The Transversity Distribution

And Its Chiral- And/Or T-Odd Friends

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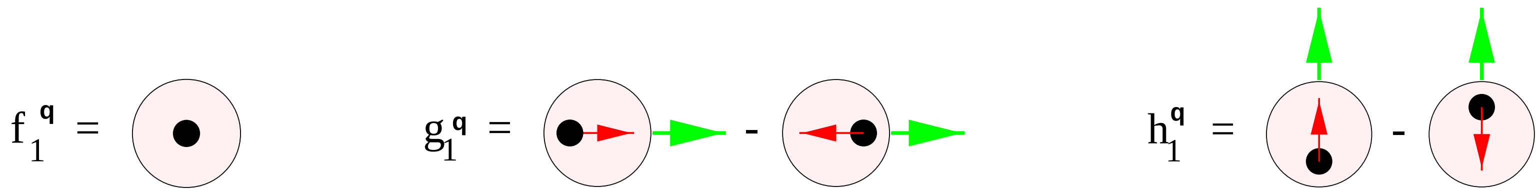
Leading-Twist Forward Quark Distributions



$\Downarrow$ $\Downarrow$ $\Downarrow$

Unpolarized quarks and nucleons

$q(x)$: spin averaged (well known)

$\Rightarrow$ Vector Charge

$\langle PS|\bar{\Psi}\gamma^{\mu}\Psi|PS\rangle$= $\int dx(q(x) - \bar{q}(x))$

Longitudinally polarized quarks and nucleons

$\Delta q(x)$: helicity difference (known)

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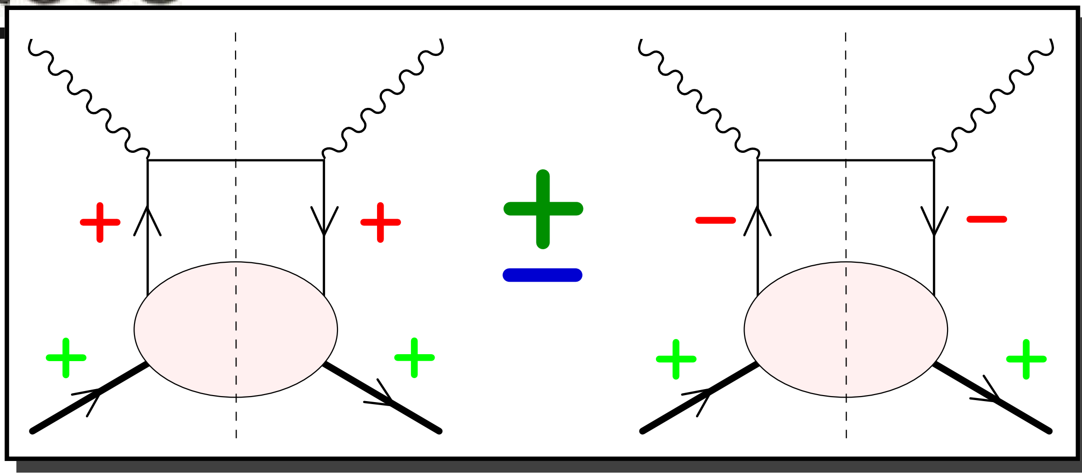

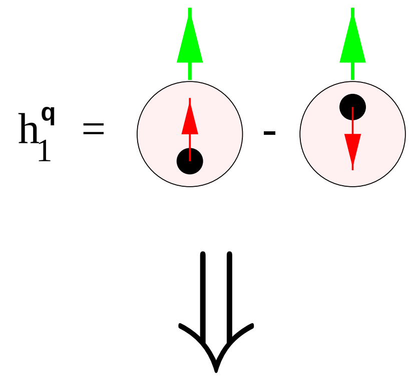

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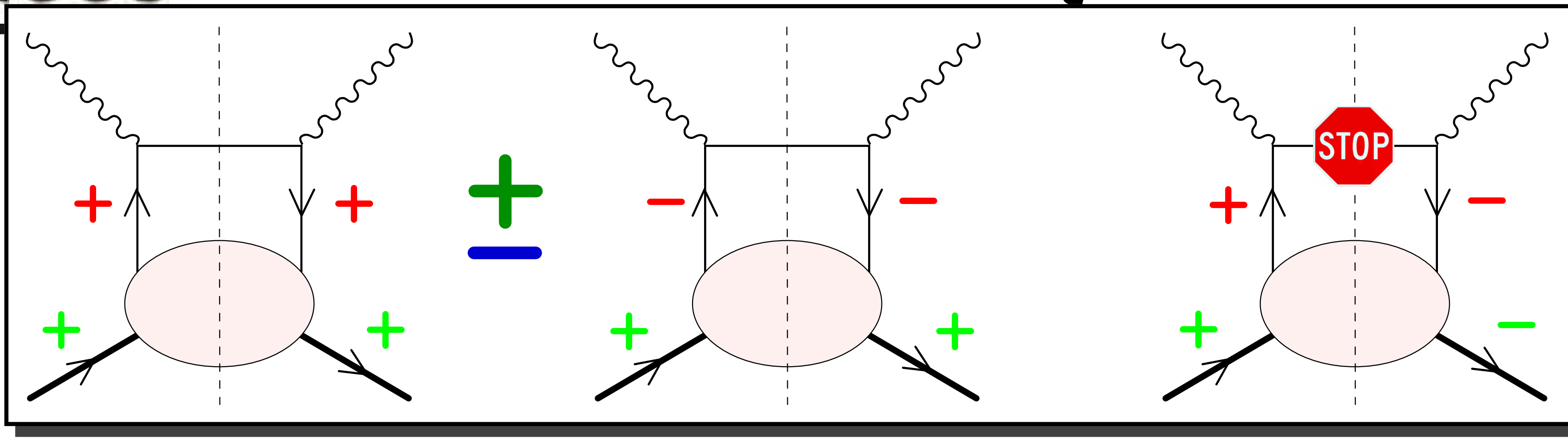

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Transverse[illegible] polarized q[illegible]s and nuc[illegible]s

$\delta q(x)$: tr[illegible]ersity (unm[illegible]red!)

$\Rightarrow$ T[illegible]r Charge

$\langle$[illegible]$^{\mu\nu}\gamma_5\Psi|PS\rangle$= $\int dx($[illegible]$q(x)-\delta\bar{q}(x))$

CHIRAL-ODD!

Transversity Distribution

- Non-relativistic quarks: $\Delta q(x) = \delta q(x)$
  $\Rightarrow \delta q$ probes relativistic nature of quark dynamics

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- transverse spin eigenstates related to helicity eigenstates via $|\perp\top\rangle = \frac{1}{2}(|+\rangle \pm \imath|-\rangle) \Longrightarrow$ transversity ($\langle\perp|\hat{O}|\perp\rangle - \langle\top|\hat{O}|\top\rangle$) flips helicity of quark and nucleon $\Rightarrow \delta q$ chiral odd

$\hookrightarrow$ No Access In Inclusive DIS!

Transversity Measurements

How can one measure transversity?

Need another chiral-odd object!

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$$\sigma^{p^\uparrow h \to l\bar{l}X} = \sum_q \delta q \otimes \sigma^{q\bar{q}\to l\bar{l}} \otimes DF$$

$\Downarrow$ **chiral-odd** DF (for δq)

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together: CHIRAL EVEN

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CHIRAL EVEN

- obvious choice: $h = p^\uparrow \rightsquigarrow \sigma \propto \delta q \otimes \delta\bar{q} \longrightarrow$ RHIC?
- slightly different: $h = \bar{p}^\uparrow \rightsquigarrow \sigma \propto \delta q \otimes \delta q \longrightarrow$ GSI?
- others (later)

Transversity in Drell-Yan

RHIC: $$A_{TT} \propto \frac{\sum_q e_q^2 \left[\delta q(x_1)\delta\bar{q}(x_2) + \delta\bar{q}(x_1)\delta q(x_2)\right]}{\sum_q e_q^2 \left[q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)\right]}$$

- **transversely polarized proton beams available**
- **large $\sqrt{s} \Rightarrow$ small NLO QCD corrections**
  but also: small-x region
- **always couples to anti-quark transversity**

$\Rightarrow A_{TT}$ small!

GSI: $$A_{TT} \propto \frac{\sum_q e_q^2 \left[\delta q(x_1)\delta q(x_2) + \delta\bar{q}(x_1)\delta\bar{q}(x_2)\right]}{\sum_q e_q^2 \left[q(x_1)q(x_2) + \bar{q}(x_1)\bar{q}(x_2)\right]}$$

- **transversely polarized anti-proton beam difficult (but possible)**
- **small $\sqrt{s} \Rightarrow$ large NLO QCD corrections to cross section**
  but: almost spin independent $\Rightarrow$ small corrections to A_{TT}
- **probing valence region**

$\Rightarrow A_{TT}$ large!

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$\Rightarrow$ Semi-Inclusive DIS

$$\sigma^{ep\rightarrow ehX} = \sum_q \delta q \otimes \sigma^{eq\rightarrow eq} \otimes FF^{q\rightarrow h}$$

$\Downarrow$ **chiral-odd** DF

$\Downarrow$ **chiral-odd** FF

CHIRAL EVEN

Transversity Measurements

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(Remember: Need another chiral-odd object!)
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$\Downarrow$ **chiral-odd** DF

$\Downarrow$ **chiral-odd** FF

CHIRAL EVEN

$\rightarrow$ chiral-odd FF as a polarimeter of transv. quark polarization

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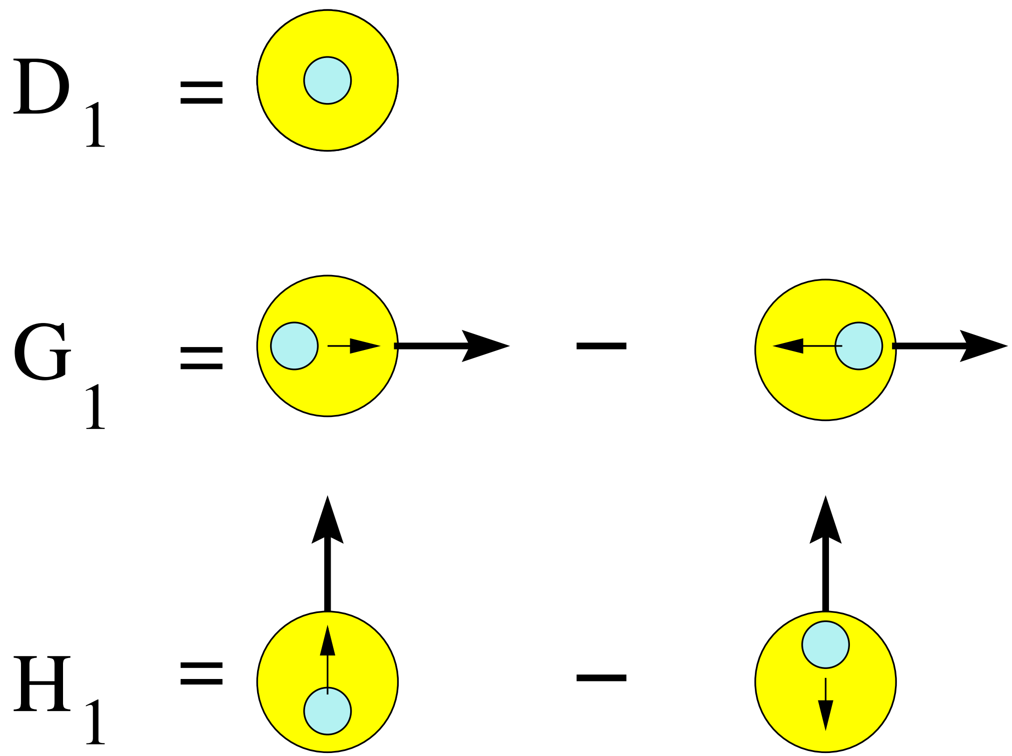
Leading-Twist Fragmentation Functions



unpolarized FF
(chiral-even)

longitudinal spin transfer FF
(chiral-even)

transverse spin transfer FF
CHIRAL-ODD!

