

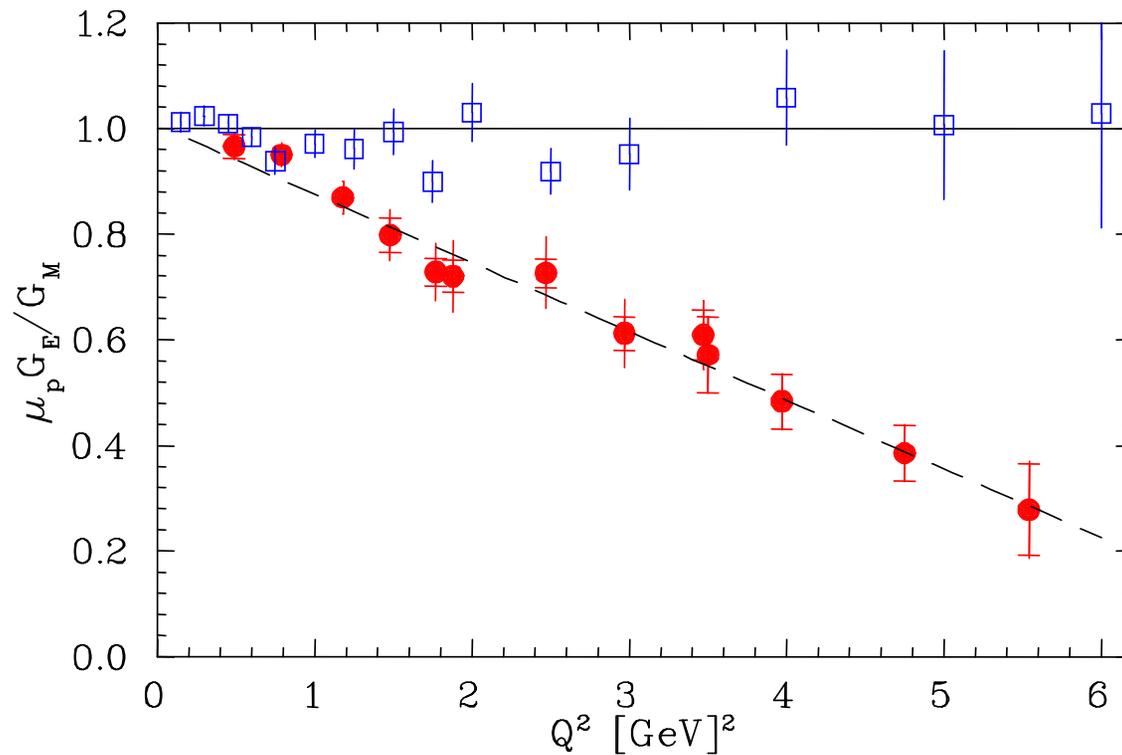


# Two-photon exchange: hadronic picture

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# Rosenbluth vs polarization transfer measurements of $G_E/G_M$ of proton



SLAC

Rosenbluth data

JLab

Polarization data

## Rosenbluth separation method

### One-photon exchange cross section

$$d\sigma_0 = A \left( G_M^2 + \frac{\epsilon}{\tau} G_E^2 \right)$$

$$\tau \equiv \frac{Q^2}{4M^2} \qquad \frac{1}{\epsilon} \equiv 1 + 2(1 + \tau) \tan^2 \frac{\theta}{2}$$

- $\epsilon$  is virtual photon polarization
  - Forward scattering  $\epsilon \rightarrow 1$
  - Backward scattering  $\epsilon \rightarrow 0$
- Extract  $G_E$  and  $G_M$  from linear  $\epsilon$  dependence
- $G_E$  suppressed as  $Q^2$  increases
  - Sensitive to small corrections linear in  $\epsilon$

## Polarization transfer method

$$\vec{e} + p \rightarrow e + \vec{p}$$

- Look at ratio of transverse ( $P_T$ ) to longitudinal ( $P_L$ ) components of recoil proton polarization using a longitudinally polarized electron beam
- Doesn't depend on absolute normalization
- Ratio relatively insensitive to radiative corrections (e.g. bremsstrahlung corrections cancel)

$$\frac{P_T}{P_L} = -\sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}} \frac{G_E}{G_M}$$

in one-photon exchange approximation

## Speculation: radiative corrections

$$d\sigma_0 \rightarrow d\sigma = d\sigma_0 (1 + \delta_{RC})$$

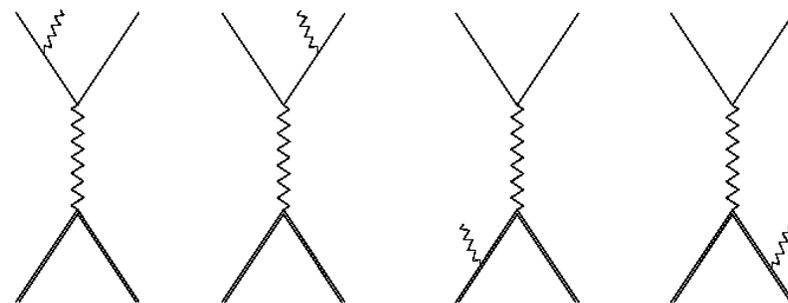
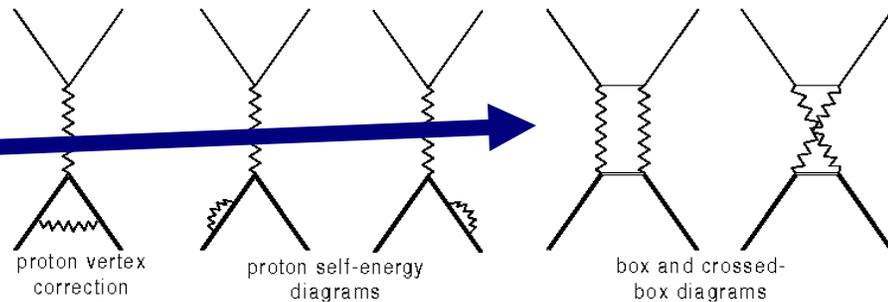
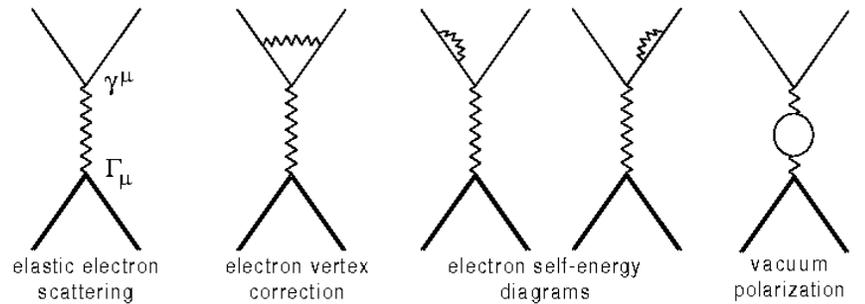
Missing effect is

- approximately linear in  $\epsilon$
- not strongly  $Q^2$  dependent

Two-photon exchange

Bremsstrahlung

- SuperRosenbluth (detect proton)



inelastic amplitudes

## Comments on radiative corrections

- Radiative corrections different depending on whether **electron** or **proton** is detected.

well understood

- **Soft bremsstrahlung**

involves long-wavelength photons

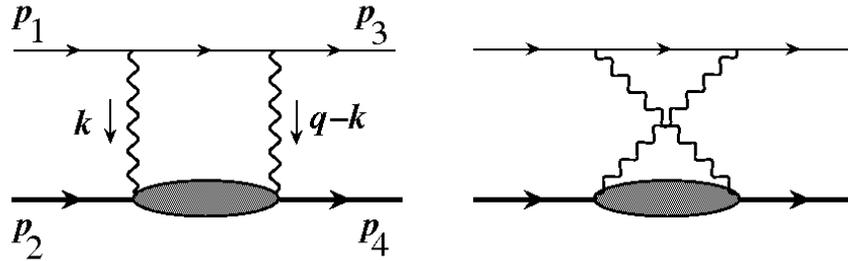
independent of hadronic structure

- **Box diagrams** (TPEX  $M_{\gamma\gamma}$ ) involve photons of all wavelengths

long wavelength (soft photon) part is included in radiative correction (IR divergence is cancelled with electron proton bremsstrahlung interference)

also independent of hadronic structure (by construction)

