Exclusive DIS

Matthias Burkardt

burkardt@nmsu.edu

New Mexico State University

Las Cruces, NM, 88003, U.S.A.

Outline

- GPDs: probabilistic interpretation as Fourier transforms of impact parameter dependent PDFs
  - $H(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $\tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\mathbf{\Delta}_\perp^2)$

    $\hookrightarrow \perp$ deformation of unpol. PDFs in $\perp$ pol. target
    - physics: orbital motion of the quarks

$\hookrightarrow$ intuitive explanation for SSAs (Sivers)

- charge density in the center of the neutron
- $2\tilde{H}_T + E_T \longrightarrow \perp$ deformation of $\perp$ pol. PDFs in unpol. target
  - correlation between quark angular momentum and quark transversity

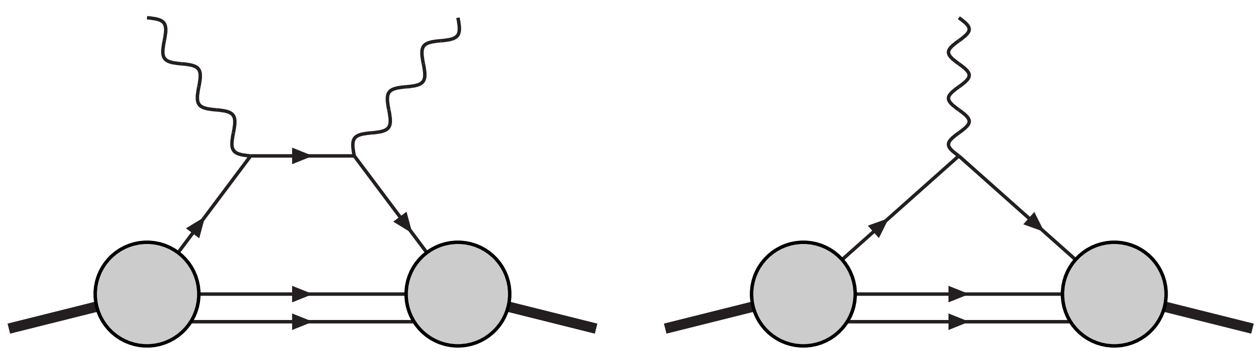
  $\hookrightarrow$ Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$
  - Are all Boer-Mulders functions alike?
- physics of $h_{1T}^\perp(x, \mathbf{k}_\perp)$
- Summary

Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$\int dx H_q(x,\xi,t) = F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t)$$
$$\int dx E_q(x,\xi,t) = F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t),$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer
  - $2\xi = x_f - x_i$
- GPDs can be probed in deeply virtual Compton scattering (DVCS)



Generalized Parton Distributions (GPDs)

- formal definition (unpol. quarks):

$$\int \frac{dx^-}{2\pi} e^{ix^- \bar{p}^+ x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

- in the limit of vanishing t and ξ, the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

$$H_q(x,0,0) = q(x) \qquad \tilde{H}_q(x,0,0) = \Delta q(x).$$

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,\xi,t)$	?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

$q(x,\mathbf{b}_\perp)$ = impact parameter dependent PDF

Impact parameter dependent PDFs

- define $\perp$ localized state [D.Soper,PRD**15**, 1141 (1977)]

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

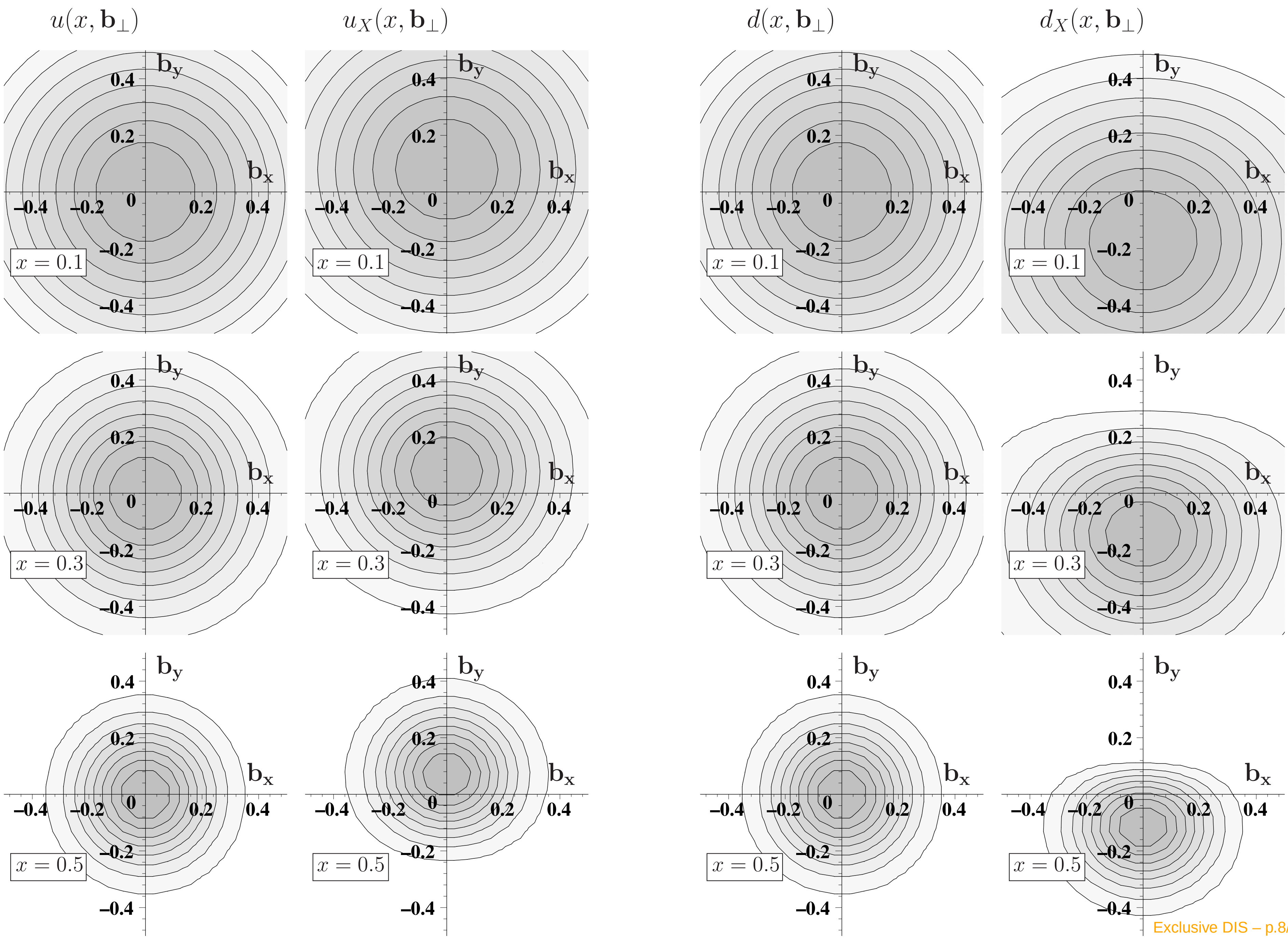
  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$\hookrightarrow$

$$\begin{aligned} q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\mathbf{\Delta}_\perp^2), \\ \Delta q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2), \end{aligned}$$


