Heavy-light mesons on a lattice

Contributors

UKQCD Collaboration

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The lattice data

- We study $Q\bar{q}$, where Q is static

  and $m_q \approx 1.3 m_{strange} \rightarrow 0.3 m_{strange}$

  i.e. $'m_\pi' \approx 750 \rightarrow 400$ MeV

- Calculate on a lattice.

  S, P+, P–, D+, D–, D+/–, F+/– energies

  S, P–, P+-wave charge and matter densities.

  Also for excited states with one node.

  L+ means J=L+1/2 and L– means J=L–1/2

  since Q-spin decouples.

- Three simplifications cw B_s data:

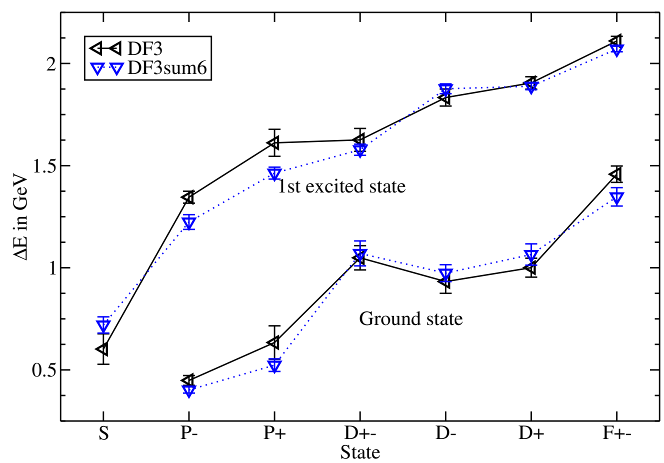
  i) We know what the data refers to – $Q\bar{q}$.

  ii) A lot of it.

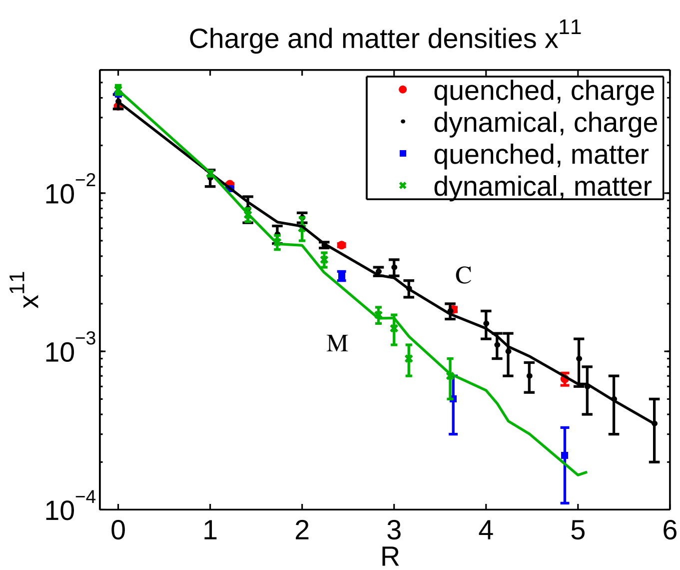
  iii) Basically it is a one-body problem.

Typical lattice data

- Energies

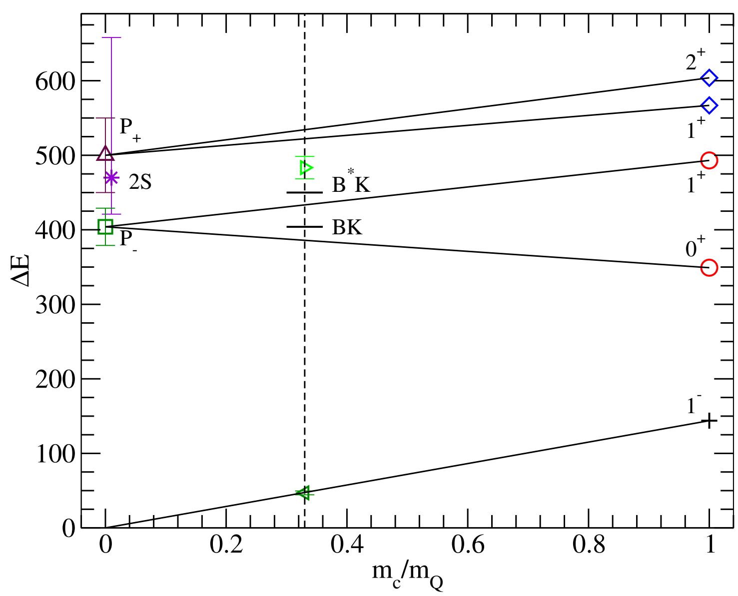


- Radial distributions– S-wave vector/scalar.