# Hadronic approach: $N, \Delta, \dots$ intermediate states



Obeys gauge invariance and crossing symmetry

Crossed box from box by  $\mathbf{p}_1 \rightarrow -\mathbf{p}_3$

Consider  $\Delta = \delta_{2\gamma} - \delta_{\text{IR}}(\text{MT})$

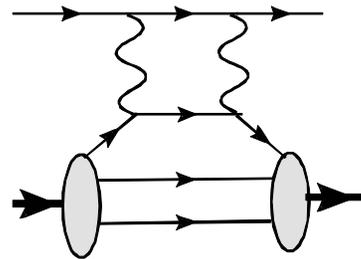
$$\delta_{2\gamma} = \frac{2\Re \{ M_\gamma^\dagger M_{2\gamma} \}}{|M_\gamma|^2}$$

$\delta_{\text{IR}}(\text{MT})$  is standard Mo & Tsai correction (soft photon exchange), which is  $\varepsilon$ -independent & IR divergent

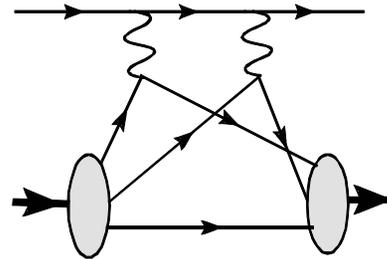
IR divergent terms cancel in  $\Delta$

# Partonic (GPD) calculation of two-photon exchange contribution

(Chen et al.)



"handbag"



"cat's ears"

valid at large  $Q^2$  :  $\delta^{\text{hard}}$

handbag diagrams (one active quark)

to reproduce the IR divergent contribution at nucleon  
correctly (Low Energy Theorem):  $\delta^{\text{soft}}$

need cat's ears diagrams (two active quarks)

## Nucleon elastic contribution (BMT)

Model form factors used as input  
in calculation

magnetic proton form factor  
Brash et al. (2002)

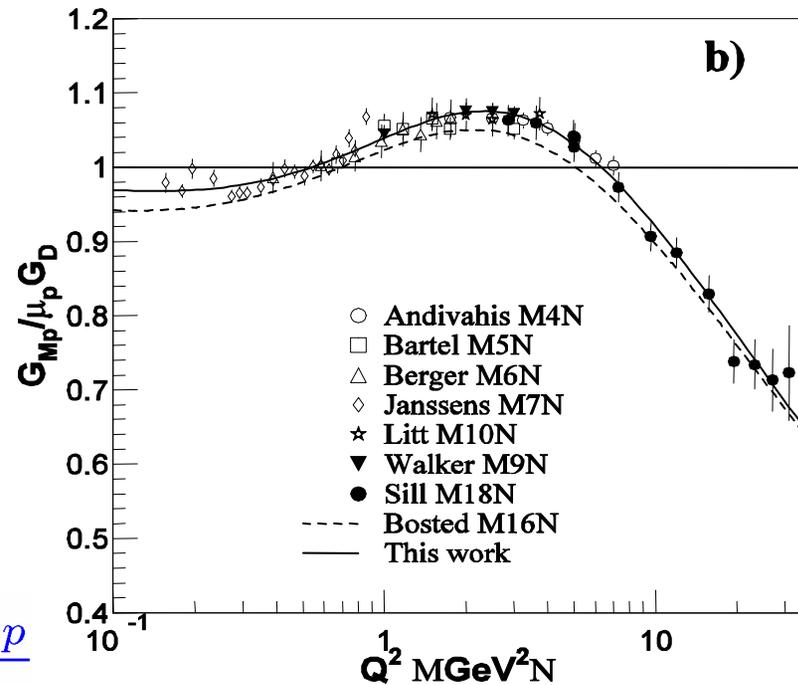
electric proton form factor :

$G_E/G_M$  of proton fixed from  
polarization data  
Gayou et al. (2002)

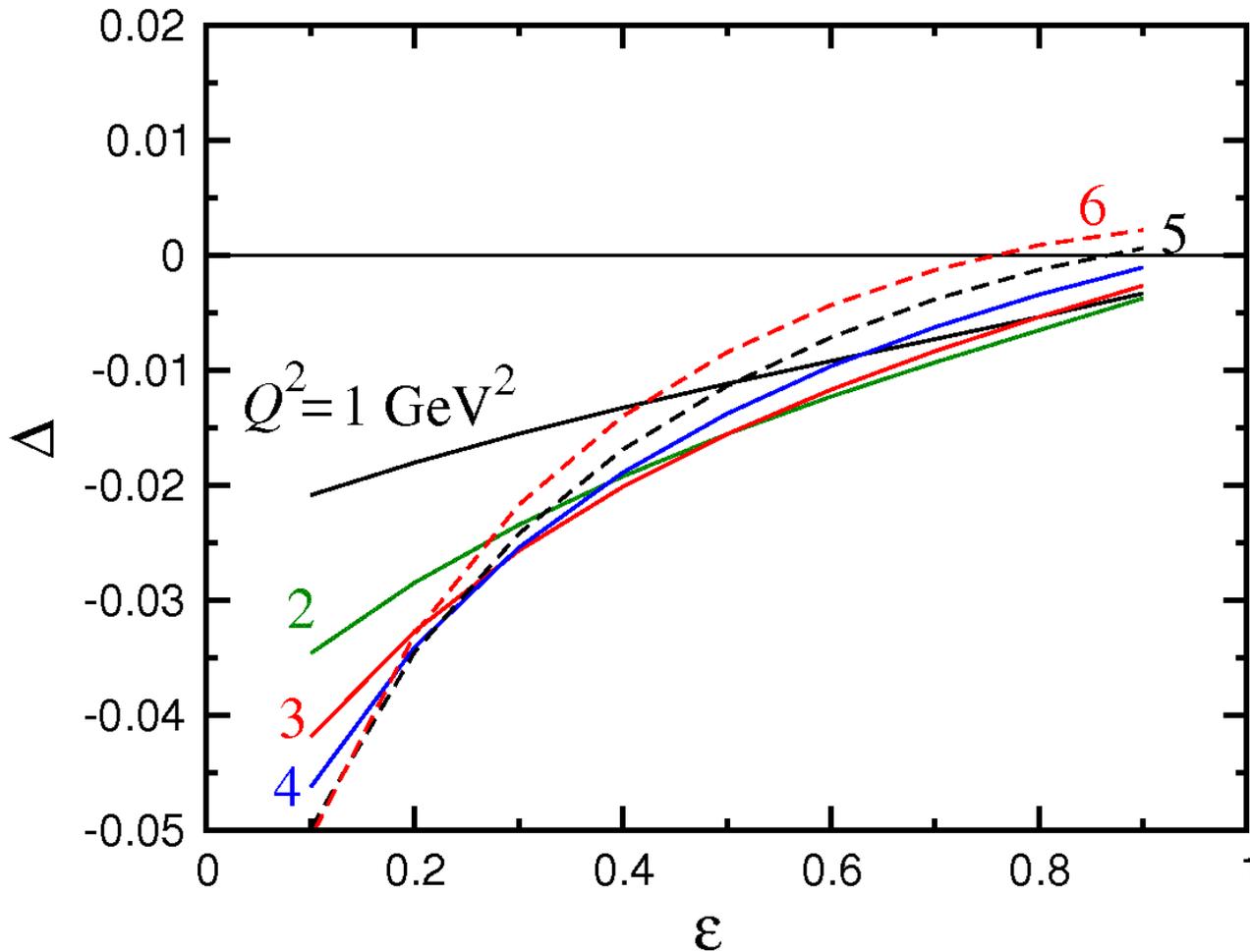
$$G_{Ep} = (1 - 0.13(Q^2 - 0.04)) \frac{G_{Mp}}{\mu_p}$$