$u(x, \mathbf{b}_\perp)$
$u_X(x, \mathbf{b}_\perp)$
$d(x, \mathbf{b}_\perp)$
$d_X(x, \mathbf{b}_\perp)$
$\mathbf{b_y}$
$\mathbf{b_x}$
$x = 0.1$
$x = 0.3$
$x = 0.5$
0.4
0.2
0
−0.2
−0.4

Transversely Deformed Distributions and $E(x, 0, -\mathbf{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\uparrow\rangle = H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\downarrow\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF) $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle$.

$\hookrightarrow$ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

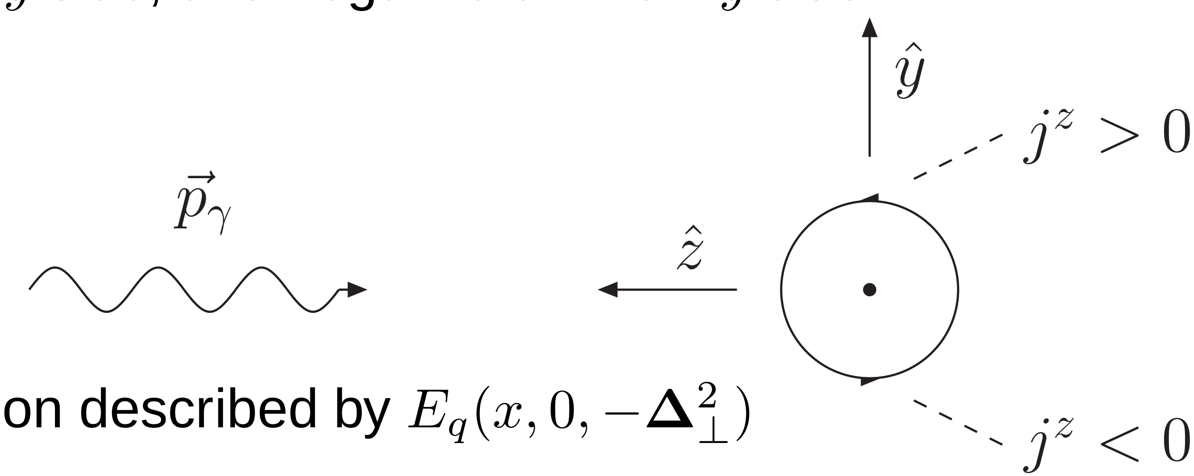
- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 ! [X.Ji, PRL **91**, 062001 (2003)]

Intuitive connection with $\vec{L}_q$

- DIS probes quark momentum density in the infinite momentum frame (IMF). Quark density in IMF corresponds to $j^+ = j^0 + j^3$ component in rest frame ($\vec{p}_{\gamma^*}$ in $-\hat{z}$ direction)

↪ j^+ larger than j^0 when quarks move towards the γ^*; suppressed when they move away from γ^*

↪ For quarks with positive orbital angular momentum in $\hat{x}$-direction, j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



- Details of $\perp$ deformation described by $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$

↪ not surprising that $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$ enters Ji relation!

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,0,0) + E_q(x,0,0) \right] x.$$

Transversely Deformed PDFs and $E(x,0,-\mathbf{\Delta}_\perp^2)$

- $q(x,\mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) b_y = \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q^p}{2M}$$

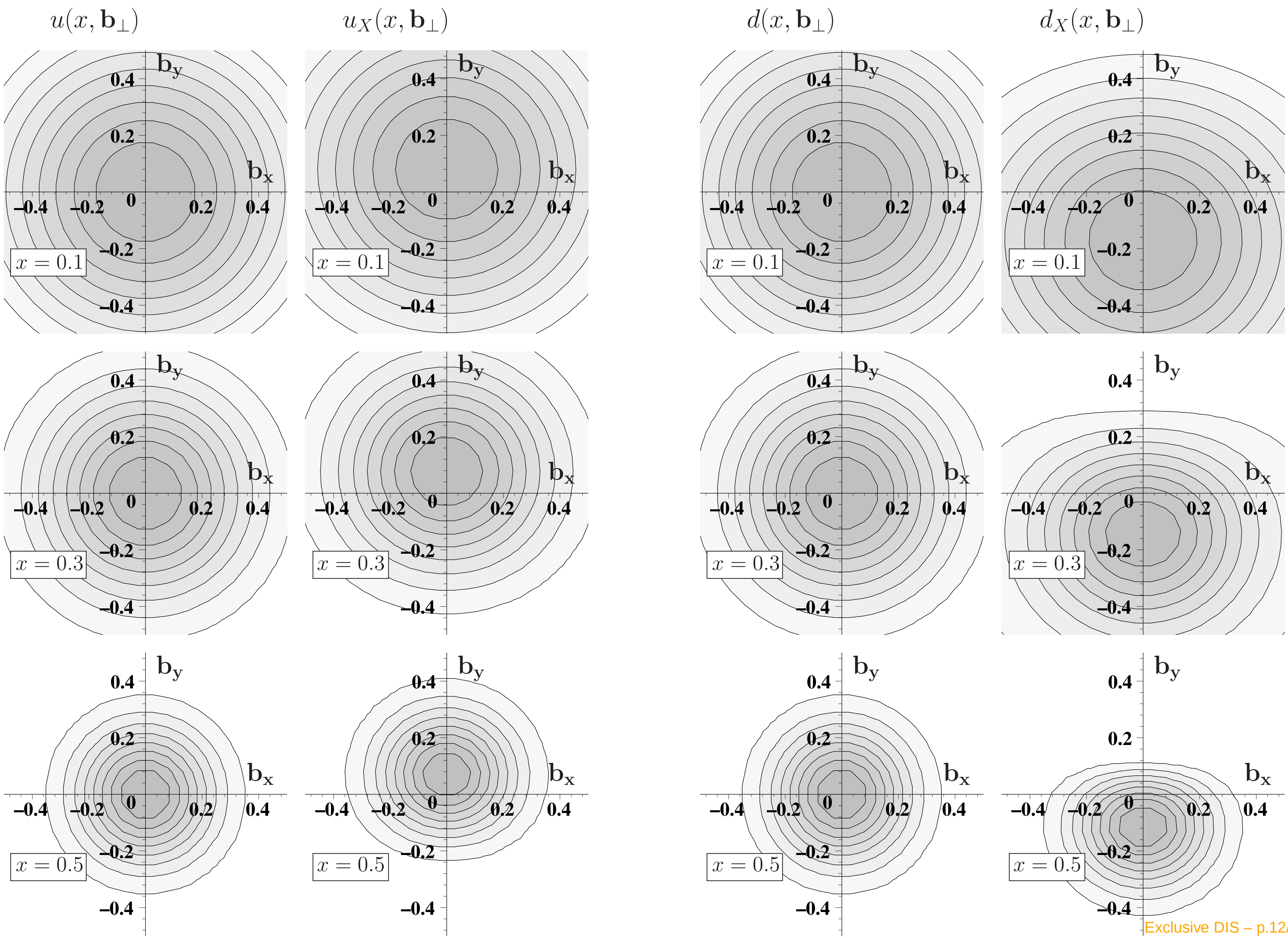
with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2 fm)$