USE 1 – B_s PREDICTIONS

	Theory	Prediction	Experiment
$m_Q =$	∞	m_b	m_c



- $1/m_Q$ interpolation seems to be good.
- Predicts several narrow B_s-meson states. Some near BK and B^*K thresholds and some decay as D-waves.
- So these could be narrow.

Use 2 – Modelling

- Basically a one-body problem
- How much can be understood/fitted with a Dirac/Shroedinger equation?
- Model input:

1) Type of equation – Dirac, Shroedinger....?

2) Potentials

i) Scalar/vector linear rising – $(b+c\gamma_0)R$

ii) One gluon exchange – e/R

iii) ???

3) Mass of quark – m_q

Lattice parameters (Typical)

- Lattice size $16^3 \times 24$ and $16^3 \times 32$
- Lattice spacings $a \approx 0.15, 0.10$ fm

  i.e. lattice extent $L = 2.4, 1.6$ fm
- Dynamical fermions

  Work on four sets of configurations

  $\kappa = 0.1395$ Tadpole improved and

  $\kappa = 0.1350, 0.1355, 0.1358$ Non-perturbative.

  Giving $m_q \approx 1.3 m_s \to 0.3 m_s$

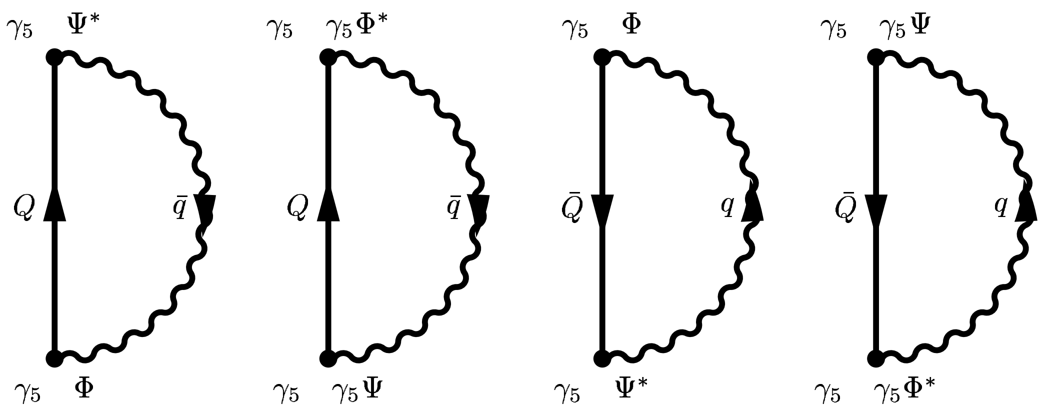
  $\frac{M_{PS}}{M_V} = 0.72 \to 0.44$ – not 0.18 as for π/ρ.

  [$\frac{M_{PS}}{M_V} = 0.68$ for m_s — near the B_s meson]
- a and m_q fixed from UKQCD $q\bar{q}$ studies.

  Therefore $Q\bar{q}$ results are predictions.