Parametrize as sum of monopoles  
→ maintains analytic form of result

Numerical results not terribly sensitive to model for  
 $G_E$ , or to details of  $G_M$



## Corrections to unpolarized cross sections for $Q^2=1$ to $6 \text{ GeV}^2$



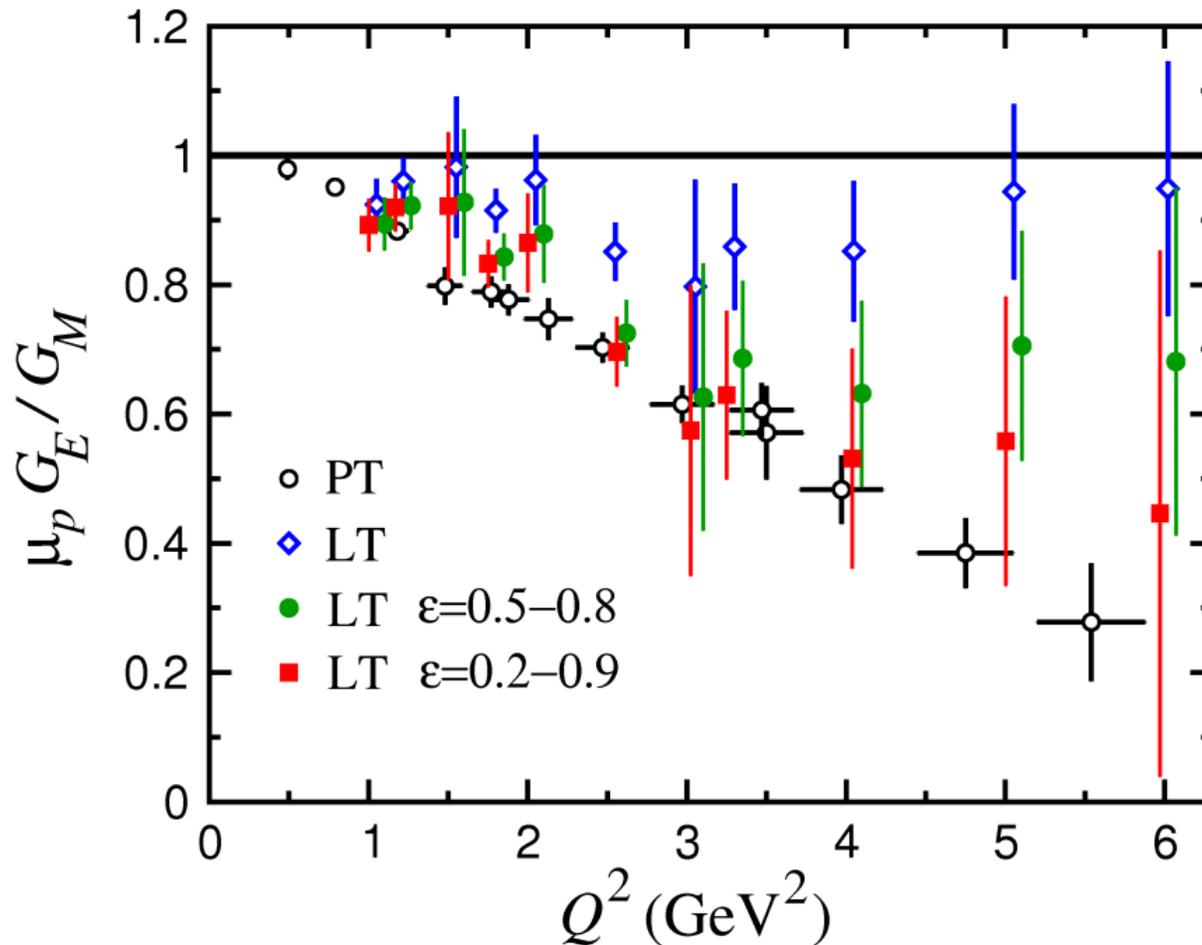
Effect largest at small  $\epsilon$  (backward angles)

Small effect as  $\epsilon \rightarrow 1$   
 $\epsilon \sim 1 - Q^2/(2 E^2)$

Nonlinearity grows with  $Q^2$

JLAB E05-017 (Arrington) will set limits on nonlinearity

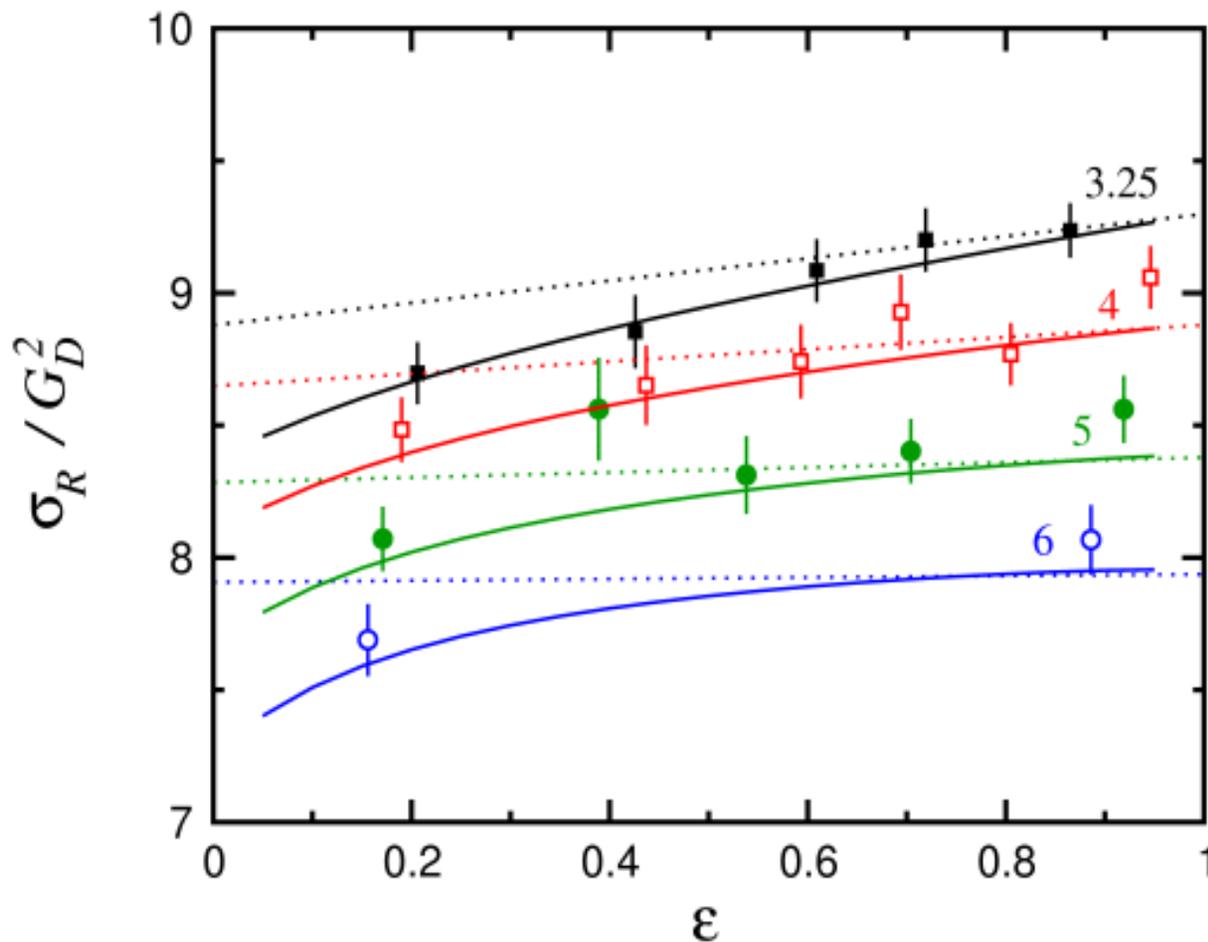
## Effect on ratio R



NOT a refit of data

Simple model: correct  
Rosenbluth data  
assuming TPEX  
correction is linear  
in  $\epsilon$  over a certain  
range

