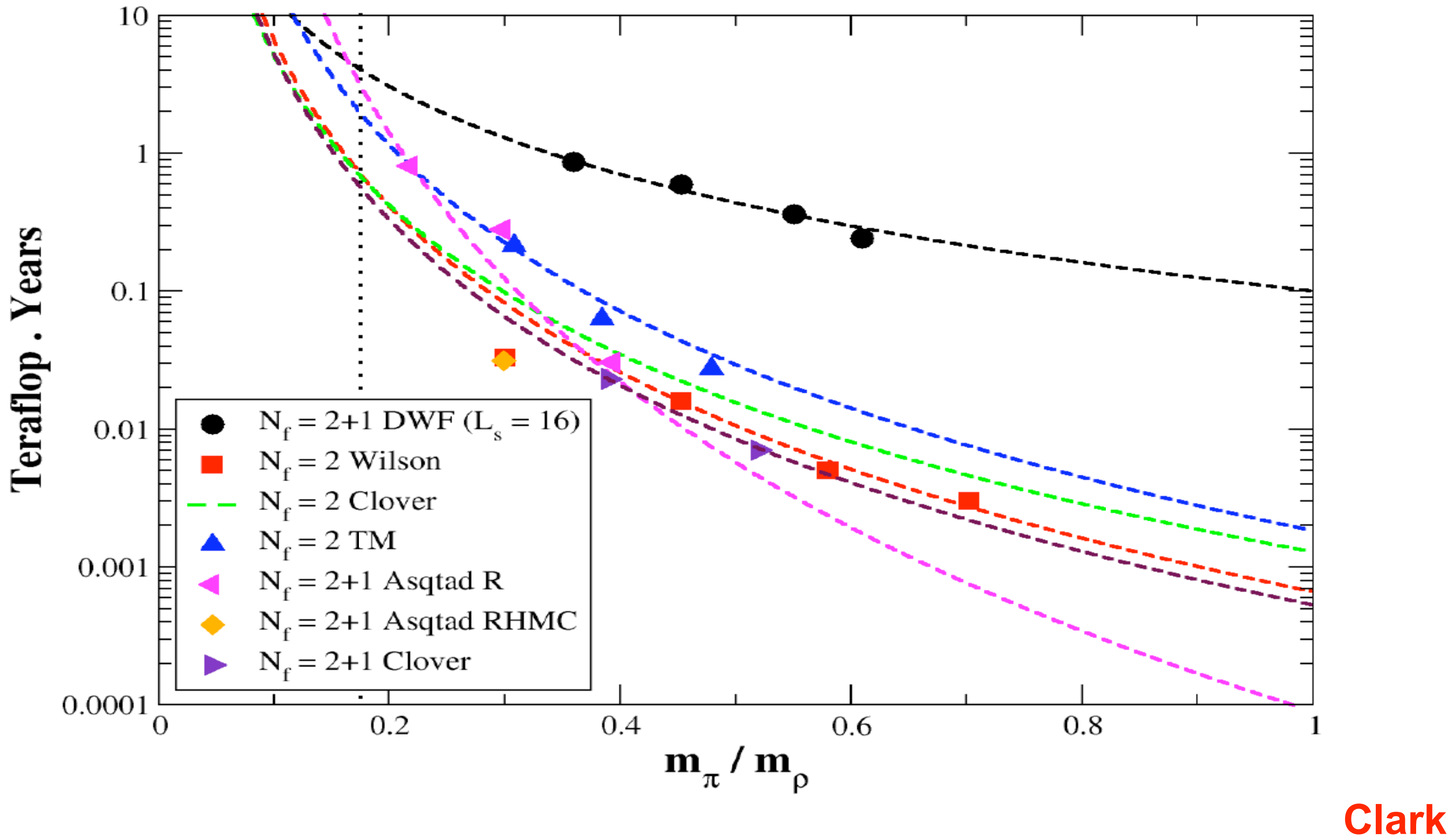
Equivalent formulations of chiral fermions on the lattice

But still expensive to simulate despite progress in algorithms

→ **Compromise:**
- **Use staggered sea (MILC) and domain wall valence (hybrid) - LHPC**
- **Twisted mass Wilson - ETMC**
- **Clover improved Wilson - QCDSF, UKQCD, PACS-CS**

Rely on new improved algorithms

Simulation of dynamical fermions



Results from different discretization schemes must agree in the continuum limit

Lattice 2007: Highlights

➢ Impressive progress in unquenched simulations using various fermions discretization schemes

With staggered, Clover improved and Twisted mass dynamical fermions results are reported with pion mass of ~300 MeV

PACS-CS reported simulations with 210 MeV pions!
But no results shown yet

- RBC-UKQCD: Dynamical N_F=2+1 domain wall simulations reaching ~300 MeV on ~3 fm volume are underway
- JLQCD: Dynamical overlap N_F=2 and N_F=2+1, $m_l \sim m_s/6$, small volume (~1.8 fm)

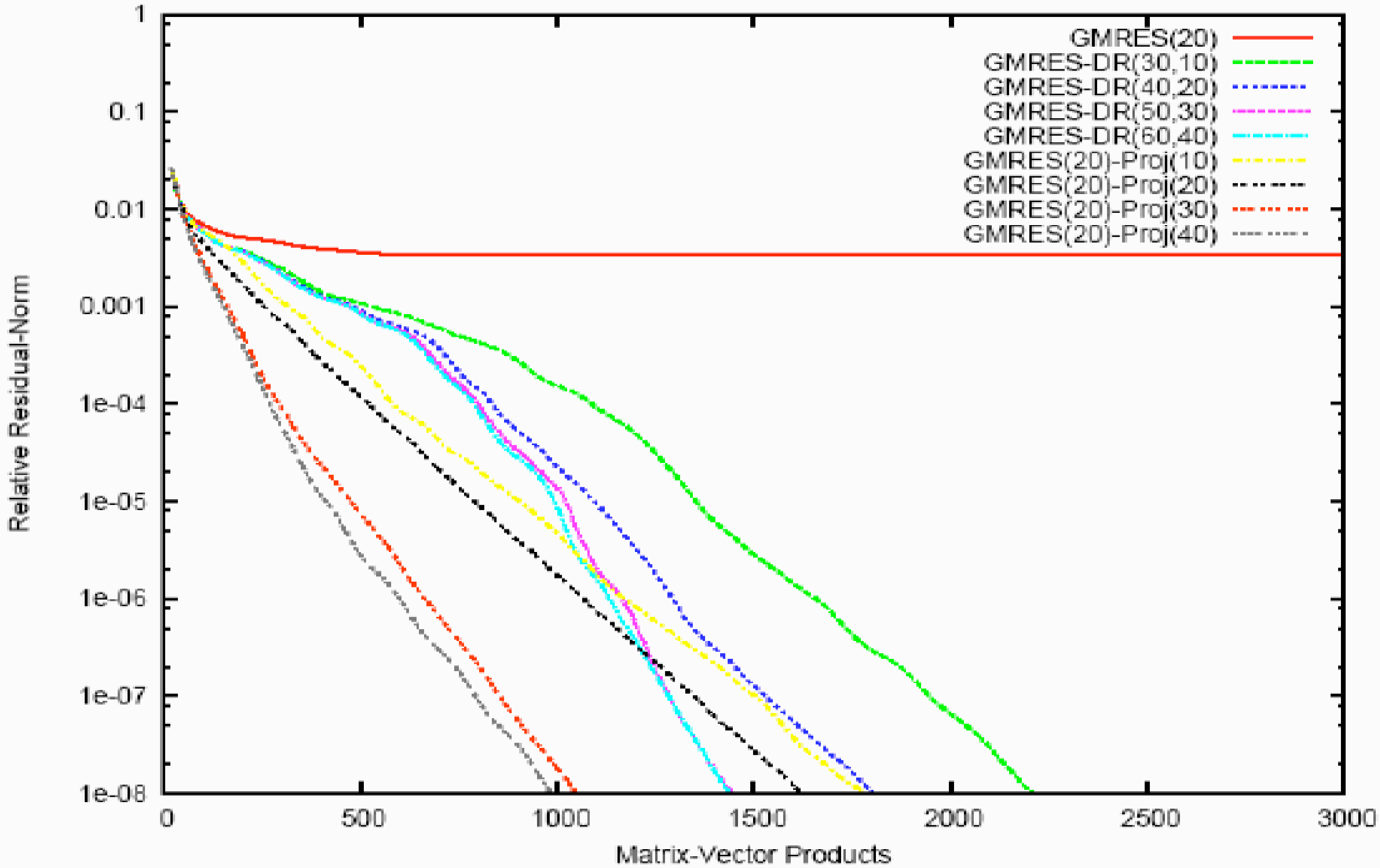
➢ Progress in algorithms for calculating D^{-1} as $m_\pi \longrightarrow$ physical value

Deflation methods: project out lowest eigenvalues
Lüscher, Wilcox, Orginos/Stathopoulos

Deflation methods in fermion inverters -Wilcox

Solve linear system $D^{a'a}_{\alpha'\alpha}(x',x)D^{-1}{}^{ab}_{\alpha\beta}(x,y) = \delta_{a',b}\delta_{\alpha',\beta}\delta_{x',y}$

Twisted mass 20^3x32 at κ_{cr}



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➢ Chiral extrapolations

Using lattice results on f_π/m_π yield LECs to unprecedented accuracy

➢ Begin to address more complex observables e.g. excited states and resonances, decays, finite density

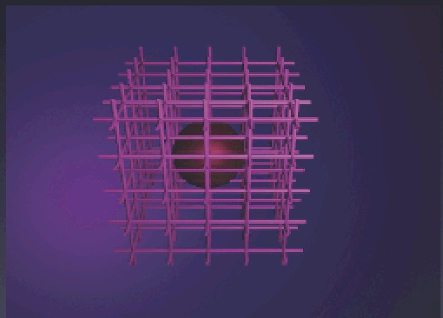
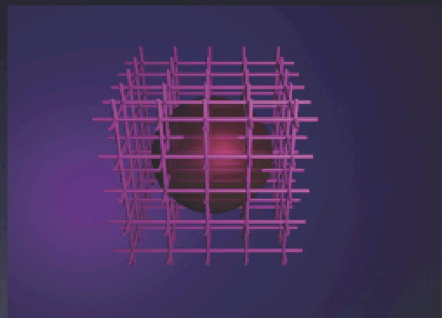
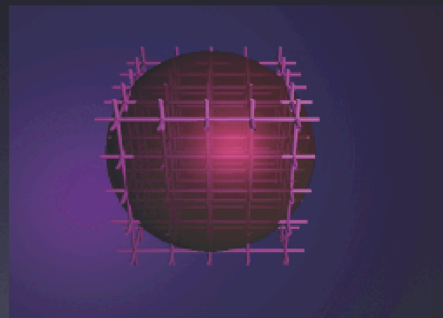
Chiral extrapolations

G. Colangelo

- Controlled extrapolations in volume, lattice spacing and quark masses: still needed for reaching the physical world
- Quark mass dependence of observables teaches about QCD - can not be obtained from experiment
- Lattice systematics are checked using predictions from χPT

Chiral perturbation theory in finite volume:

Correct for volume effects depending on pion mass and lattice volume

		
$m_\pi L \gg 1$	$\mu_{\text{had}} L \gg 1$	$\mu_{\text{had}} L \lesssim 1$
p-regime **Like in infinite volume**	ε-regime (pion zero modes become non-perturbative)	"Out of luck"-regime W. Detmold

Pion sector

Applications to pion and nucleon mass and decay constants

e.g.

$$\frac{m_\pi - m_\pi(L)}{m_\pi} = -\sum_{|\vec{n}|\neq 0} \frac{x}{2\lambda}\left[I^{(2)}_{m_\pi}(\lambda) + x I^{(4)}_{m_\pi}(\lambda)\right]$$

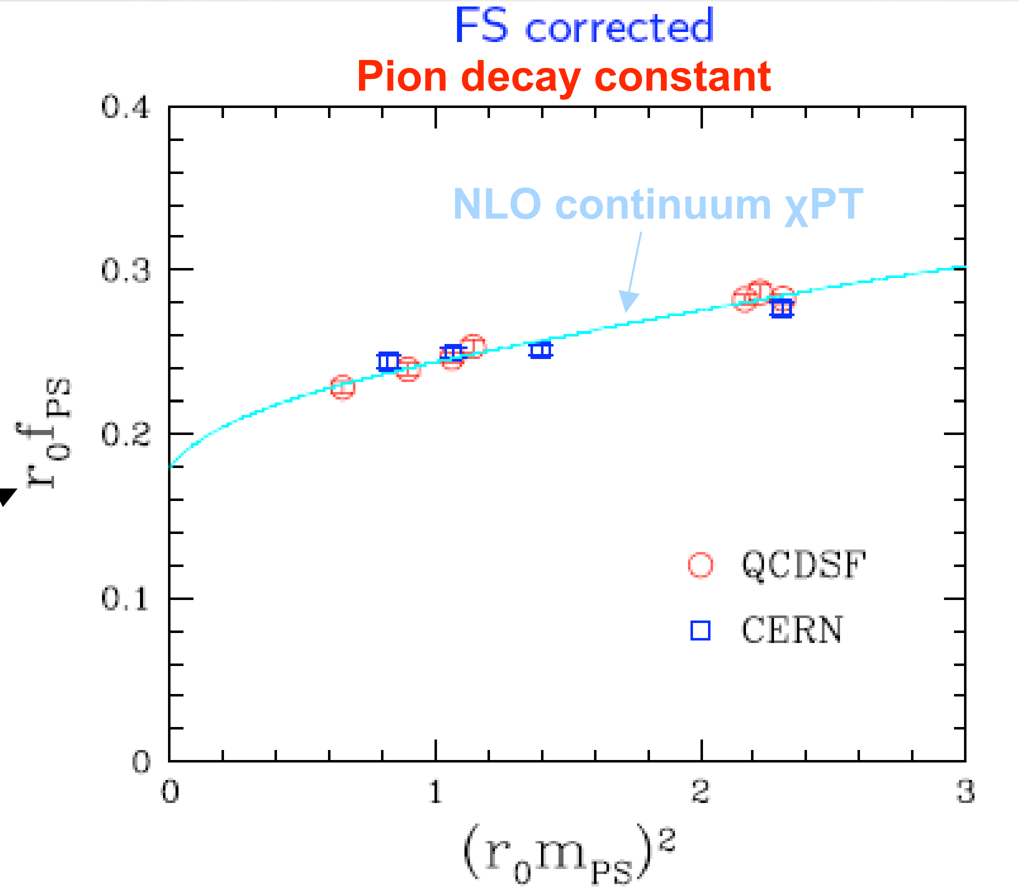
G. Colangelo, St. Dürr and Haefeli, 2005

$$\frac{f_\pi - f_\pi(L)}{f_\pi} = -\sum_{|\vec{n}|\neq 0} \frac{x}{\lambda}\left[I^{(2)}_{f_\pi}(\lambda) + x I^{(4)}_{f_\pi}(\lambda)\right]$$

$$x = \frac{m_0^2}{(4\pi f_\pi^0)^2}, \qquad \lambda = m_\pi |\vec{n}| L$$

Applied by QCDSF to correct lattice data

$r_0=0.45(1)$fm



Twisted mass dynamical fermions

C. Urbach

Consider two degenerate flavors of quarks: Action is

$$S_F = a^4 \sum_x \bar{\psi}(x)\left(D_w[U] + m_0 + i\mu\gamma_5\tau_3\right)\psi(x)$$

Wilson Dirac operator → $D_w[U]$

untwisted mass → m_0

Twisted mass μ

τ_3 acts in flavor space

Advantages: - Automatic $\mathcal{O}(a)$ improvement (at maximal twist)

- Only one parameter to tune (zero PCAC mass at smallest μ)

- No additional operator improvement

Disadvantages: - explicit chiral symmetry breaking (like in all Wilson)

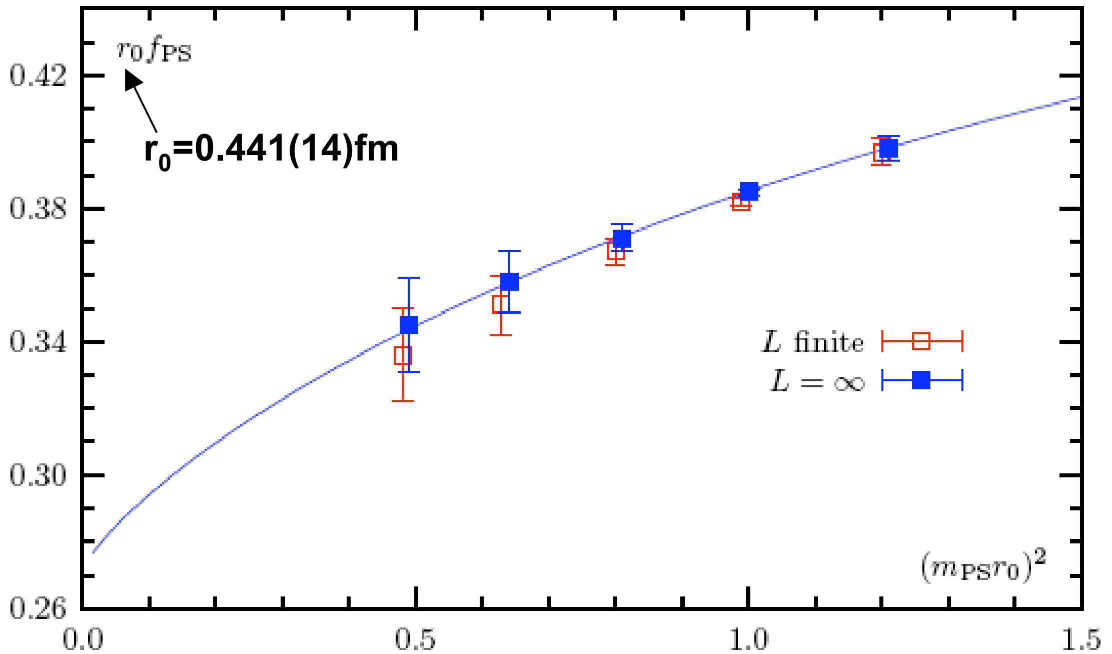
- explicit breaking of flavour symmetry ← appears only at $\mathcal{O}(a^2)$ in practice only affects π_0

Physics results reported for ~300 MeV pions on spatial size of about 2.7 fm with a < 0.1fm

Pion decay constant with twisted mass fermions

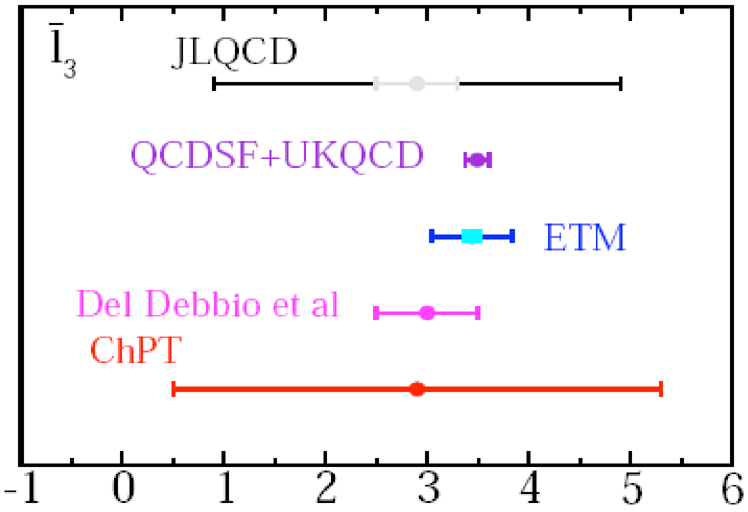
G. Herdoiza

Simultaneous chiral fits to m_π and f_π → Extract low energy constants

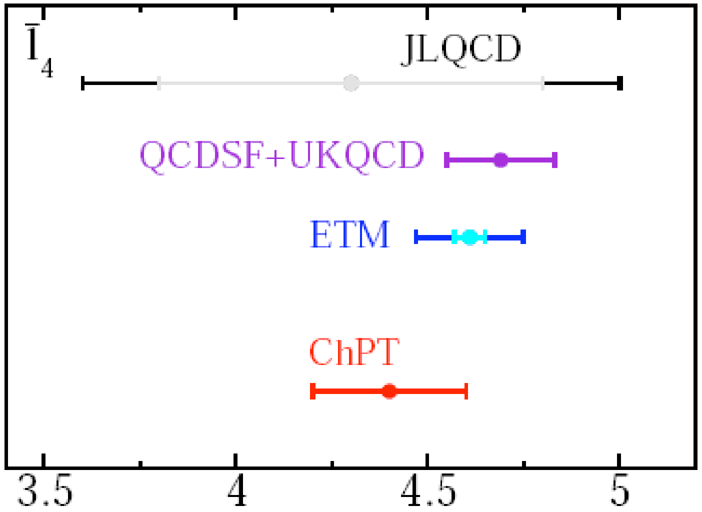


$$\bar{l}_{3,4} \equiv \log\left(\frac{\Lambda_{3,4}^2}{m_{\pi^\pm}^2}\right)$$

$$\bar{l}_3 = 3.44(28), \quad \bar{l}_4 = 4.61(9)$$



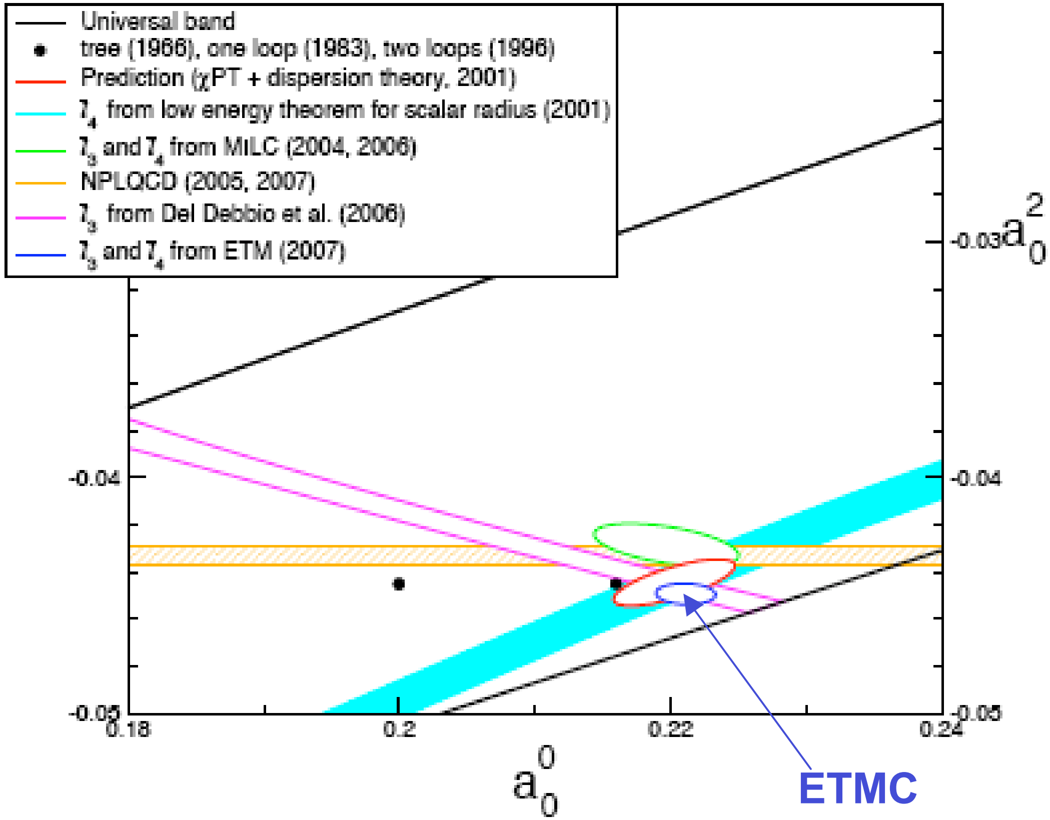
$N_F=2$



S. Necco, Lat07

ππ-scattering lengths

ππ s-wave length a_0^0 and a_0^2



Leutwyler (hep-ph/0612112)

$$a_0^0 = 0.220 \pm 0.005$$
$$10 \cdot a_0^2 = -0.444 \pm 0.01$$
$$a_0^0 - a_0^2 = 0.265 \pm 0.004$$