Extraction of Energies

- **2-point correlation functions $C(2)$**



a) b) c) d)

- Fit $C(2,T)$ with $\tilde{C}(2,T)$

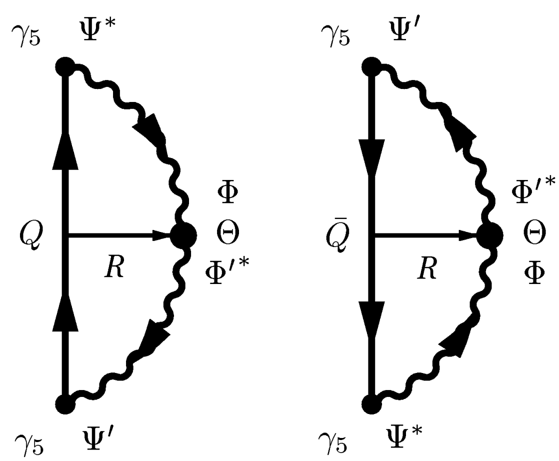
$\tilde{C}(2,T)_{ij} = \sum_{\alpha=1}^{M_2} v_i^\alpha \exp(-E_\alpha T) v_j^\alpha$

with $M_2 \approx 3$ and $i,j \leq 3$

to give E_α and v_i^α for use in distributions.

Extraction of Radial Distributions

- **3-point correlation functions** $C(3, R)$



a) b)

$\Theta = \gamma_4$ – Vector/Charge

$=1$ – Scalar/Matter

- **Radial correlations** $x^{\alpha\beta}(R)$

Fit $C(3, -t_2,\ t_1,\ \mathbf{R})$ with $\tilde{C}(3, -t_2,\ t_1,\ \mathbf{R})$

$\sum_{\alpha,\beta}^{M_3} v_i^{\alpha}\ e^{[-E_\alpha t_1]}\ x^{\alpha\beta}(R)\ e^{[-E_\beta(T-t_1)]}\ v_j^{\beta}.$

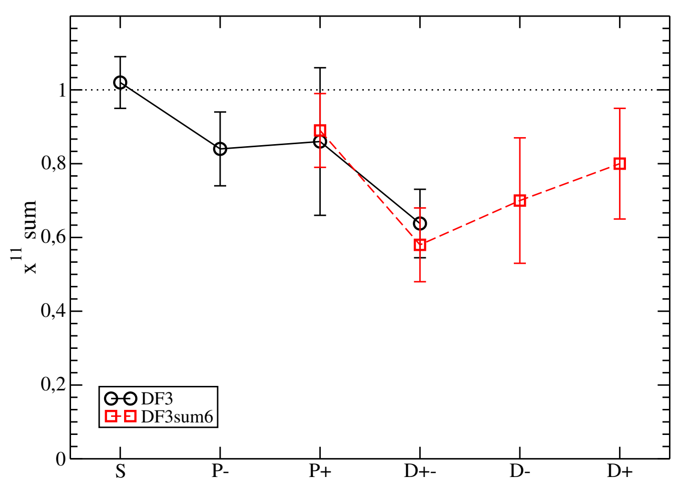
Charge Sumrule – Direct

- Directly from lattice the sum

$C(3, -t_2,\ t_1) = \sum_R C(3, -t_2,\ t_1,\ \mathbf{R})$

- Fit $C(3, -t_2,\ t_1)$ with $\tilde{C}(3, -t_2,\ t_1)$

$\sum_{\alpha,\beta}^{M_3} v_i^\alpha\ e^{[-E_\alpha t_1]}\ x^{\alpha\beta}\ e^{[-E_\beta (T-t_1)]}\ v_j^\beta.$



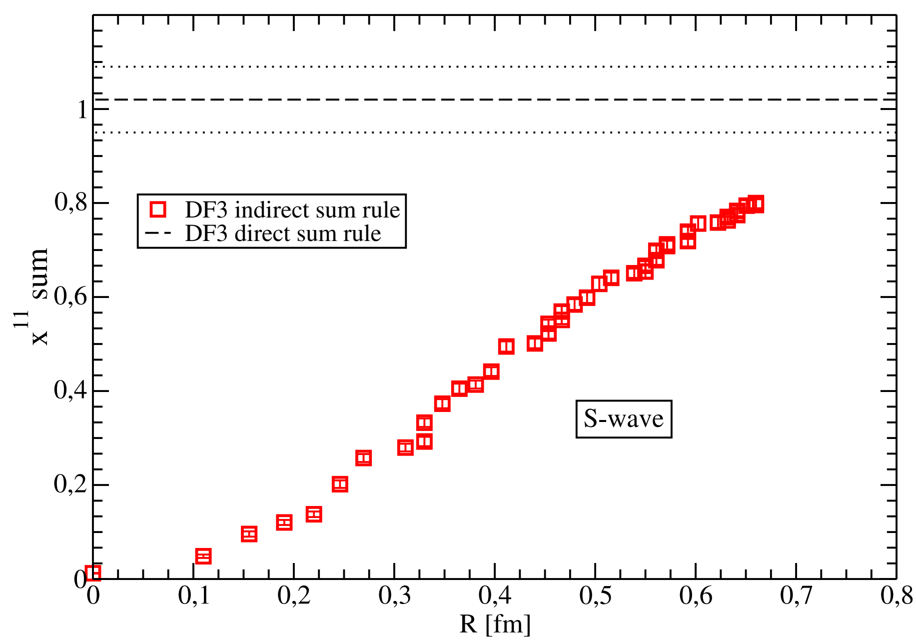
- The $x^{\alpha\beta}$ are model independent
- All should give unity