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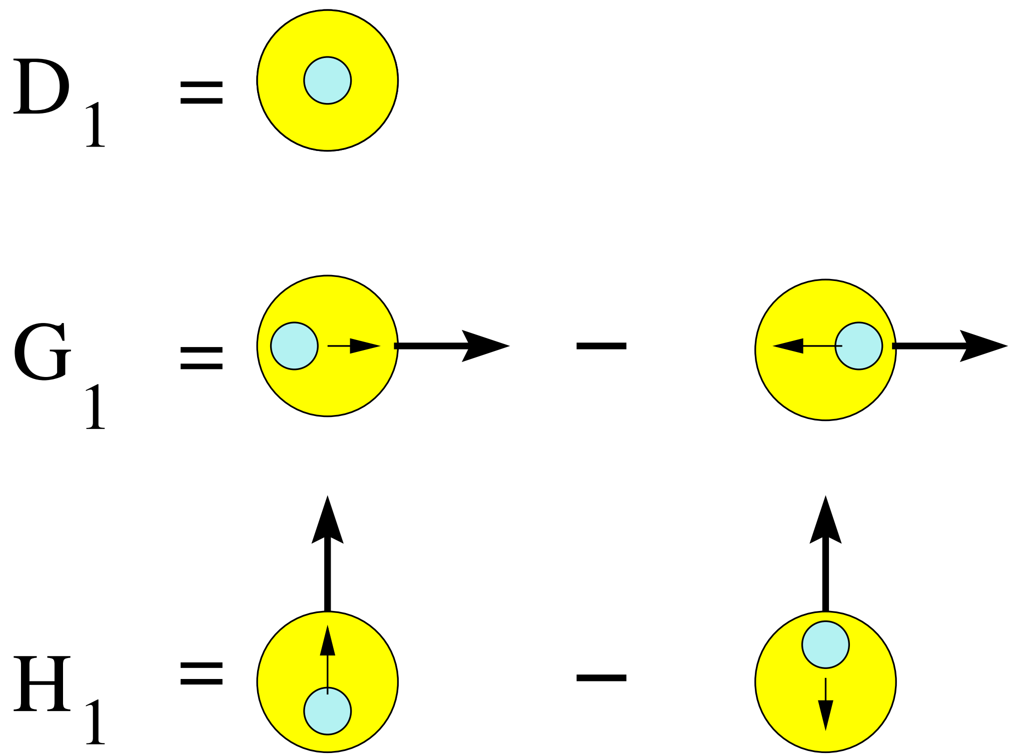

unpolarized FF
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longitudinal spin transfer FF
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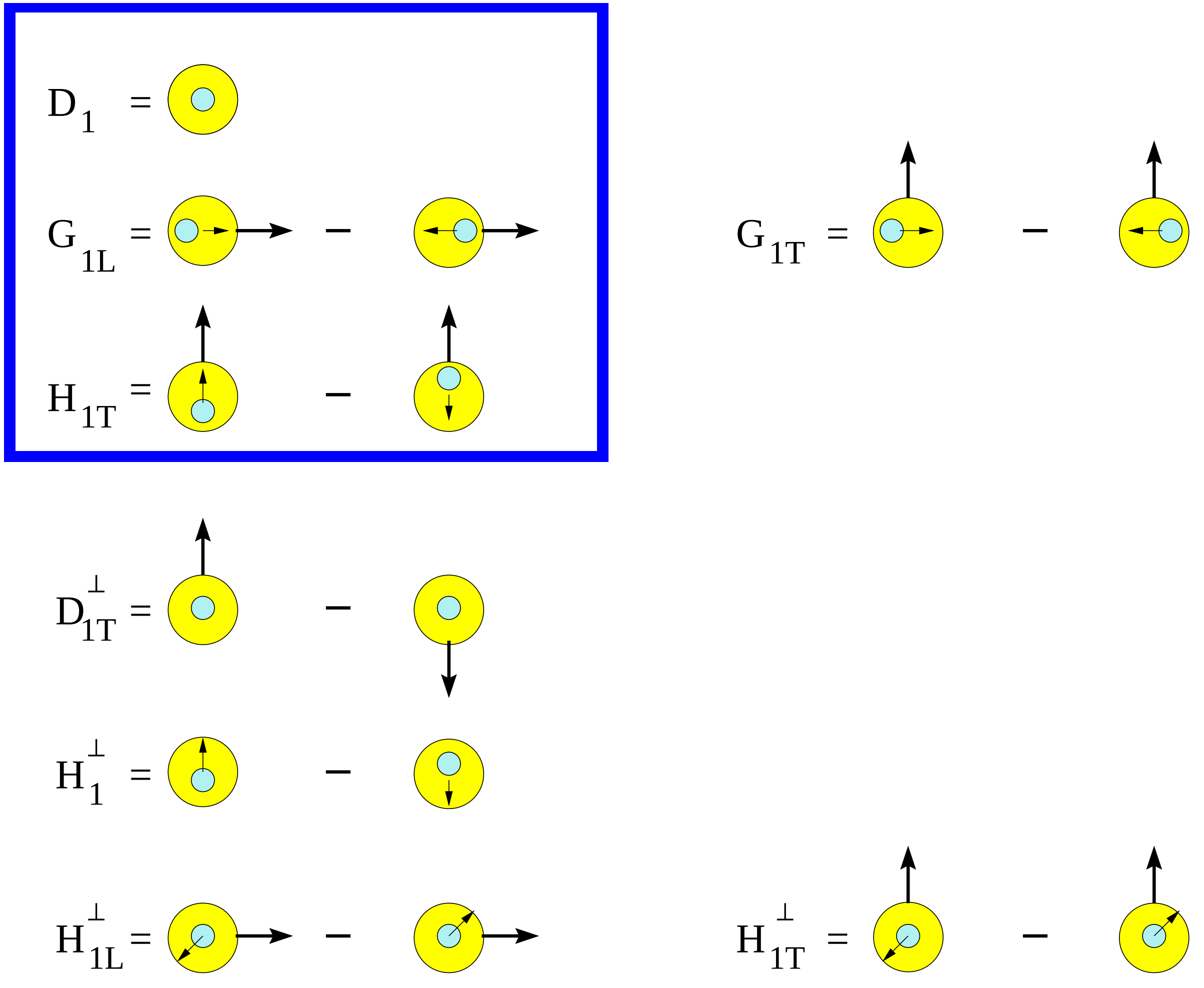
transverse spin transfer FF
CHIRAL-ODD!

- need to observe final hadron spin $\rightsquigarrow$ transverse Λ polarization
- relatively easy to measure (parity-violating decay of Λ)
- SIDIS u-quark dominated, BUT: u-quark presumably weakly polarized in Λ

Transverse-Momentum-Unintegrated Fragmentation Functions

Functions surviving integration over intrinsic transverse momentum



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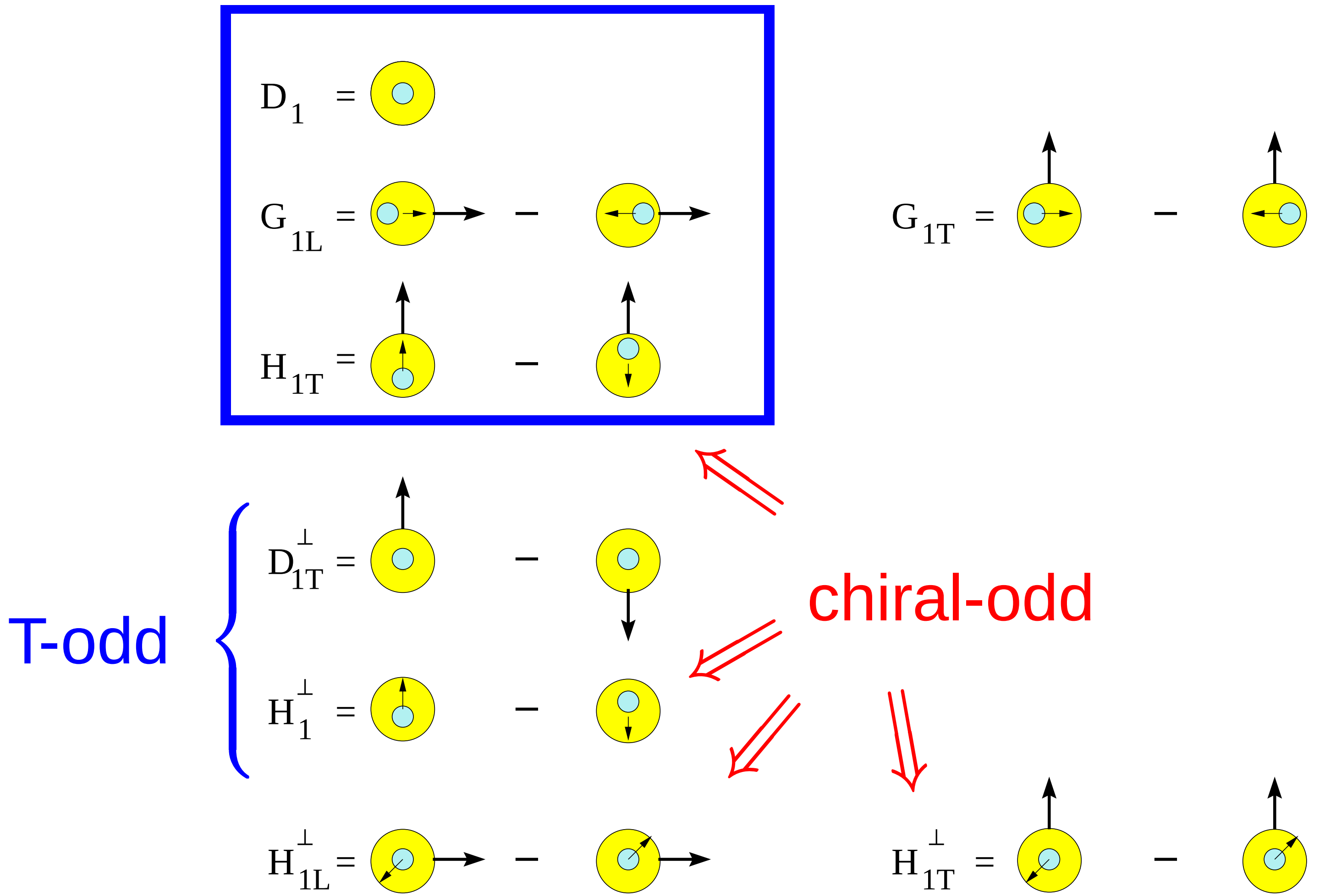

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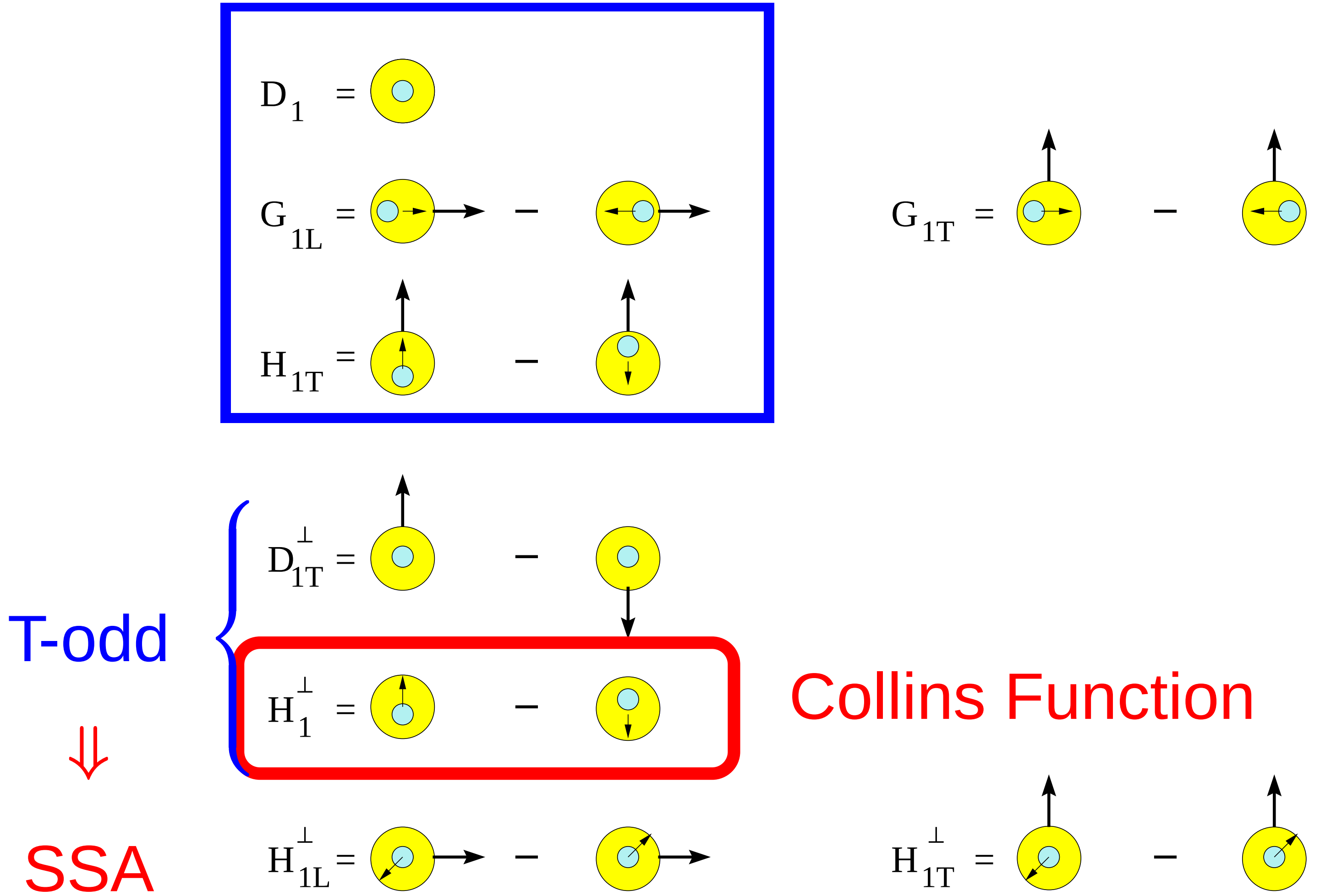
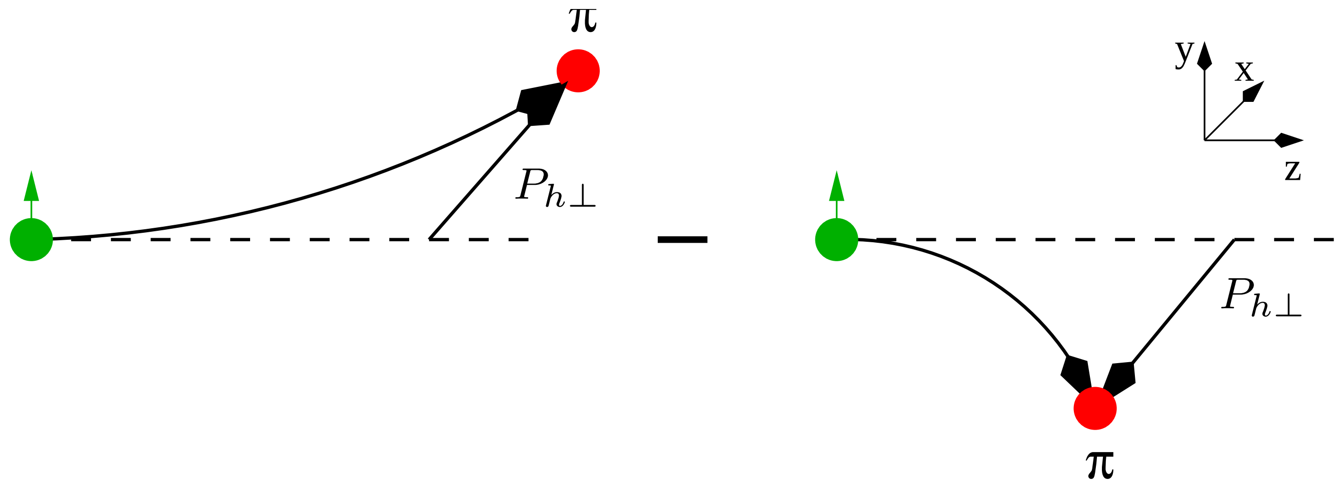