π-π scattering

A. Walker-Loud

Lüscher's finite volume approach:

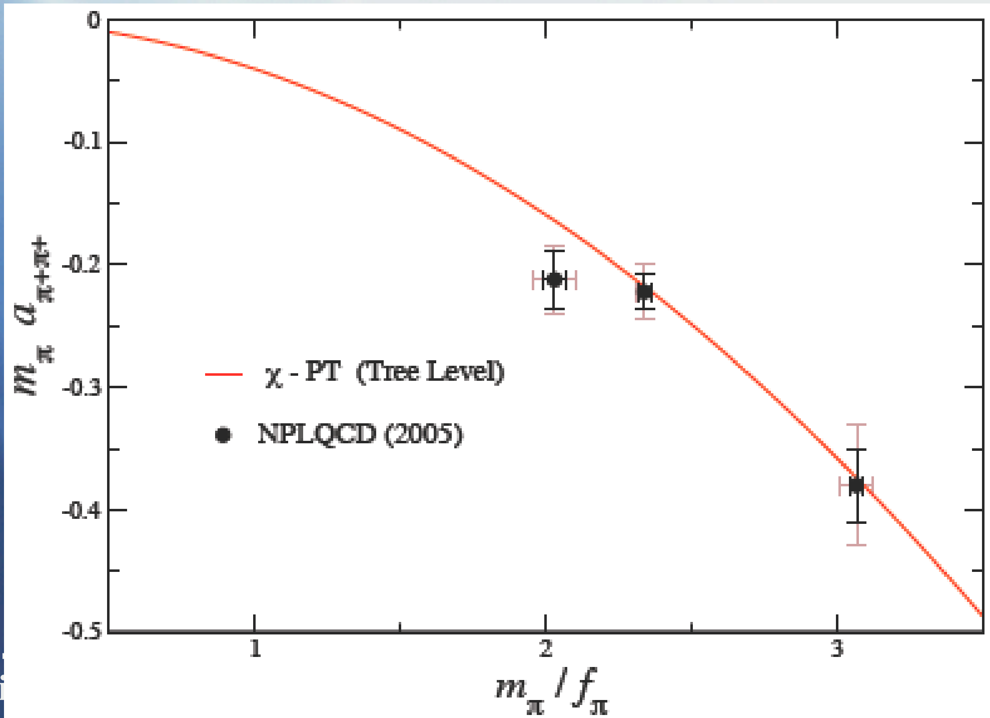
Calculate the energy levels of two particle states in a finite box

$$\Delta E_n = E_n - 2m = 2\sqrt{p_n^2 + m^2} - 2m$$ with p_n given by $$p\cot\delta(p) = \frac{1}{\pi L}\sum_{\vec{n}} \frac{f(\vec{n})}{\vec{n}^2 - \left(\frac{pL}{2\pi}\right)^2}$$

Expand ΔE_0 about p~0: $$\Delta E_0 = -\frac{4\pi a}{mL^3}\left[1 + c_1\frac{a}{L} + c_2\left(\frac{a}{L}\right)^2\right] + \mathcal{O}\left(\frac{1}{L^6}\right)$$

where we use particle definition for the scattering length: $a = \lim_{p\to 0}\frac{\tan\delta(p)}{p}$

Applied to I=2 two-pion system within the hybrid approach (staggered sea and domain wall fermions)



$m_\pi a_0^2$ =-0.0426(27)

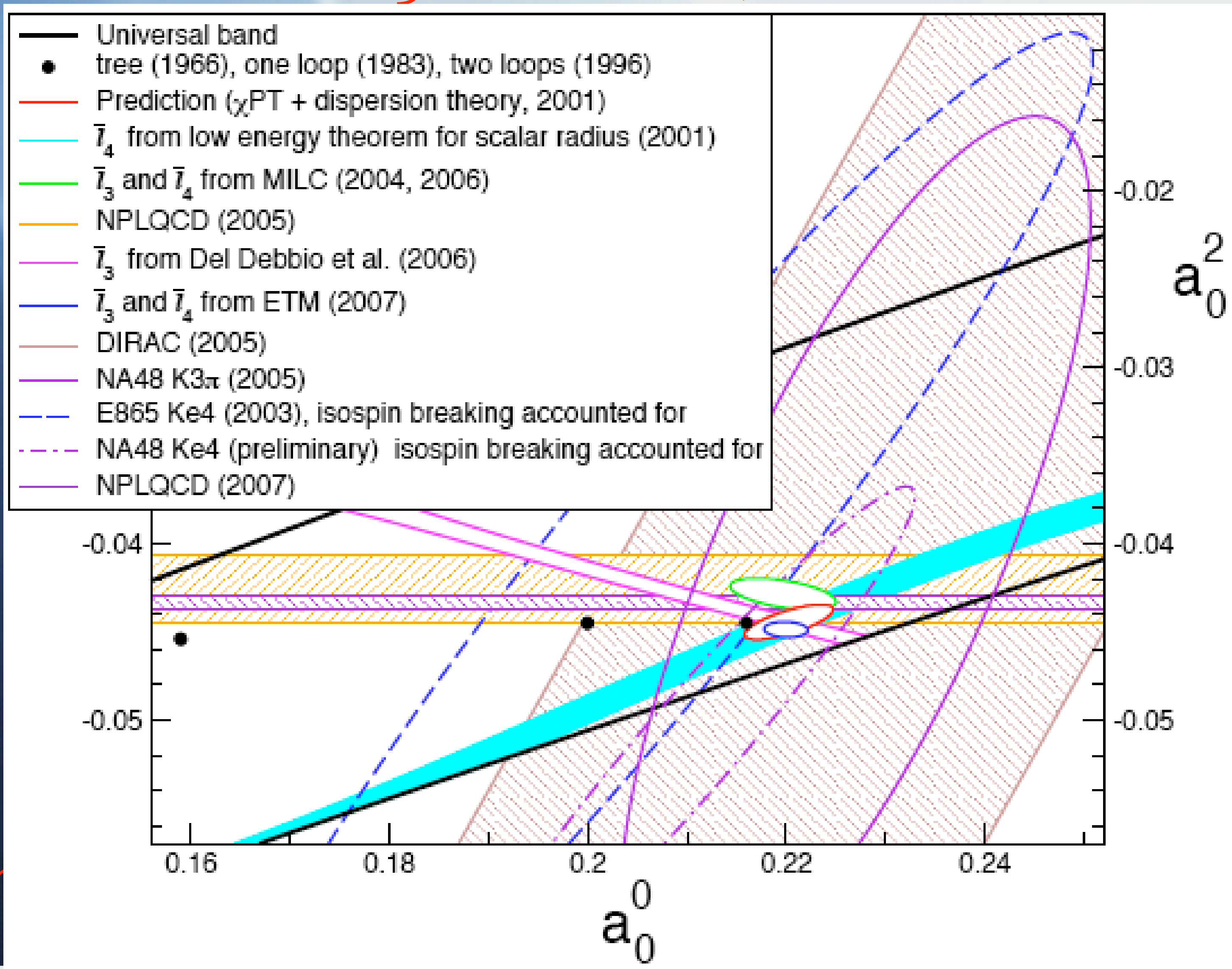
S. Beans et al. PRD73 (2006)

π-π scattering - improved

- More statistics
- mixed action χPT formula

$m_\pi a_0^2$ =-0.04330(42)

A. Walker-Loud, arXiv:0706.3026



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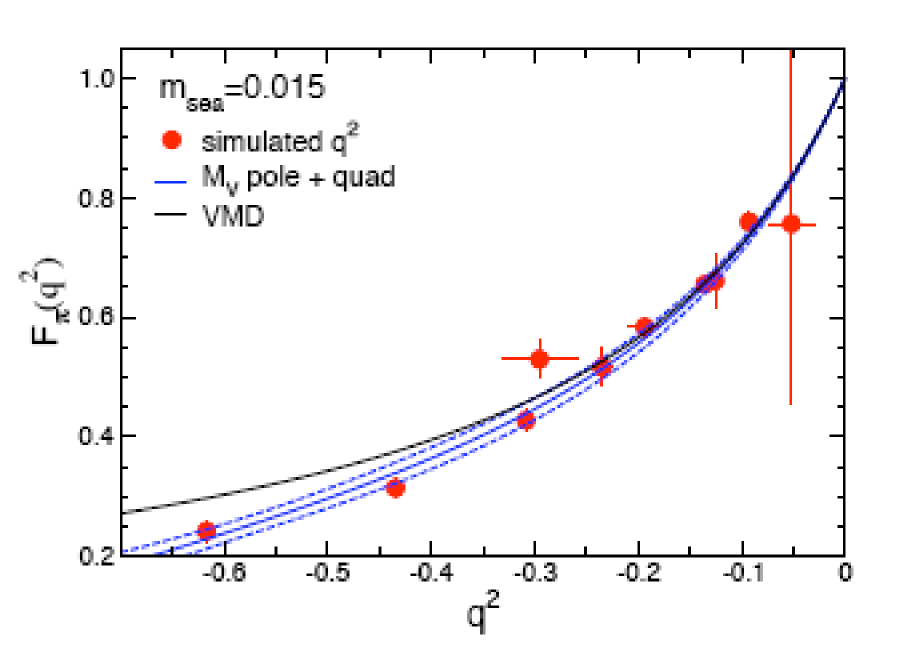
Oth

Pion form factor

Sh. Hashimoto

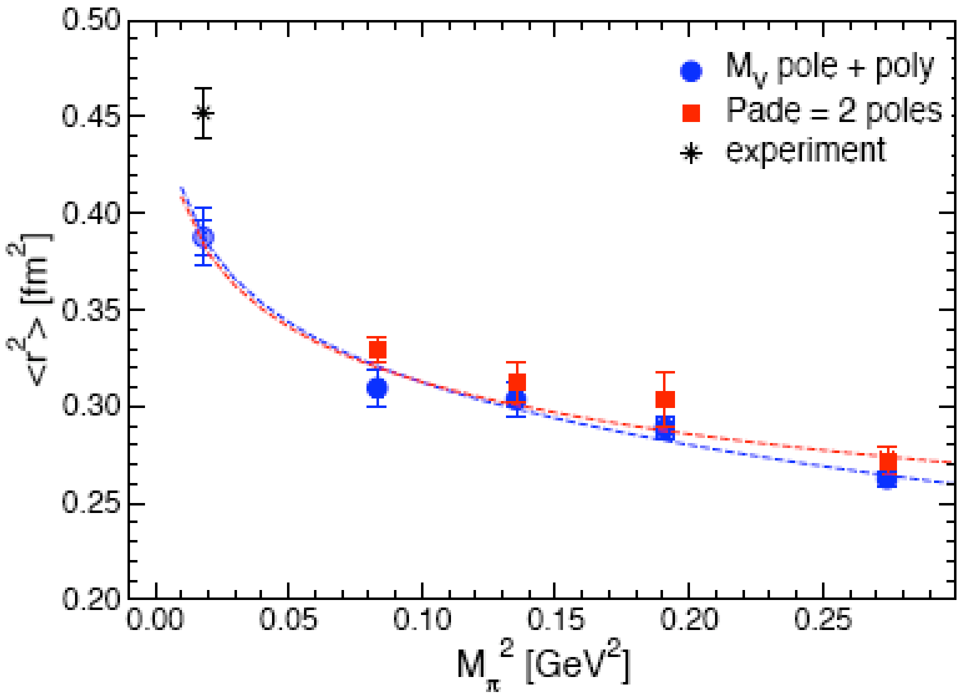
Dynamical overlap, L~1.8 fm



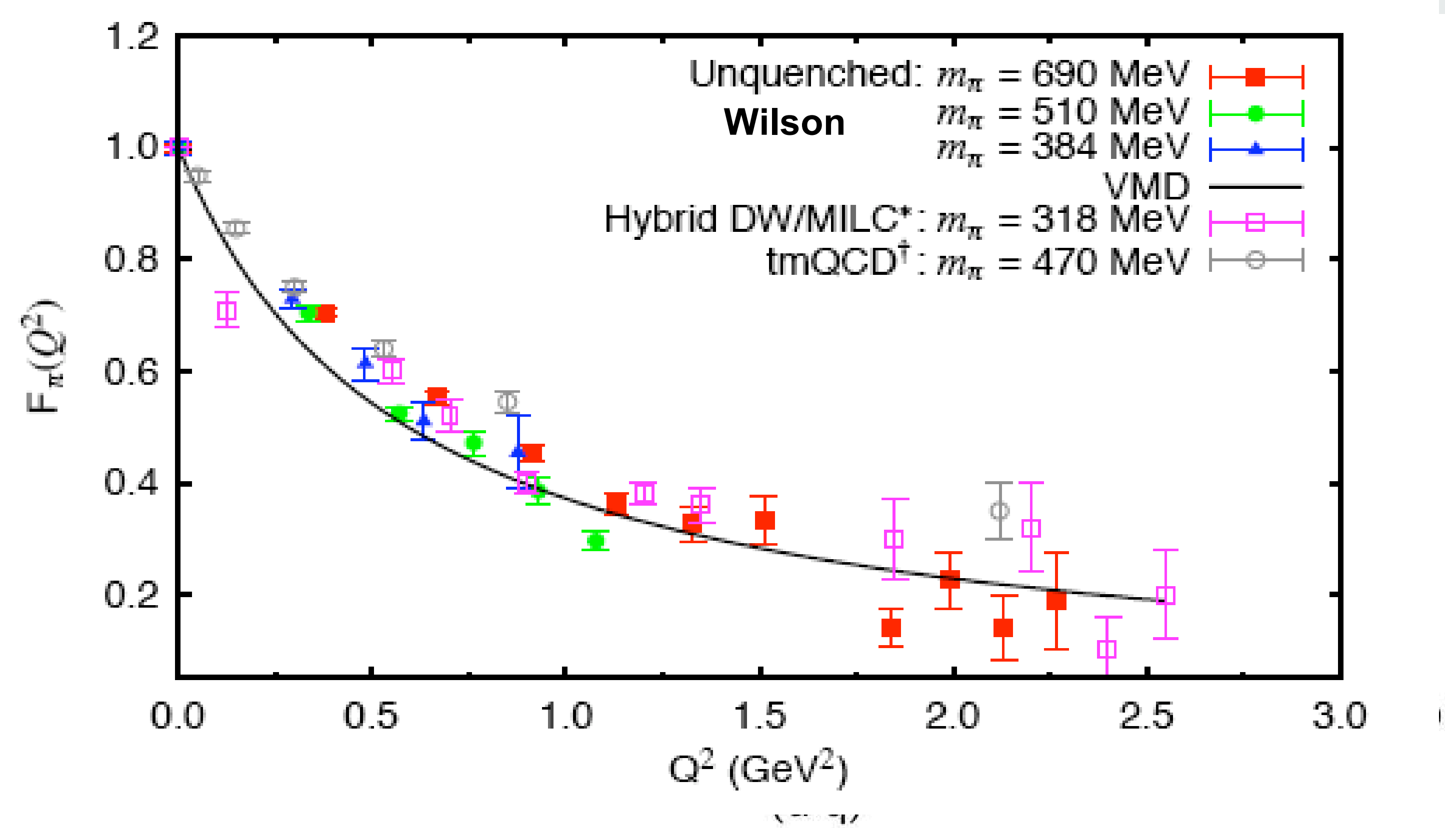
$$F_\pi(q^2) = \frac{1}{1 - q^2/m_\rho^2} + d_1 q^2 + \ldots$$

$$F_\pi(q^2) = 1 - \frac{1}{6} < r^2 > q^2 + \ldots$$

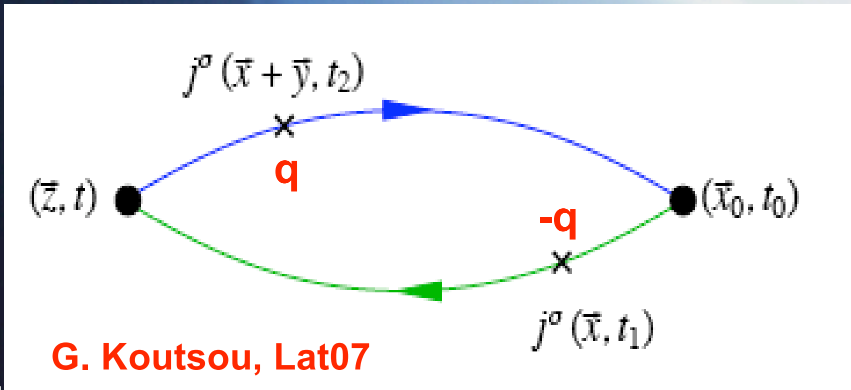
$$< r^2 > = c_0 - \frac{1}{(4\pi f_\pi)^2} \ln\left(\frac{m_\pi^2}{(4\pi f_\pi)^2}\right) + c_1 m_\pi^2$$



Pion form factor

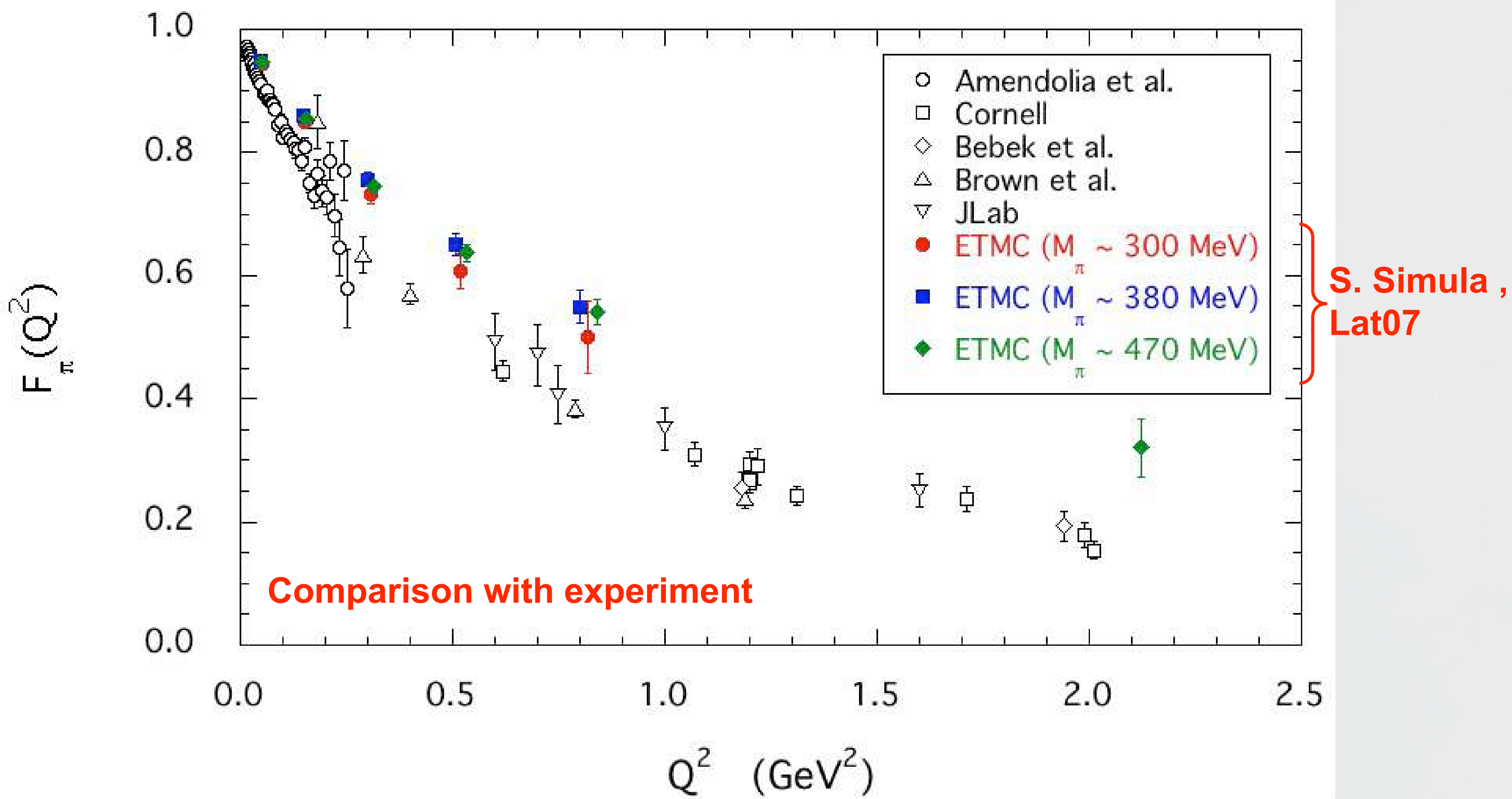


4-point function: first application of the one-end trick (C. Michael):



Set initial and final pion momentum to zero --> no disadvantage in applying the one-end trick