after $Z_V = 0.773$ vertex correction

Bakeyev et al. PLB 580 (2004) 197

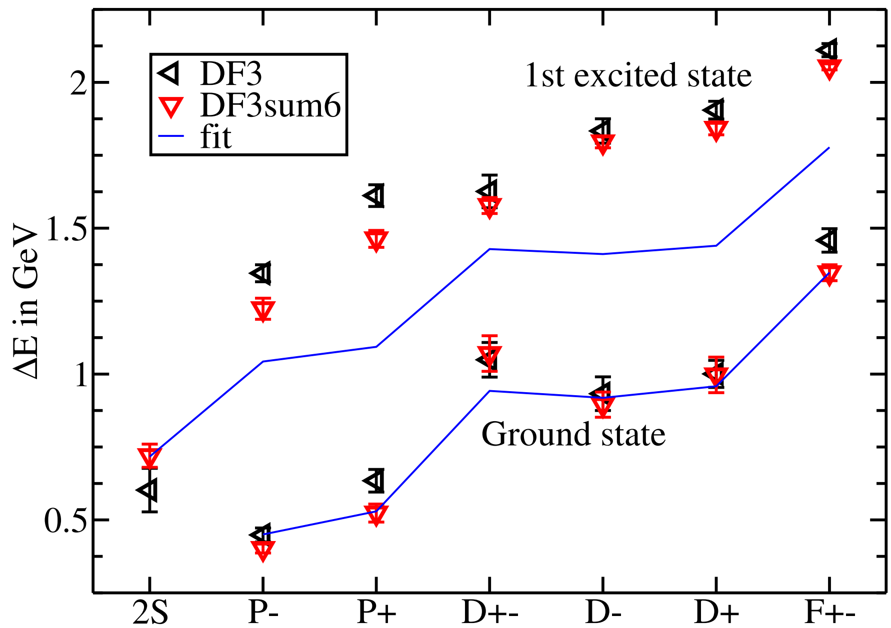
Charge sumrule – Indirect

- Consistency check
- First extract distribution $x^{\alpha\beta}(R)$.
- Calculate $x^{\alpha\beta}[R(max)] = \sum_{R(max)} x^{\alpha\beta}(R)$ with R(max) upto 6a.



- 20% deficit mainly due to $R > 6a$.