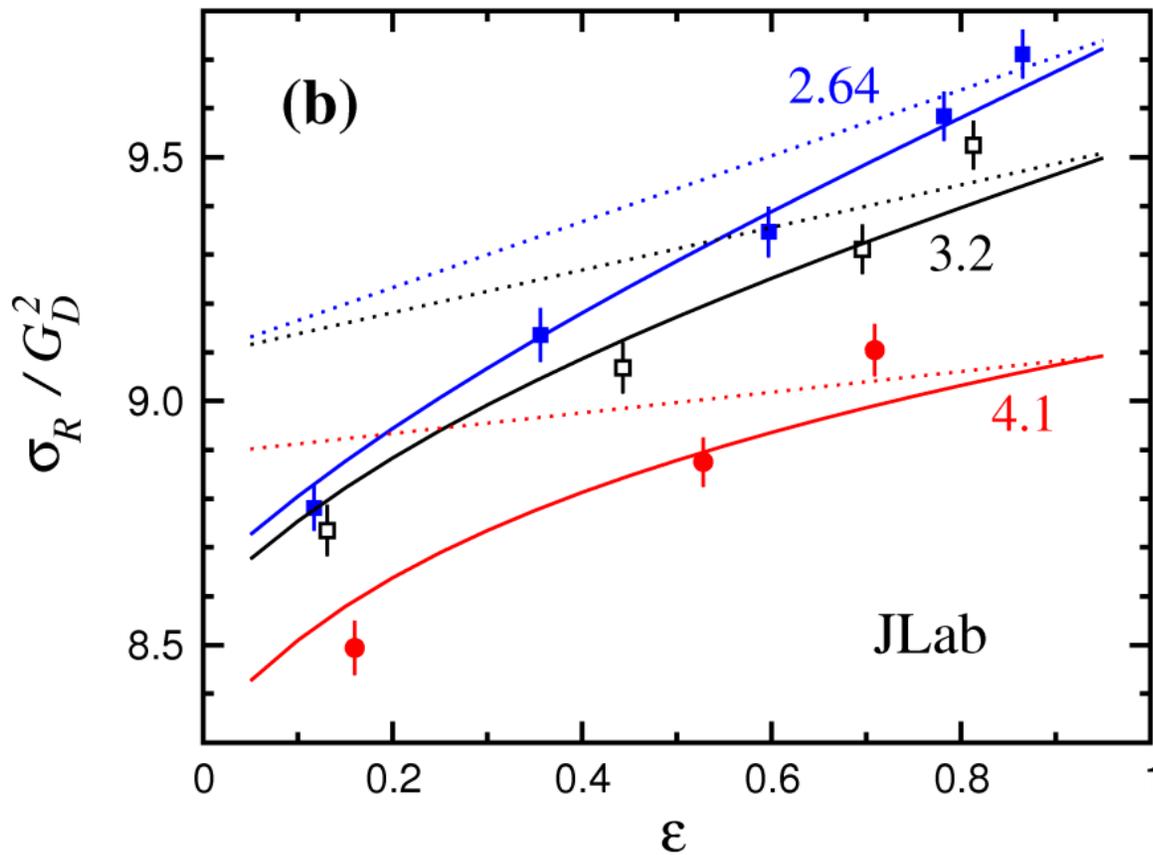
Effect on SLAC reduced cross sections at different  $Q^2$   
(normalized to dipole  $G_D^2$ )



Nonlinearity in  $\epsilon$  is displayed here

JLAB proposals to measure nonlinearity

## SuperRosenbluth (JLAB) data

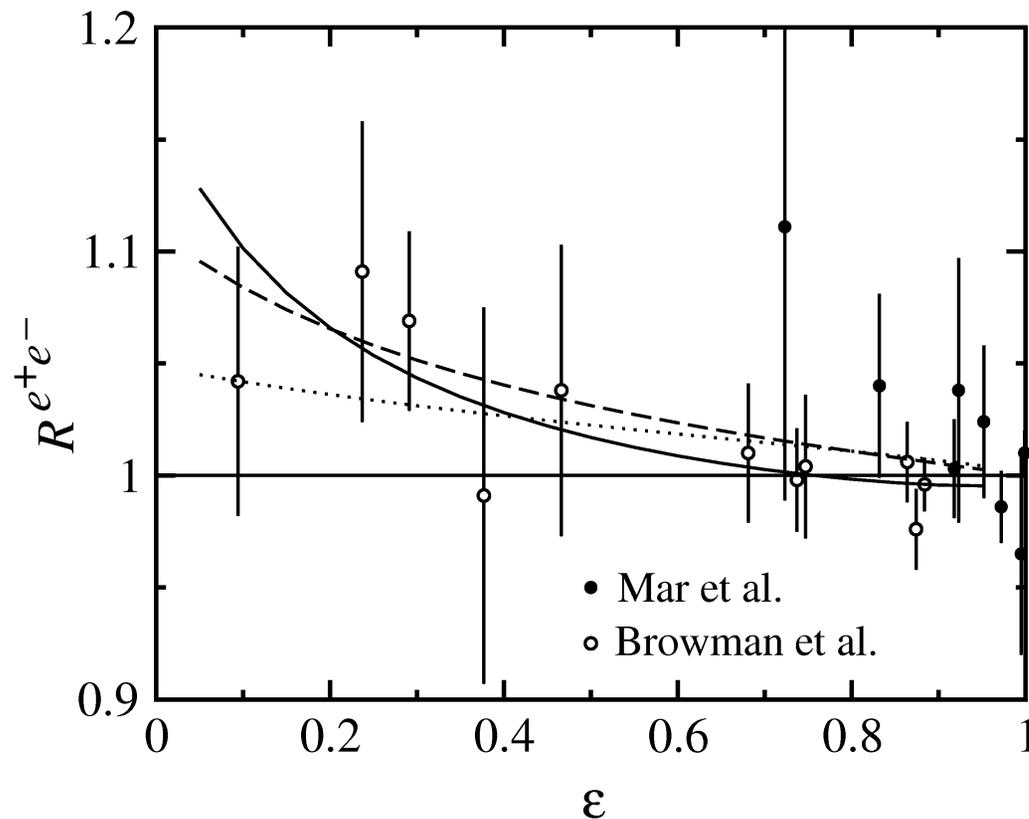


Curves shifted by

+1.0%	2.64
+2.1%	3.20
+3.0%	4.10

(Effect on  
determination of  $G_M$ )

Effect on ratio of  $e^+p$  to  $e^-p$  cross sections (SLAC,  $Q^2$  from 0.01 to 5  $\text{GeV}^2$ )