Pion form factor



Rho form factors

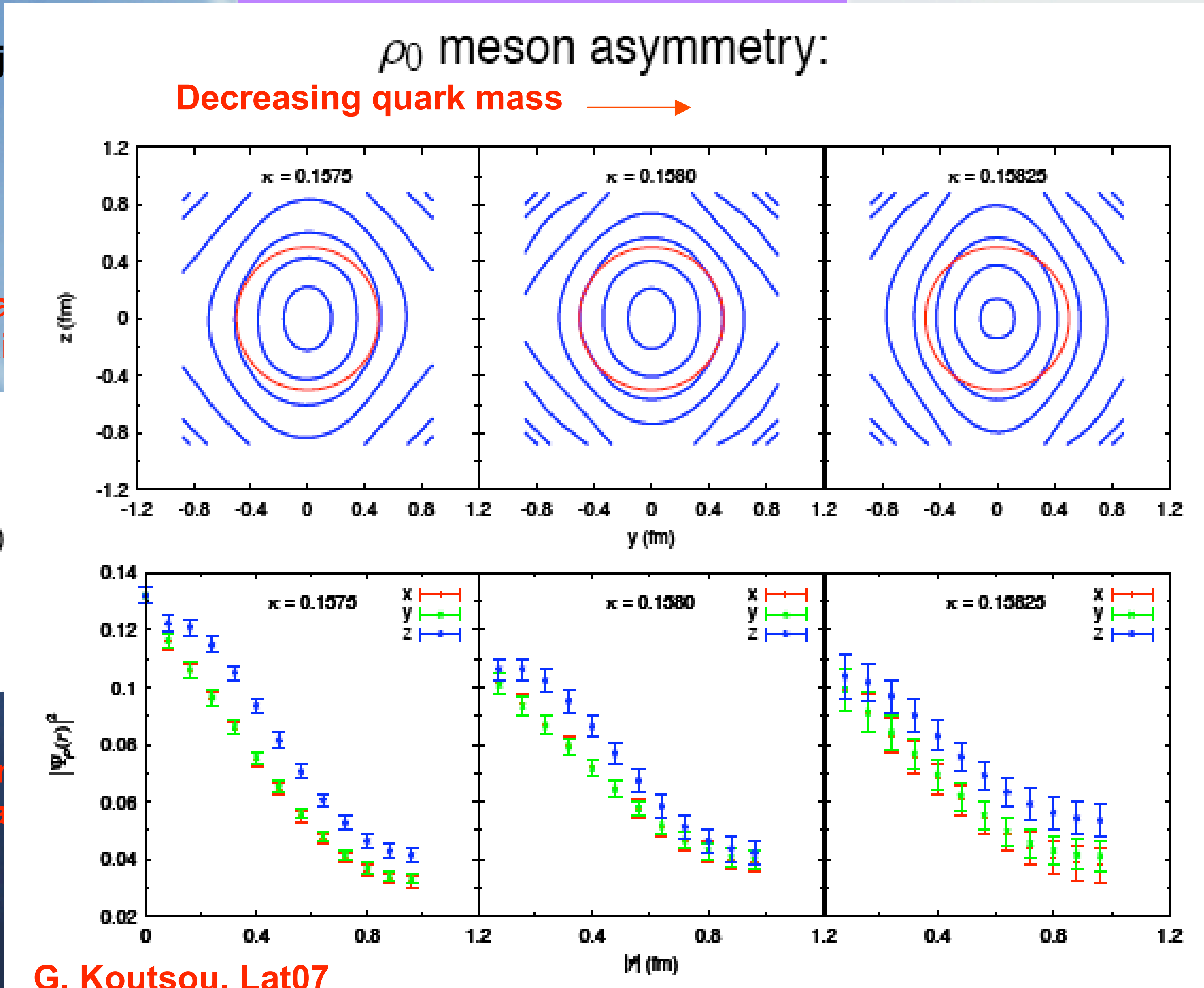
<ρ(p')|j

Adela
functi

$(\vec{z}, t)$

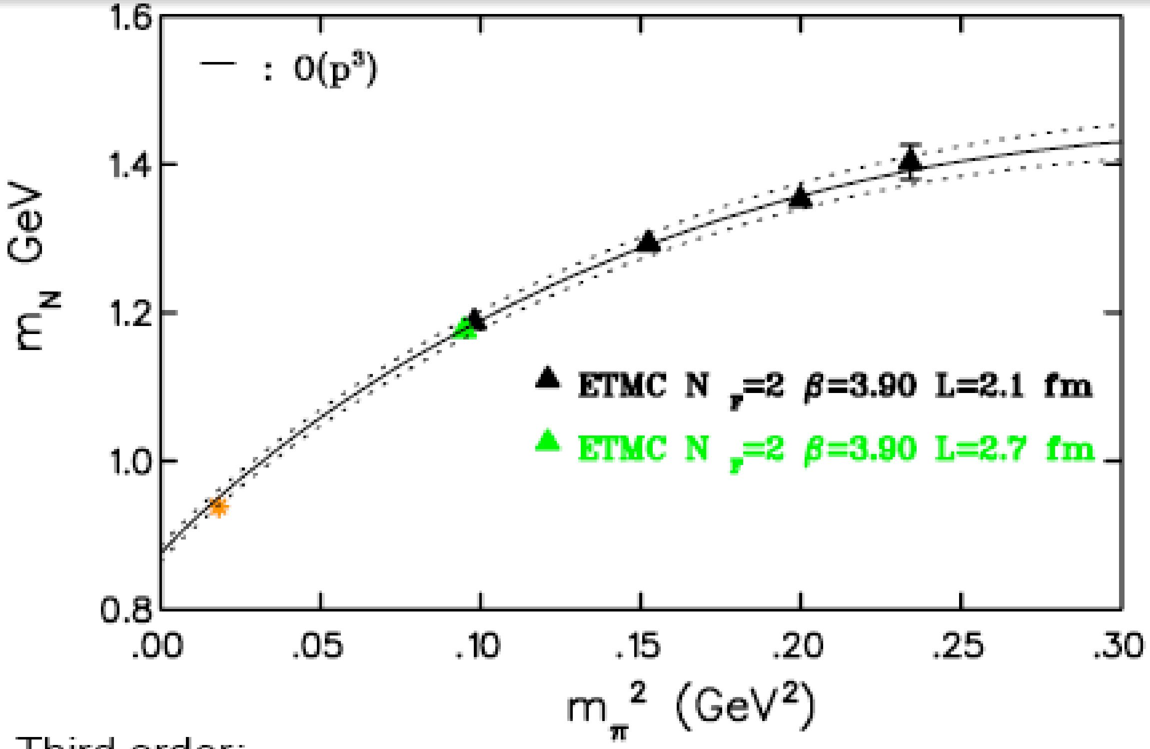
Or
ba

rupole



Baryon sector

G. Herdoiza



- Third order:

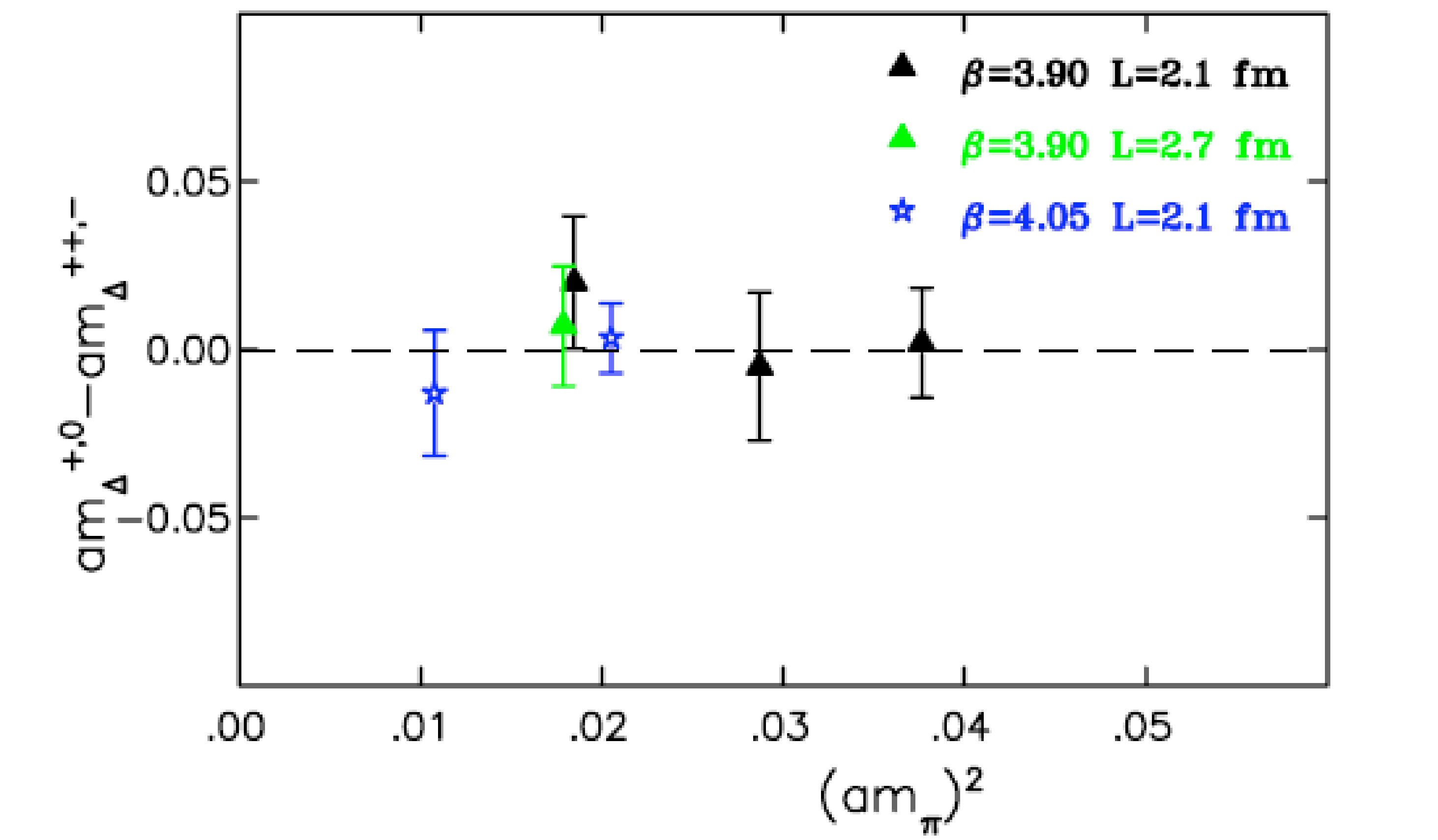
$$m_N = m_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3$$

Two fit parameters: m_0=0.865(10), c_1=-1.224(17) GeV^{-1} to be compared with ~-0.9 GeV^{-1} (π-N sigma term)

Delta mass splitting

Symmetry: u ↔ d --> Δ^{++} is degenerate with Δ^{-} and Δ^{+} with Δ^{0}

Check for flavour breaking by computing mass splitting between the two degenerate pairs

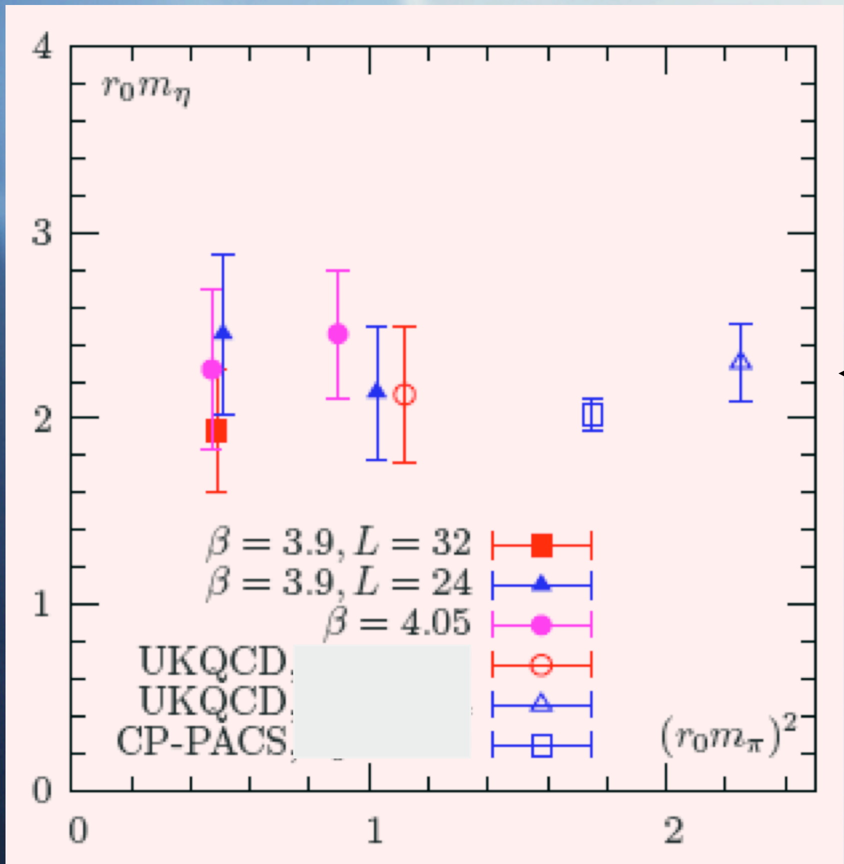


Splitting consistent with zero in agreement with theoretical expectations that isospin breaking only large for neutral pions (~16% on finest lattice)

Flavour singlets

C. Urbach

N_F=2 : flavour singlet pseudoscalar "η" - need disconnected contributions



~ constant w.r.t. m_π
--> m~880 MeV

N_F=2+1: Wilson Clover

m_η=0.454(16) GeV $m_{\eta'}$=0.871(46) GeV

Hadron structure

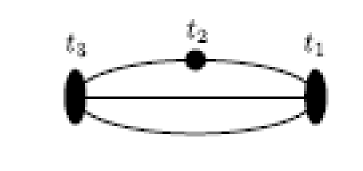
J. W. Negele - hybrid

G. Schierholz - Wilson clover

- Coupling constants: g_A, $g_{\pi NN}$, $g_{\pi N\Delta}$, …
- Form factors: $F_\pi(Q^2)$, $G_A(Q^2)$, $G_p(Q^2)$, $F_1(Q^2)$,….
- Parton distribution functions: $q(x)$, $\Delta q(x)$,…
- Generalized parton distribution functions: $H(x,\xi,Q^2)$, $E(x,\xi,Q^2)$,…

Masses: two-point functions $<J(t_{sink})\, J^\dagger(0)>$

Need to evaluate three-point functions: $<J(p',t_{sink})\; \mathcal{O}(t,q)\; J^\dagger(p,0)>$



$$\mathcal{O}(t,\vec{q}) = \int d^3x\, e^{-i\vec{x}\cdot\vec{q}}\; \bar{\psi}(x)\left(\Gamma D^{\mu_1} D^{\mu_2} \ldots\right)\psi(x)$$

In addition:

Four-point functions: yield detailed info on quark distribution

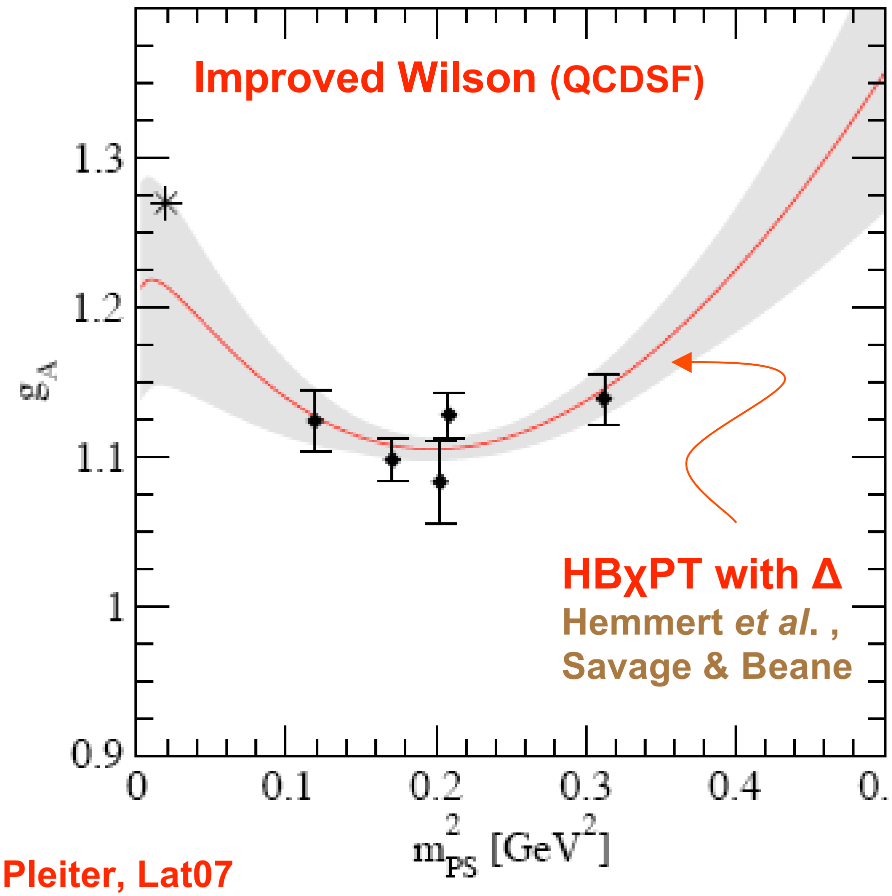
only simple operators used up tp now - need all-to-all propagators

nucleon structure

Nucleon axial charge

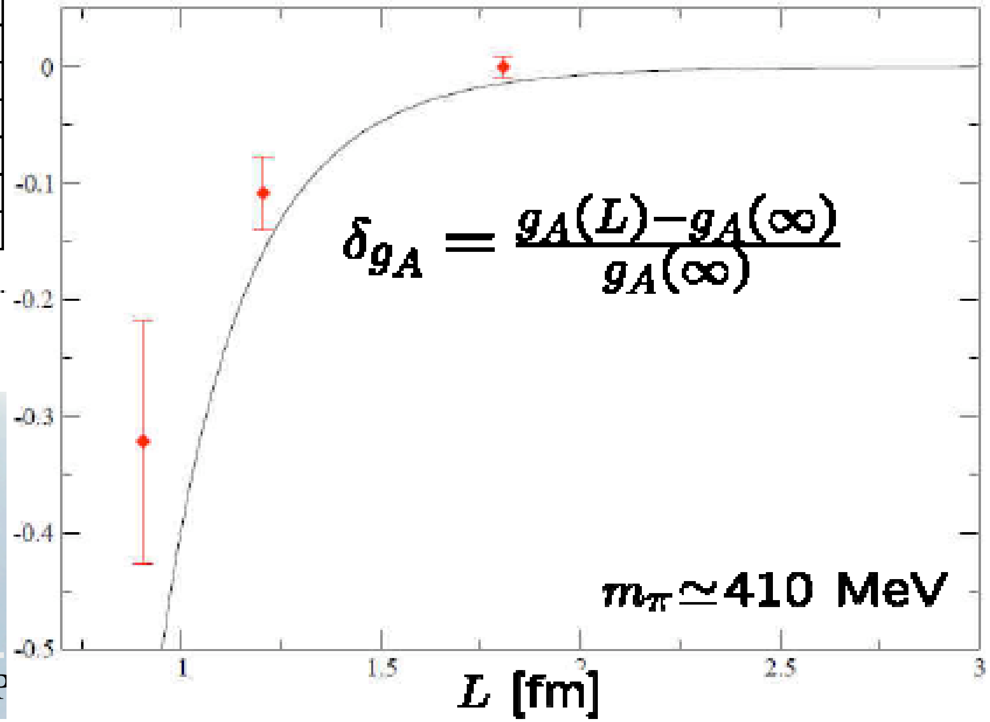
Forward nucleon axial vector matrix element: $<N|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|N>= g_A \bar{v}(p)\gamma_\mu\gamma_5 v(p)$

- accurately measured
- no-disconnected diagrams
- chiral PT

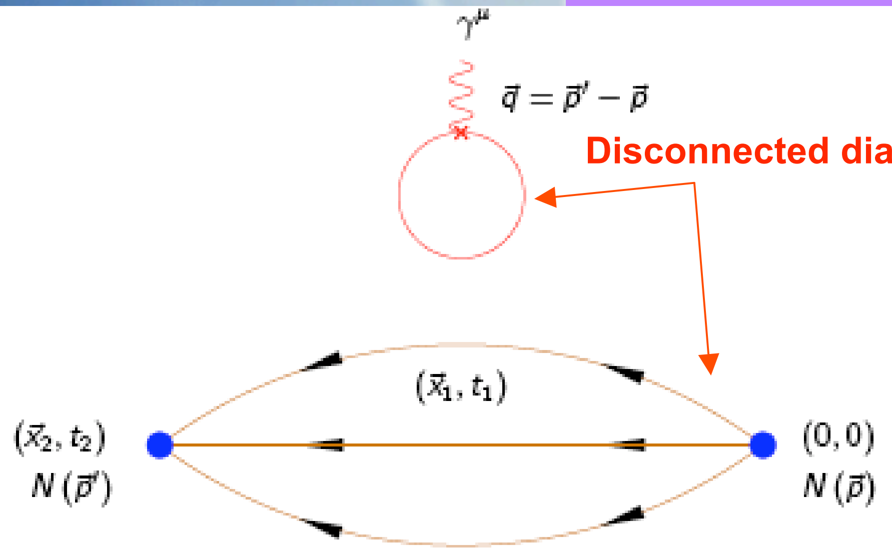


Pleiter, Lat07

Finite volume dependence



Nucleon form factors

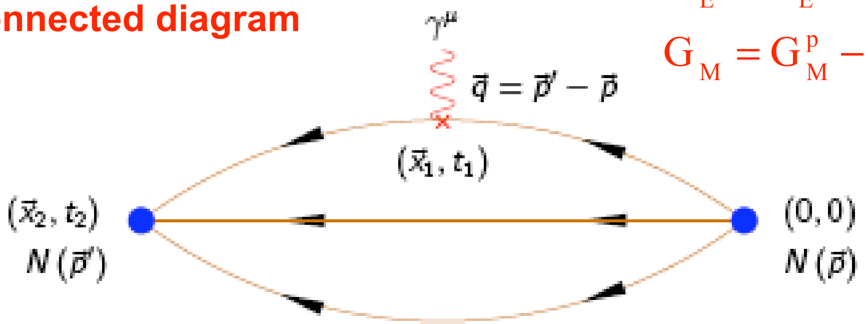


Disconnected diagram not evaluated

In last couple of years better methods have become available, dilution, one-end trick,…

Isovector form factors obtained from connected diagram → Isovector Sachs form factors:

$$G_E = G_E^p - G_E^n$$
$$G_M = G_M^p - G_M^n$$



with G_E and G_M given in terms of F_1 and F_2

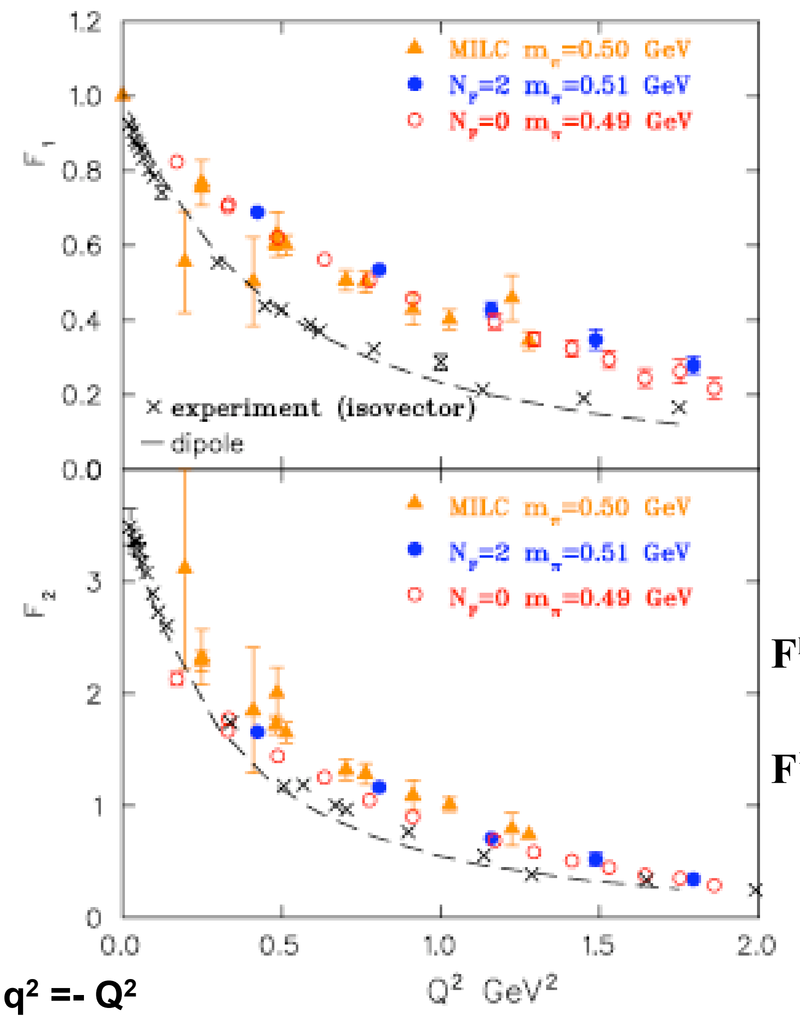
$$G_E(q^2) = F_1(q^2) + \frac{q^2}{(2m_N)^2} F_2(q^2)$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

Evaluation of 3pt function using the fixed sink approach --> any current to be inserted with no additional computational cost

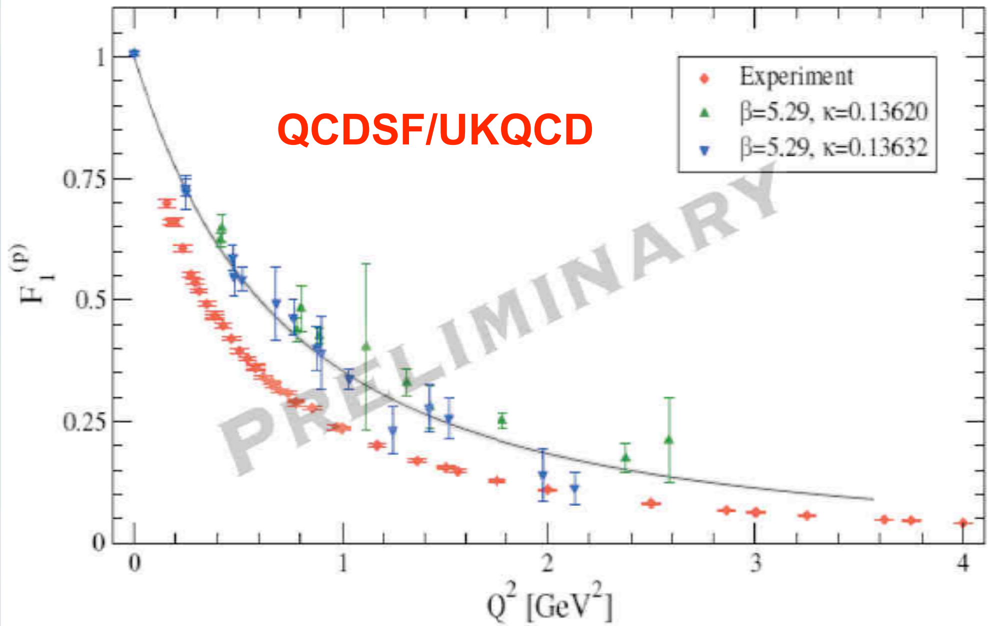
$$\langle N(p', s') | j_\mu | N(p, s) \rangle = \left(\frac{m_N^2}{E_N(\mathbf{p}')\, E_N(\mathbf{p})} \right)^{1/2} \bar{u}(p', s') \left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} F_2(q^2) \right] u(p, s)$$