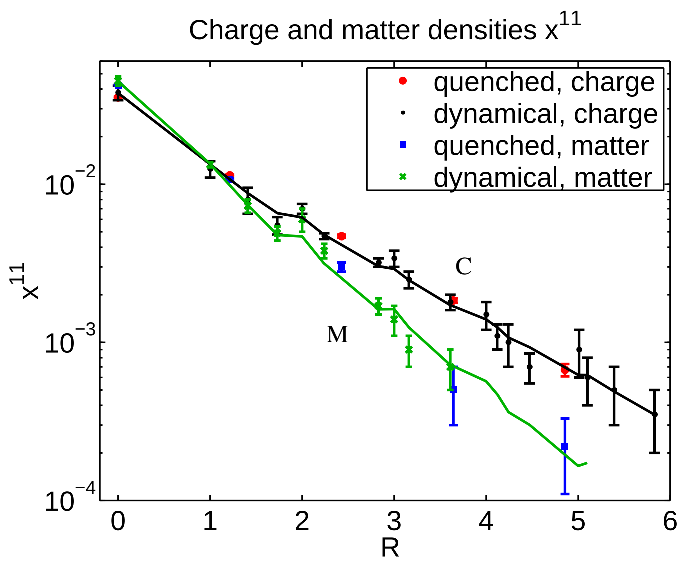
FIT TO ENERGIES



- Perfect fit to eight energy differences.
- Needs "about" three parameters – later.
- Now we live with these for distributions.
- Small 1D and 2D-wave spin-orbit splitting.

  i.e. No Inversion seen (Schnitzer 1989)

x^{11} S-wave distributions – I



- $x_C(0) \approx x_M(0)$ and $x_C(R) \neq x_M(R)$

  follows from Dirac equation

  $x_C(R) = G(R)^2 + F(R)^2$

  $x_M(R) = G(R)^2 - F(R)^2$

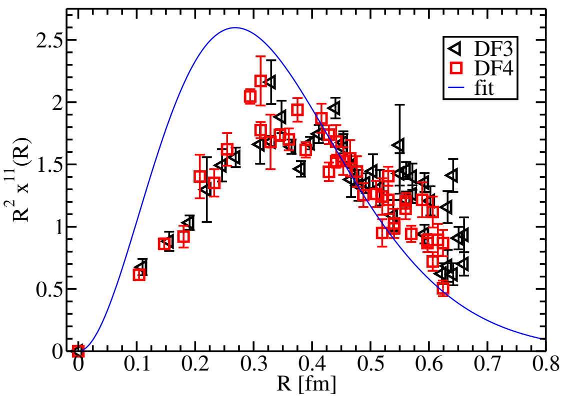
  R small – $G \gg F$ so $x_C(0) = x_M(0)$

  R large – $F \sim G$ so that $x_C(R) > x_M(R)$

- Fit with discretized Yukawa/Exponentials.

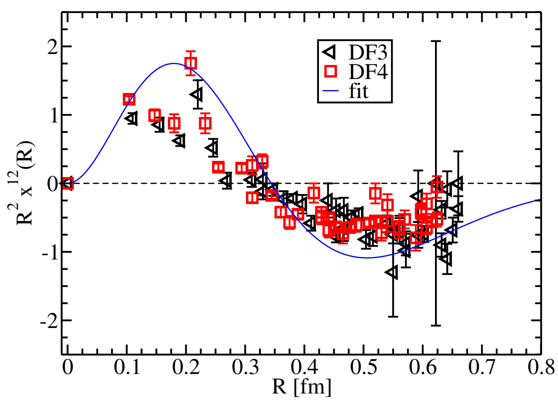
x^{11} S-wave distributions – II

- More informative to plot $R^2 x^{11}$ etc.

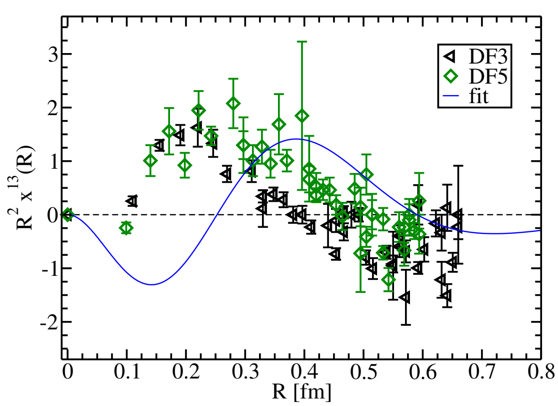


- x^{11}– not well given by Dirac.
- Needs some repulsion.

S-wave distributions - excited states

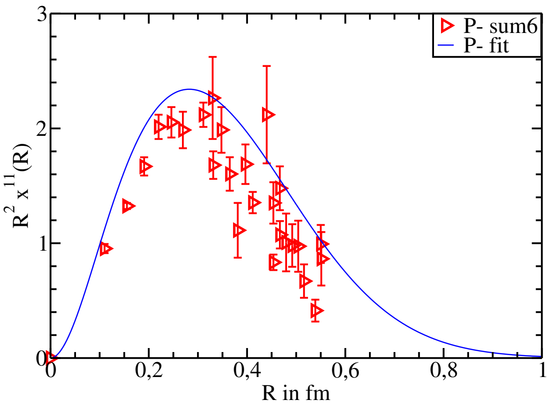


- x^{12}– node consistent with Dirac.