Collins Fragmentation Function

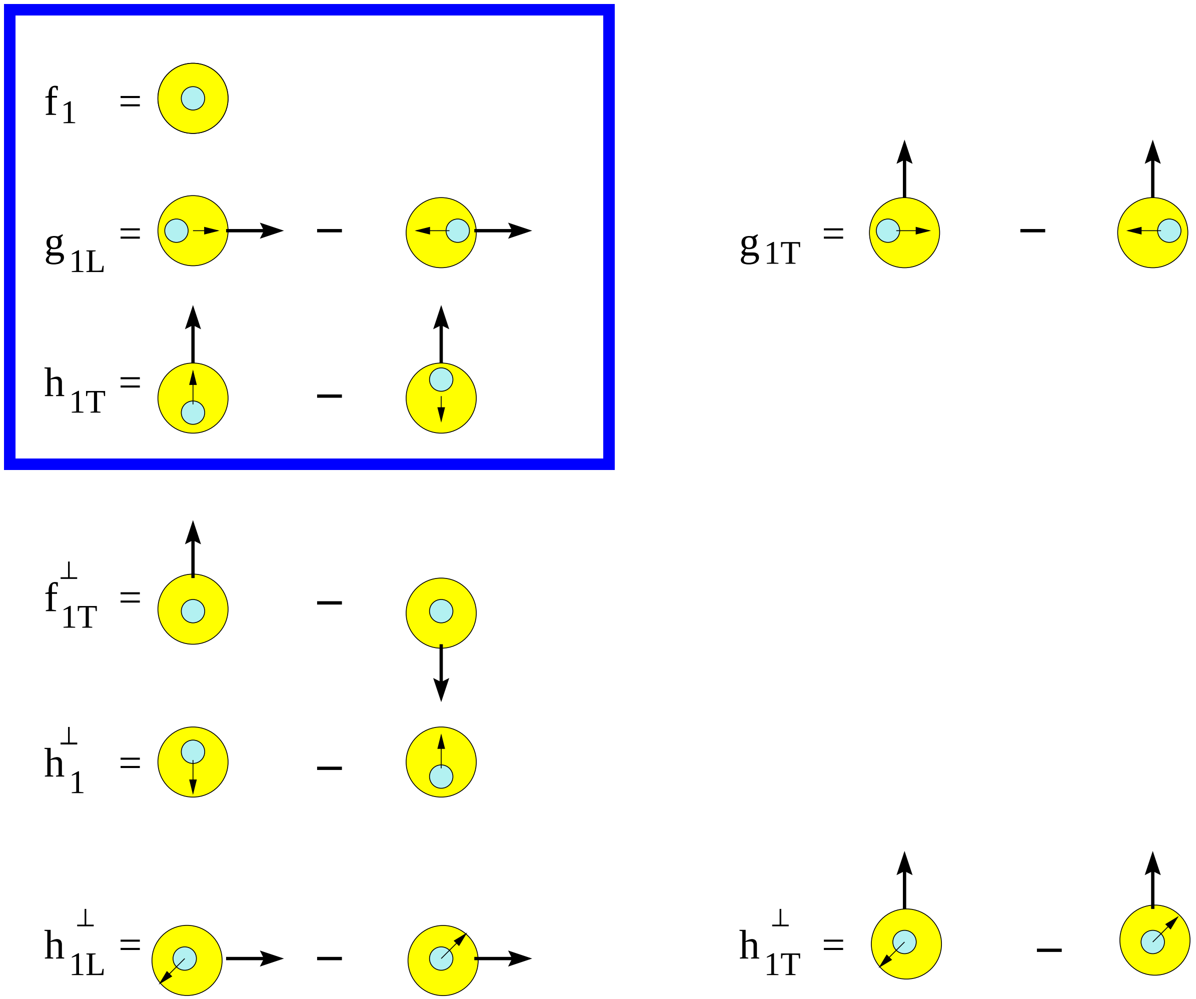


- Collins function $\boldsymbol{H_1^\perp}$ describes left-right asymmetry in the direction of outgoing hadron
- Originally proposed by Collins (& Heppelman)
- T-odd $\Rightarrow$ need interference of amplitudes
- Schäfer-Teryaev Sum Rule: $\sum_h \int dz H_1^{\perp,h} = 0$
- first data from Belle supports non-zero $\boldsymbol{H_1^\perp}$

Caution!

Other Spin-Momentum-Correlations exist!

Functions surviving integration over intrinsic transverse momentum



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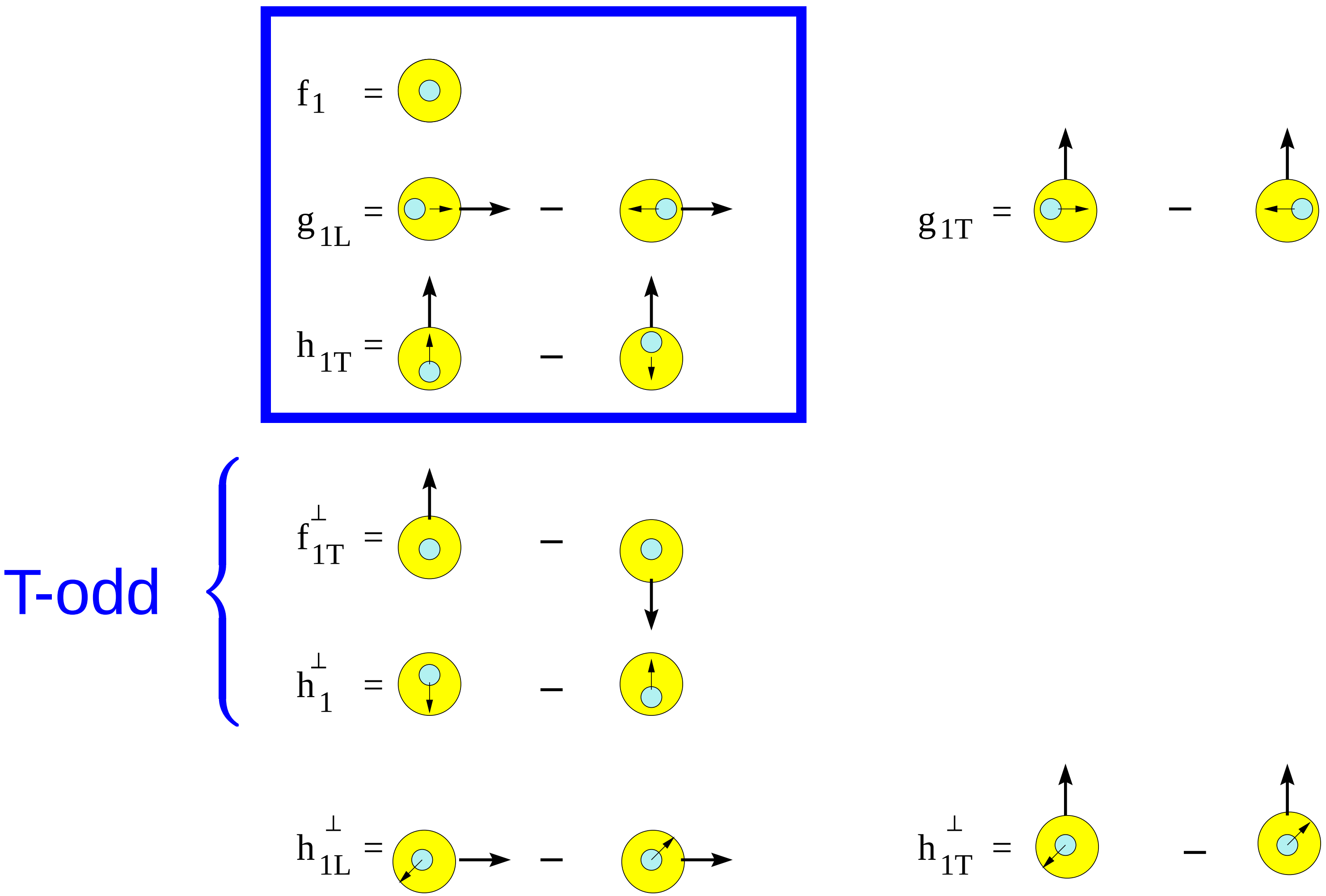
Unintegrated Quark Distributions

Functions surviving integration over intrinsic transverse momentum



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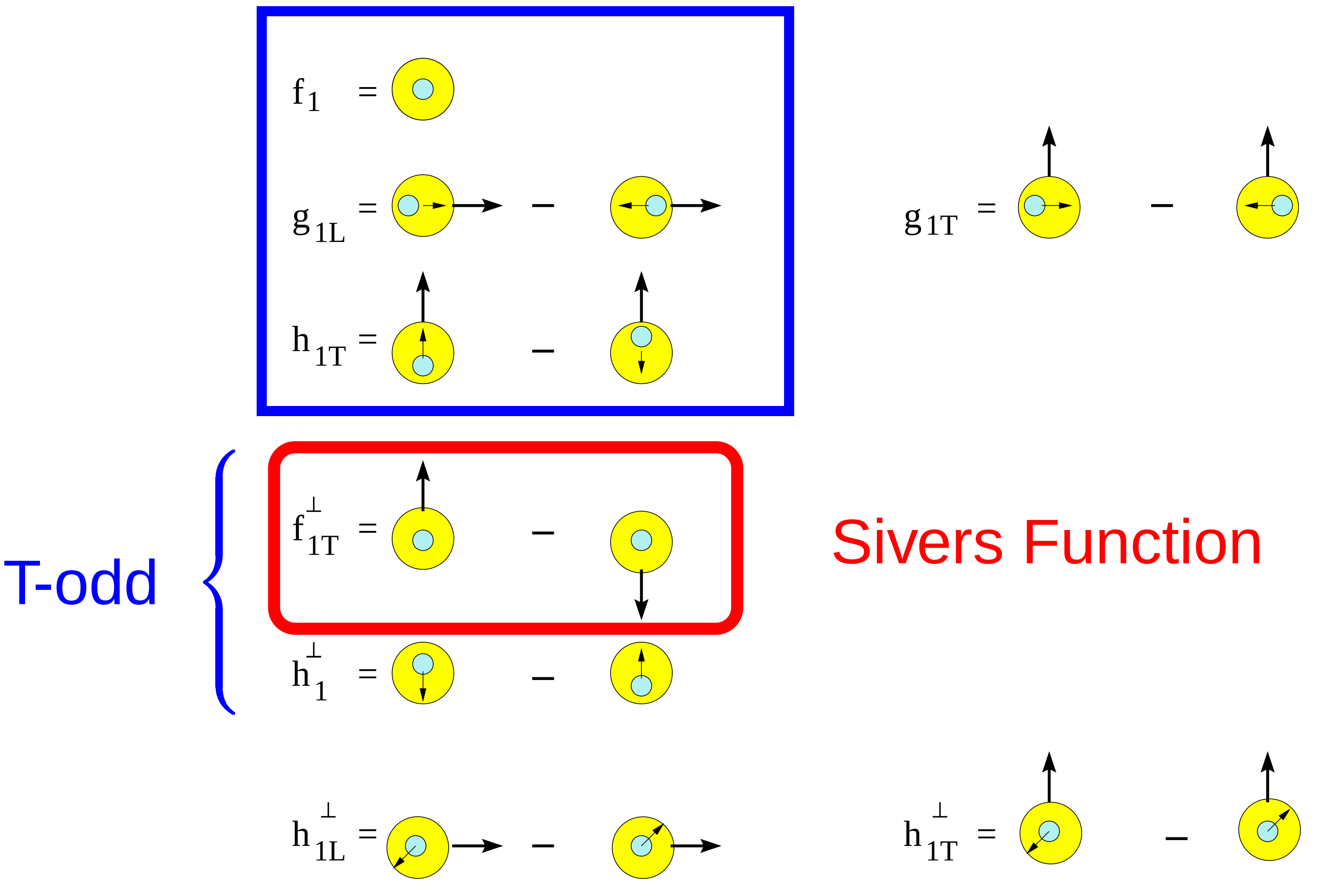
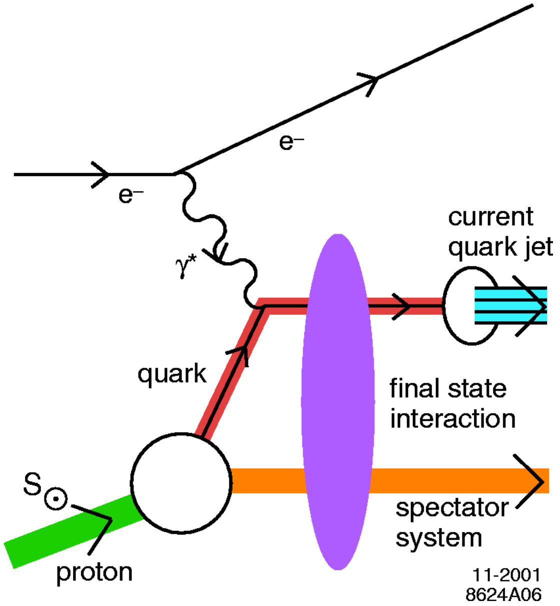