- simple model: for simplicity, make ansatz where $E_q \propto H_q$

$$\begin{aligned} E_u(x,0,-\mathbf{\Delta}_\perp^2) &= \frac{\kappa_u^p}{2} H_u(x,0,-\mathbf{\Delta}_\perp^2) \\ E_d(x,0,-\mathbf{\Delta}_\perp^2) &= \kappa_d^p H_d(x,0,-\mathbf{\Delta}_\perp^2) \end{aligned}$$

with $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$

- Model too simple but illustrates that anticipated deformation is very significant since κ_u and κ_d known to be large!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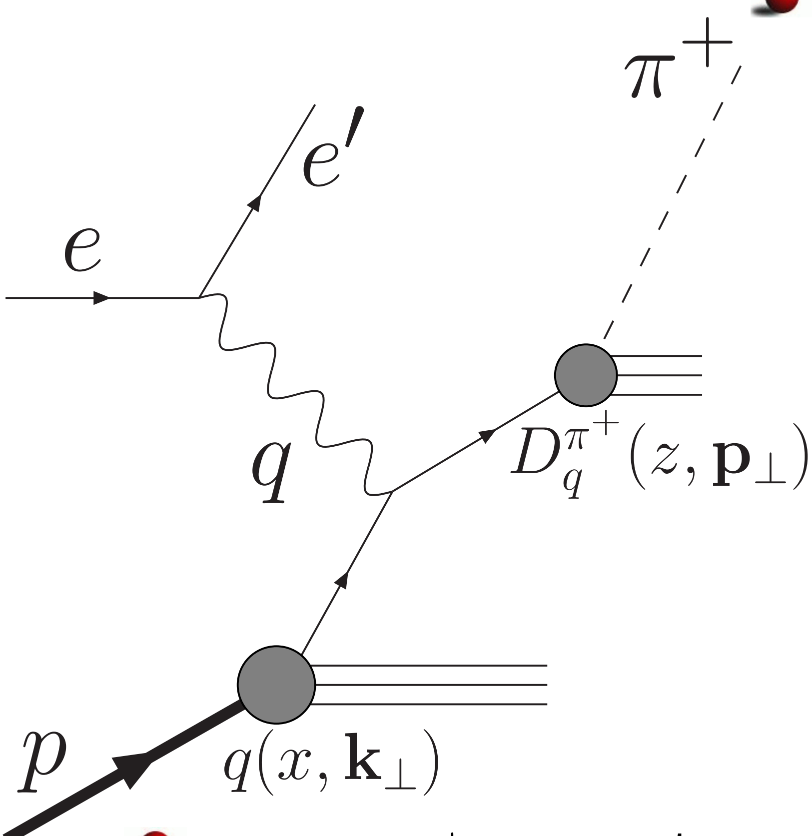
SSAs in SIDIS ($\gamma + p\uparrow \longrightarrow \pi^+ + X$)



- use factorization (high energies) to express momentum distribution of outgoing π^+ as convolution of
  - momentum distribution of quarks in nucleon

  $\hookrightarrow$ unintegrated parton density $f_{q/p}(x, \mathbf{k}_\perp)$
  - momentum distribution of π^+ in jet created by leading quark q

  $\hookrightarrow$ fragmentation function $D_q^{\pi^+}(z, \mathbf{p}_\perp)$
- average $\perp$ momentum of pions obtained as sum of
  - average $\mathbf{k}_\perp$ of quarks in nucleon (Sivers effect)
  - average $\mathbf{p}_\perp$ of pions in quark-jet (Collins effect)

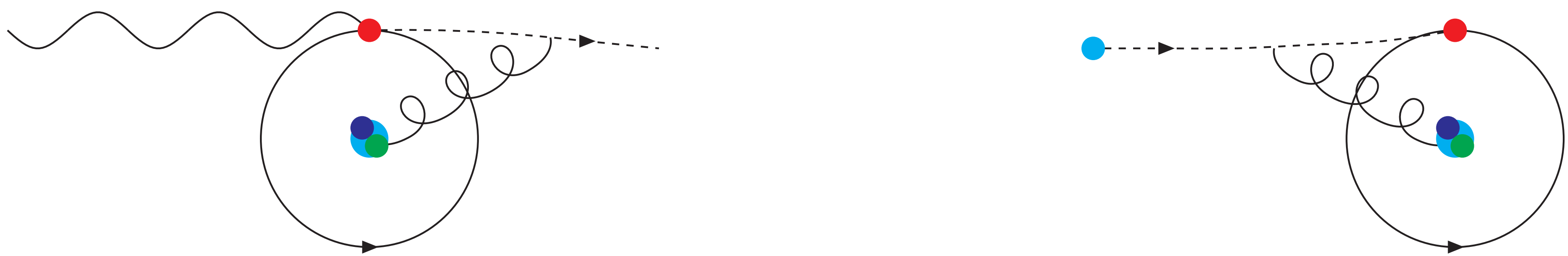
GPD $\longleftrightarrow$ SSA (Sivers)

- Sivers: distribution of **unpol.** quarks in $\perp$ pol. proton

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S}{M}$$

- without FSI, $\langle \mathbf{k}_\perp \rangle = 0$, i.e. $f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2) = 0$

$f_{1T}^{\perp}(x, \mathbf{k}_\perp)_{DY} = -f_{1T}^{\perp}(x, \mathbf{k}_\perp)_{SIDIS}$



a) b)

- time reversal: FSI $\leftrightarrow$ ISI
- compare FSI for 'red' q that is being knocked out with ISI for an anti-red $\bar{q}$ that is about to annihilate that bound q