Cyprus/MIT Collaboration, C. A., G. Koutsou, J. W. Negele, A. Tsapalis

Nucleon EM form factors

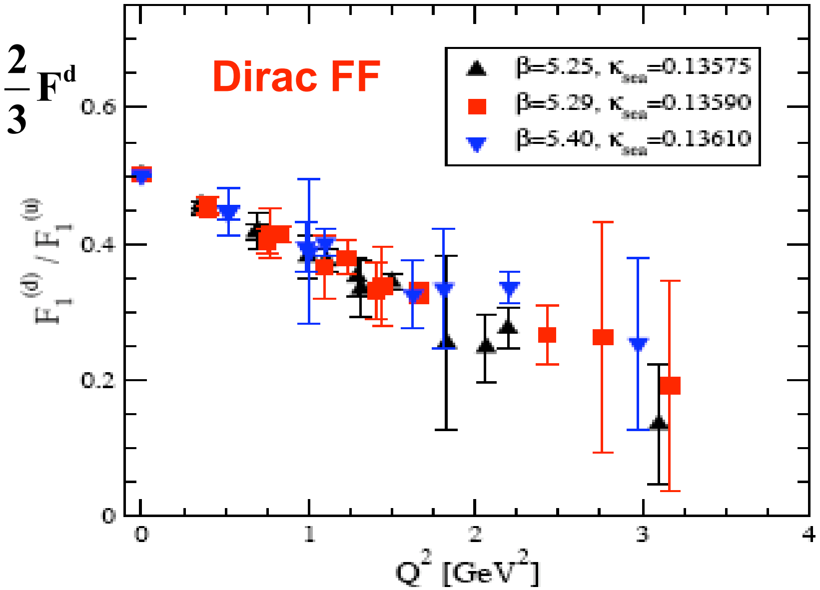


$q^2 = - Q^2$



$$F^p = \frac{2}{3}F^u - \frac{1}{3}F^d$$

$$F^n = -\frac{1}{3}F^u + \frac{2}{3}F^d$$



Results using dynamical Wilson fermions and domain wall valence on staggered sea (from LHPC) are consistent

Nucleon axial form factors

Th. Korzec

Within the fixed sink method for 3pt function we can evaluate the nucleon matrix element of any operator with no additional computational cost

- use axial vector current $A^a_\mu = \bar{\psi}\gamma_\mu\gamma_5 \frac{\tau^a}{2}\psi$ to obtain the axial form factors
- pseudoscalar density $P^a = \bar{\psi} i\gamma_5 \frac{\tau^a}{2}\psi$ to obtain $G_{\pi N}(q^2)$

$$< N(\vec{p}\,',s')\,|\,A^3_\mu\,|\,N(\vec{p},s) >= i\left(\frac{m_N^2}{E_N(p')E_N(p)}\right)^{1/2} \bar{u}(p')\left[G_A(Q^2)\gamma_\mu\gamma_5 + \frac{q_\mu\gamma_5}{2m_N}G_p(Q^2)\right]\frac{\tau^3}{2}u(p)$$

$$2m_q < N(\vec{p}\,',s')\,|\,P^3\,|\,N(\vec{p},s) >= \left(\frac{m_N^2}{E_N(p')E_N(p)}\right)^{1/2} \frac{f_\pi m_\pi^2 G_{\pi NN}(Q^2)}{m_\pi^2+Q^2}\bar{u}(p')\, i\gamma_5\frac{\tau^3}{2}u(p)$$

PCAC relates G_A and G_p to $G_{\pi NN}$:

$$G_A(Q^2) - \frac{Q^2}{4m_N^2}G_p(Q^2) = \frac{1}{2m_N}\frac{f_\pi m_\pi^2 G_{\pi NN}(Q^2)}{m_\pi^2+Q^2}$$

Generalized Goldberger-Treiman relation

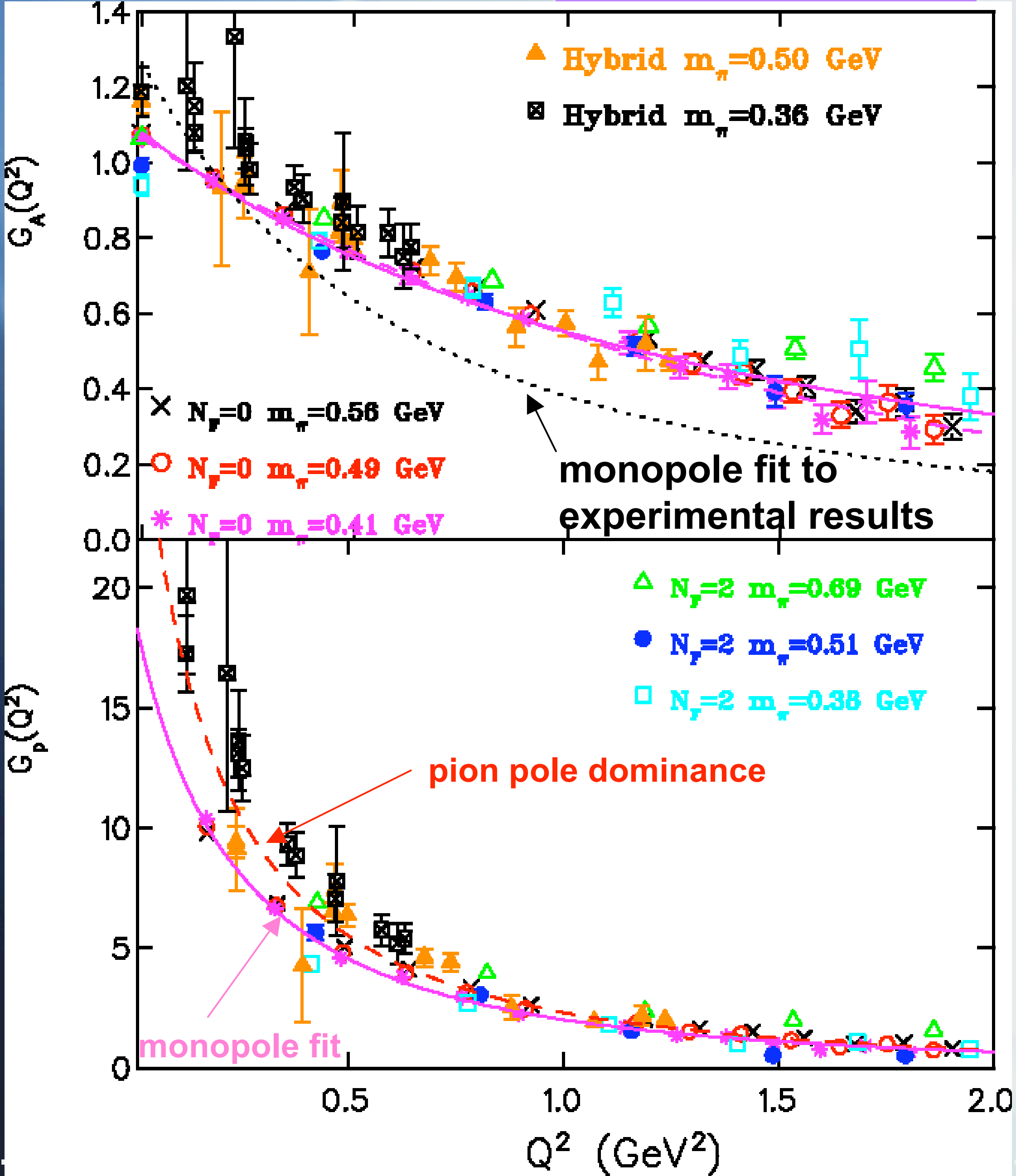
Pion pole dominance:

$$\frac{1}{2m_N}G_p(Q^2) \sim \frac{2G_{\pi NN}(Q^2)f_\pi}{m_\pi^2+Q^2}$$

$$\rightarrow \quad G_{\pi NN}(Q^2)f_\pi = m_N G_A(Q^2)$$

Goldberger-Treiman relation

G_A and G_p

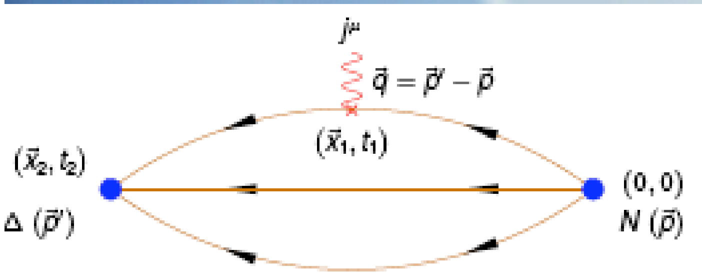


Results in the hybrid approach by the LPHC in agreement with dynamical Wilson results

Unquenching effects large at small Q^2 in line with theoretical expectations that pion cloud effects are dominant at low Q^2

Cyprus/MIT collaboration

N to Δ transition form factors



Electromagnetic N to Δ: Write in term of Sachs form factors:

$$<\Delta(\vec{p}',s')\,|\,j_\mu\,|N(\vec{p},s)>= i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta m_N}{E_\Delta(p')E_N(p)}\right)^{1/2}\bar{u}^\sigma(\vec{p}',s')\mathcal{O}_{\sigma\mu}\,u(\vec{p},s)$$

$$\mathcal{O}_{\sigma\mu} = G_{M1}(q^2)K^{M1}_{\sigma\mu} + G_{E2}(q^2)K^{E2}_{\sigma\mu} + G_{C2}(q^2)K^{C2}_{\sigma\mu}$$

Magnetic dipole: dominant — **Electric quadrupole** — **Coloumb quadrupole**

Axial vector N to Δ: four additional form factors, $C_3^A(Q^2)$, $C_4^A(Q^2)$, $C_5^A(Q^2)$, $C_6^A(Q^2)$

Dominant: $C_5^A \leftrightarrow G_A$
$C_6^A \leftrightarrow G_p$

PCAC relates C_5^A and C_6^A to $G_{\pi N\Delta}$:

$$C_5^A(Q^2) - \frac{Q^2}{m_N^2}C_6^A(Q^2) = \frac{1}{2m_N}\frac{f_\pi m_\pi^2 G_{\pi N\Delta}(Q^2)}{m_\pi^2+Q^2}$$

Generalized non-diagonal Goldberger-Treiman relation

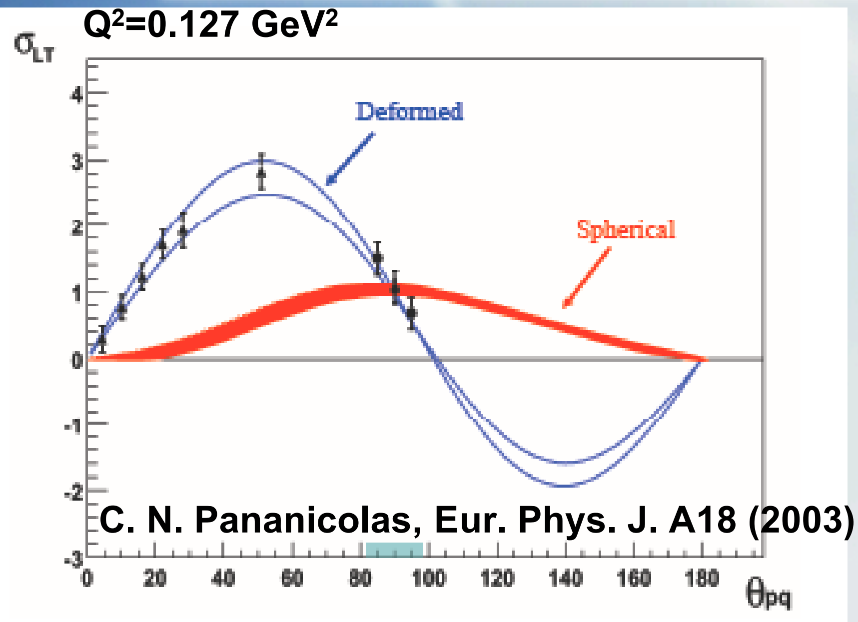
Pion pole dominance:

$$\frac{1}{m_N}C_6^A(Q^2) \sim \frac{1}{2}\frac{G_{\pi N\Delta}(Q^2)f_\pi}{m_\pi^2+Q^2}$$

$$\rightarrow \quad G_{\pi N\Delta}(Q^2)f_\pi = 2m_N C_5^A(Q^2)$$

Non-diagonal Goldberger-Treiman relation

Quadrupole form factors and deformation



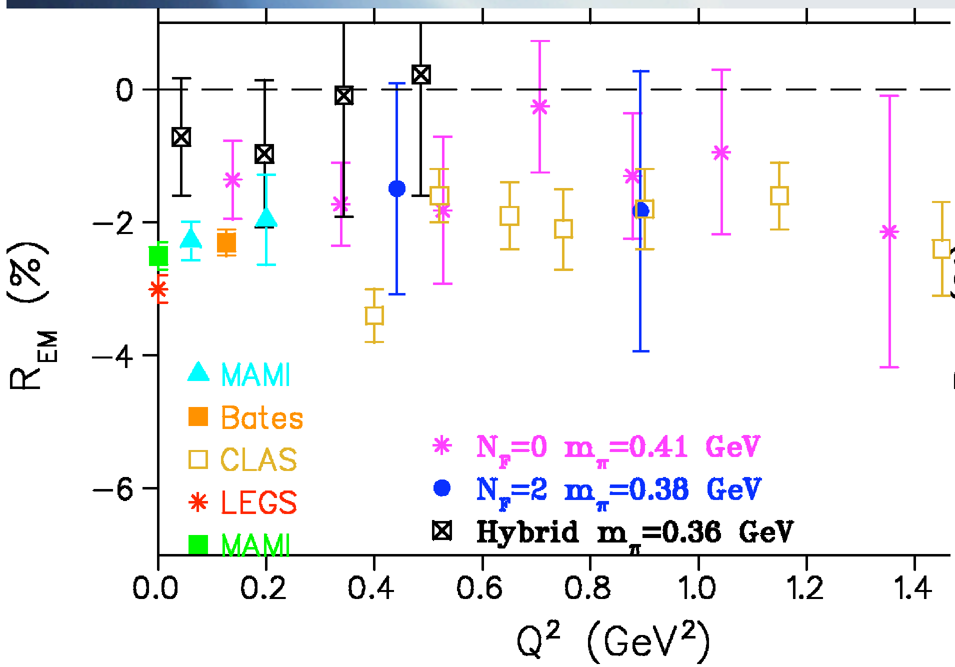
- Quantities measured in the lab frame of the Δ are the ratios:

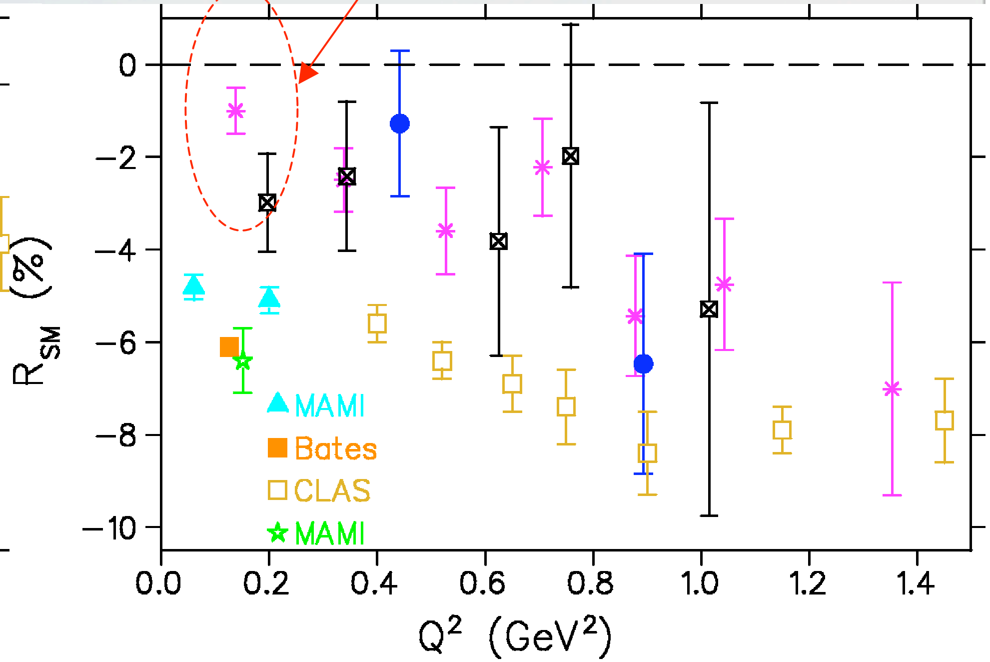
$$R_{EM}(EMR) = -\frac{G_{E2}(Q^2)}{G_{M1}(Q^2)}$$

$$R_{SM}(CMR) = -\frac{|\vec{q}\,|}{2m_\Delta}\frac{G_{C2}(Q^2)}{G_{M1}(Q^2)}$$

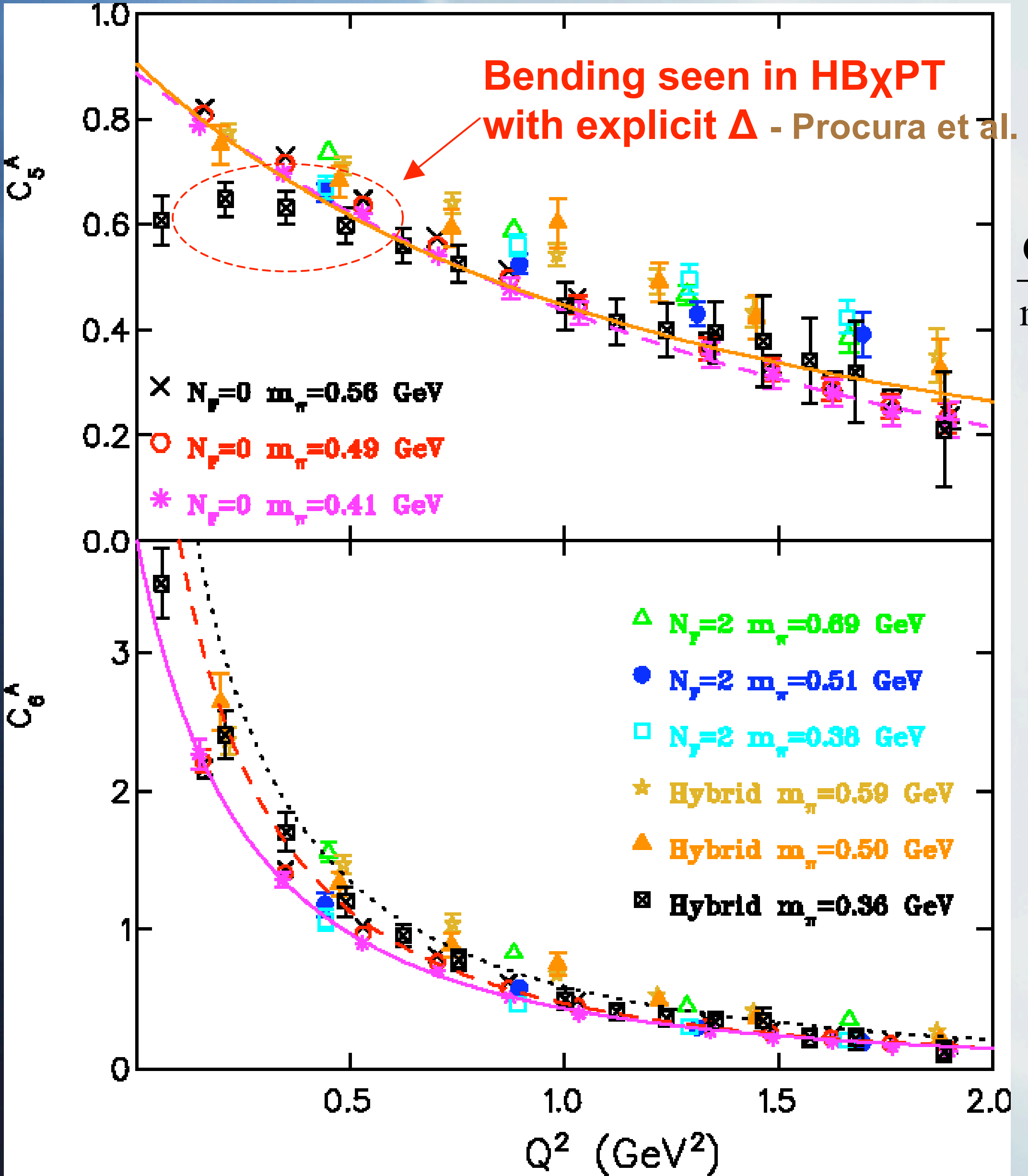
- Precise experimental data strongly suggest deformation of Nucleon/Δ
- First conformation of non-zero EMR and CMR in full QCD

Large pion effects





Axial N to Δ



SSE to $\mathcal{O}(\varepsilon^3)$:

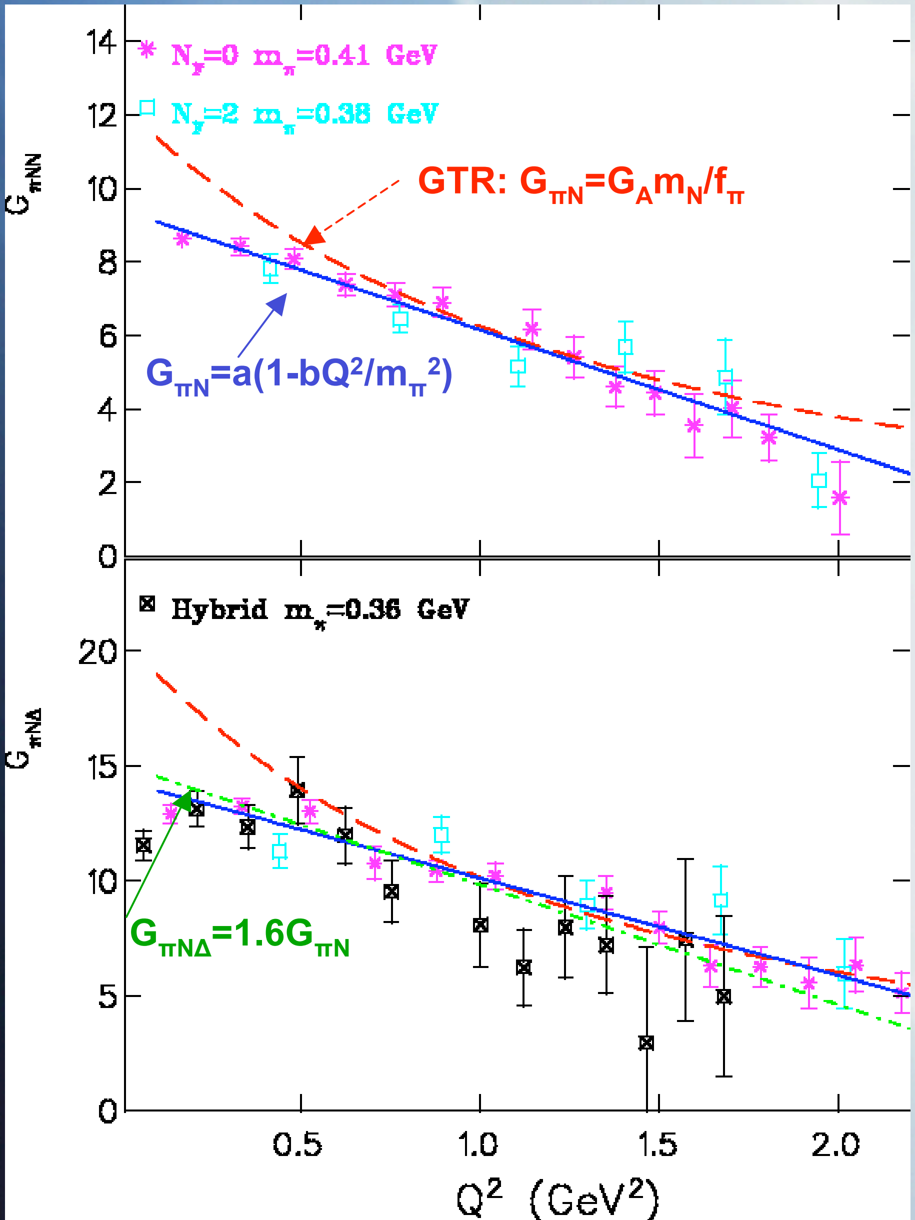
$C_5^A(Q^2,m_\pi)=a_1+a_2m_\pi^2+a_3Q^2+\text{loop}$

$$\frac{C_6^A}{m_N^2}=a_4-\frac{1}{m_\pi^2+Q^2}\left[\begin{array}{l}a_5+a_6m_\pi^2+a_7Q^2\\+\text{loop}(m_\pi,c_A,g_A,g_1,f_\pi,\Delta)\end{array}\right]$$