Some words about Sivers Effect



Thanks to Brodsky, Hwang, Schmidt:

- quark rescattering via soft gluon exchange
- correlates transverse spin with direction of outgoing hadron
- requires L_z of quarks

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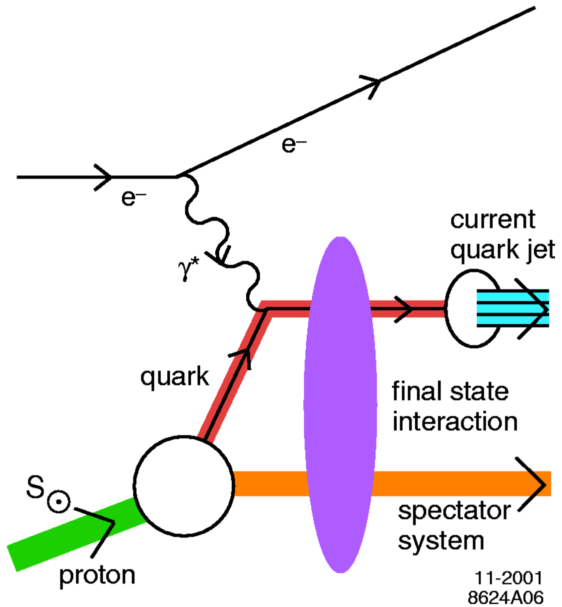

Thanks to Brodsky, Hwang, Schmidt:

- quark rescattering via soft gluon exchange
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Thanks to Collins, Ji, Yuan, Belitzky ...:

- Soft gluon is model for gauge link needed for gauge invariance
- Gauge links provide necessary complex phase for interference
- T-Symmetry of QCD requires opposite sign of Sivers function in DIS and DY
- slightly different approach by Burkardt using impact parameter dependent PDF's ("chromodynamic lensing")

SSA & Unintegrated Distribution and Fragmentation Functions

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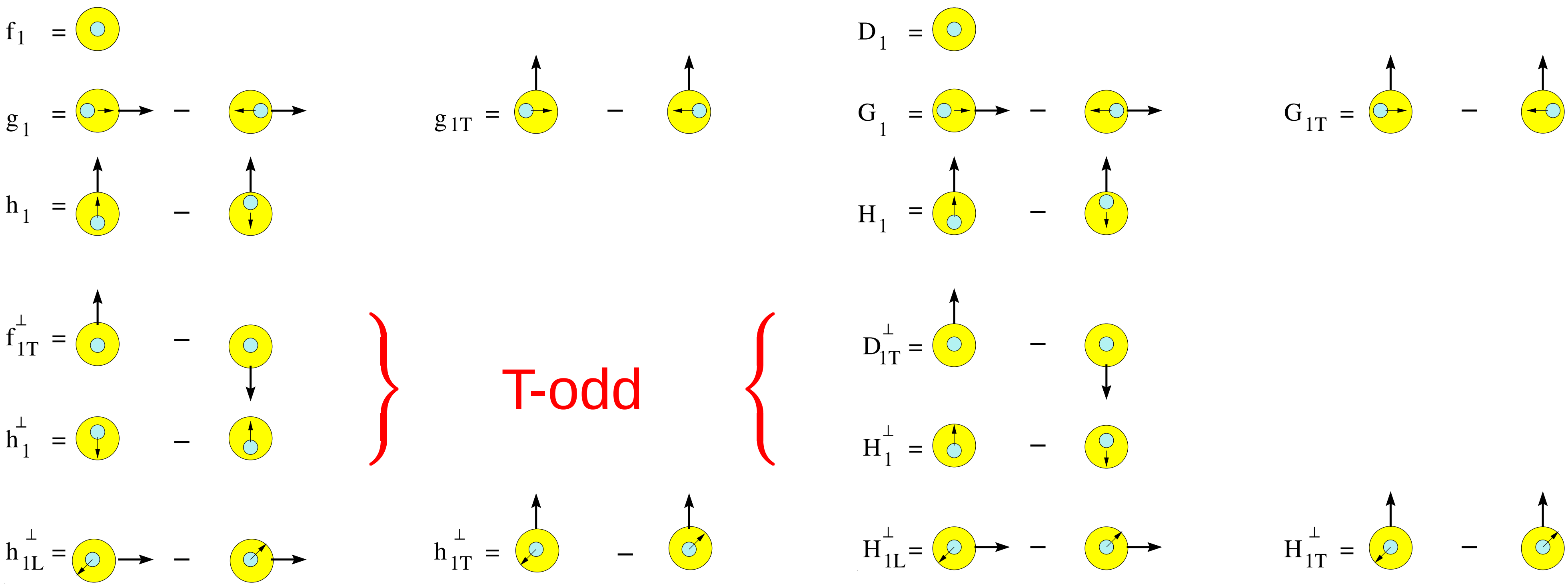
Distribution Functions | **Fragmentation Functions**



SSA require one and only one T-odd function

SSA & Unintegrated Distribution and Fragmentation Functions

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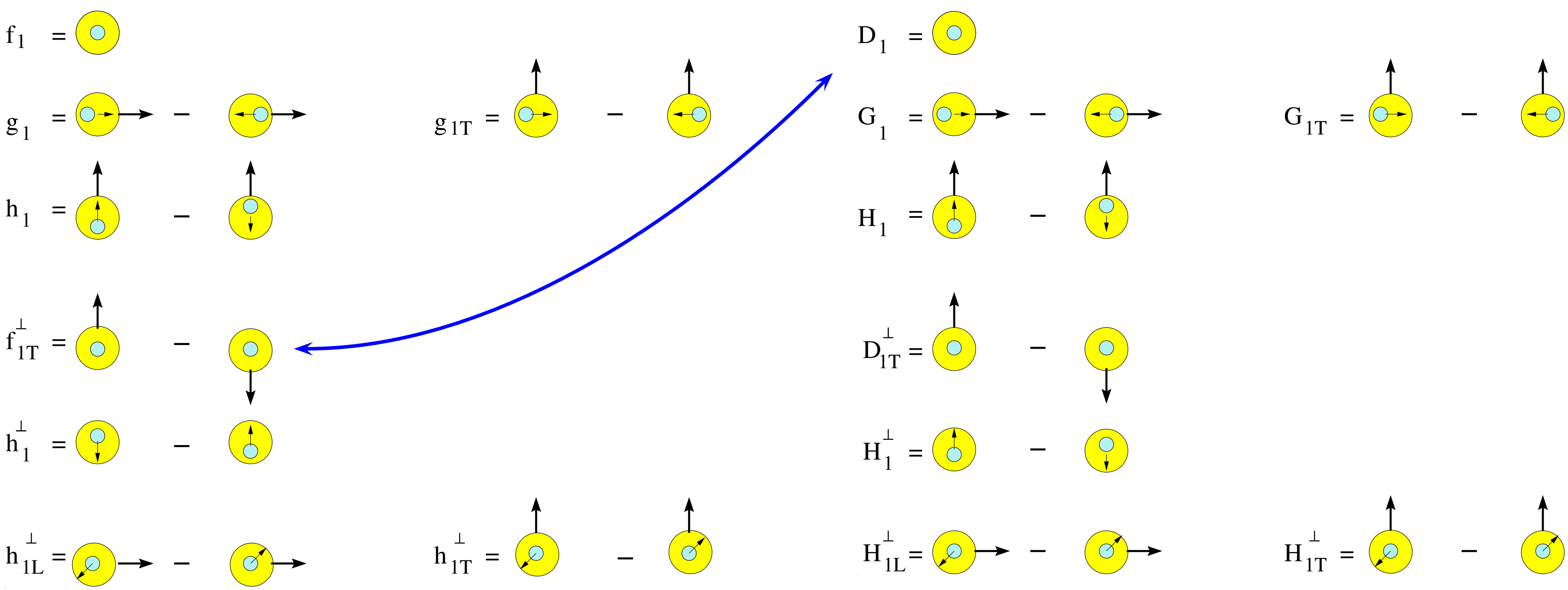
Distribution Functions | Fragmentation Functions



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$\Rightarrow$ SSAs through Sivers function

SSA & Unintegrated Distribution and Fragmentation Functions

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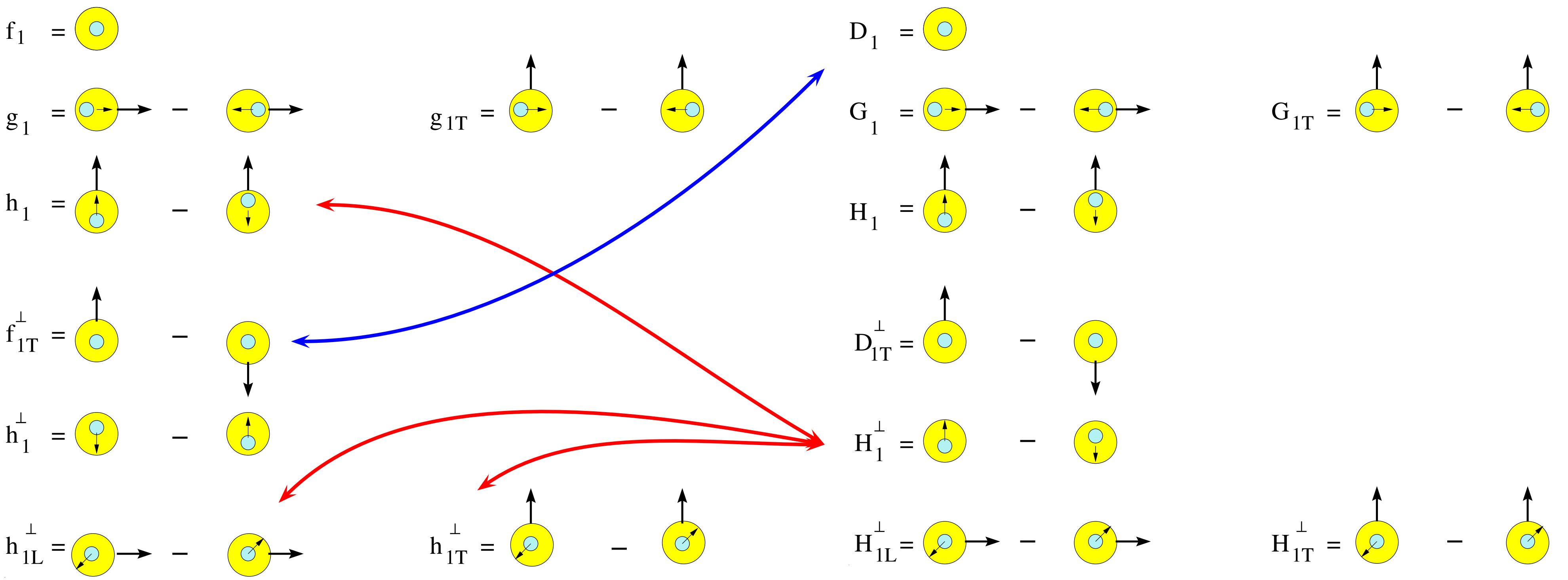