$\hookrightarrow$ FSI for knocked out q is attractive

- nucleon is color singlet $\rightarrow$ when to-be-annihilated q is 'red', the spectators must be anti-red

$\hookrightarrow$ ISI with spectators is repulsive

GPD $\longleftrightarrow$ SSA (Sivers)

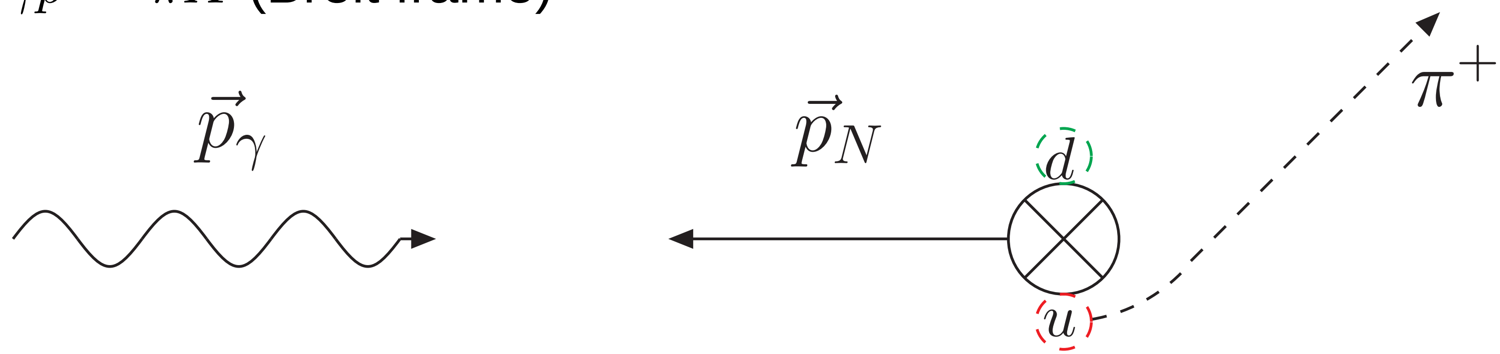
- Sivers: distribution of **unpol.** quarks in $\perp$ pol. proton

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S}{M}$$

- without FSI, $\langle \mathbf{k}_\perp \rangle = 0$, i.e. $f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2) = 0$
- with FSI, $\langle \mathbf{k}_\perp \rangle \neq 0$ (Brodsky, Hwang, Schmidt)
- FSI formally included by appropriate choice of Wilson line gauge links in gauge invariant def. of $f_{q/p}(x,\mathbf{k}_\perp)$
- What should we expect for Sivers effect in QCD ?

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \to \pi X$ (Breit frame)



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
-

  attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q^p and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q^p$

- $f_{1T}^{\perp q} \sim -\kappa_q^p$ confirmed by analysis of pion-data (HERMES). Also consistent with COMPASS $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$

GPD $\longleftrightarrow$ SSA (Sivers)

- $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$ also consistent with sum rule [M.B., PRD**69**, 091501 (2004)],

$$\int dx \sum_{i \in q,g} f_{1T}^{\perp q}(x, \mathbf{k}_\perp)\mathbf{k}_\perp^2 = 0.$$

provided net gluon Sivers is small

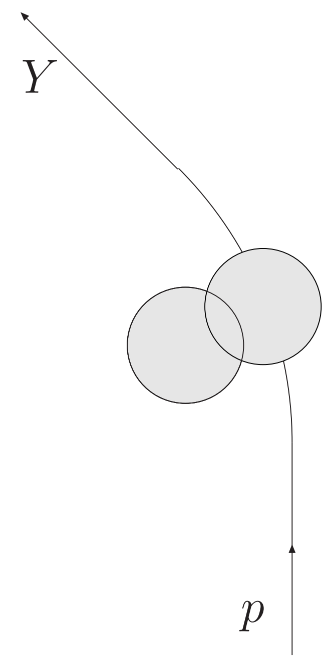
$\perp$ hyperon polarization

model for polarization in $pp \to Y + X$ ($Y \in \Lambda, \Sigma, \Xi$) at high energy:

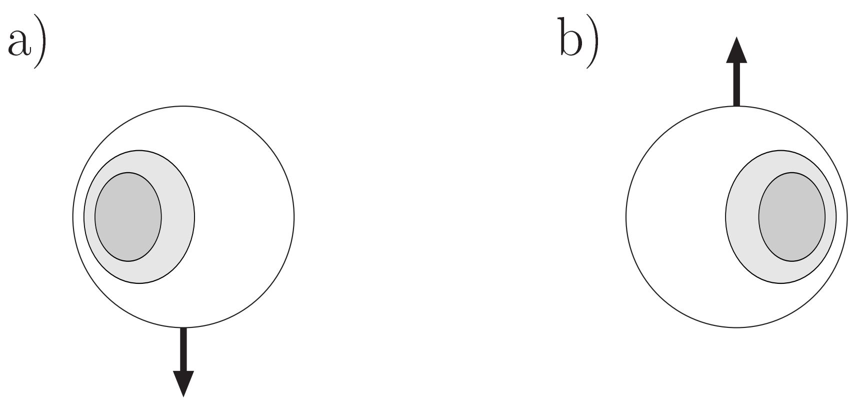
- peripheral scattering
- $s\bar{s}$ produced in overlap region, i.e. on “inside track”

$\hookrightarrow$ if Y deflected to left then s produced on left side of Y (and vice versa)

$\hookrightarrow$ if $\kappa_s > 0$ then intermediate state has better overlap with final state Y that has spin down



$\hookrightarrow$ remarkable prediction: $\boxed{\vec{P}_Y \sim -\kappa_s^Y \vec{p}_P \times \vec{p}_Y\,.}$



$\perp$ hyperon polarization

- SU(3) analysis for κ_s^B yields (assuming $|\kappa_s^p| \ll |\kappa_u^p|, |\kappa_d^p|$)

$$\begin{aligned} \kappa_s^\Lambda &= \kappa^p + \kappa_s^p = 1.79 + \kappa_s^p \\ \kappa_s^\Sigma &= \kappa^p + 2\kappa^n + \kappa_s^p = -2.03 + \kappa_s^p \\ \kappa_s^\Xi &= 2\kappa^p + \kappa^n + \kappa_s^p = 1.67 + \kappa_s^p. \end{aligned}$$

$\hookrightarrow$ expect (polarization $\mathcal{P}$ w.r.t. $\vec{p}_P \times \vec{P}_Y$)

$$\mathcal{P}_\Lambda < 0 \qquad \mathcal{P}_\Sigma > 0 \qquad \mathcal{P}_\Xi < 0$$

consistent with exp. observed pattern

- similar reasoning 'explains' sign of SSA in $p + p\uparrow \longrightarrow h + X$ in those cases where h contains some valence quarks from initial proton and large x_F