Again large unquenching effects at small Q^2

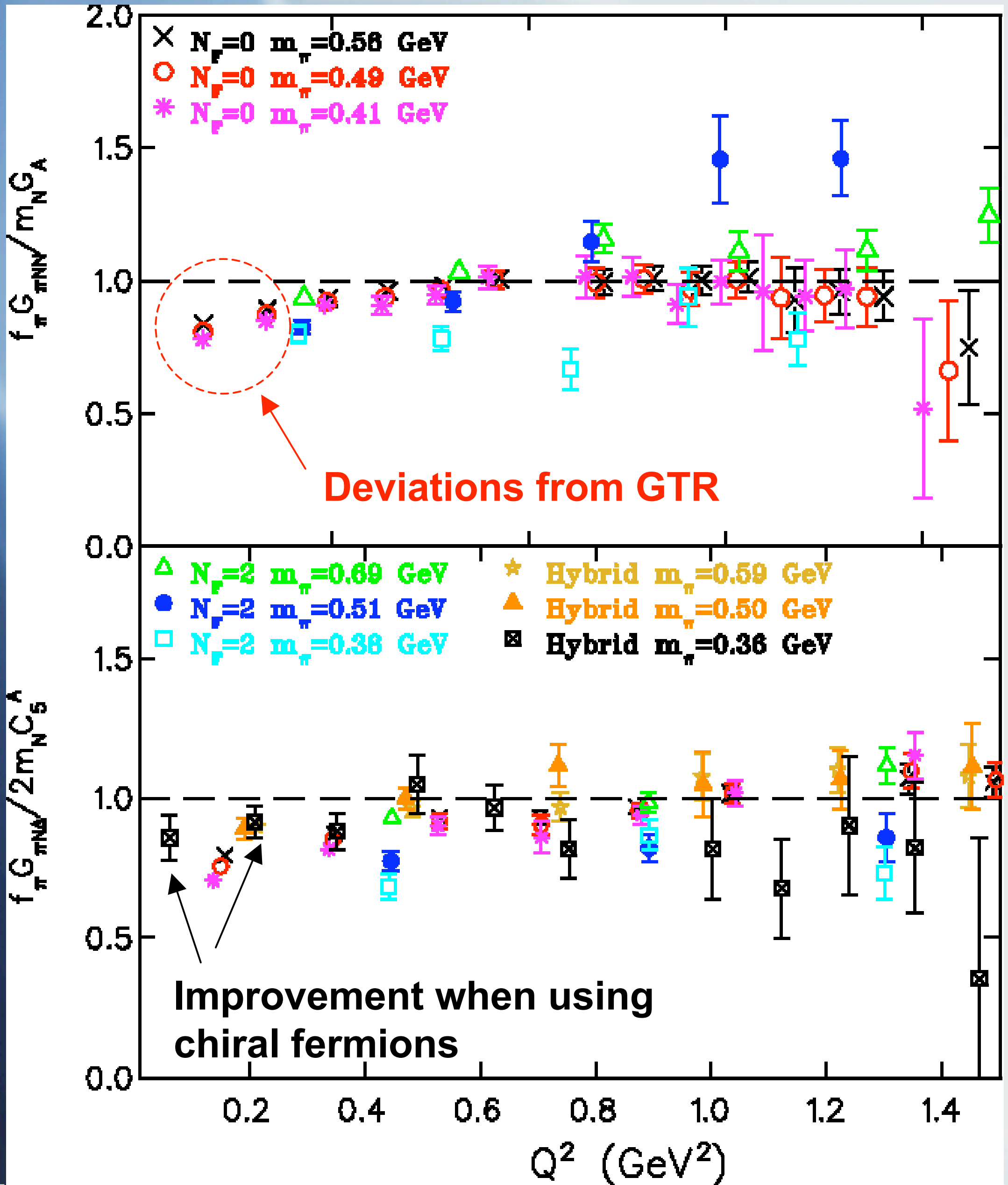
$G_{\pi NN}$ and $G_{\pi N\Delta}$

Curves refer to the quenched data



- Q^2-dependence for $G_{\pi N}$ ($G_{\pi N\Delta}$) extracted from G_A (C_5^A) using the Goldberger-Treiman relation deviates at low Q^2
- Small deviations from linear Q^2-dependence seen at very low Q^2 in the case of $G_{\pi N\Delta}$
- $G_{\pi N\Delta}/G_{\pi N} = 2C_5^A/G_A$ as predicted by taking ratios of GTRs
- $G_{\pi N\Delta}/G_{\pi N}=1.60(1)$

Goldberger-Treiman Relations



$$\frac{1}{2m_N} G_p(Q^2) \sim \frac{2G_{\pi N}(Q^2) f_\pi}{m_\pi^2+Q^2}$$

$$\rightarrow \quad G_{\pi N}(Q^2) f_\pi = m_N G_A(Q^2)$$

$$\frac{1}{m_N} C_6^A(Q^2) \sim \frac{1}{2}\frac{G_{\pi N\Delta}(Q^2) f_\pi}{m_\pi^2+Q^2}$$

$$\rightarrow \quad G_{\pi N\Delta}(Q^2) f_\pi = 2m_N C_5^A(Q^2)$$

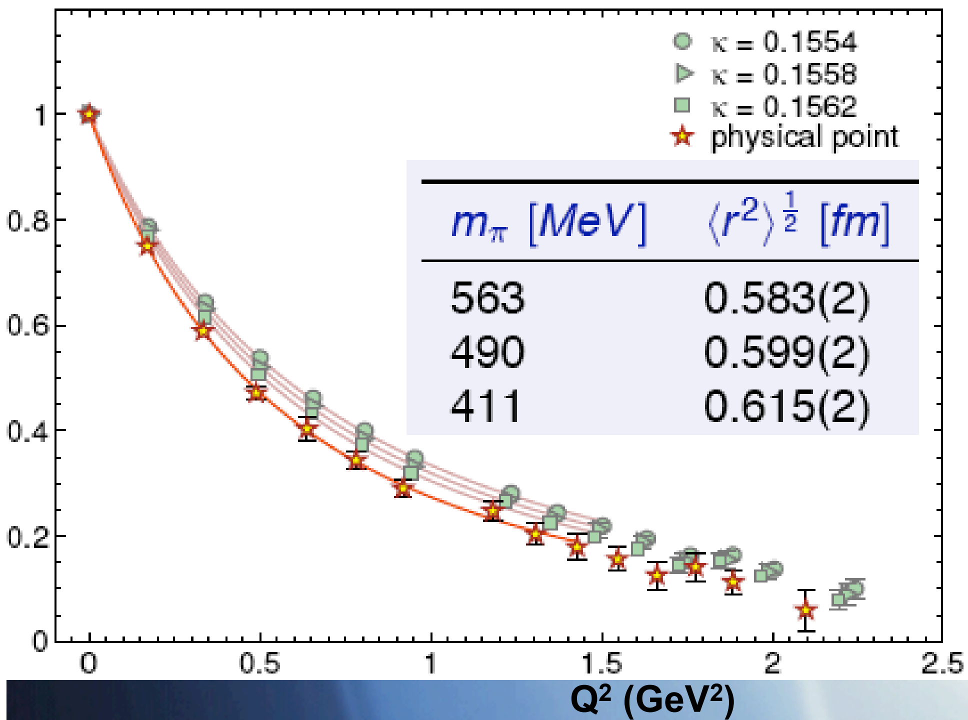
Δ electromagnetic form factors

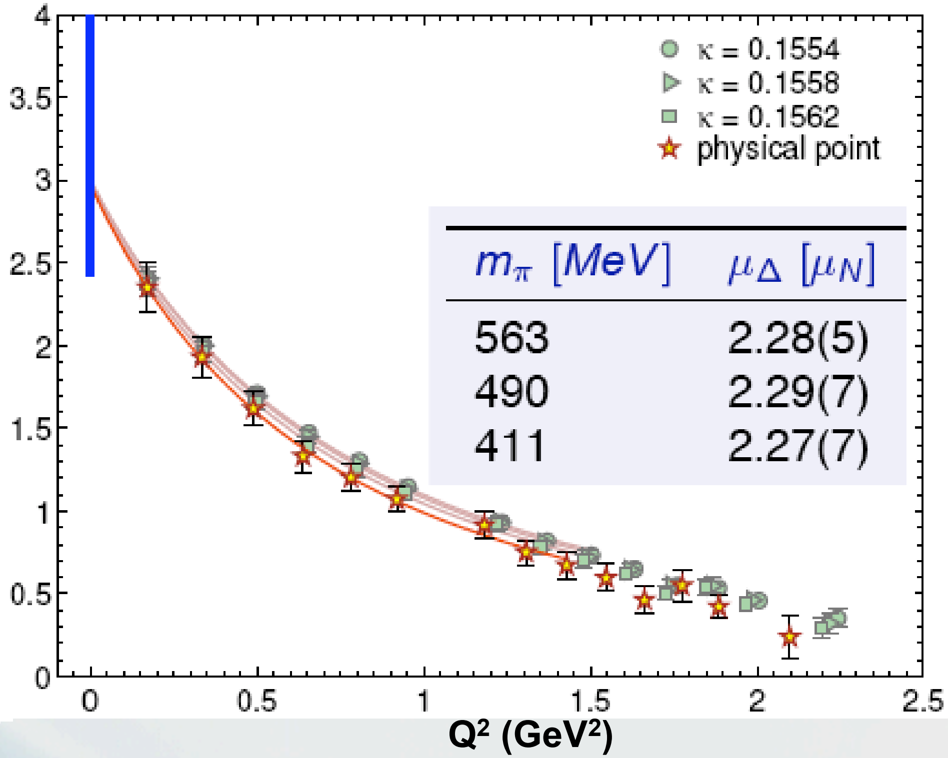
Four form factors: G_{E0}, G_{E2}, G_{M1}, G_{M3} given in terms of a_1, a_2, c_1, c_2

$$<\Delta(\vec{p}',s')\,|\,j^\mu\,|\,\Delta(\vec{p},s)>= i\sqrt{\frac{m_\Delta^2}{E_\Delta(\vec{p}')E_\Delta(\vec{p})}}\,\bar{u}_\sigma(p',s')O^{\sigma\mu\tau}u_\tau(p,s)$$

Like for the nucleon form factors only the connected diagram is evaluated --> isovector part

$$O^{\sigma\mu\tau} = \delta^{\sigma\tau}\left[a_1 i\gamma^\mu + \frac{a_2}{2m_\Delta}(p'^\mu + p^\mu)\right] - \frac{q^\sigma q^\tau}{2m_\Delta^2}\left[c_1 i\gamma^\mu + \frac{c_2}{2m_\Delta}(\vec{p}'^\mu + \vec{p}^\mu)\right]$$

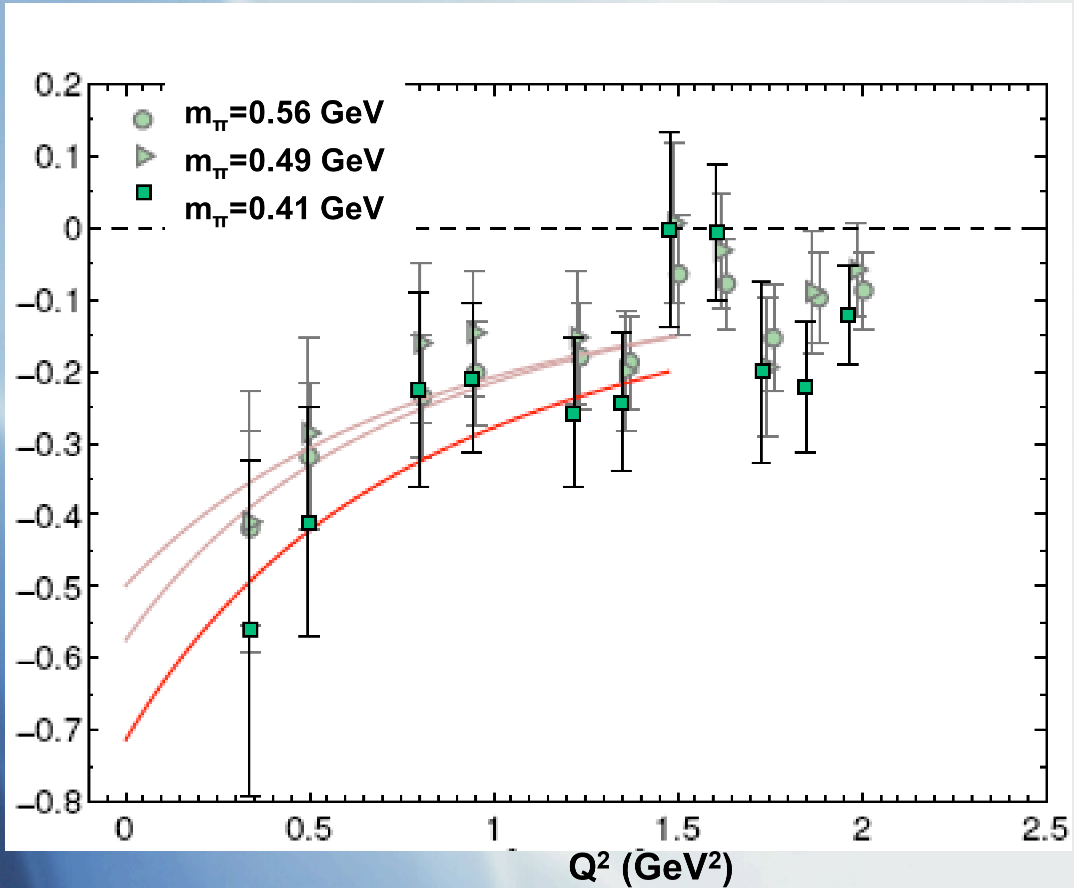




Accurate results for the dominant form factors calculated in the quenched approximation. Unquenched evaluation is in progress - Cyprus/MIT Collaboration, C. A., Th. Korzec, Th. Leontiou J. W. Negele, A. Tsapalis

Electric quadrupole Δ form factor

G_{E2} connected to deformation of the Δ: $G_{E2}(Q^2=0) \sim m_\Delta^2 \int d^3r\, \bar{\psi}(r)\left[3z^2 - r^2\right]\psi(r)$

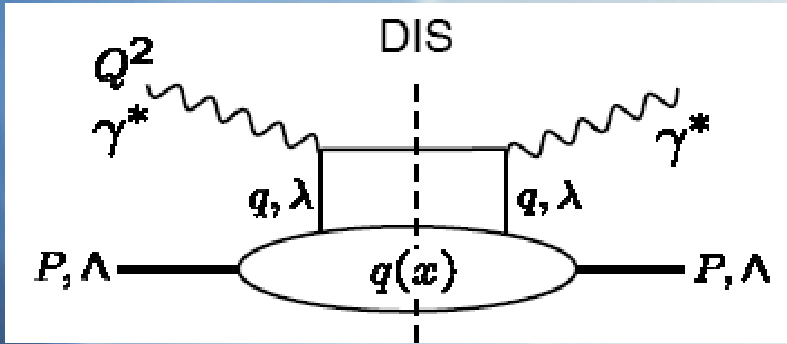


$G_{E2} < 0$ ➡ Δ oblate in agreement with what is observed using density correlators to extract the wave function, C. A., Ph. de Forcrand and A. Tsapalis, PRD (2002)

G_{M3} also evaluated, small and negative

Moments of parton distributions

Parton distributions measured in deep inelastic scattering



Forward matrix elements:

$$< N(p,s) | O_q^{\{\mu_1 \cdots \mu_n\}} | N(p,s) > \sim \int dx\, x^{n-1} q(x)$$

3 types of operators:

Moments of parton distributions

$$O_q^{\mu_1 \cdots \mu_n} = \bar{q}\left[\gamma^{\mu_1}\, i\overleftrightarrow{D}^{\mu_2} \ldots i\overleftrightarrow{D}^{\mu_n\}}\right] q$$

unpolarized moment →

$$< x^n >_q = \int_{-1}^{1} dx\, x^n \left[q(x) + (-1)^{n+1} \bar{q}(x) \right] \quad q = q_\downarrow + q_\uparrow$$

$$O_{5q}^{\mu\mu_1 \cdots \mu_n} = \bar{q}\left[\gamma_5 \gamma^{\mu}\, i\overleftrightarrow{D}^{\mu_1} \ldots i\overleftrightarrow{D}^{\mu_n\}}\right] q$$

polarized moment →

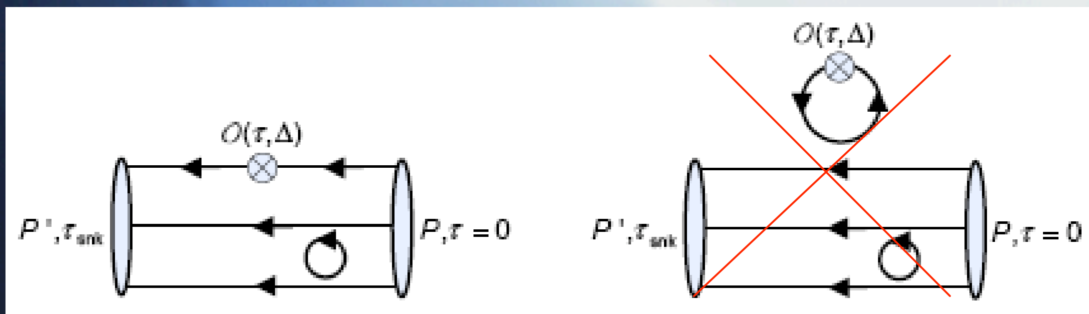
$$< x^n >_{\Delta q} = \int_{-1}^{1} dx\, x^n \left[\Delta q(x) + (-1)^{n} \Delta\bar{q}(x) \right] \quad \Delta q = q_\downarrow - q_\uparrow$$

$$O_{Tq}^{\mu\nu\mu_1 \cdots \mu_n} = \bar{q}\left[\gamma_5 \sigma^{\mu\nu}\, i\overleftrightarrow{D}^{\mu_2} \ldots i\overleftrightarrow{D}^{\mu_n\}}\right] q$$

transversity →

$$< x^n >_{\delta q} = \int_{-1}^{1} dx\, x^n \left[\delta q(x) + (-1)^{n+1} \delta\bar{q}(x) \right] \quad \delta q = q_T + q_\perp$$

$$\overleftrightarrow{D} = \frac{1}{2}(\vec{D} - \overleftarrow{D})$$



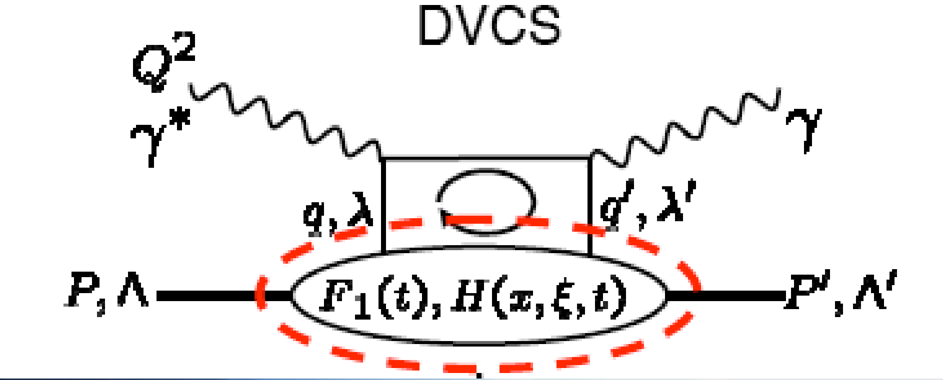
Moments of Generalized parton distributions

Off diagonal matrix element → Genelarized parton distributions

$$< N(p',s') | O_q^{\{\mu_1 \cdots \mu_n\}} | N(p,s) >= \bar{u}(p') \left[\sum_{\substack{i=0 \\ even}}^{n-1} \left\{ A_{ni}^q(t) K_{ni}^A + B_{ni}^q(t) K_{ni}^B \right\} + \delta_{n,even} C_{ni}^q(t) K_n^C \right] u(p)$$

GPD measured in Deep Virtual Compton Scattering

Generalized FF



$$p = \frac{1}{2}(p+p'), \quad q = (x+\xi)p, \quad q' = (x-\xi)p$$

$$t = -Q^2 = \Delta^2$$

$$H^n(\xi,t) \equiv \int_{-1}^{1} dx\, x^{n-1} H(x,\xi,t)$$

$$E^n(\xi,t) \equiv \int_{-1}^{1} dx\, x^{n-1} E(x,\xi,t)$$

GPDs

$$H^n(0,t) = A_{n0}(t) \qquad A_{10}(t) = F_1(t)$$

$$E^n(0,t) = B_{n0}(t) \qquad B_{10}(t) = F_2(t)$$

t=0 (forward) yield moments of parton distributions

Transverse quark densities

Fourier transform of form factors/GPDs gives probability

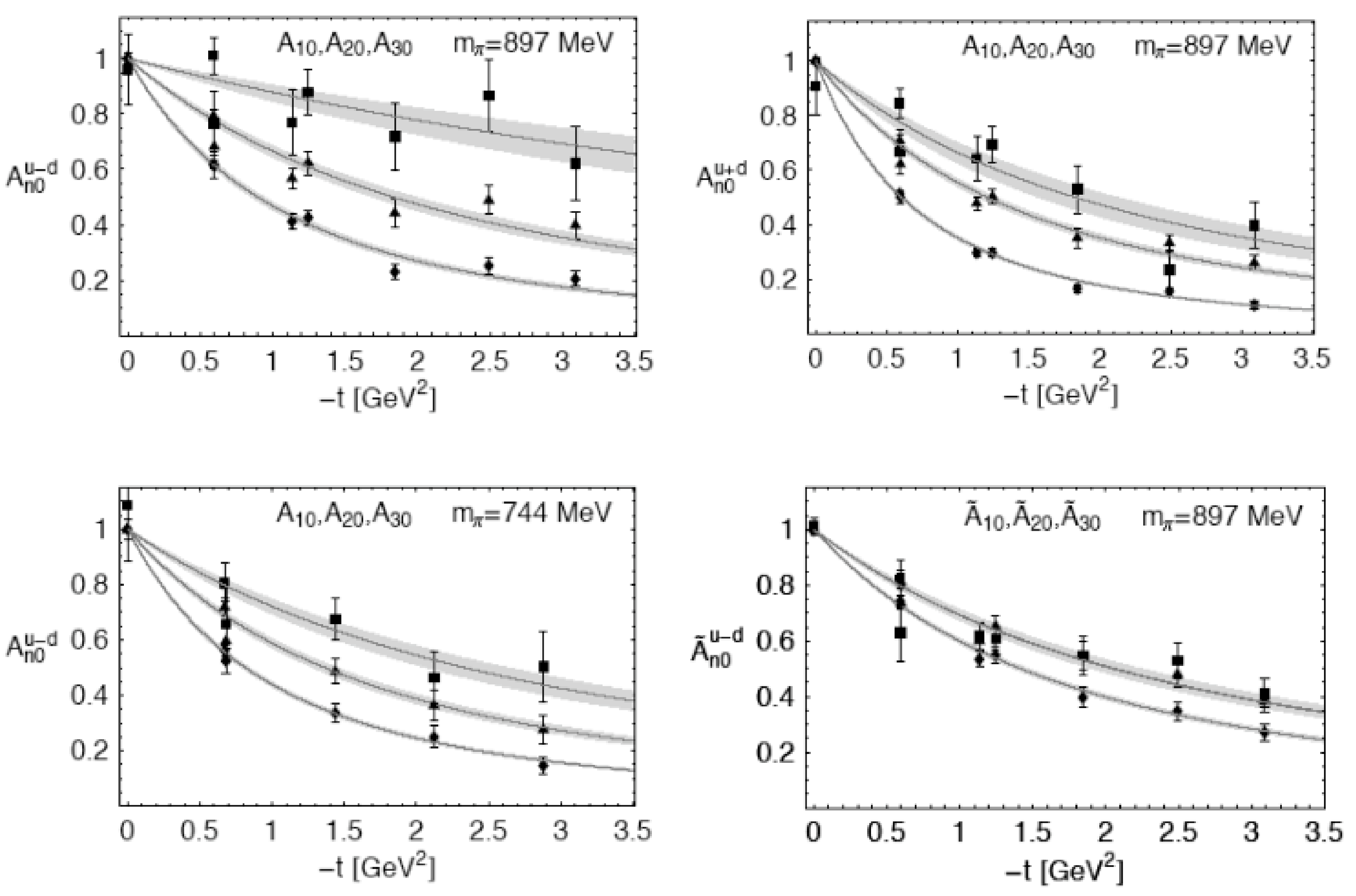
Transverse quark distributions:

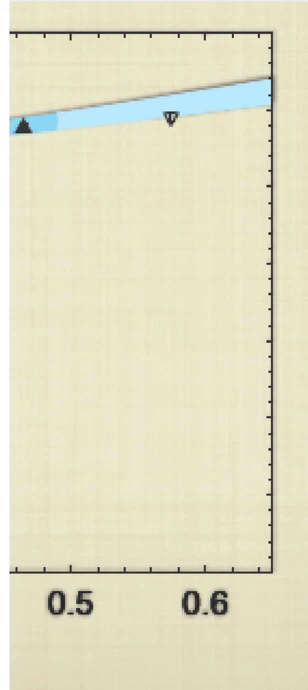
$$\int_{-1}^{1} dx\, x^{n-1}\, q(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}\, A^q_{n0}(-\vec{\Delta}^2_\perp)$$

RD62 (2000)

$), -\Delta^2_\perp)$

$_{n0}$ decreases





Spin structure of the nucleon

Total spin of quark: $J_q = \frac{1}{2}\left(<x>_q + B^q_{20}(0)\right)$ $L_q = J_q - \frac{1}{2}\Delta\Sigma_q$ $\Delta\Sigma = \tilde{A}^q_{10}(0)$

($<x>_q$ ← $A_{20}(0)$)

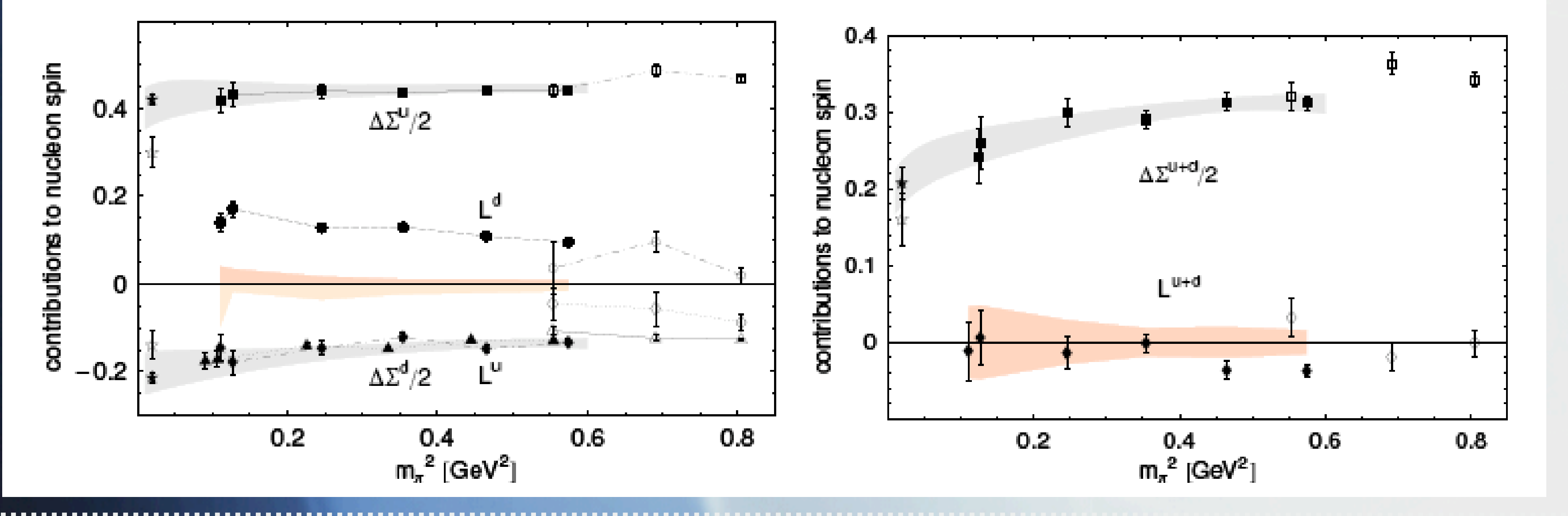
Spin of the nucleon: $\frac{1}{2} = L^{u+d} + \frac{1}{2}\Delta\Sigma^{u+d} + J^g$

QCDSF/UKQCD Dynamical Clover

For m_π~340 MeV: J_u=0.279(5) J_d=-0.006(5)

L_{u+d}=-0.007(13) $\Delta\Sigma_{u+d}$=0.558(22)

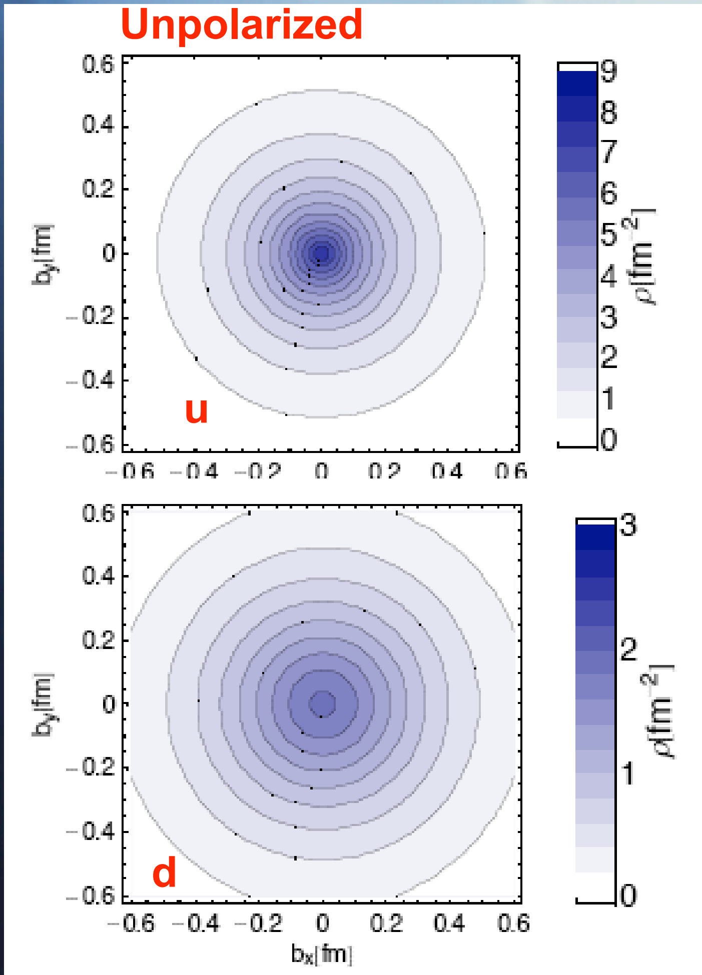
In agreement with LPHC, Ph. Hägler et al., hep-lat/07054295

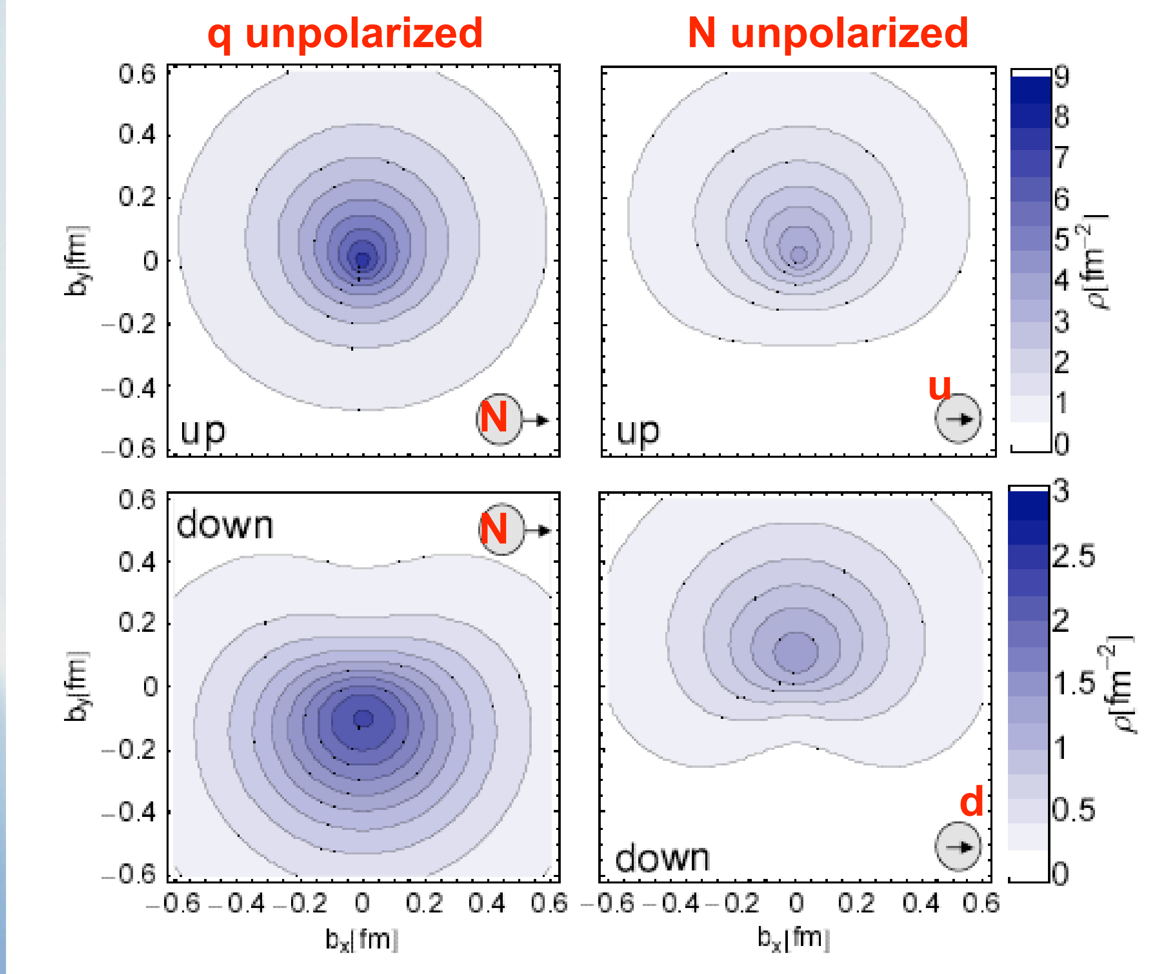


Transverse spin density

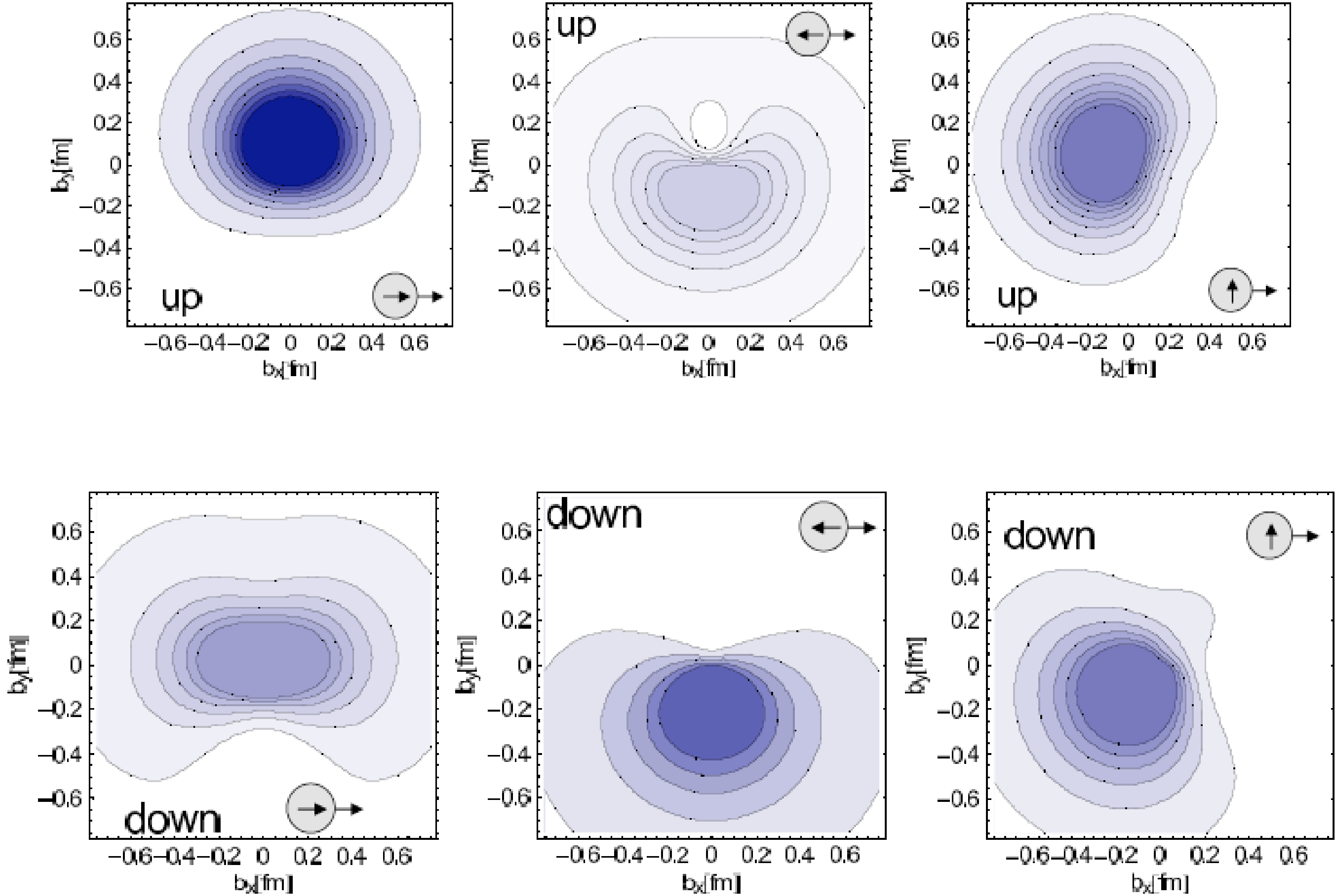
$$\langle p_+, s_\perp | \bar{q}(\mathrm{b}_\perp) \left[\gamma_+ - \lambda_{\perp i}\, \sigma_{+j}\, \gamma_5 \right] q(\mathrm{b}_\perp) | p_+, s_\perp \rangle$$

$\lambda_\perp$ quark spin
$s_\perp$ nucleon spin





Nucleon and quarks both polarized



Quark masses

R. Sommer

Fundamental parameters of L_{QCD}: Use experimental input to fix, then predict other observables

$$\begin{bmatrix} f_\pi \\ m_\pi \\ m_K \\ m_D \\ m_B \end{bmatrix} \xrightarrow{L_{QCD}(g_0, m_f)} \begin{bmatrix} \Lambda_{QCD} \\ \hat{M} = (M_u + M_d)/2 \\ M_s \\ M_c \\ M_b \end{bmatrix}$$

Asymptotic theory: know the large energy behaviour

For energies>>Λ_{QCD} use perturbation theory

At low energy scale use lattice QCD

} need a scheme to connect

Quark masses

Define running mass using PCAC:

$$\partial_\mu A^{su}_\mu = (m_s + m_u)P^{su}$$

Take matrix elements:

- u and d quarks: $$\bar{m}_{ud}(\mu) = \frac{Z_A}{Z_P(\mu)} \frac{\overbrace{m_\pi < 0 | A_0^a | \pi^a(0) >}^{\text{bare}}}{2 < 0 | P^a | \pi^a(0) >}$$

- For s-quark use K:, $$\bar{m}_u(\mu) + \bar{m}_s(\mu) = \frac{Z_A}{Z_P(\mu)} \frac{m_K < 0 | A_0^{su} | K(0) >}{< 0 | P^{su} | K(0) >}$$

renormalization constants Z_A and Z_P are calculable non-perturbatively, Alpha-Collaboration

For c-quark and for b-quark additional complication of very different scales between the light and heavy quarks

- for charm use D: large cut off effects $\sim(m_c a)^2$
- for b-quark need a different method

Various approaches exist e.g. Fermilab approach which resumes all mass dependences into the coefficients of the action, HQET ← match to QCD using a "small box" with fine lattice spacing

Results

- $N_F = 2, \quad \bar{m}_{ud}^{\overline{MS}}(2\,\text{GeV}) = 3.82(13)(24)\,\text{MeV}$ G. Herdoiza, ETMC 2007
- $N_F = 0, \quad \bar{m}_{s}^{\overline{MS}}(2\,\text{GeV}) = 97(3)\,\text{MeV}$ R. Sommer, Alpha Collaboration

N_F=2, Preliminary unquenched results from Alpha, ETM, QCDSF, CP-PACS - need smaller errors and continuum extrapolation

- $N_F = 0, \quad \bar{m}_{c}^{\overline{MS}}(\bar{m}_c) = 1.301(34)\,\text{GeV}$ R. Sommer, Alpha Collaboration
- $N_F = 0, \quad \bar{m}_{b}^{\overline{MS}}(\bar{m}_b) = 4.347(48)\,\text{GeV}$ R. Sommer

But expect progress in unquneched calculations in next ~3 years

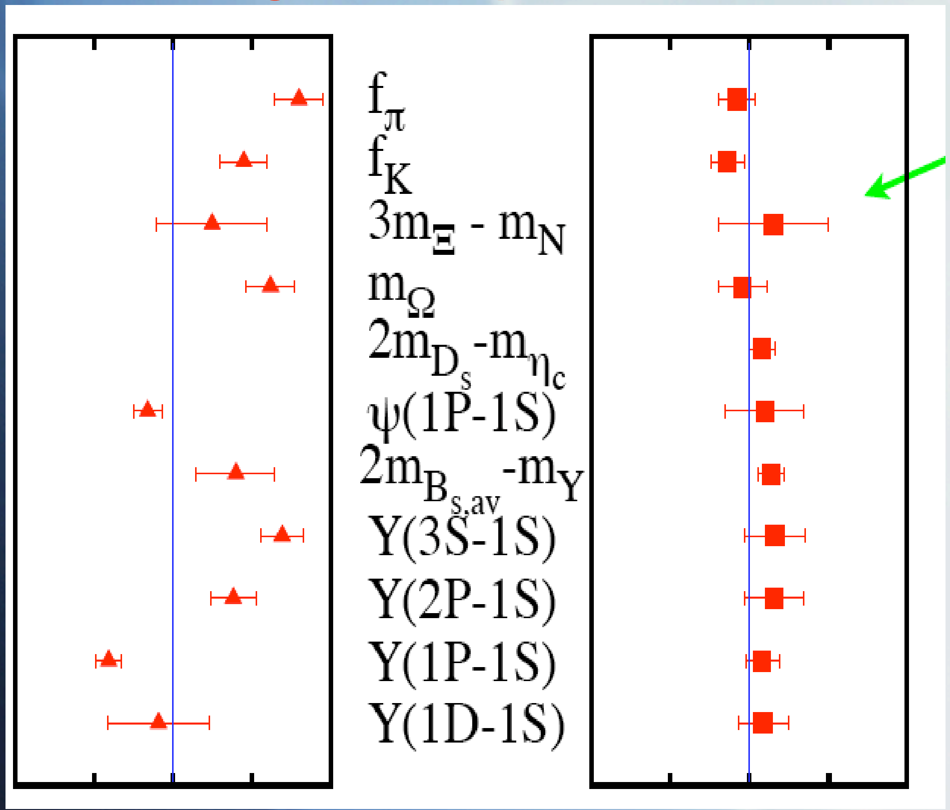
Charm Physics

C. Davies

Determination of CKM matrix: Test unitary triangle

Weak decays - calculate in lattice QCD

First message: Use unquenched QCD

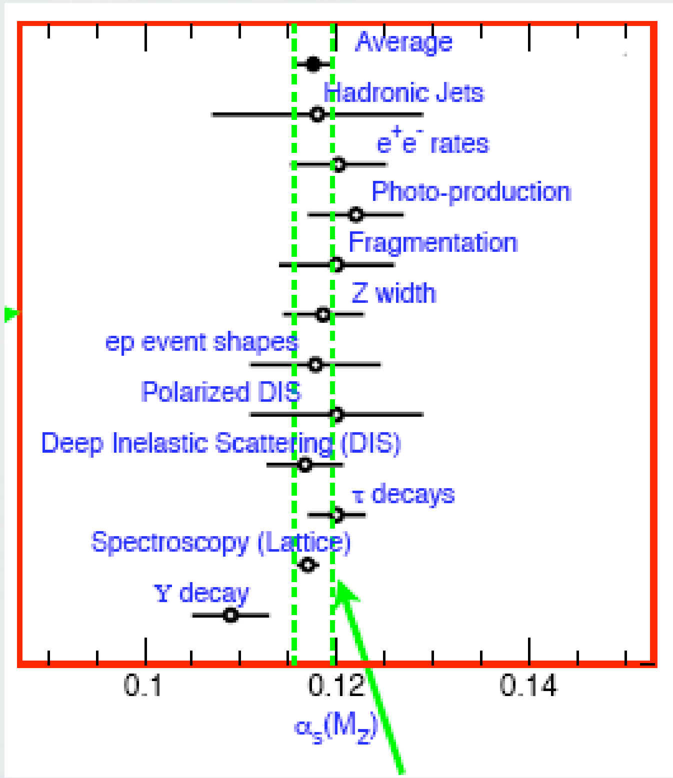


0.9 1.0 1.1 Quenched

Bench mark

0.9 1.0 1.1

N_f=2+1 dynamical staggered fermions



Lattice QCD

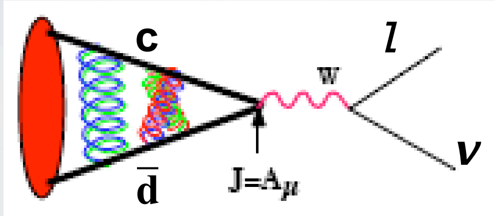
Full QCD results show impressive agreement with experiment across the board - from light to heavy quarks

Weak Decays and CKM matrix

For charm use highly improved staggered action remove further cut off effects at $\alpha(am_c)^2$ and $(am_c)^4$

$$
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
\pi \to l\nu & K \to l\nu & B \to \pi l\nu \\
 & K \to \pi l\nu & \\
V_{cd} & V_{cs} & V_{cb} \\
D \to l\nu & D_s \to l\nu & B \to D l\nu \\
D \to \pi l\nu & D \to K l\nu & \\
V_{td} & V_{ts} & V_{tb} \\
\langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle &
\end{pmatrix}
$$