$M_{\text{Born}}$  opposite sign for  $e^+p$  vs.  $e^-p$ , so enhancement instead of suppression as  $\epsilon \rightarrow 0$

$$R(e^+p/e^-p) \approx 1 - 2\Delta$$

Curves are elastic results for  $Q^2=1, 3, 6 \text{ GeV}^2$

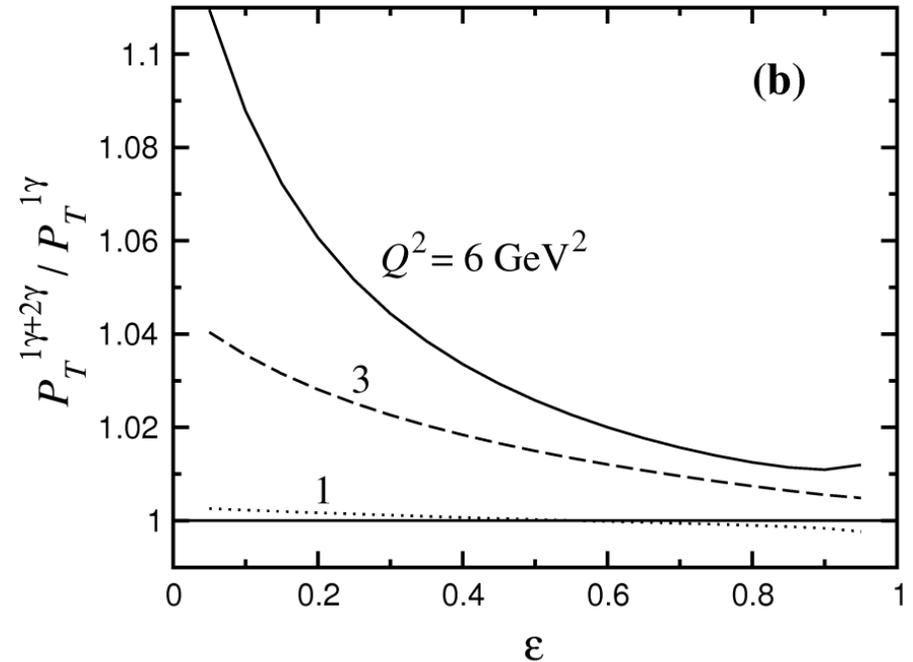
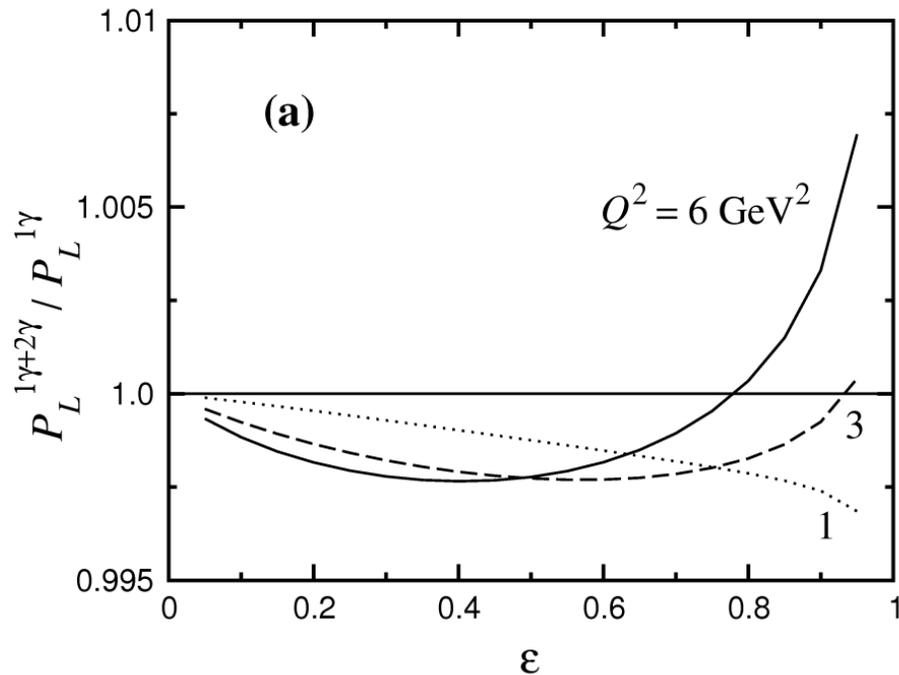
Proposed expts.

E04-116  $Q^2 < 2 \text{ GeV}^2$

VEPP-3  $Q^2=1.6 \text{ GeV}^2, \epsilon \approx 0.4$

$$\vec{e} + p \rightarrow e + \vec{p}$$

Corrections to  $P_L$  and  $P_T$  at  $Q^2=1, 3, \text{ and } 6 \text{ GeV}^2$



$P_T/P_L$  will show some variation with  $\epsilon$ , esp. at low  $\epsilon$   
 JLab data taken at  $\epsilon \sim 0.7$

JLAB expt (Gilman) will measure  $P_T/P_L$  at low  $\epsilon$   
 GPD calculation predicts suppression of  $P_T/P_L$

## Resonance ( $\Delta$ ) contribution:

$$\gamma(q^\alpha) + \Delta(p^\mu) \rightarrow N$$



- Lorentz covariant form
- Spin  $\frac{1}{2}$  decoupled
- Obeys gauge symmetries

$$p_\mu \Gamma^{\alpha\mu}(p, q) = 0$$

$$q_\alpha \Gamma^{\alpha\mu}(p, q) = 0$$

$$\begin{aligned} \Gamma_{\gamma\Delta\rightarrow N}^{\alpha\mu}(p, q) = & \frac{ieF_\Delta(q^2)}{2M_\Delta^2} \{ g_1 (g^{\alpha\mu} \not{p} \not{q} - p^\alpha \gamma^\mu \not{q} - \gamma^\alpha \gamma^\mu p \cdot q + \gamma^\alpha \not{p} q^\mu) \\ & + g_2 (p^\alpha q^\mu - g^{\alpha\mu} p \cdot q) \\ & + (g_3/M_\Delta) (q^2 (p^\alpha \gamma^\mu - g^{\alpha\mu} \not{p}) + q^\alpha (q^\mu \not{p} - \gamma^\mu p \cdot q)) \} \gamma_5 T_3 \end{aligned}$$

3 coupling constants  $g_1$ ,  $g_2$ , and  $g_3$

At  $\Delta$  pole:

$g_1$	magnetic
$(g_2 - g_1)$	electric
$g_3$	Coulomb

Take dipole FF  $F_\Delta(q^2) = 1/(1 - q^2/\Lambda_\Delta^2)^2$  with  $\Lambda_\Delta \approx 0.84$  GeV



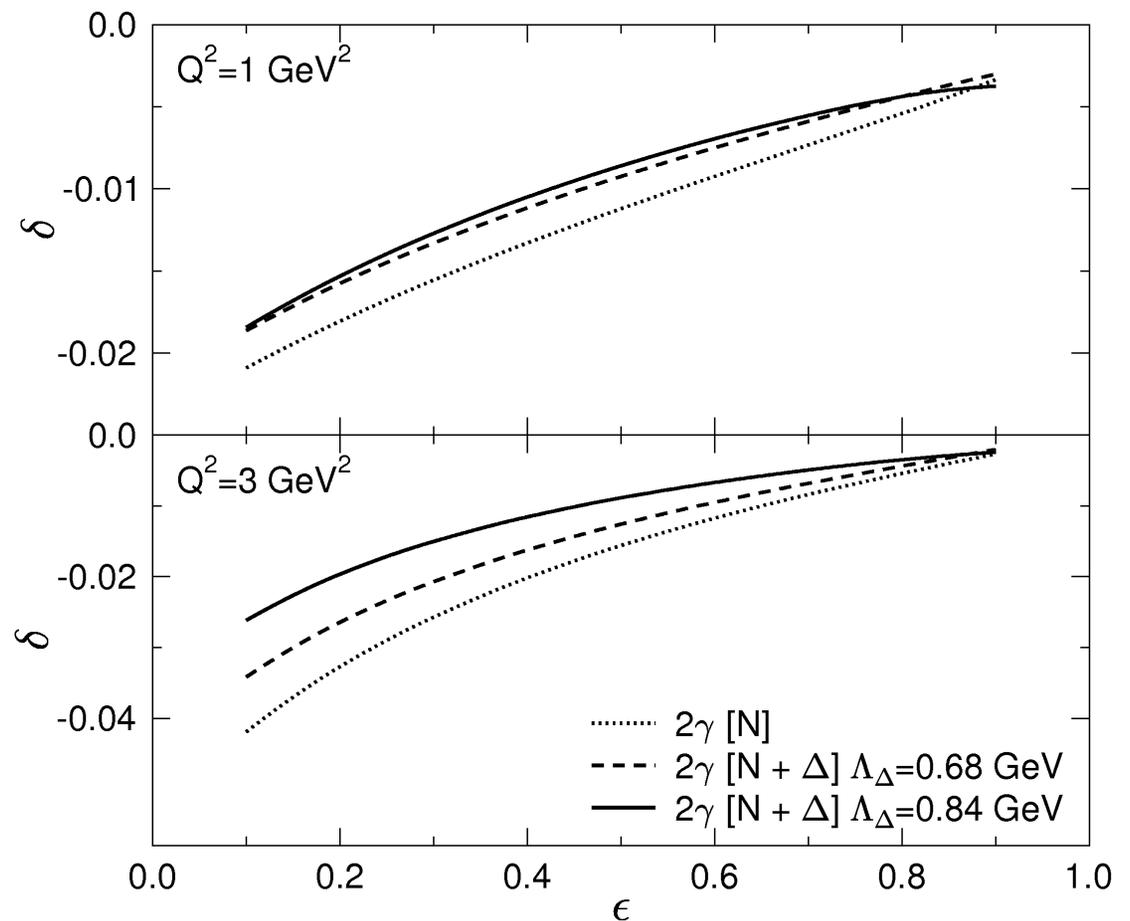
No infrared divergences (since  $M_\Delta > M_N$ )