SSA require one and only one T-odd function

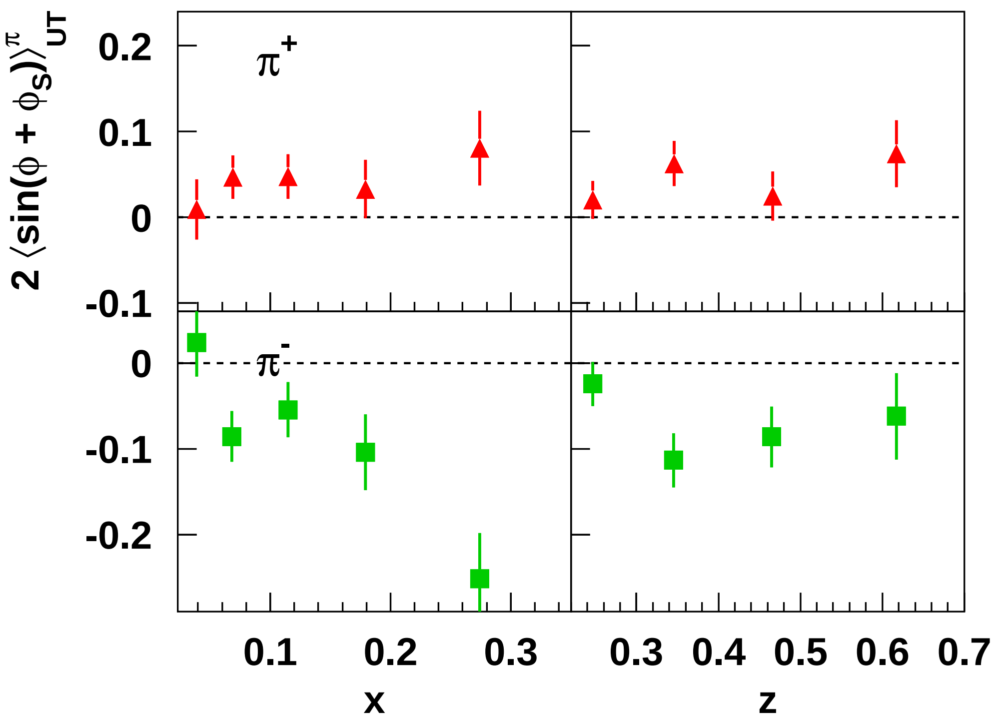
⇒ SSAs through Sivers function or Collins function

Collins Asymmetries in SIDIS

The HERMES Data

hermes : $ep^{\uparrow} \rightarrow e\pi^{\pm}X$

$$A_{UT} \propto \mathcal{I}[\ldots h_1(x,p_T^2)H_1^{\perp}(z,k_T^2)]$$

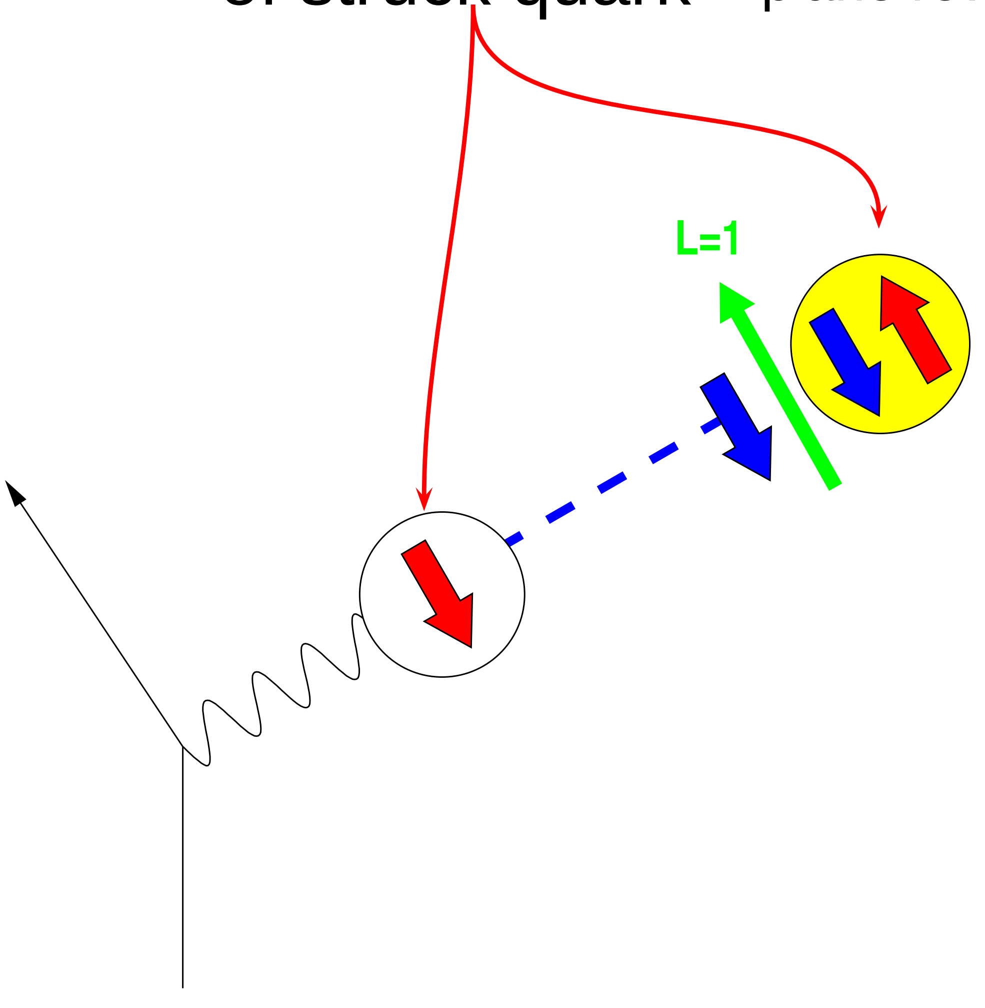


- non-zero Collins effect!
- both Collins FF and transversity sizeable
- surprisingly large π^- asymmetry
  $\Rightarrow$ large contribution (with opposite sign) from unfavored fragmentation, i.e. $u \longrightarrow \pi^-$

[*A. Airapetian et al*, **Phys. Rev. Lett. 94 (2005) 012002**]

Understanding the Collins FF - String Model Interpretation (Artru)

transverse spin of struck quark (polarization component in lepton scattering plane reversed by photoabsorption)



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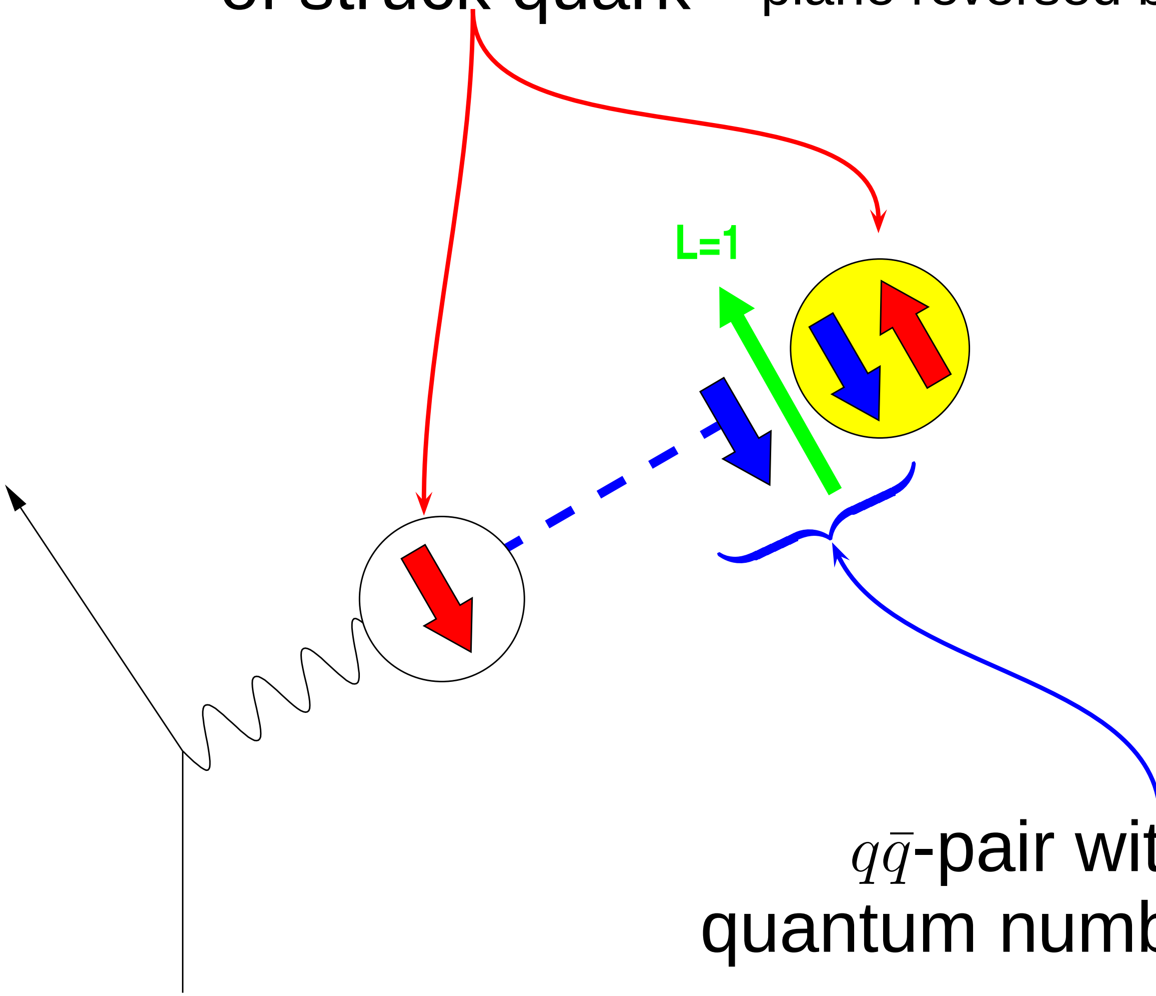

$q\bar{q}$-pair with vacuum quantum numbers ($^3P_0$-state)

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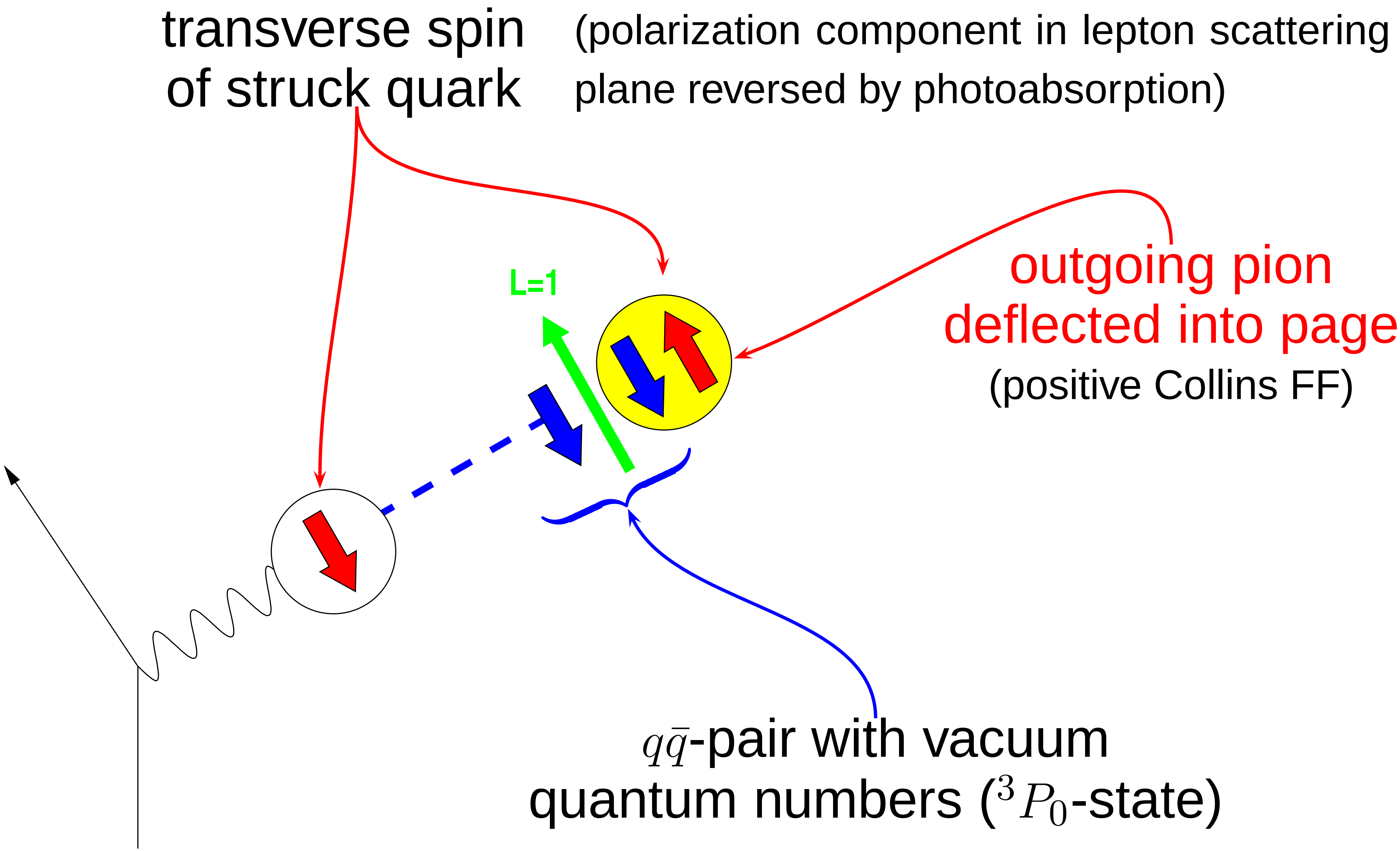
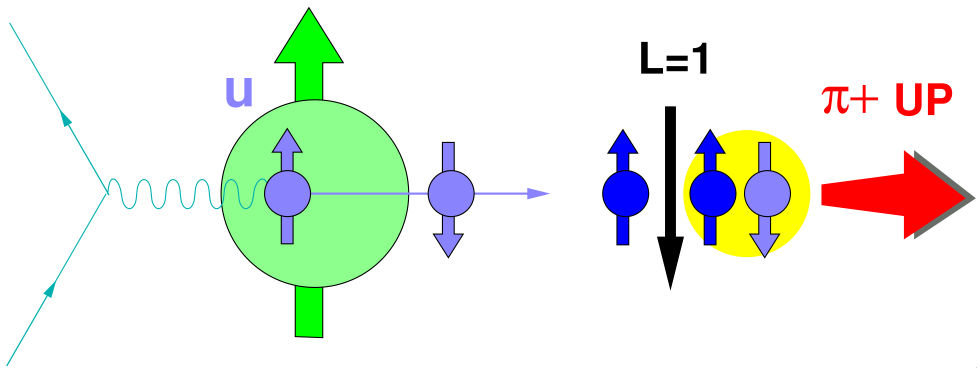