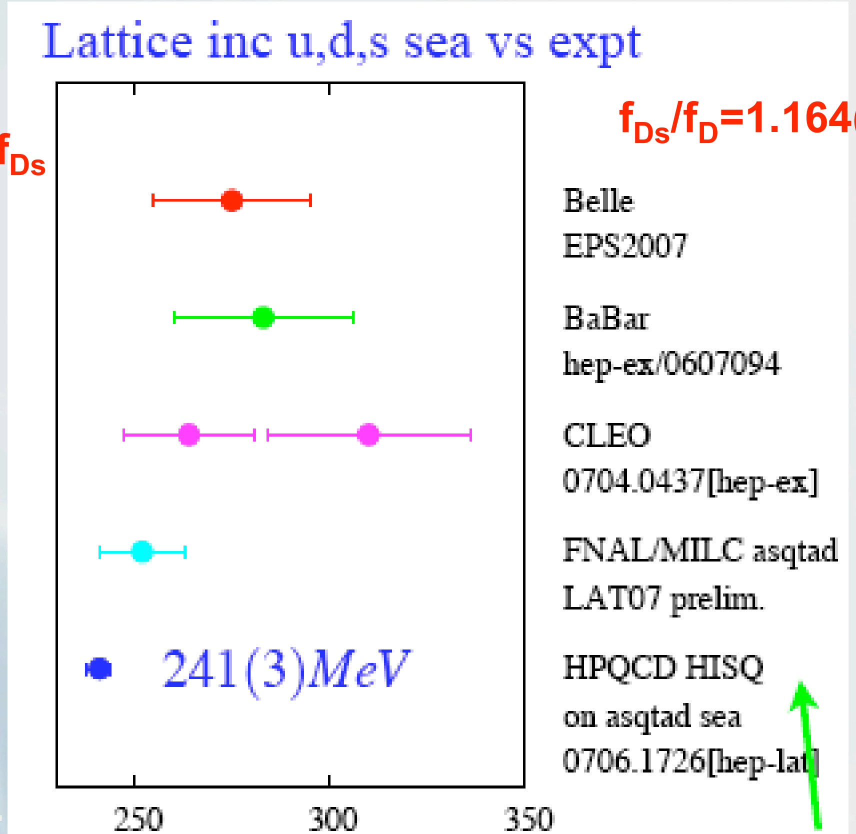
Leptonic D-meson decays:



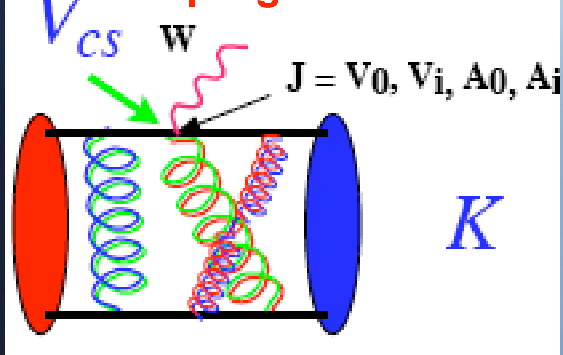
Lattice results versus experiment - CLEO-c

f_{Ds}

$f_{Ds}/f_D = 1.164(11)$

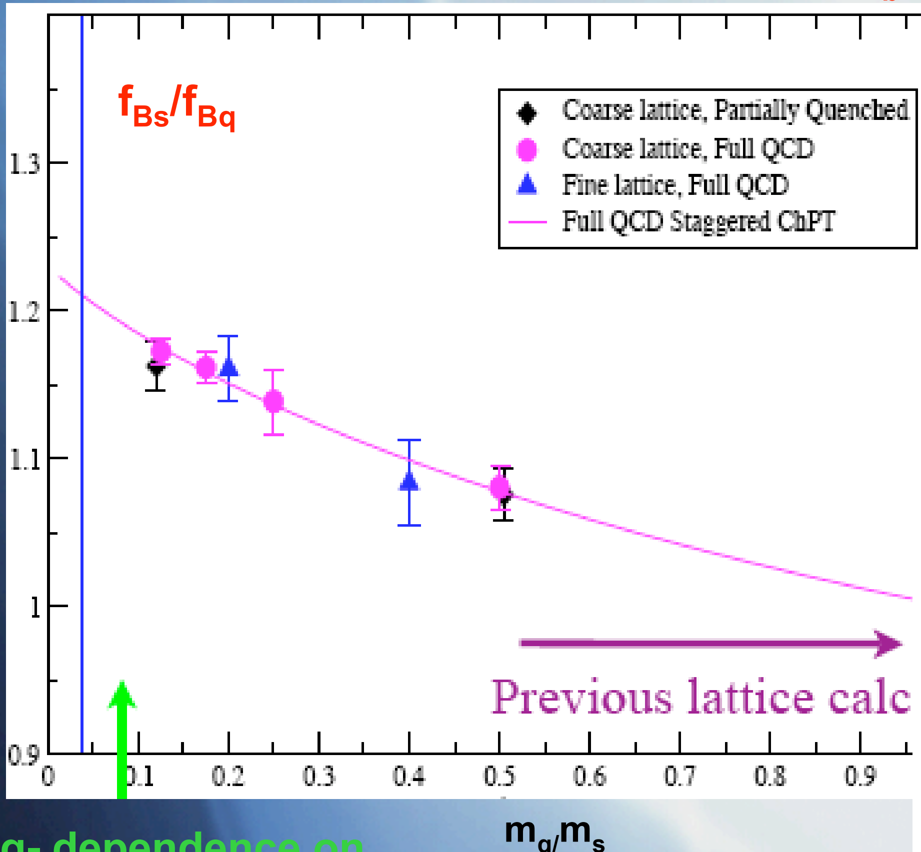


Semi-leptonic: in progress



B-decays

Use non-relativistic QCD for b-quark with m_b fixed from m_Υ

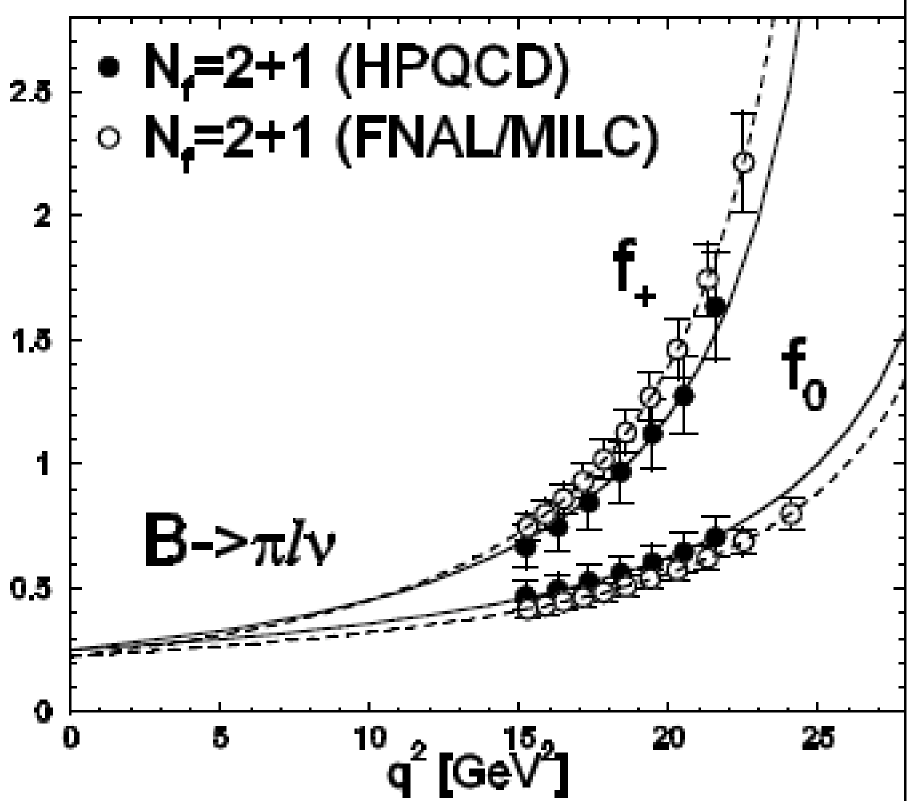


Log- dependence on light quark mass

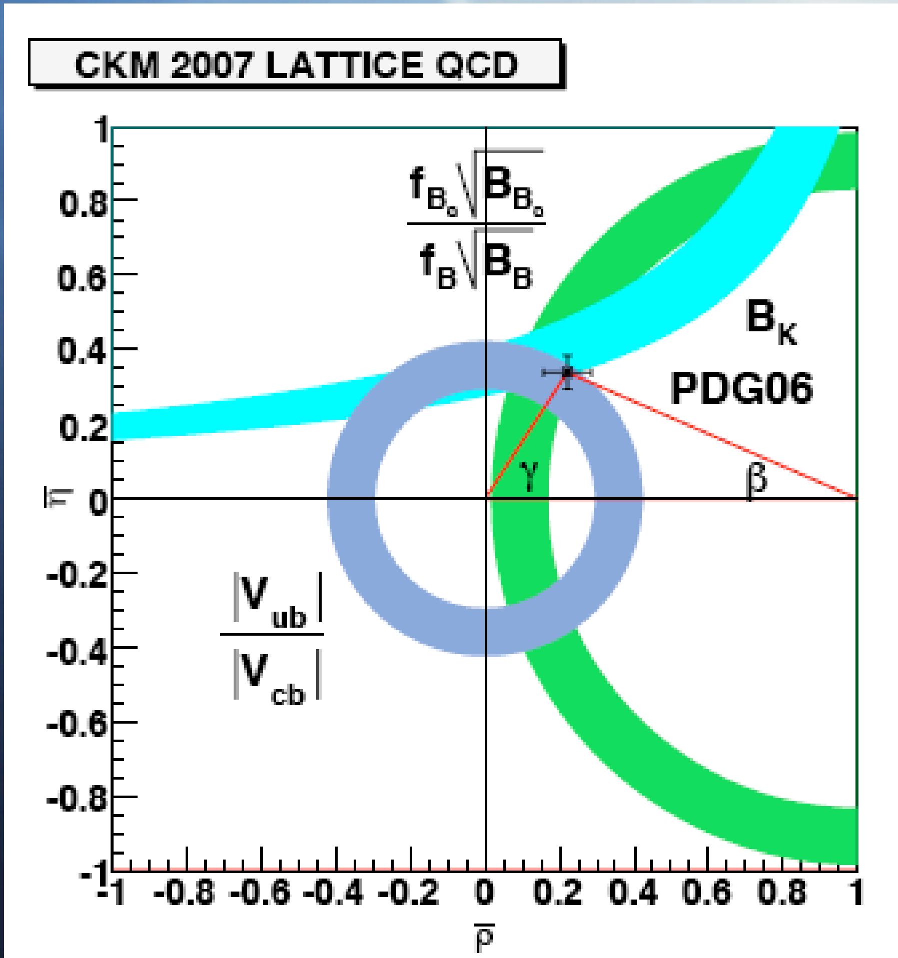
$$f_B = 216(22)\ \text{MeV}, \qquad \frac{f_{B_s}}{f_B} = 1.20(3)$$

to be compared with experiment ICHEP 06: f_B=229(36)(34)MeV

Semileptonic B-decays



Unitarity triangle



Using lattice input

B_K

f_K/f_π , $f_+(K \to \pi l \nu)$

$F(B \to D^* l \nu)$

$f_+(B \to \pi l \nu)$

$$\frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}}$$

Expectation: errors on lattice results should halve in the next two years

Excited states

C. Morningstar

- for a given $N \times N$ correlator matrix $C_{\alpha\beta}(t) = \langle 0 | O_\alpha(t) O_\beta^+(0) | 0 \rangle$ one defines the N *principal correlators* $\lambda_\alpha(t,t_0)$ as the eigenvalues of

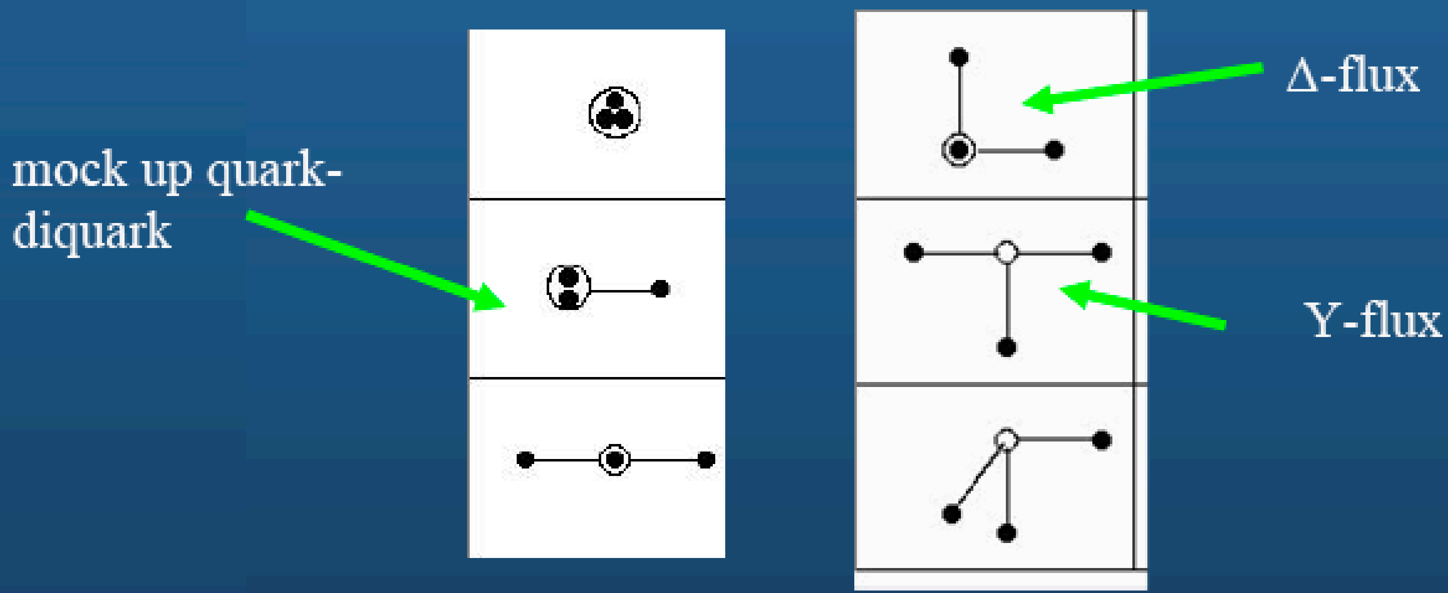
$$C(t_0)^{-1/2} C(t)\, C(t_0)^{-1/2}$$

where t_0 (the time defining the "metric") is small

- can show that $\lim_{t\to\infty} \lambda_\alpha(t,t_0) = e^{-(t-t_0)E_\alpha}(1+e^{-t\Delta E_\alpha})$
- N principal effective masses defined by $m_\alpha^{\text{eff}}(t) = \ln\left(\frac{\lambda_\alpha(t,t_0)}{\lambda_\alpha(t+1,t_0)}\right)$ now tend (plateau) to the N lowest-lying stationary-state energies

Incorporating orbital and radial structure

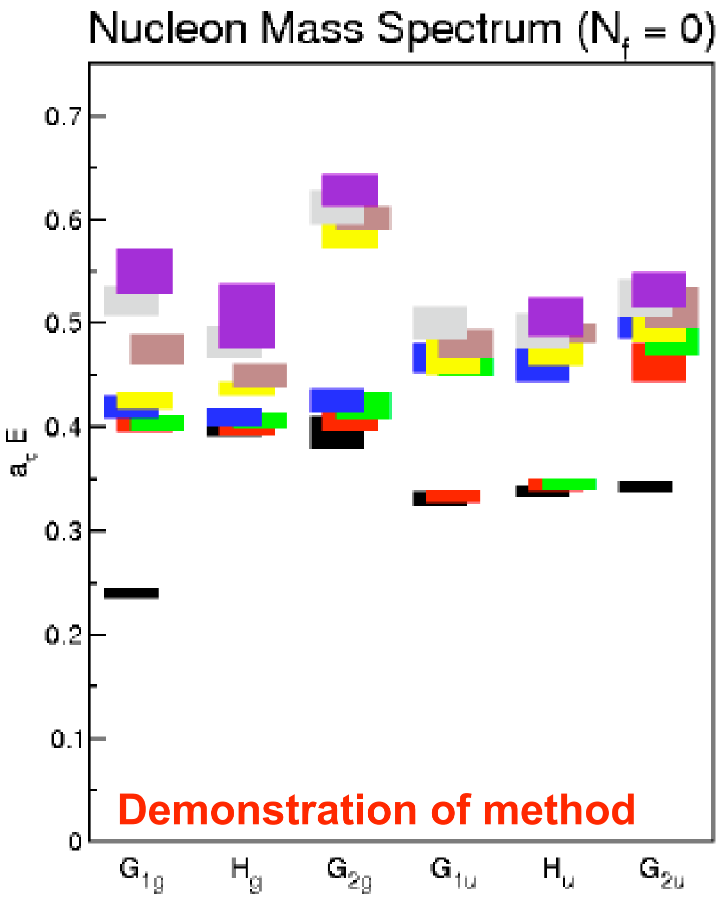
- displacements of different lengths build up radial structure
- displacements in different directions build up orbital structure

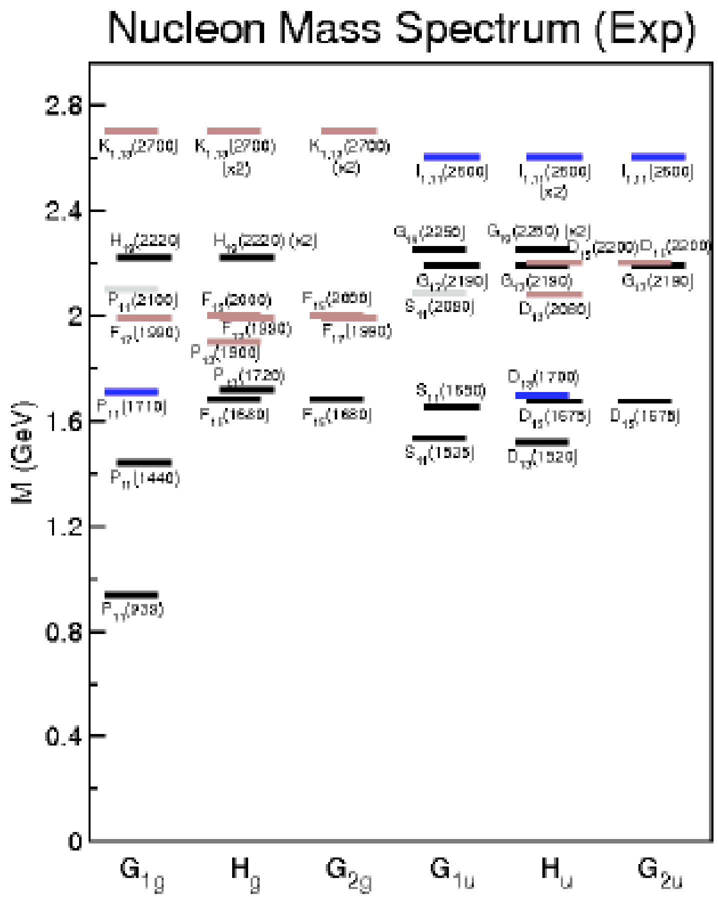


- operator design minimizes number of sources for quark propagators
- useful for mesons, tetraquarks, pentaquarks even!
- can even incorporate hybrid meson operators

Nucleon spectrum

- 200 quenched configs, $12^3\times48$ anisotropic Wilson lattice, $a_s\sim0.1$ fm, $a_s/a_t\sim3$, $m_\pi\sim700$ MeV





Light quark-hybrids

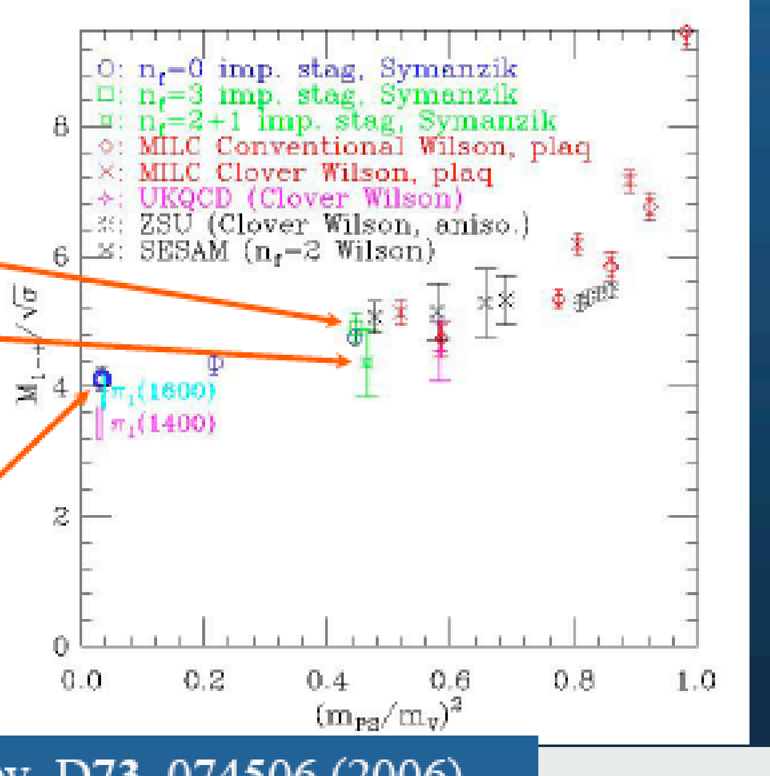
- determinations of exotic 1^{-+} hybrid meson from 2003 and earlier
  - improved staggered fermions (lighter quark masses)
  - quenched and unquenched, Wilson gluon action
  - $a \approx 0.09$ fm
  - lightest mass still above experiment

MILC, hep-lat/0301024

$N_f = 3, \quad m_u = m_d = m_s$
(around strange quark mass)

$m_u = m_d = 0.4 m_s$

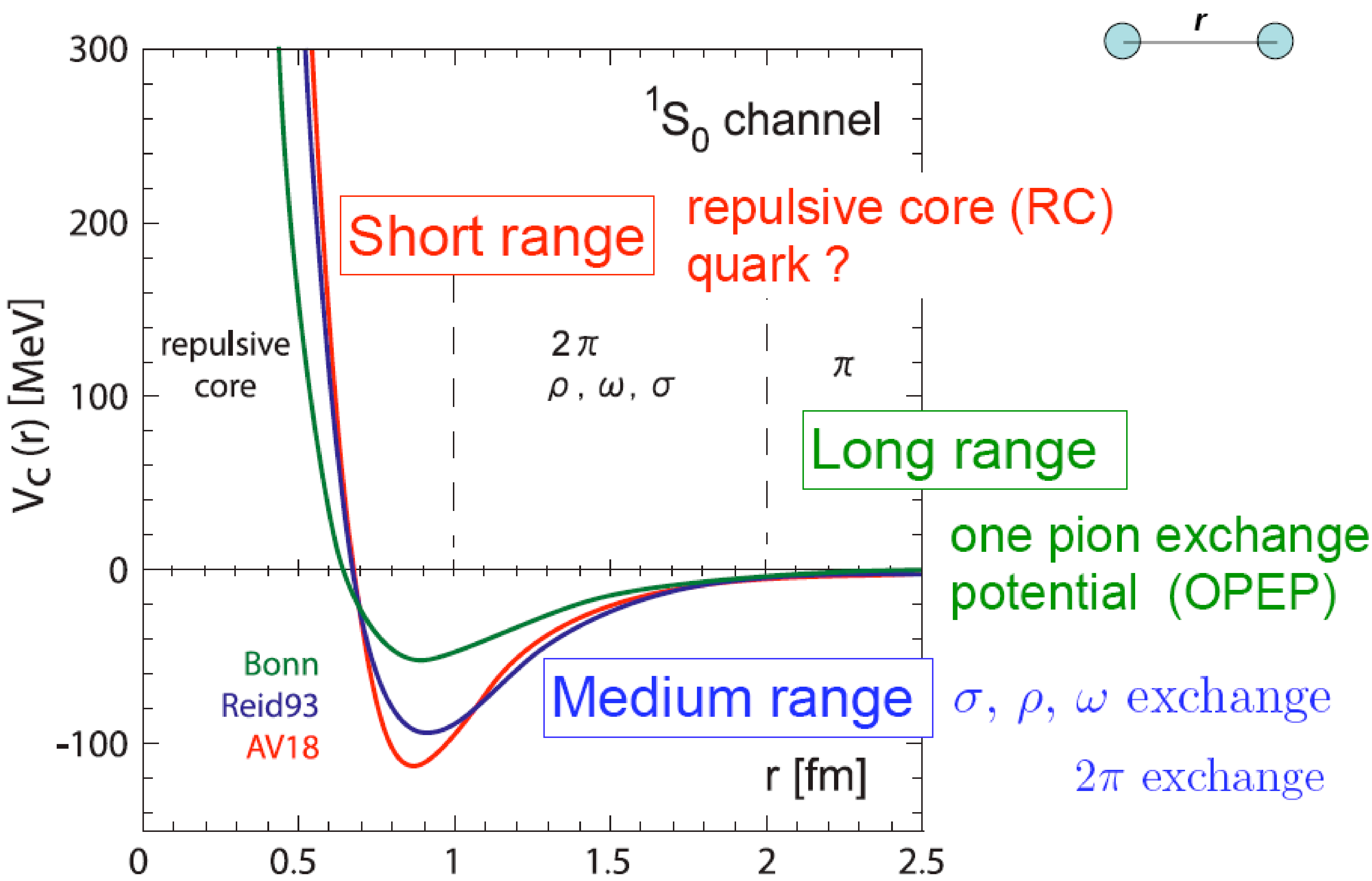
quenched continuum limit



- McNeile et al. (UKQCD) Phys.Rev. D73, 074506 (2006)
- N_f=2 dynamical clover fermions
- found hybrid 1^{-+} mass 2.2(2) GeV

Nuclear force

N. Ishii

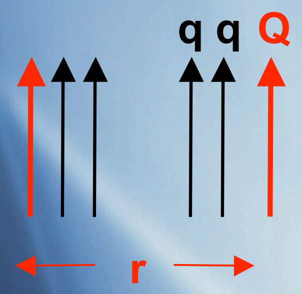


Nucleon-nucleon potential

Traditional approach: potential between static quarks

For NN: one static quark and two light

Calculate energy of Qqq+Qqq minus twice the mass of Qqq as a function of r between Q-Q



Q: static quark

q: light quark

$V(r) = E_{HH} - 2m_H$

finite volume effects???