Charge Density in the Center of the Neutron

- Galilean subgroup of $\perp$ boosts in IMF

$\hookrightarrow$ Interpretation of 2-D Fourier trafo of GPDs as impact parameter dependent PDFs $q(x, \mathbf{b}_\perp)$ relativistically correct

- $F_1(-\mathbf{\Delta}_\perp^2) = \int dx H(x, 0, -\mathbf{\Delta}_\perp^2)$

$\hookrightarrow$ interpretation of $\rho(\mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} F_1(-\mathbf{\Delta}_\perp^2) e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp}$ as charge density accross the nucleon-pizza also relativistically correct

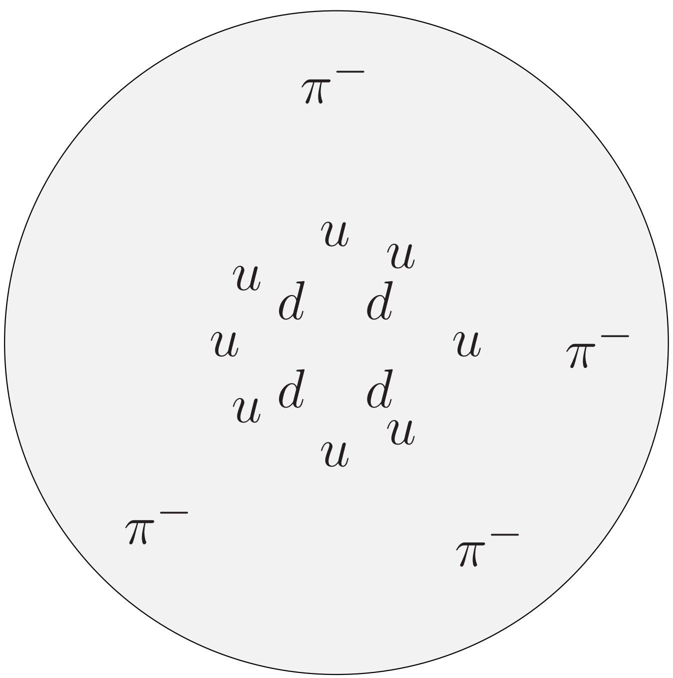
- similar for 2-D Fourier trafo of G_A

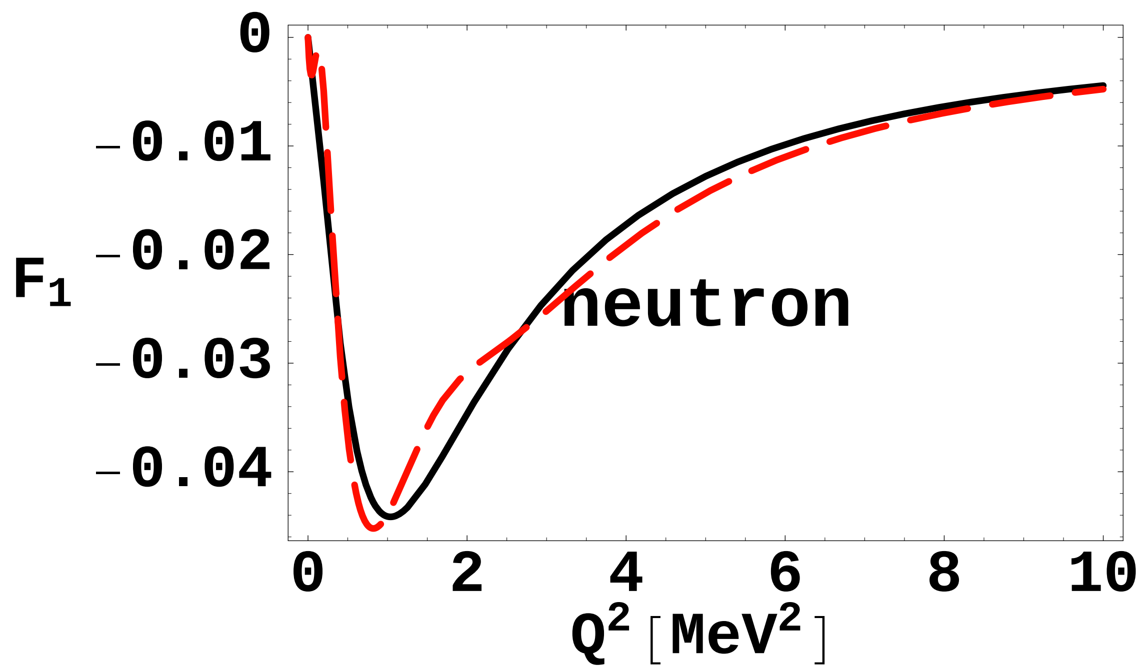
$\hookrightarrow$ distribution of polarization density accross pizza $\Delta\rho(\mathbf{b}_\perp)$

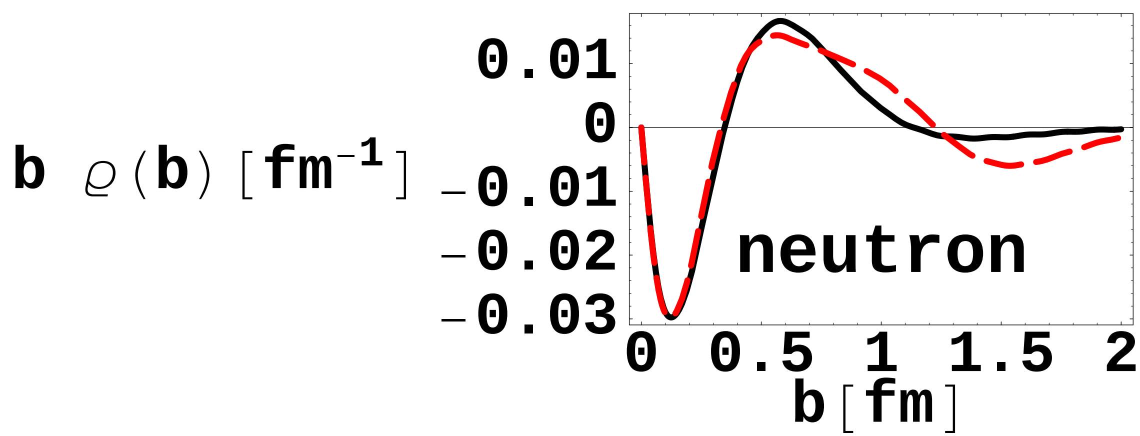
Charge Density in the Center of the Neutron

- 2d FT of F_1^n (G.A.Miller)

The neutron pizza







Charge Density in the Center of the Neutron

- suppression of u quarks/enhancement of d quarks in center of neutron-pizza (in IMF)
- Explanation: several indications that, in proton, d-quarks in proton have larger p-wave component than u-quarks
  - after charge factors taken out, contribution from d quarks to anomalous magnetic moment of proton larger than from u quarks ($\kappa_u^p = 1.673$, $\kappa_d^p = -2.033$)
  - HERMES: Sivers function for d quarks (in proton) at least as large as for u quarks
  - lattice: $L_u \approx -L_d$
  - all despite the fact that proton contains more u than d quarks!!!!