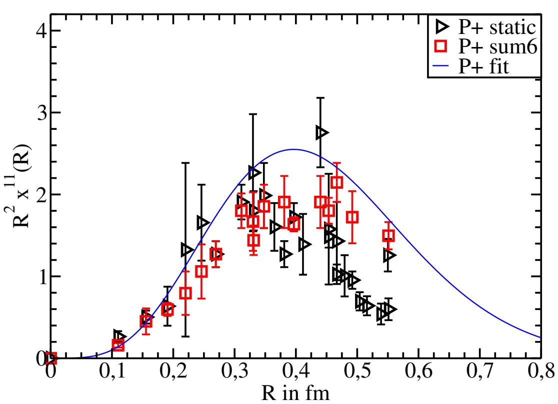


- x^{13}– pushing our luck.
- See two nodes.
- Needs more attraction

P-wave distributions

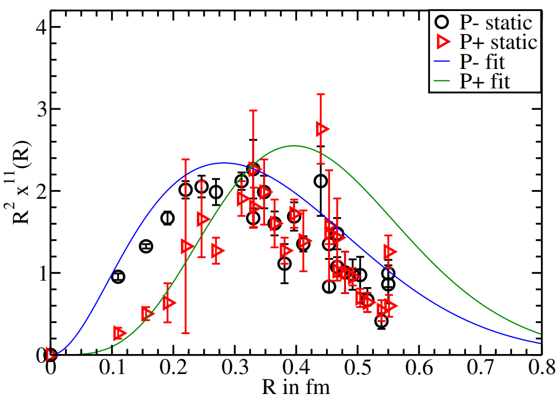


- x^{11}(P−) – agrees with Dirac.



- x^{11}(P+) – agrees with Dirac.

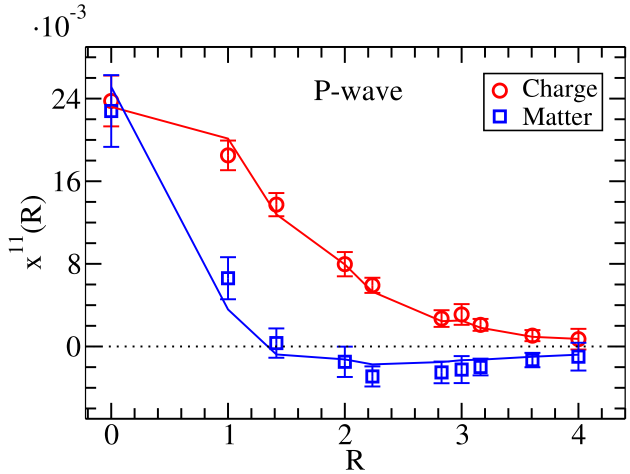
P− versus P+

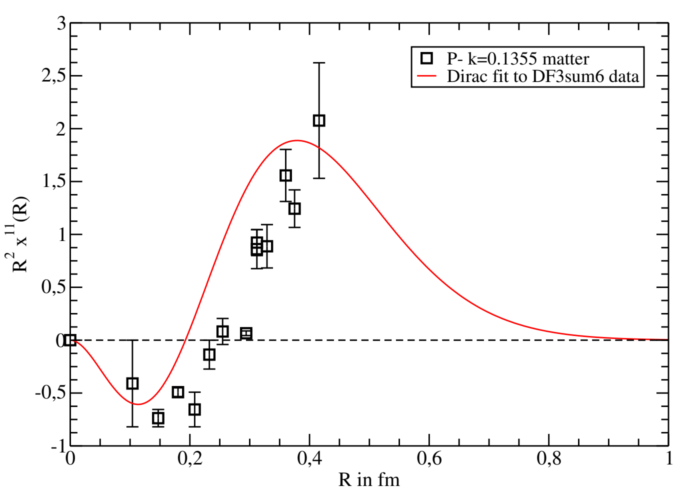


- x^{11}(P−) versus x^{11}(P+) .