The  $\gamma N \Delta$  vertex was used in Dressed K-matrix model (Kondratyuk and Scholten) to describe pion photoproduction,  $\pi N$  scattering, Compton scattering at low to medium energies

$g_1$  and  $g_2$  taken from fits to E2/M1 ratio

Coulomb contribution  $\sim (g_3)^2$  and is small, independent of sign

- Smaller than nucleon contribution for reasonable range of parameters
- Becomes more important as  $Q^2$  increases
- Partially cancels the nucleon only contribution at backward angles
- Reduces nonlinear  $\epsilon$  dependence somewhat

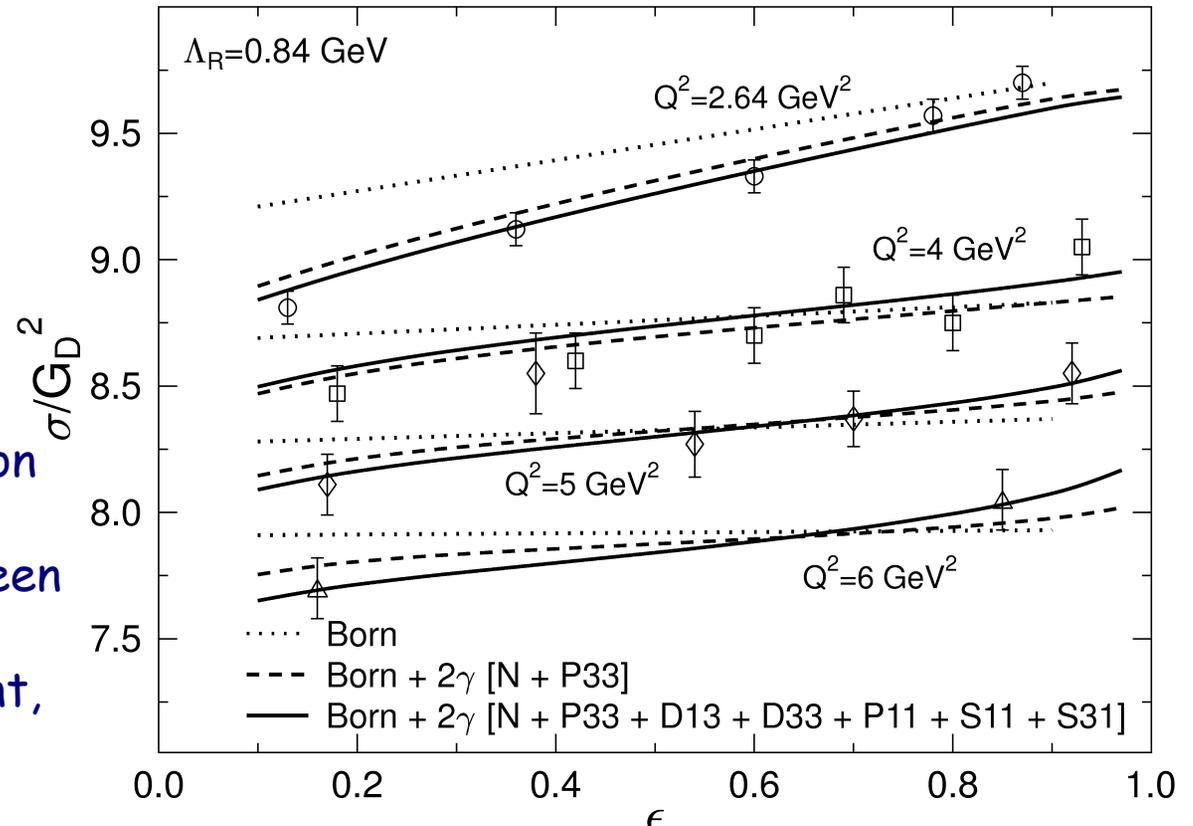


## Other resonances

- **N (P11),  $\Delta$  (P33) + D13, D33, P11, S11, S31**
- Parameters from dressed K-matrix model

### Results

- contribution of heavier resonances much smaller than **N** and  **$\Delta$**
- **D13** next most important (consistent with second resonance shape of Compton scattering cross section)
- partial cancellation between spin 1/2 and spin 3/2
- leads to better agreement, especially at high  $Q^2$



## Global Analysis (Arrington et al, nucl-ex/0707.1861)

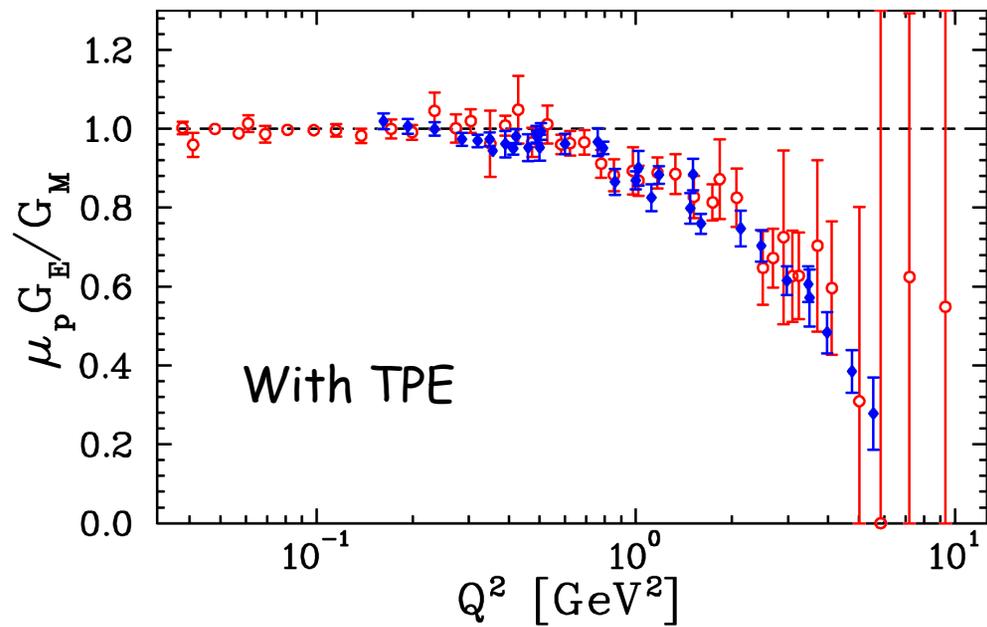
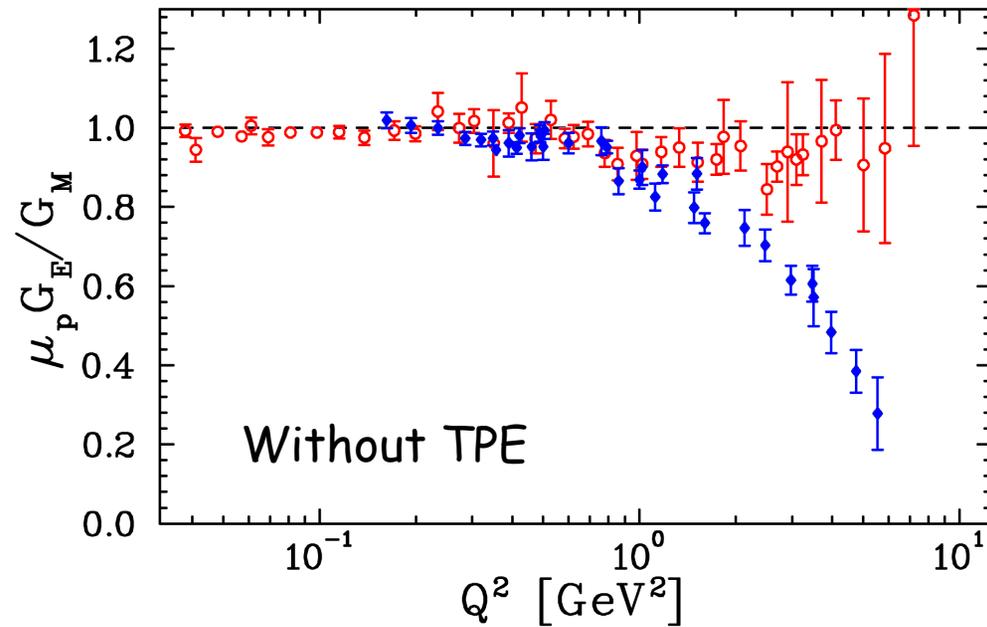
- Incorporate TPE effects directly into analysis of Rosenbluth and PT data
- Extract  $G_E$  and  $G_M$  over range of  $Q^2$
- Input: Estimate of  $Q^2$  dependence of higher resonances from hadronic and GPD calculations