$\hookrightarrow$ (in neutron), u quarks should have larger p-wave component than d quarks

- p wave suppressed at origin!

$\hookrightarrow$ **suppression of u quarks at center of neutron due to larger p-wave component**

Chirally Odd GPDs

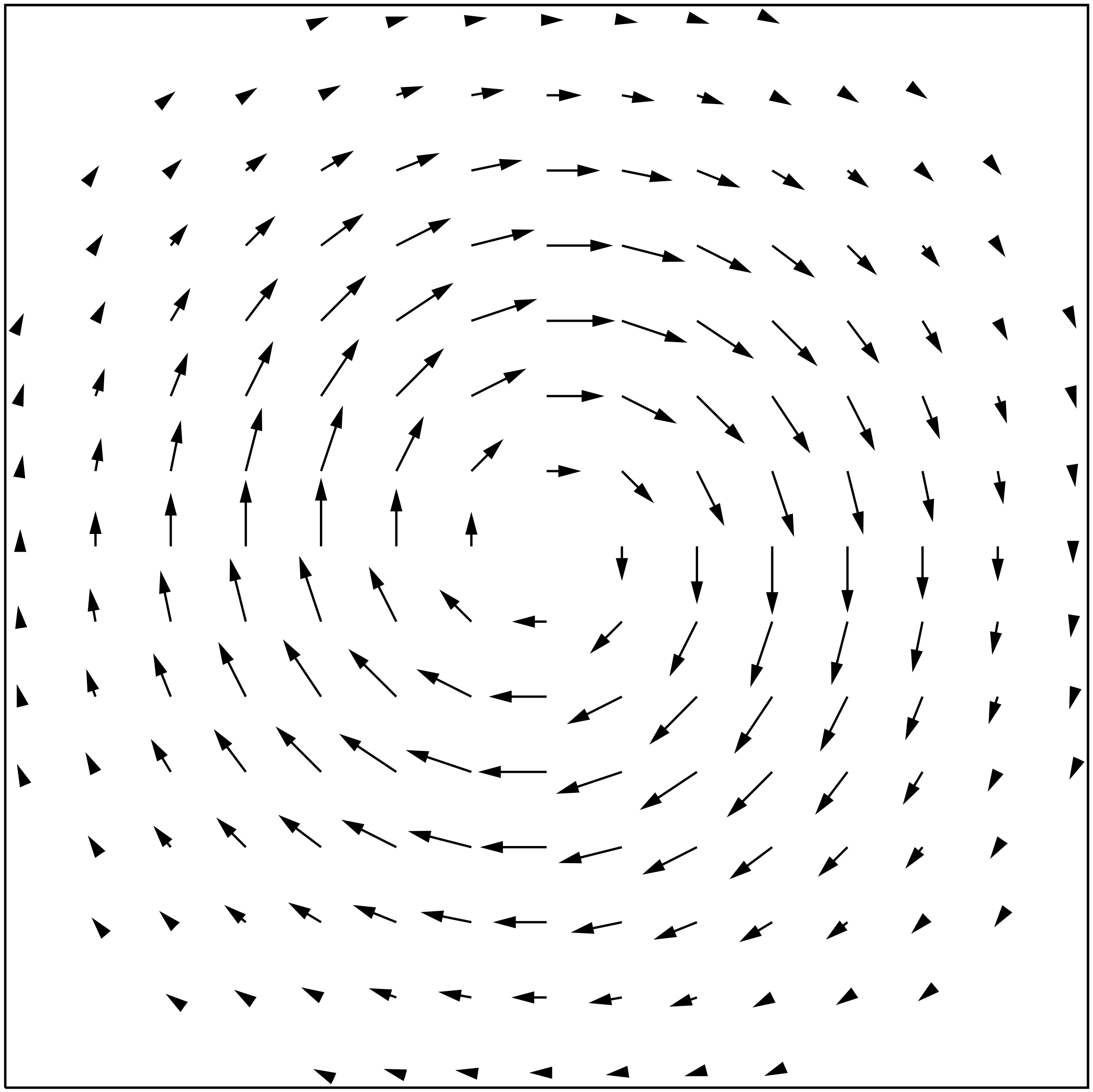
$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \sigma^{+j}\gamma_5 q\left(\frac{x^-}{2}\right) \right| p \right\rangle = \quad H_T \bar{u}\sigma^{+j}\gamma_5 u + \tilde{H}_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha P_\beta}{M^2} u$$
$$+ E_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha \gamma_\beta}{2M} u + \tilde{E}_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}P_\alpha \gamma_\beta}{M} u$$

- See also M.Diehl+P.Hägler, hep-ph/0504175.
- Fourier trafo of $\bar{E}_T^q \equiv 2\tilde{H}_T^q + E_T^q$ for $\xi = 0$ describes distribution of transversity for unpolarized target in $\perp$ plane

$$q^i(x, \mathbf{b}_\perp) = \frac{\varepsilon^{ij}}{2M} \frac{\partial}{\partial b_j} \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \bar{E}_T^q(x, 0, -\mathbf{\Delta}_\perp^2)$$

- origin: correlation between quark spin (i.e. transversity) and angular momentum

Transversity Distribution in Unpolarized Target



Boer-Mulders Function

- SIDIS: attractive FSI expected to convert position space asymmetry into momentum space asymmetry

$\hookrightarrow$ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

$\hookrightarrow$ (qualitative) connection between Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$ and the chirally odd GPD $\bar{E}_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x, \mathbf{k}_\perp)$ and the GPD E.