P– Charge versus Matter I

- $x^{11}(0) \neq 0$ and node in Matter distribution.





- Again follows from Dirac equation

P– Charge versus Matter II

- $x_C(R) = F(R)^2 + G(R)^2$

  $x_M(R) = F(R)^2 - G(R)^2$

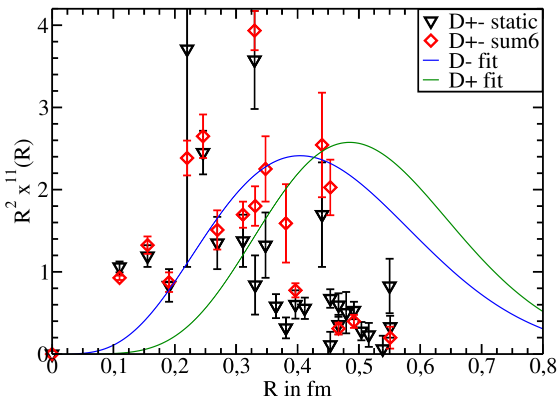
- For the P- state i.e. $P_{1/2}$

  R small – $F \gg G \neq 0$ so $x_C(0) \approx x_M(0)$

- R larger $F < G$

  Therefore a node in $x_M(R)$

D-wave distributions



- x^{11}(D+–) – lattice data poor.
- x^{11}(D–) and x^{11}(D+) are underway.

DIRAC PARAMETERS – I

- Quark mass m_q=0.088 GeV

  Weak dependence – not a parameter.

- Linear rising potential $(b + c\gamma_0)R$.

  Scalar term b=1.14 GeV/fm – sensible.

  Vector term c=1.12 GeV/fm !!!!

  Can well use c=b so that:

$$V(R) = -\frac{a}{R} + bR(1 + \gamma_0) \quad \text{giving} \qquad (1)$$

$$V_{\mathrm{Spin-Orbit}}(R) = \frac{1}{4m^2}(\frac{a}{R^3} + 0) \qquad (2)$$

- So small Spin-orbit splittings (Page et al.)
- So far, out of m_q, b, c there seems to be

  one truly free parameter - $b = 1.14$ GeV/fm.

Dirac parameters – II

- One Gluon Exchange
- Make replacement:

$$\frac{e}{r} \rightarrow \frac{f_0(r)}{r} \quad \text{where} \tag{3}$$

$$f_0(r) = g\frac{2}{\pi}\int_0^\infty dk \frac{\sin(kr)}{k}\alpha_s(k^2). \tag{4}$$

$$\alpha_s(k^2) = \frac{12\pi}{27}\frac{1}{ln[(k^2+4m_g^2)/\Lambda_0^2]}, \tag{5}$$

- $m_g = 290$ MeV, $\Lambda_0 = 260$ MeV, to fit e.g. $c\bar{c}$, $b\bar{b}$ spectra and e^+e^- annihilation.
- We need $g = 0.81$ – reasonable.

Dirac parameters – III

- Quark mass L-dependence
- Make replacement:

  $m_q \to m_q[1 + \omega L(L+1)]$
- We seem to need $\omega = 0.028$

  i.e. $m_q(S) = \ 88, \ m_q(P) = \ 93,$

  $m_q(D) = 103, m_q(F) = 117$ MeV
- Interpret as flux tube rotational energy.
- Is same needed for nodal excitations?
- Outcome: Three important parameters

$$b, \ g, \ \omega$$

Conclusion

- Dirac equation can give a qualitative understanding of the lattice data.
- Seems to need:

  i) A large vector linear rising potential

  i.e. $c \approx b$ in $(b + c\gamma_0)R$.

  ii) A mass L-dependence – $m_q[1 + \omega L(L+1)]$

- But can this become quantitative?

  i) A mass n-dependence?

  ii) Lots of data not yet fitted:

  e.g. Charge and Matter radial distributions

  involving not only ground states

  but nodal excited states.