Artru Model vs. HERMES



$$\left.\begin{aligned}\phi_S &= 0\\ \phi &= \pi/2\end{aligned}\right\}\sin(\phi+\phi_S) > 0$$

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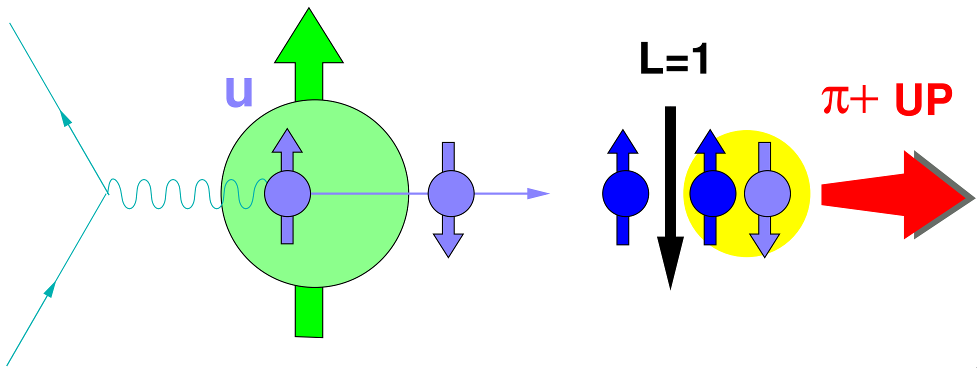

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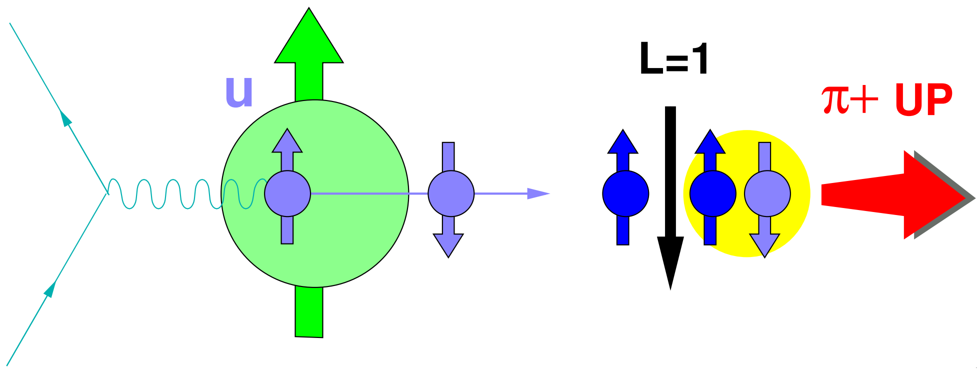

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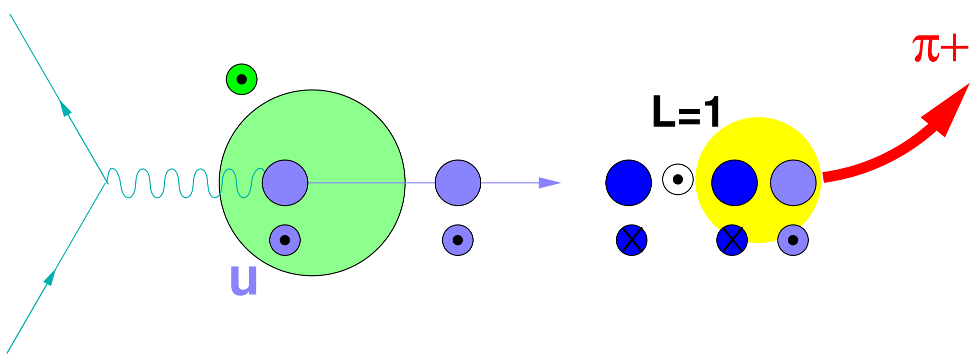

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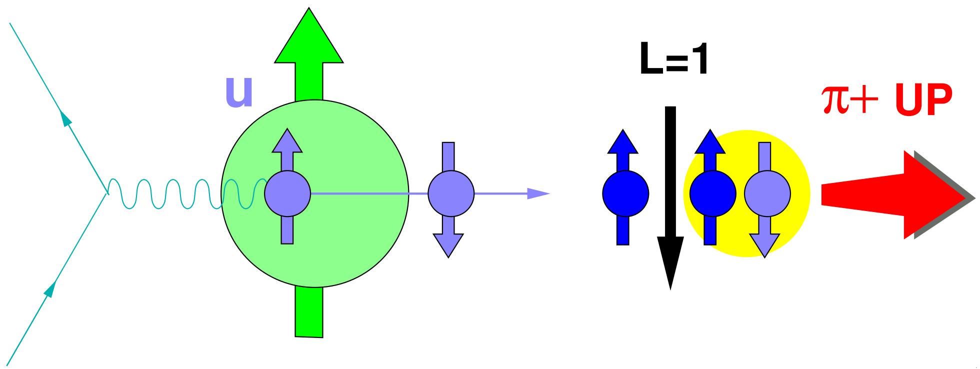

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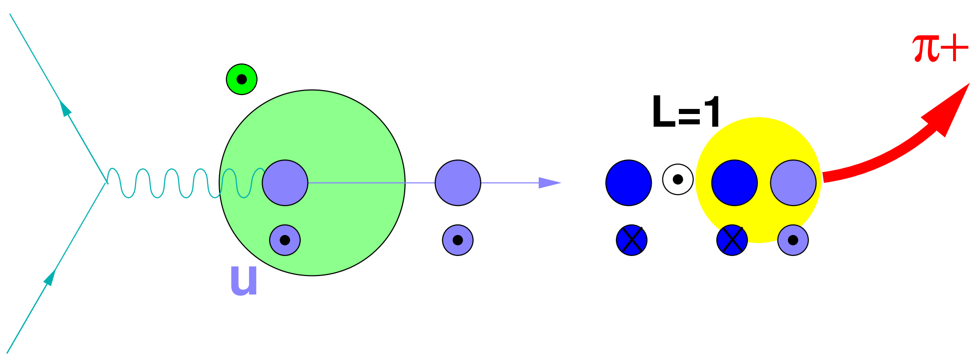

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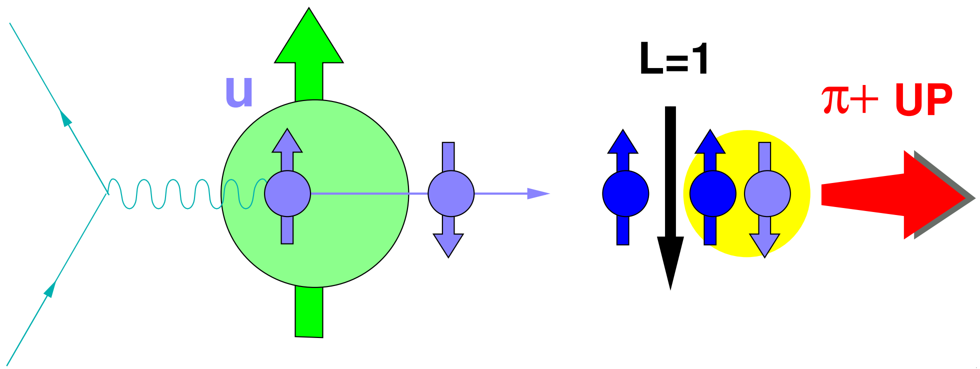

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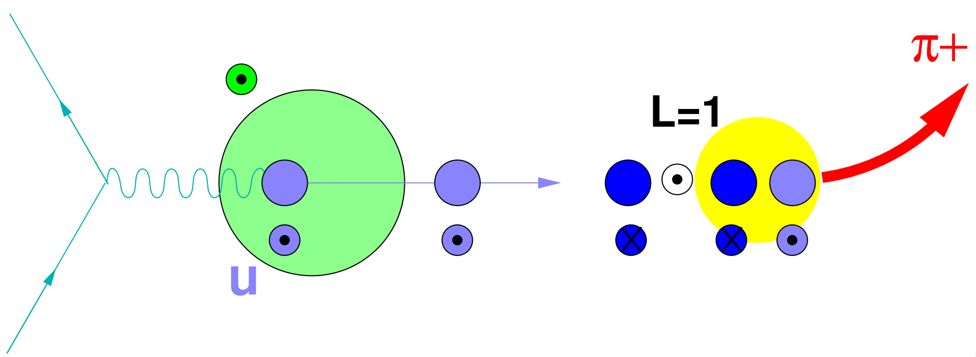

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✓

Artru model and HERMES results in agreement!

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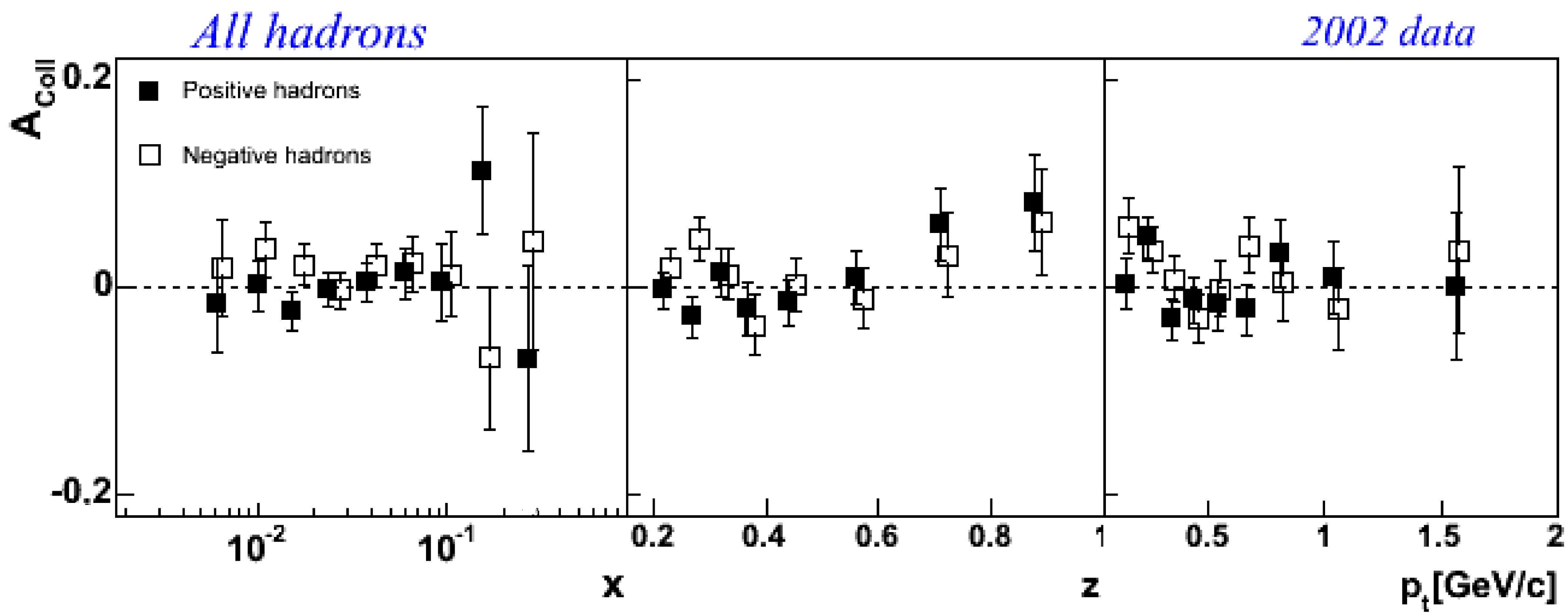
Collins Asymmetries in SIDIS