- Boer-Mulders: distribution of $\perp$ **pol.** quarks in **unpol.** proton

$$f_{q^\uparrow/p}(x, \mathbf{k}_\perp) = \frac{1}{2}\left[f_1^q(x, \mathbf{k}_\perp^2) - h_1^{\perp q}(x, \mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot S_q}{M}\right]$$

- $h_1^{\perp q}(x, \mathbf{k}_\perp^2)$ can be probed in DY (RHIC, J-PARC, GSI) and tagged SIDIS (JLab, eRHIC), using Collins-fragmentation: $\cos(2\pi)$ asymmetry $\propto$ Boer-Mulders $\times$ Collins

$\hookrightarrow$ more π's normal to lepton scattering plane than in it

probing BM function in tagged SIDIS

- consider semi-inclusive pion production off unpolarized target
- spin-orbit correlations in target wave function provide correlation between (primordial) quark transversity and impact parameter

$\hookrightarrow$ (attractive) FSI provides correlation between quark spin and $\perp$ quark momentum $\Rightarrow$ BM function

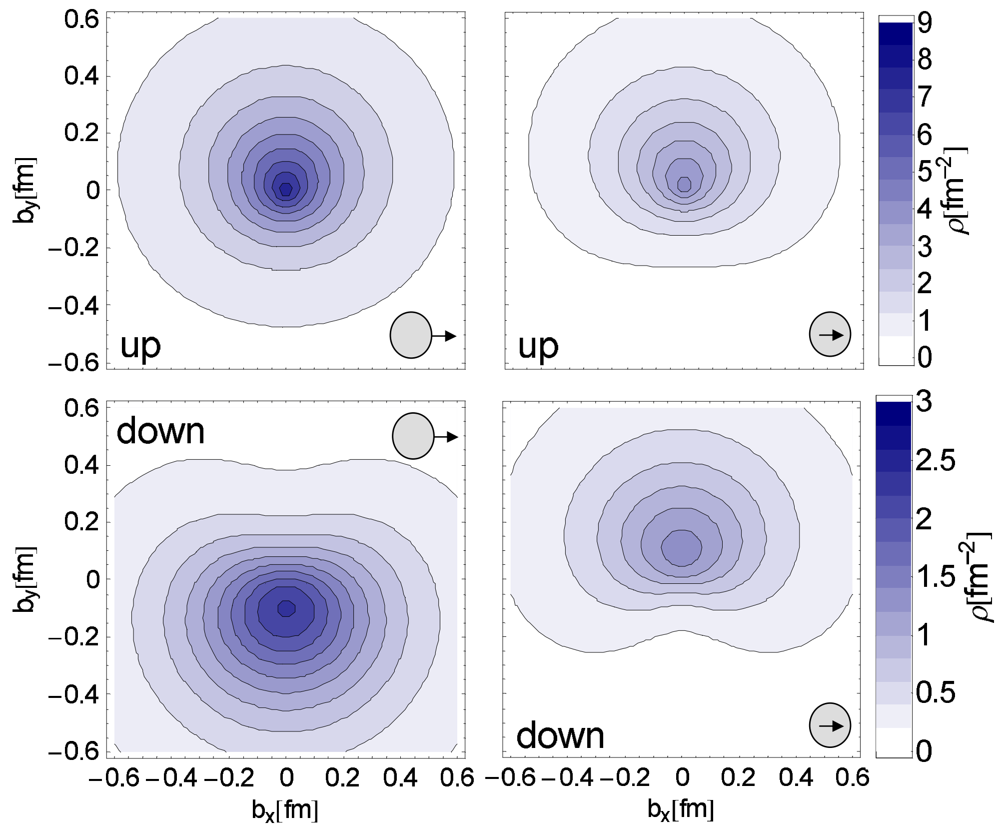
- Collins effect: left-right asymmetry of π distribution in fragmentation of $\perp$ polarized quark $\Rightarrow$ 'tag' quark spin

$\hookrightarrow$ $\cos(2\phi)$ modulation of π distribution relative to lepton scattering plane

$\hookrightarrow$ $\cos(2\phi)$ asymmetry proportional to: Collins $\times$ BM

Chirally Odd GPDs: sign

- models: $h_1^{\perp q}(x, \mathbf{k}_\perp^2) < 0$, $q = u, d$
- lattice: $\bar{E}_T > 0$



- All models & lattice agree on sign! There seems to be a fundamental reason for this sign...

Chirally Odd GPDs: sign

[M.B.+B.Hannafious, hep-ph/0705.1573]

- matrix element for $\bar{E}_T$ involves quark helicity flip

↪ requires interference between wave function components that differ by one unit of OAM (e.g. s-p interference)

↪ sign of $\bar{E}_T$ depends on rel. sign between s & p components

- bag model: p-wave from lower component

$$\Psi_m = \begin{pmatrix} if\chi_m \\ -g(\vec{\sigma}\cdot\hat{\vec{x}})\chi_m \end{pmatrix},$$

(relative sign from free Dirac equation $g = \frac{1}{E}\frac{d}{dr}f$)

- $\bar{E}_T \propto -f \cdot g$. Ground state: f peaked at $r = 0 \Rightarrow \bar{E}_T > 0$
- more general potential model: $\frac{1}{E} \longrightarrow \frac{1}{E-V_0(r)+m+V_S(r)}$

↪ sign of $\bar{E}_T$ same as in Bag model!

Chirally Odd GPDs: sign

[M.B.+B.Hannafious, hep-ph/0705.1573]

- relativistic constituent model: spin structure from SU(6) wave functions plus “Melosh rotation”
  $\hookrightarrow$ $\bar{E}_T > 0$ (B.Pasquini et al.)
  - origin of sign: “Melosh rotation” is free Lorentz boost

  $\hookrightarrow$ relative sign between upper and lower component same as for free Dirac eq. (bag)
- diquark models: nucleon structure from perturbative splitting of spin $\frac{1}{2}$ ‘nucleon’ into quark & scalar/a-vector diquark: $\bar{E}_T > 0$
  - origin of sign: interaction between q and diquark is point-like

  $\hookrightarrow$ except when q & diquark at same point, q is noninteracting

  $\hookrightarrow$ relative sign between upper and lower component same as for free Dirac eq. (bag)
- NJL model (pion): $\bar{E}_T > 0$
  origin of sign: NJL model also has contact interaction!
- lattice QCD (u, d in nucleon; pion): $\bar{E}_T > 0$ ($\longrightarrow$ P.Hägler)

Chirally Odd GPDs: magnitude

- large N_C: $\bar{E}_T^u = \bar{E}_T^d$
- Bag model/potential models: correlation between quark orbit and quark spin same for all quark states (regardless whether $j_z = +\frac{1}{2}$ or $j_z = -\frac{1}{2}$)

$\hookrightarrow$ all quark orbits contribute coherently to $\bar{E}_T$

- compare E (anomalous magnetic moment), where quark orbits with $j_z = +\frac{1}{2}$ and $j_z = -\frac{1}{2}$ contribute with opposite sign

$\hookrightarrow$ E, which describes correlation between quark OAM and nucleon spin smaller than $\bar{E}_T$, which describes correlation between quark OAM and quark spin: $\bar{E}_T > |E|$