- Quenched, m_π~400 MeV, a~0.1 fm L~1.6 fm, W. Detmold, K. Orginos and M. Savage, hep-lat/0703009

$$V_{I,s} = V_1(r) + \sigma_1.\sigma_2 V_\sigma(r) + \sigma_1.\sigma_2 \tau_1.\tau_2 V_{\sigma\tau}(r) + \tau_1.\tau_2 V_\tau(r)$$

σ_l, τ_l: for light quarks

only this involves m_H

Short range repulsion observed for all V's

- Quenched, a~0.19 fm L~3.8 fm, thre light quark masses, Takahashi, Doi, Suganuma, hep-la/0601006

No r-dependence seen!!

preliminary results, further study required

Nucleon-nucleon potential

"Wave function" approach:

Define a potential through the Schrödinger equation

$$\left[H_0 + V(x)\right]\varphi_E(x) = E\varphi_E(x) \qquad H_0 = -\frac{\nabla^2}{2\mu}$$

reduced mass

non-relativistic approximation - valid?

Extract potential as $$V(x) = \frac{\left(E - H_0\right)\varphi_E(x)}{\varphi_E(x)}$$

"Wave function": $\varphi_E(\vec{r}) = \langle 0 | N(\vec{x},0)N(\vec{y},0) | 2N;E \rangle \qquad \vec{r} = \vec{x} - \vec{y}$ 4pt correlator

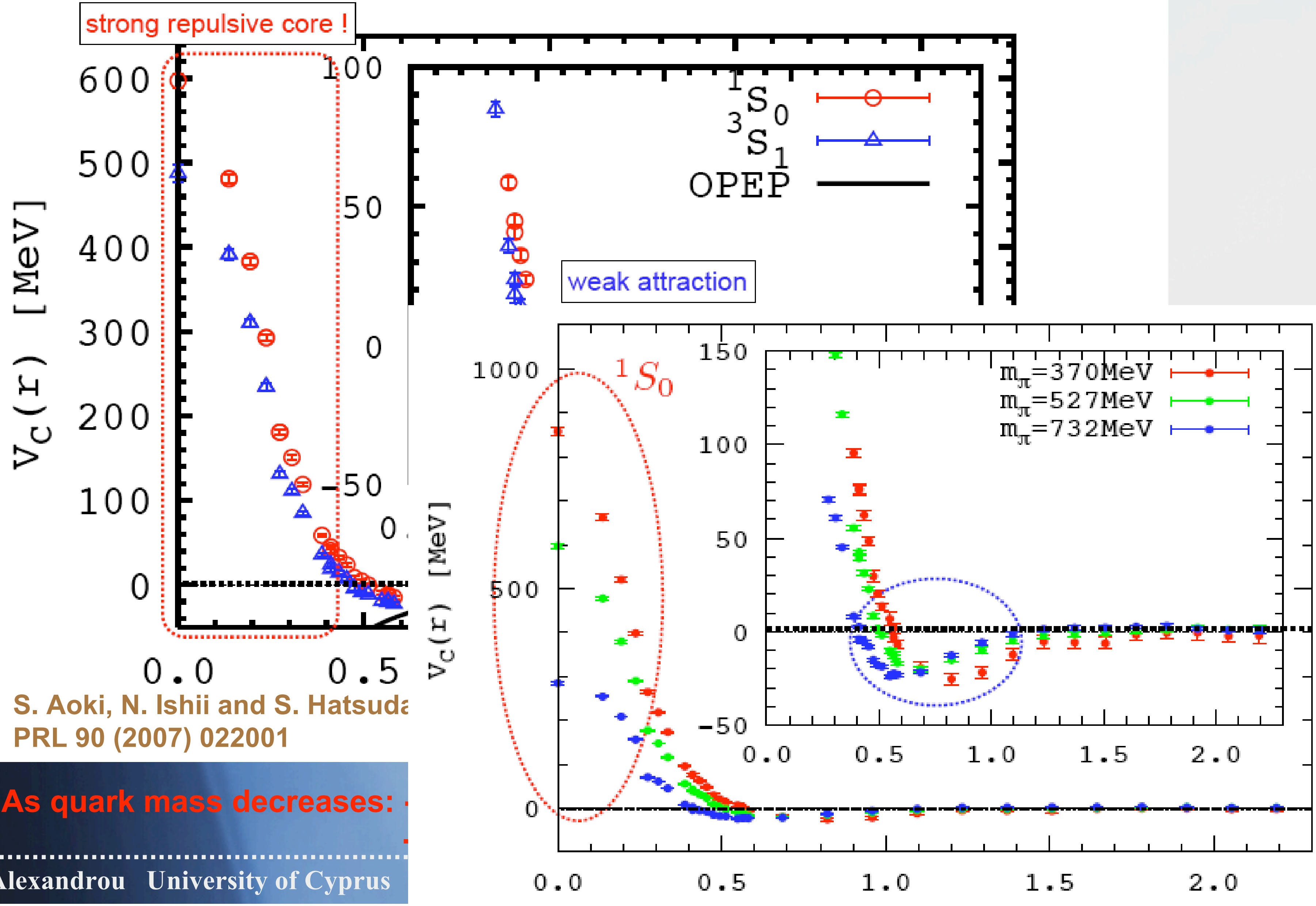
2N state with energy E (use wall source in Coulomb gauge)

Two L=0 channels:

- Singlet $^1S_0$: Central potential only $V_C(r)$
- Triplet $^3S_1$: Central potential $V_C(r)$ and tensor $V_T(r)$ --> $V_C^{eff}(r)$

Central NN potential

Quenched, a=.137 fm, L=4.4 fm, m_π=370, 527, 732 MeV



S. Aoki, N. Ishii and S. Hatsuda
PRL 90 (2007) 022001

As quark mass decreases:

Electric and magnetic polarizability of hadrons

W. Detmold

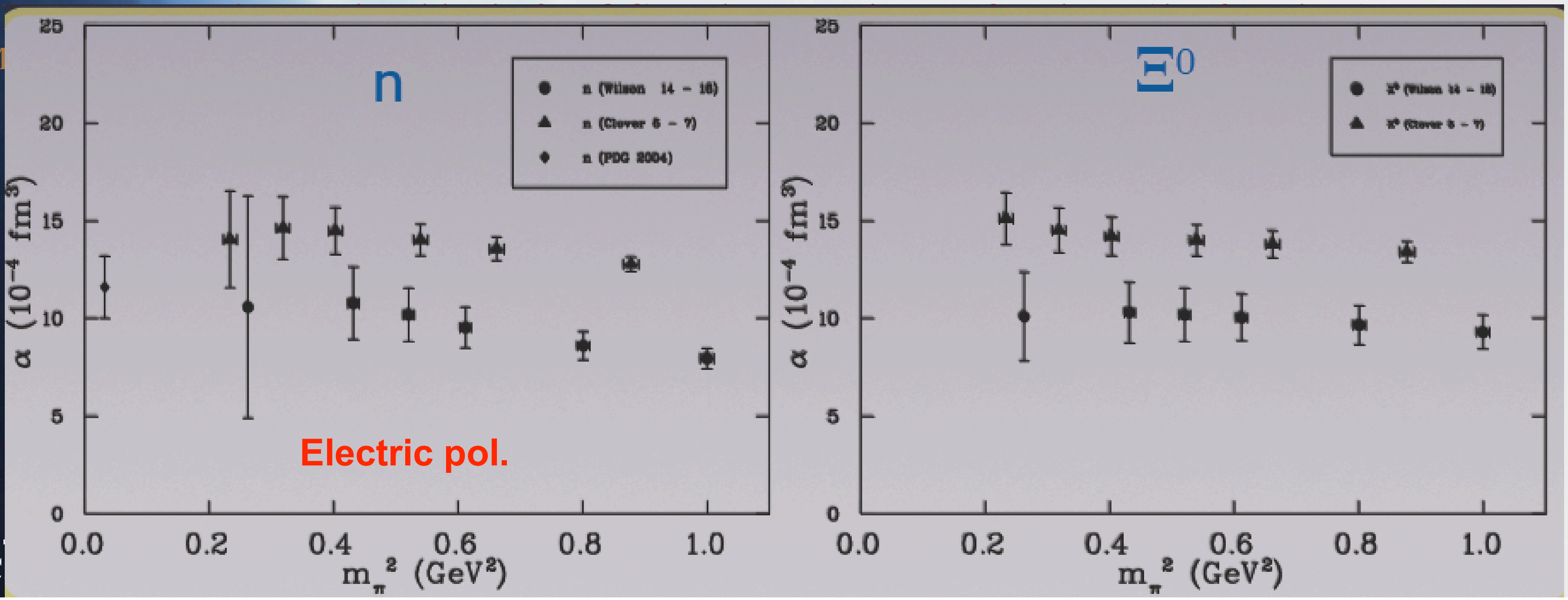
Response of hadron to an external EM field

Calculate the quadratic mass shift in the presence of an external uniform electric or magnetic field:

$$\Delta m = -\frac{1}{2}\alpha\,\vec{E}^2 - \frac{1}{2}\beta\,\vec{B}^2$$

electric polarizability (α), magnetic polarizability (β)

- Electric polarizability calculated for neutral hadrons in the **quenched approximation-value for neutron consistent with experimental value,** J. Christensen et al., PRD72 (2005) 034503



Electric dipole moment of the neutron

S. Simula

Definition: first moment of neutron charge distribution

$$\vec{d}_n = \int d^3x\, \vec{x} < n \,|\, J^0(x) \,|\, n >$$

Zeroth component of EM current

Non-zero electric dipole moment --> time reversal violation --> important check of standard model predictions

Standard model: $d_n < 10^{-31}$ e cm

Electroweak baryongenesis: $d_n < 10^{-26}$ e cm

The asymmetry matter/antimatter observed in the Universe requires time reversal violation many order of magnitude larger

Current limit: $|d_n| < 2.9 \times 10^{-26}$ e cm C. A. Baker et al. PRL 98 (2007)

many accurate experiments in progress aiming at increasing accuracy by 3 orders

Lattice: Involves the topological charge operator --> Difficult to evaluate

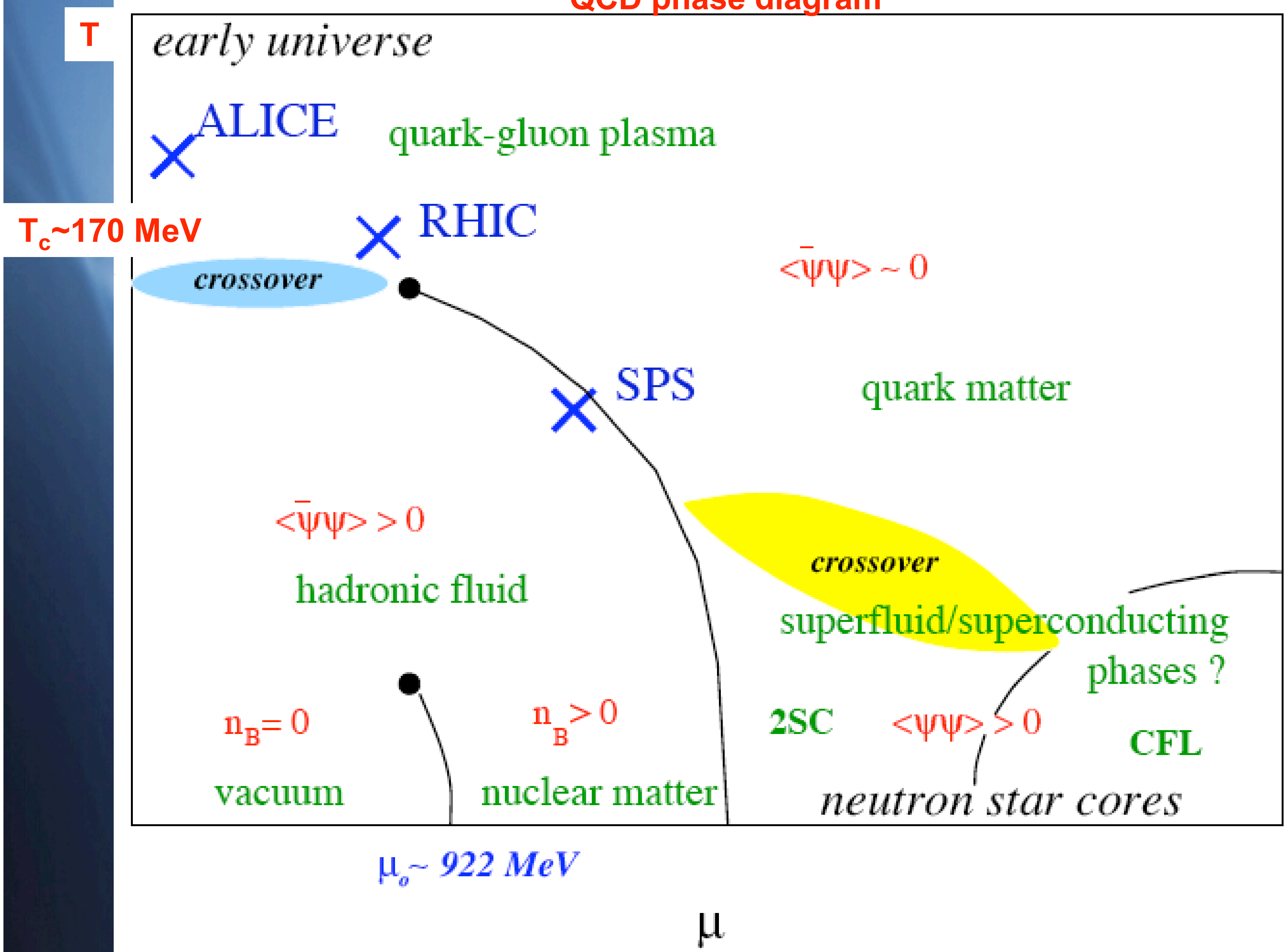
Dynamical domain wall fermions with a θ-term in the action, d_n is found to be, within the statistical errors, consistent with zero, F. Berruto, et al. PRD 73 (2006))

But strategy is well defined….

QCD in extreme conditions

C. Allton

QCD phase diagram



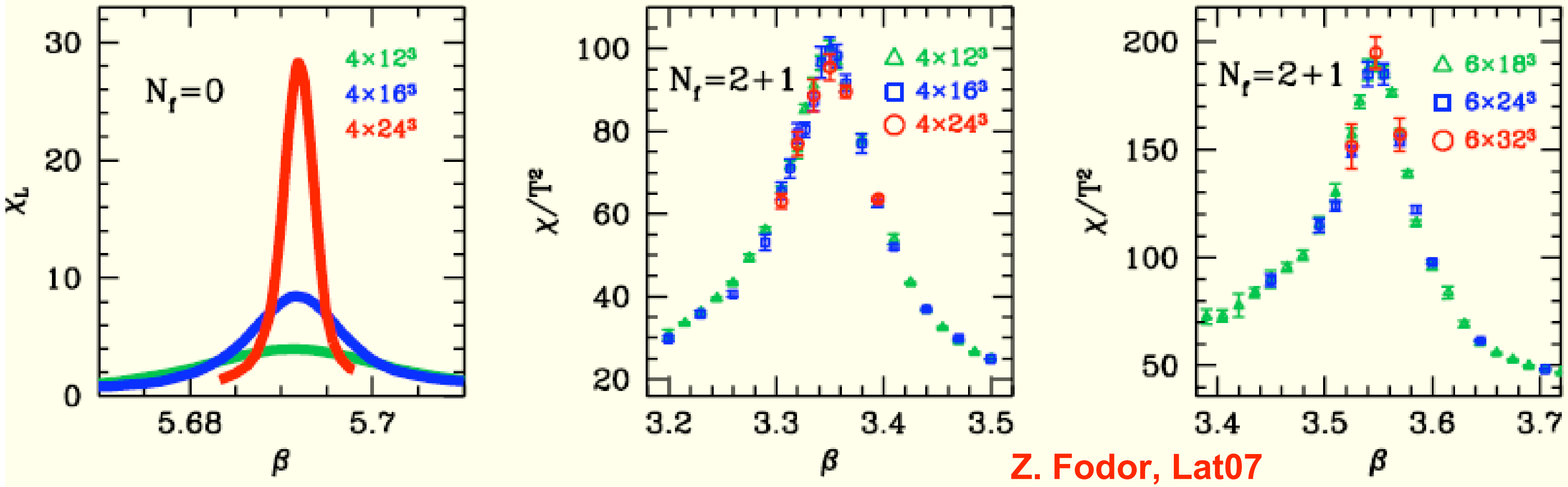
Nature of QCD transition

Impressive progress at zero density finite temperature

The nature of the QCD transition depends on masses of quarks --> simulate along line of constant physics: m_π=135 MeV and m_K=500 MeV, Y. Aoki et al. Nature 443 (2006)

Finite size scaling for chiral susceptibility $\chi = \frac{T}{V}\frac{\partial^2 \log Z}{\partial m^2}$

- first order transition: peak width ~1/V, peak height ~ V
- cross-over: peak width~constant, peak height~ constant

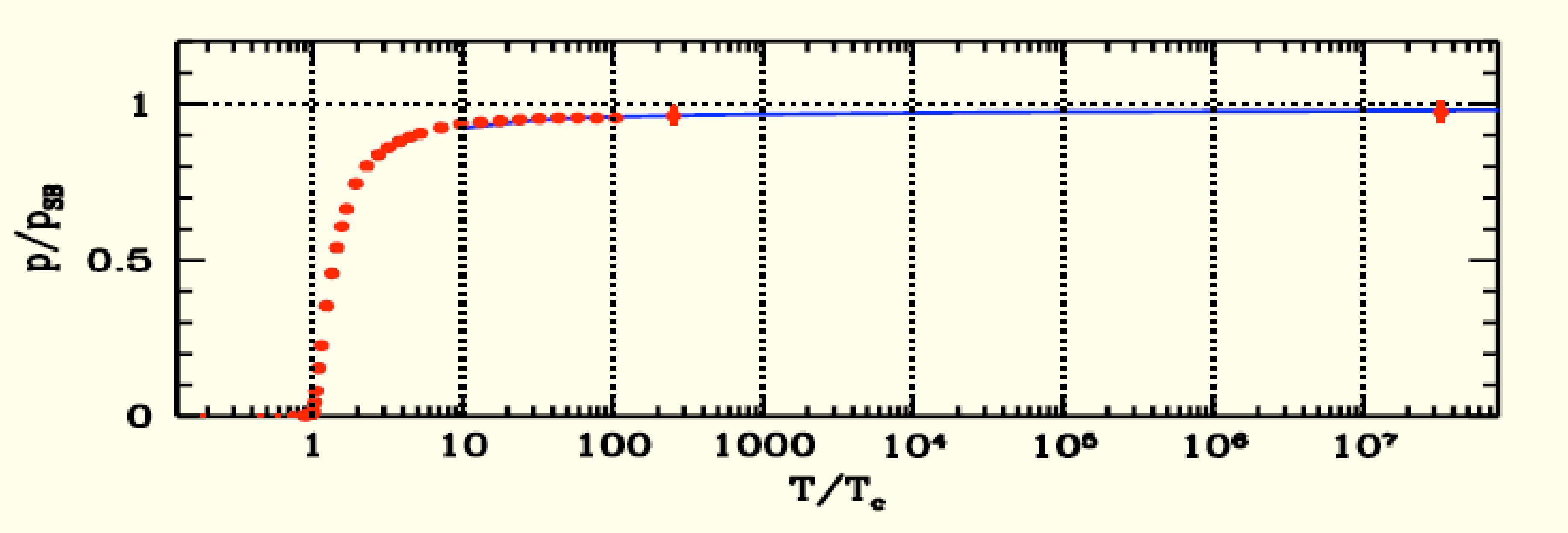


Z. Fodor, Lat07

- With an eight-fold increase in volume scaling volume independent --> cross-over
Finite a-effects checked
- Different transition temperatures for deconfinement and chiral restoration

Link to perturbation theory

Z. Fodor, Lat07



Long awaited link between lattice thermodynamics and perturbation theory

In medium modification of hadrons

What happens to hadrons in the limit of high temperature and density?

- at T=0 quarks dress with gluons → constituent quarks
  bare quark mass m_q~0 → constituent quark mass M_q~300 MeV
- in hot medium dressing melts M_q -->0 for m_q=0 → chiral restoration
- diquark matter (Cooper pairs of QCD) → diquark condensate~ color superconductor

Energy density: - ideal gas of massless pions, $\varepsilon_H \sim T^4$
- idela gas of massless quarks (N_f=2), $\varepsilon_{QGP} \sim 12T^4$

⇒ Sudden increase in energy density - latent heat of deconfinement

Quark-gluon plasma:

- not weakly coupled → Measure spectral functions (MEM)
- very low viscosity

$$G(t,\vec{p}) = \int \rho(\omega,\vec{p})\, K(t,\omega)\, d\omega$$

correlator: $G(t,\vec{p})$; known kernel: $K(t,\omega)$

Main conclusion: pseudoscalar and vector states melt between 1.3 T_c and 2T_c

Conclusions

- Lattice QCD is entering an era where it can make significant contributions in the interpretation of current experimental results.
- A valuable method for understanding hadronic phenomena

  - Accurate results on coupling constants, Form factors, moments of generalized parton distributions are becoming available close to the chiral regime (m_π~300 MeV).
  - More complex observables are evaluated e.g excited states, polarizabilities, scattering lengths, resonances, finite density
  - First attempts into Nuclear Physics e.g. nuclear force

- Computer technology and new algorithms will deliver 100´s of Teraflop/s in the next five years

  - Provide dynamical gauge configurations in the chiral regime
  - Enable the accurate evaluation of more involved matrix elements

Expect lots and lots of Physics


Many Thanks
and
enjoy Milos!