The COMPASS Data

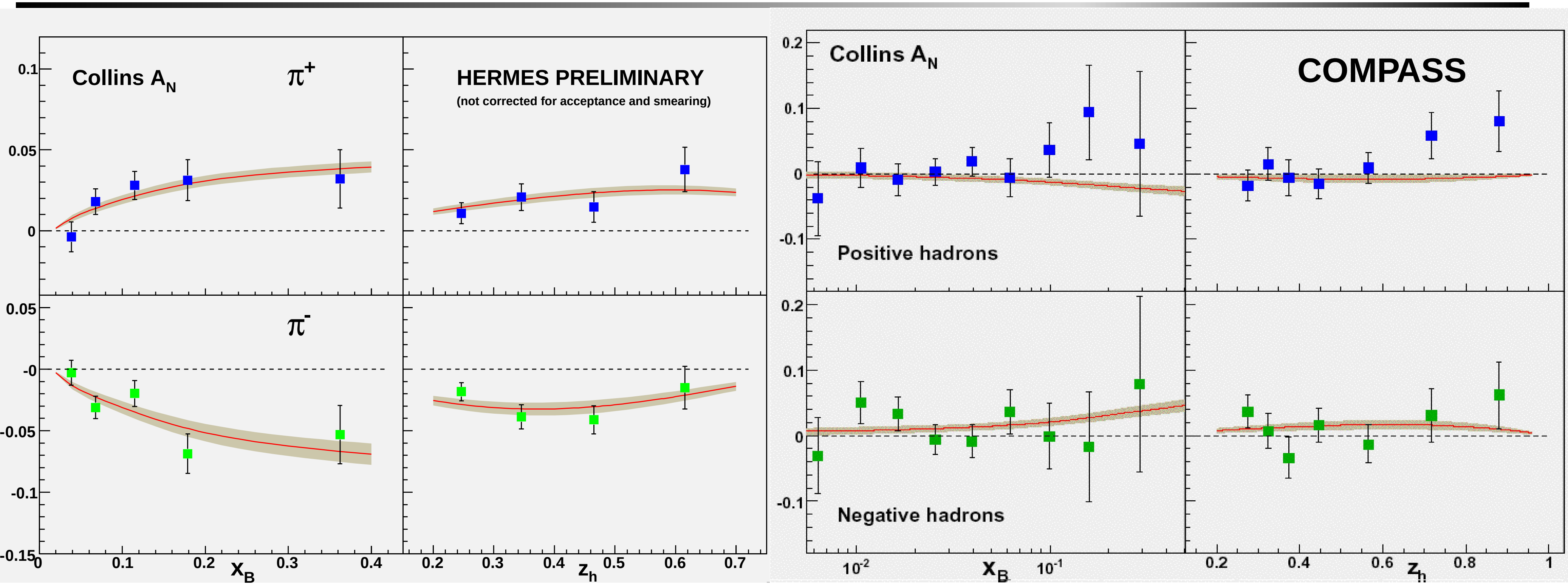
COMPASS : $\mu d^{\uparrow} \rightarrow \mu h^{\pm} X$



- Collins effect consistent with zero!
- cancellations because of deuteron target possible and probable

[*V.Yu. Alexakhin et al*, **Phys. Rev. Lett. 94 (2005) 202002**]

Collins Function Fits to Data (W. Vogelsang, F. Yuan)

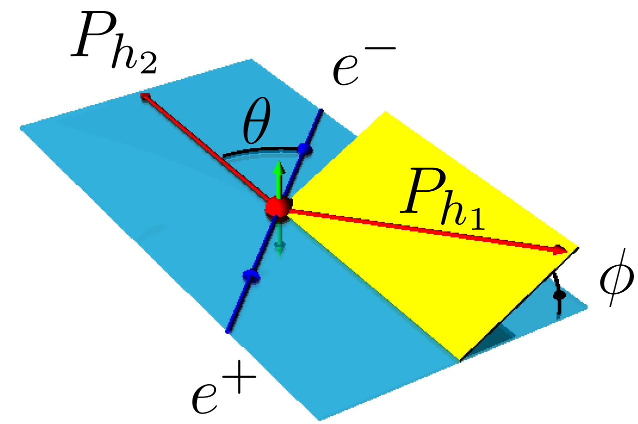


- Soffer bound for transversity saturated
- get Collins fragmentation function via fit to HERMES data
- consistent results for HERMES and COMPASS data

[hep-ph/0507266]

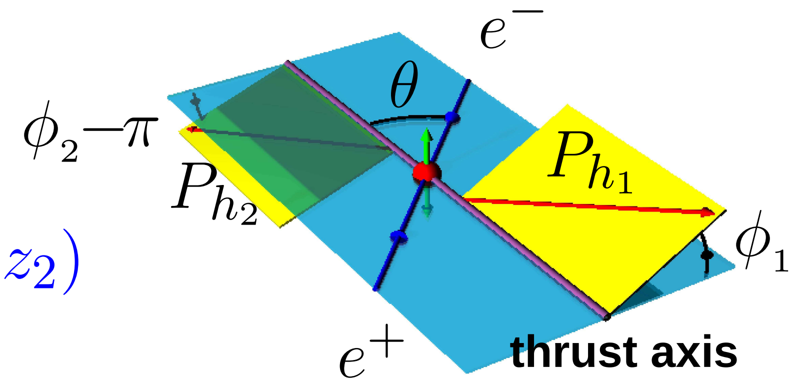
Results on Collins FF from BELLE

$$\sigma^{e^+e^-\to h_1 h_2 X} \propto \cos(2\phi)\mathcal{I}[\ldots H_1^\perp(z_1)H_1^\perp(z_2)]$$



$\mathcal{I}[\ldots]$ **convolution integral over intrinsic transverse momenta**

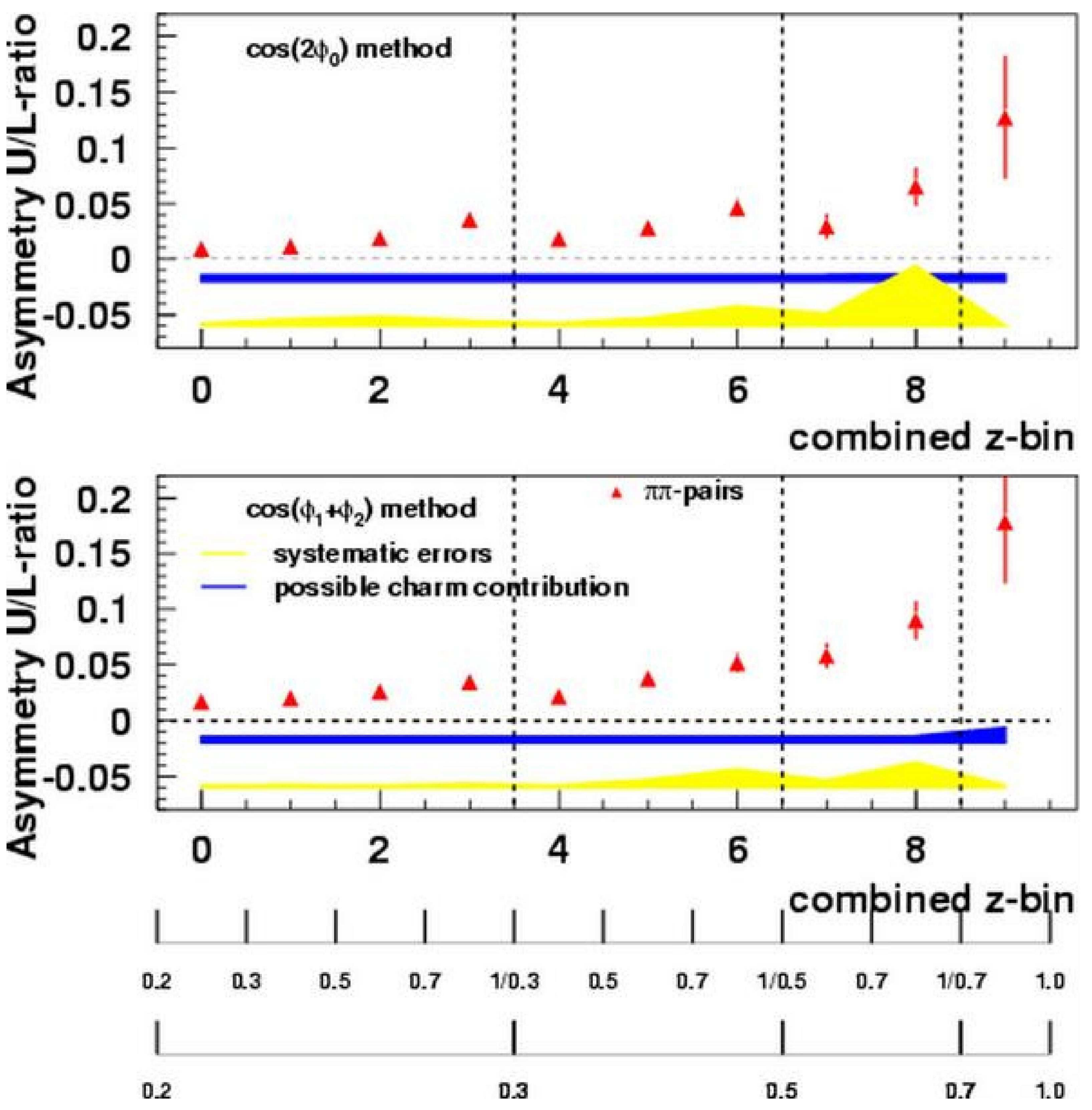
- independent of thrust axis
- involves convolution integral

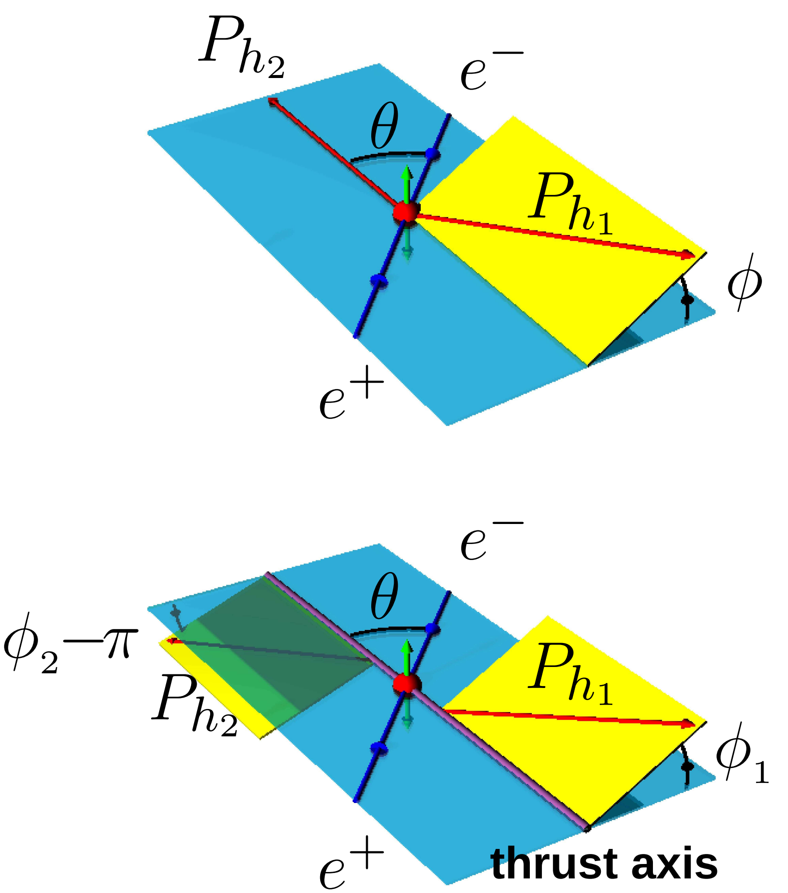


$$\sigma^{e^+e^-\to h_1 h_2 X} \propto \cos(\phi_1+\phi_2)H_1^\perp(z_1)H_1^\perp(z_2)$$