$$\delta_{2\gamma}^* = 0.01 (\varepsilon-1) \ln Q^2 / \ln 2.2; \quad Q^2 > 1 \text{ GeV}^2$$

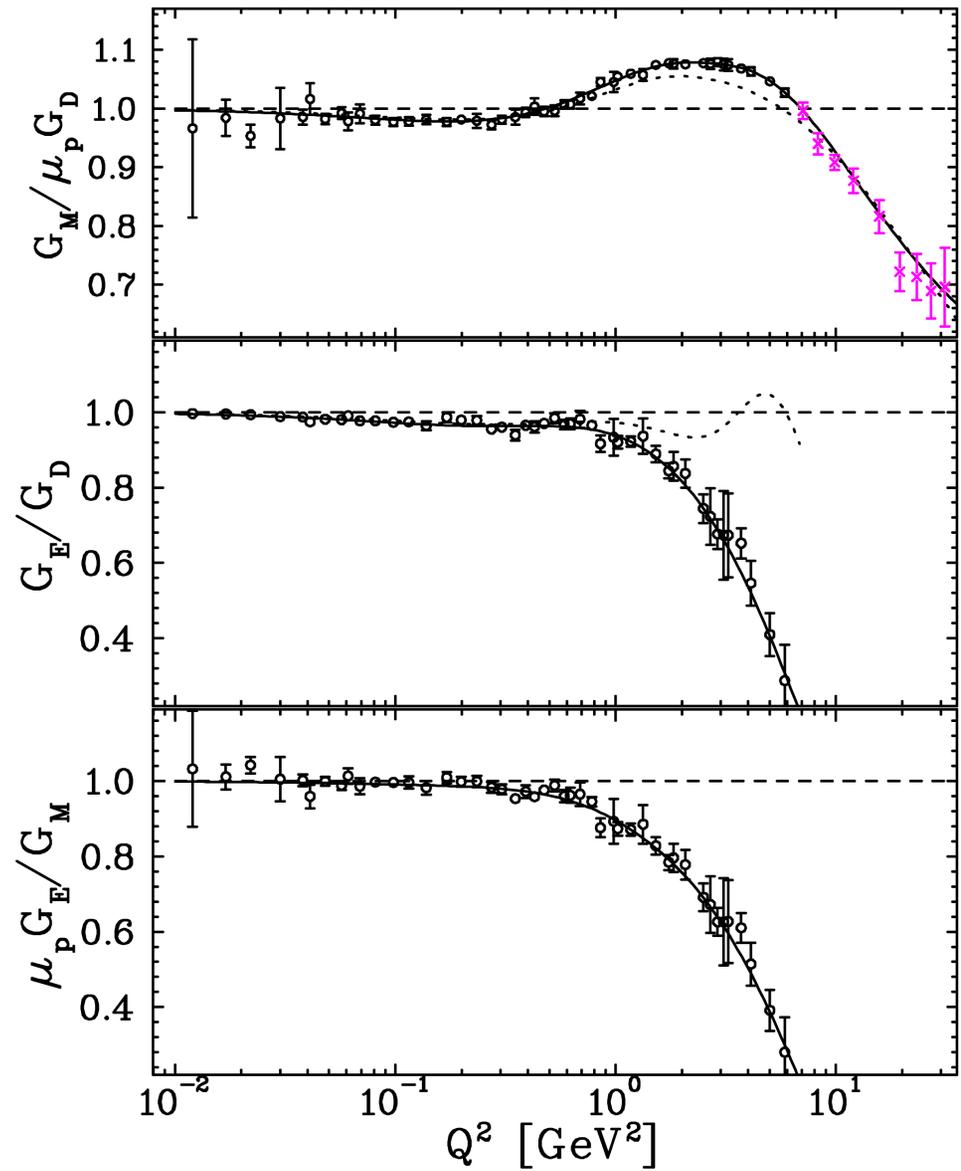
together with nucleon elastic contribution, with 100% uncertainty

- linear in  $\varepsilon$
- decreases cross section by 1% at  $Q^2 = 2.2 \text{ GeV}^2$
- Hadronic and GPD agree TPE corrections to PT data are small ( $\sim 2\%$ ), but give opposite signs
  - Don't include in analysis of PT data

# Effect on ratio R



# Extraction of $G_M$ and $G_E$



## Effect on Parity-violating asymmetry in elastic e+p

$$A_{PV} = \frac{2\Re \{ M_\gamma^\dagger M_Z \}}{|M_\gamma|^2}$$

Weak radiative corrections  
interfere with  $M_\gamma$   
Electromagnetic radiative  
corrections interfere with  $M_Z$

Afanasev and Carlson used generalized form factors to analyze effect on A (GPD model)

$$A_{PV} = -\frac{G_F Q^2}{e^2 \sqrt{2}} \frac{A_E + A_M + A_A + A'_M + A'_A}{\sigma_R}$$

$A'_M$  and  $A'_A$  are new terms

What is effect at low  $Q^2$  (e.g. G0, Qweak, SAMPLE)?

Qweak At low  $Q^2$ , forward angles ( $\varepsilon \rightarrow 1$ )