- potential models: $\bar{E}_T \propto$ # of q $\quad \Rightarrow \quad \bar{E}_T^u = 2\bar{E}_T^d$

$\hookrightarrow$ expect $2\bar{E}_T^d > \bar{E}_T^u > \bar{E}_T^d$

- all of the above consistent with LGT results ($\longrightarrow$ P.Hägler)

Transversity decomposition of J_q

[M.B., PRD72, 094020 (2006); PLB639, 462 (2006)]

- $J^i = \frac{1}{2}\varepsilon^{ijk}\int d^3x \left[T^{0j}x^k - T^{0k}x^j\right]$
- J_q^x diagonal in transversity, projected with $\frac{1}{2}(1 \pm \gamma^x\gamma_5)$, i.e. one can decompose

$$J_q^x = J_{q,+\hat{x}}^x + J_{q,-\hat{x}}^x$$

where $J_{q,\pm\hat{x}}^x$ is the contribution (to J_q^x) from quarks with positive (negative) transversity

$\hookrightarrow$ derive relation quantifying the correlation between $\perp$ quark spin and angular momentum (nucleon polarized in $+\hat{y}$ direction)

$$\left\langle J_{q,\pm\hat{y}}^{+\hat{y}}\right\rangle = \frac{1}{4}\int dx\, x \left[q(x) + E^q(x,0,0)\right] \pm \frac{1}{4}\int dx\, x\left[h_1^q(x) + \bar{E}_T^q(x,0,0)\right]$$

For unpolarized target only second term.

$\hookrightarrow$ learn about $\vec{L}_q \cdot \vec{S}_q$ correlations

Physics of $h_{1T}^\perp$

- consider $\mathbf{k}_\perp$-dependence of PDFs for quarks with $\perp$ spin $\mathbf{s}$

$$q(x,\mathbf{k}_\perp,\mathbf{s},\mathbf{S}) = \frac{1}{2}\Big[f_1 + s^i S^i h_1 + \frac{1}{M} S^i \varepsilon^{ij} k^j f_{1T}^\perp + \frac{1}{M} s^i \varepsilon^{ij} k^j h_1^\perp + \frac{1}{M}\Lambda s^i k^i h_{1L}^\perp + \frac{1}{2M^2} s_i S_j \left(2k^i k^j - \mathbf{k}_\perp \delta^{ij}\right) h_{1T}^\perp\Big],$$

where Λ is the longitudinal nucleon polarization and $\mathbf{S}$ its $\perp$ spin.

- similar structures can be defined in impact parameter space
- if $h_{1T}^\perp > 0$ then, for $\mathbf{s} = \mathbf{S} = \hat{x}$, enhancement along $\hat{x}$ axis

$\hookrightarrow$ naturally arises from p-wave component, with $L_x = 0$

- if $h_{1T}^\perp < 0$ then, for $\mathbf{s} = \mathbf{S} = \hat{x}$, enhancement $\perp$ to $\hat{x}$ axis

$\hookrightarrow$ naturally arises from p-wave component, with $L_x = \pm 1$

- intuitive expectation: u quarks, expect lower component to have quarks polarized opposite to nucleon spin, with OAM in direction of nucleon spin $\longrightarrow$ $h_{1T}^{\perp u} > 0$ (d quarks: $h_{1T}^{\perp d} < 0$)

Physics of $h_{1T}^{\perp}$

- $h_{1T}^{\perp} >$ tells us about components of quark OAM for different polarization combinations
- relation to tensor-term in impact parameter space:
  - p wave in momentum space same shape as p wave in position space

  $\hookrightarrow$ same interpretation and same expectation for signs

Summary

- GPDs $\stackrel{FT}{\longrightarrow}$ PDFs in impact parameter space
- $E(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target

$\hookrightarrow$ origin for deformation: orbital motion of the quarks

$\hookrightarrow$ simple mechanism (attractive FSI) to predict sign of f_{1T}^q

$$f_{1T}^u < 0 \qquad\qquad f_{1T}^d > 0$$

- neg. charge density in center of neutron $\Rightarrow L_u$
- distribution of $\perp$ polarized quarks in unpol. target described by chirally odd GPD $\bar{E}_T^q = 2\bar{H}_T^q + \tilde{E}_T^q$

$\hookrightarrow$ origin: correlation between orbital motion and spin of the quarks

$\hookrightarrow$ attractive FSI $\Rightarrow$ measurement of $h_1^\perp$ (DY,SIDIS) provides information on $\bar{E}_T^q$ and hence on spin-orbit correlations

- expect: $h_1^{\perp,q} < 0 \qquad\qquad |h_1^{\perp,q}| > |f_{1T}^q|$
- $h_{1T}^\perp$: info on $L_x = \pm 1$ vs. $L_x = 0$

$\perp$ Single Spin Asymmetry (Sivers)

- Naively (time-reversal invariance) $f(x, \mathbf{k}_\perp) = f(x, -\mathbf{k}_\perp)$
- However, including the final state interaction (FSI) results in nonzero asymmetry of the ejected quark! (Brodsky, Hwang, Schmidt)
- Gauge invariant definition requires quark to be connected by gauge link. Choice of path not arbitrary but must be chosen along path of outgoing quark to incorporate FSI

$$f(x, \mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip\cdot\xi} \left\langle P, S \left| \bar{q}(0) U_{[0,\infty]} \gamma^+ U_{[\infty,\xi]} q(\xi) \right| P, S \right\rangle \Big|_{\xi^+=0}$$

with $U_{[0,\infty]} = P \exp\left(ig \int_0^\infty d\eta^- A^+(\eta)\right)$

Sivers Mechanism in $A^+ = 0$ gauge

- Gauge link along light-cone trivial in light-cone gauge

$$U_{[0,\infty]} = P \exp\left(ig \int_0^\infty d\eta^- A^+(\eta) \right) = 1$$

$\hookrightarrow$ Puzzle: Sivers asymmetry seems to vanish in LC gauge (time-reversal invariance)!

- X.Ji: fully gauge invariant definition for $P(x, \mathbf{k}_\perp)$ requires additional gauge link at $x^- = \infty$

$$\begin{aligned} f(x, \mathbf{k}_\perp) &= \int \frac{dy^- d^2\mathbf{y}_\perp}{16\pi^3} e^{-ixp^+y^- + i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \\ &\times \left\langle p, s \left| \bar{q}(y)\gamma^+ U_{[y^-, \mathbf{y}_\perp; \infty^-, \mathbf{y}_\perp]} U_{[\infty^-, \mathbf{y}_\perp, \infty^-, \mathbf{0}_\perp]} U_{[\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]} q(0) \right| p, s \right\rangle \end{aligned}$$

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