$$A_{PV} \approx -\frac{G_F Q^2}{e^2 \sqrt{2}} (A + B Q^2)$$

$A = (1 - 4 \sin^2 \theta_W)$  independent of hadron structure

$B$  = hadronic correction

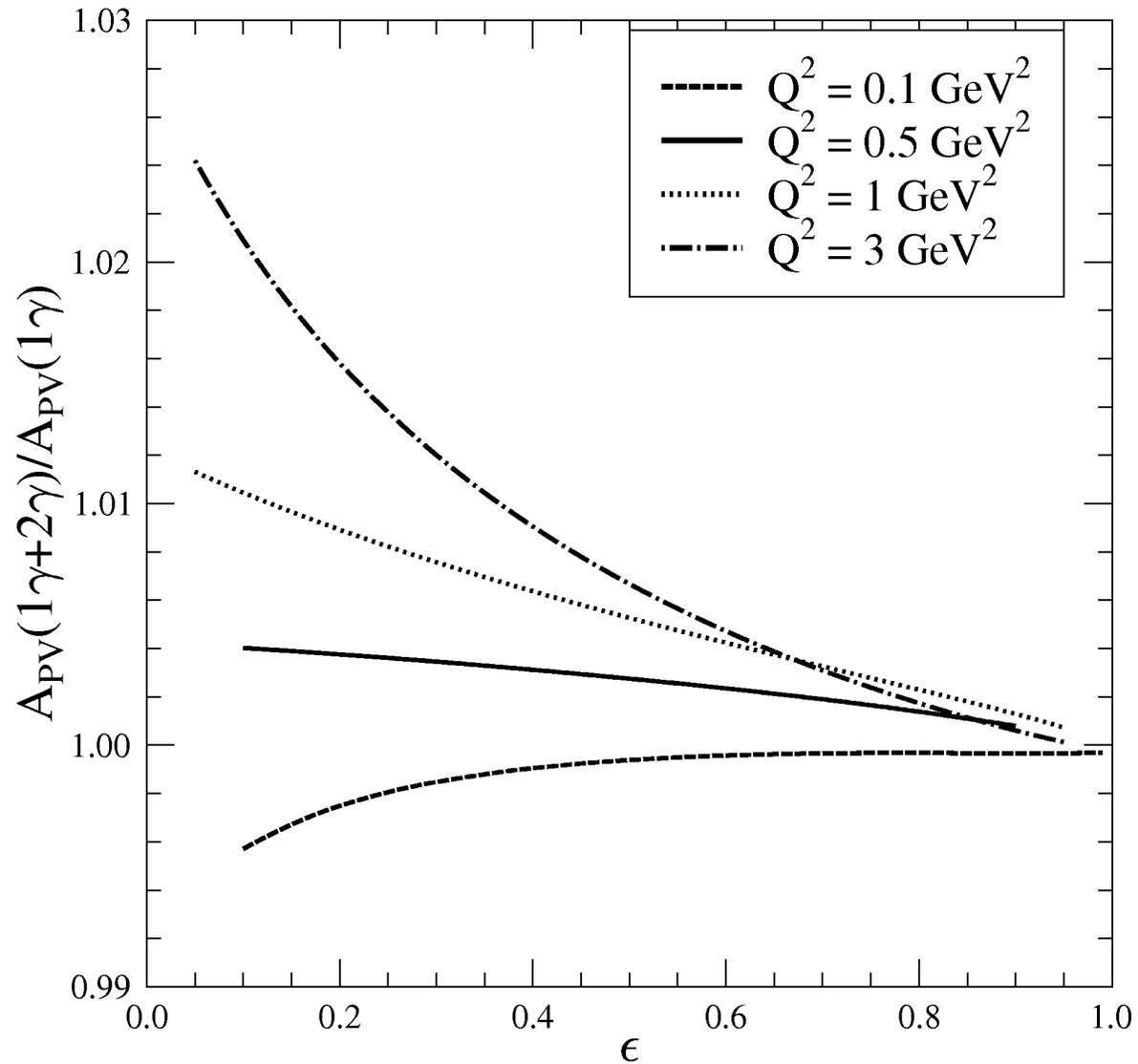
Qweak aims for a 2% measurement of  $A_{PV}$

Though not obvious at first glance,  $A_M'$  and  $A_A'$  are of order  $Q^2$

Our corrections to  $A$  vanish as  $\varepsilon \rightarrow 1$

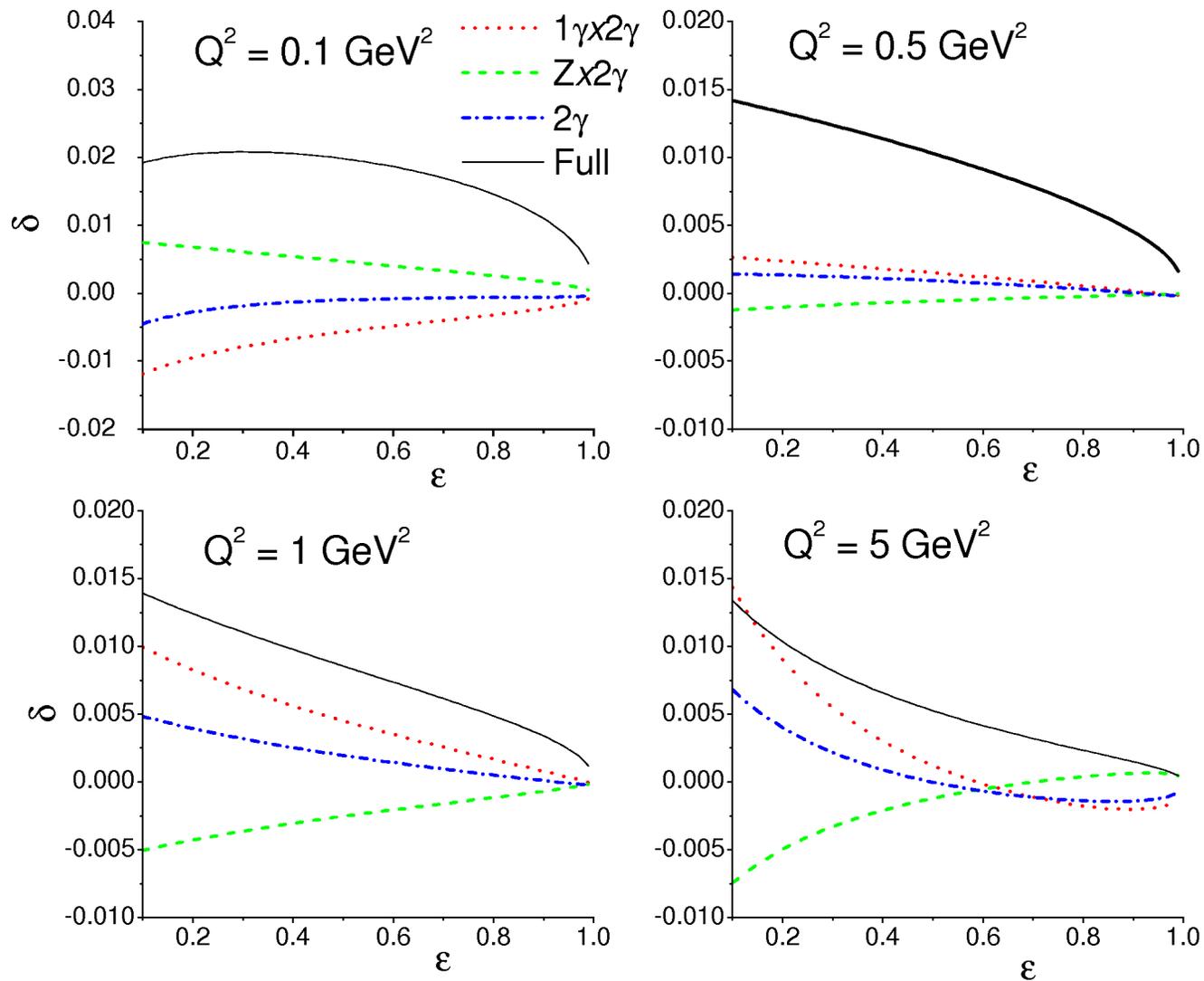
At Qweak kinematics, TPEX correction is **-0.05%**

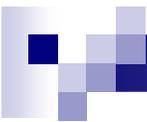
$A_{pV}$  vs.  $\epsilon$  for  $Q^2 = 0.1, 0.5, 1.0, 3.0 \text{ GeV}^2$



# $\gamma Z$ electroweak as well as TPE

Hadronic model, Zhou et al. (hep-ph/0708.4297)





# Outlook

## Theory

- Connect real and imaginary parts of TPEX amplitude
  - more work needs to be done on hadronic models
- Look at sensitivity to off-shell form factors (preliminary work indicates probably not a large effect)
- Use phenomenological input from Compton scattering at high  $Q^2$  to constrain high mass spectrum and/or merge with GPD

## Experiment

- $e^+p/e^-p$  ratio
- look for nonlinearity in  $\varepsilon$ 
  - E04-019/E04-108 for PT
  - E05-017 for cross section (recently completed)

Collaborators: Melnitchouk, Tjon + Kondratyuk ( $N+\Delta$ ), Kondratyuk (resonances) + Scholte ( $A_{pV}$ )