- need to know thrust axis
- model-independent interpretation possible

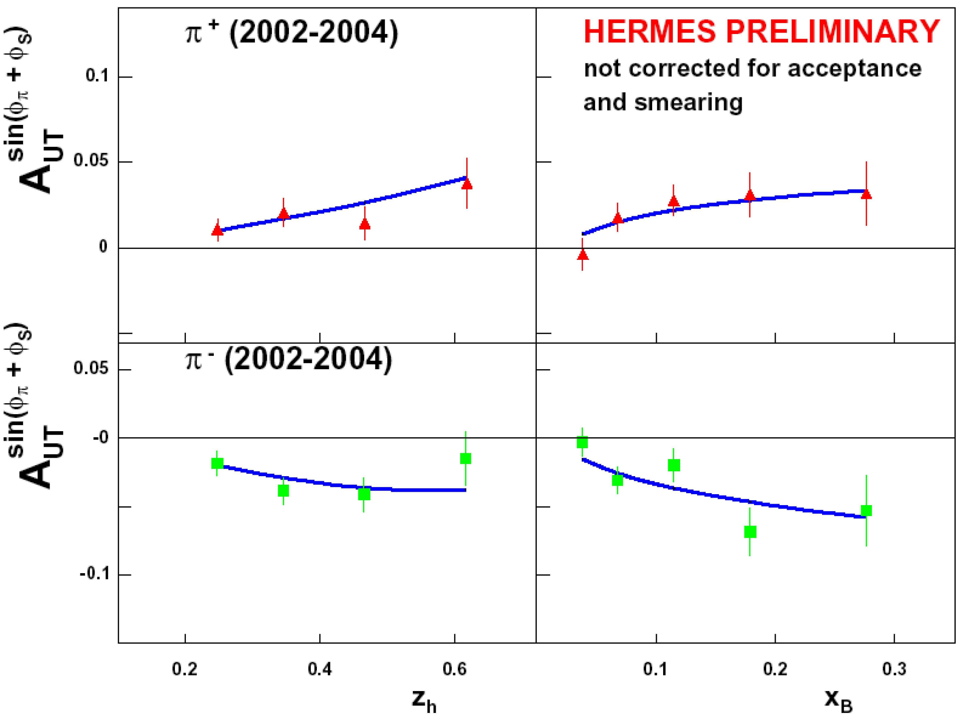
Results on Collins FF from BELLE

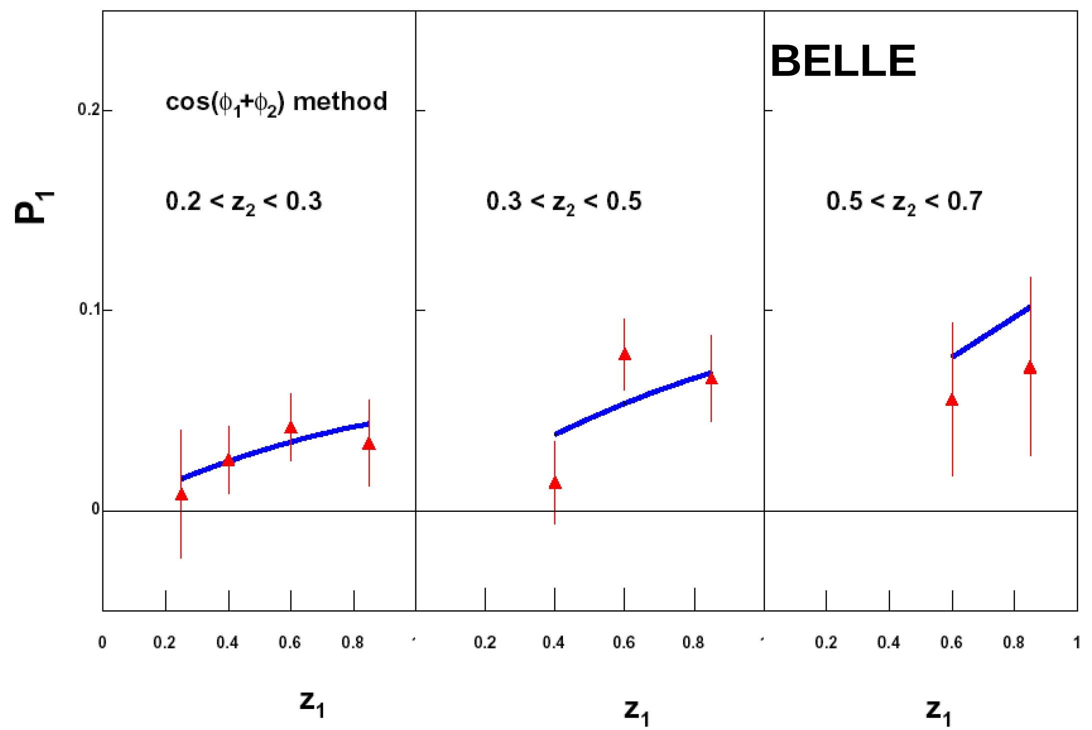




non-zero Collins FF!

Collins Function from SIDIS and BELLE (M. Anselmino et al)





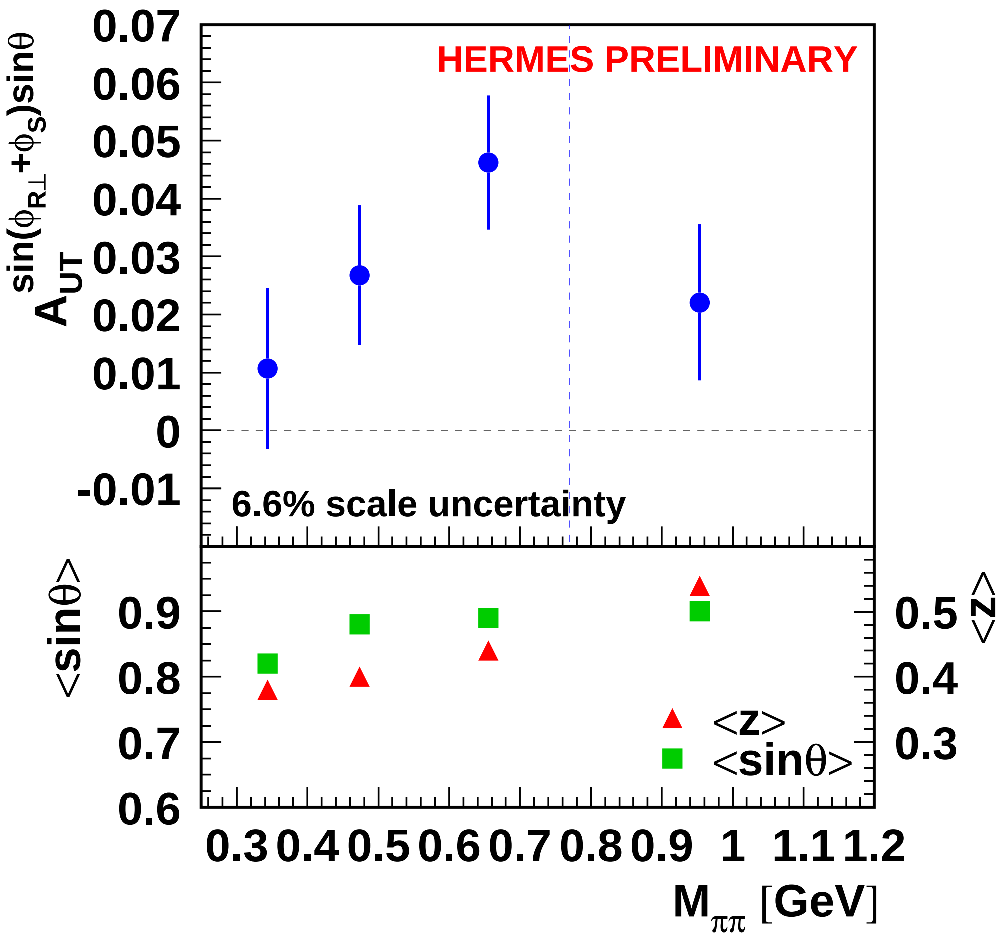
consistent results for HERMES and BELLE data

[talk presented at Transversity'05,Como]

Another Chiral-Odd Friend

Interference Fragmentation

hermes $: ep^{\uparrow} \rightarrow e\pi^+\pi^- X$



$$A_{UT} \propto \delta q(x) H_1^{\sphericalangle, sp}(z, M_{\pi\pi}^2)$$

- caused by interference of s- and p-waves
- first evidence for non-zero interference fragmentation
- COMPASS data using deuterium consistent with zero

[see talk by U. Elschenbroich]

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Alternative Ways to Measure Transversity

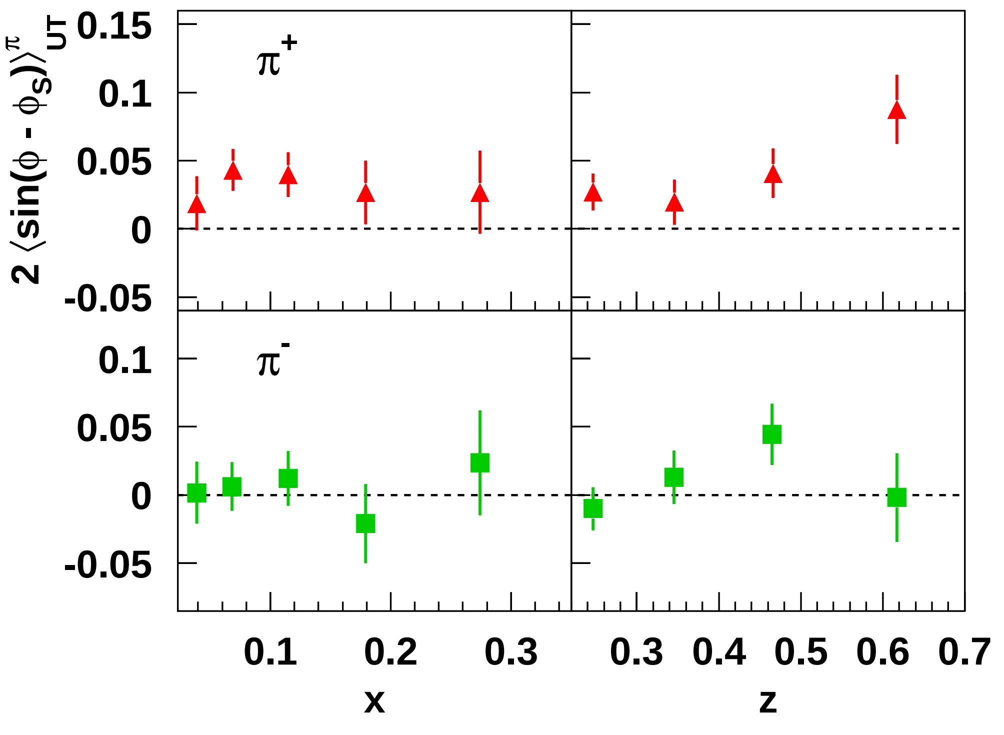
- Semi-Inclusive DIS:
  - SSA with twist-3 fragmentation function $\tilde{H}$
  - DSA with twist-3 fragmentation function E
  - spin-1/2 fragmentation
  - spin-1 fragmentation
- Drell-Yan $p^{\uparrow}p \rightarrow l\bar{l} + X$: transversity in conjunction with (chiral- and T-odd) Boer-Mulders Function $h_1^{\perp}$ (transversity distribution in an unpolarized nucleon)
- single- or double-polarized proton-proton scattering $p^{\uparrow}p^{(\uparrow)} \rightarrow \pi + X$: transversity in conjunction with Collins function, Boer-Mulders function or ... **[talk by M. Anselmino]**

Sivers – The Other T-Odd Effect

Friend (or Enemy?) of Transversity

Sivers Asymmetries in SIDIS

hermes : $ep^{\uparrow} \rightarrow e\pi^{\pm}X$



$$A_{UT} \propto -\mathcal{I}[\ldots f_{1T}^{\perp}(x, p_T^2) D_1(z, k_T^2)]$$

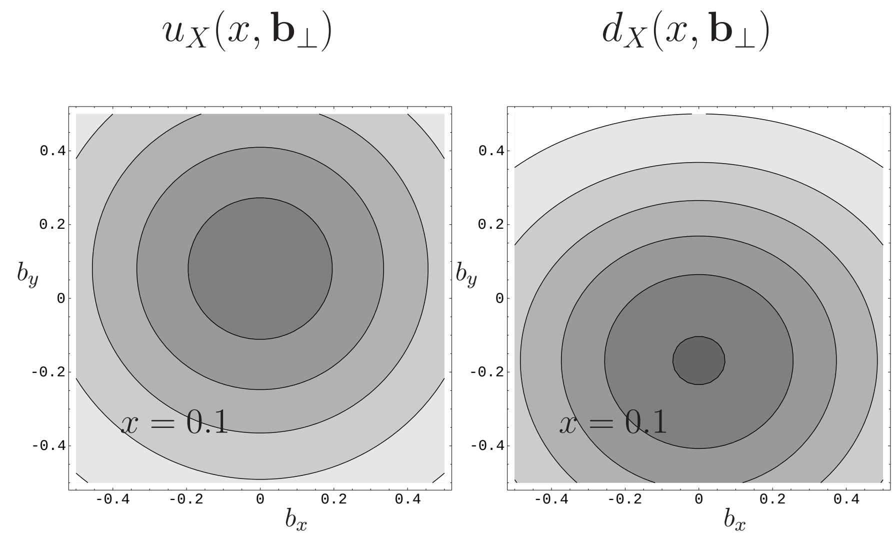
- first observation of non-zero Sivers effect in SIDIS!
- u-quark dominance and positive π^+ asymmetry suggests

$$f_{1T}^{\perp,u} < 0$$

[*A. Airapetian et al*, **Phys. Rev. Lett. 94 (2005) 012002**] approach by M. Burkardt: **[hep-ph/0309269]**

spatial distortion of q-distribution

(obtained using anom. magn. moments & impact parameter dependent PDFs)



Chromodynamic Lensing

Understanding the Sivers Moments

approach by M. Burkardt: **[hep-ph/0309269]**

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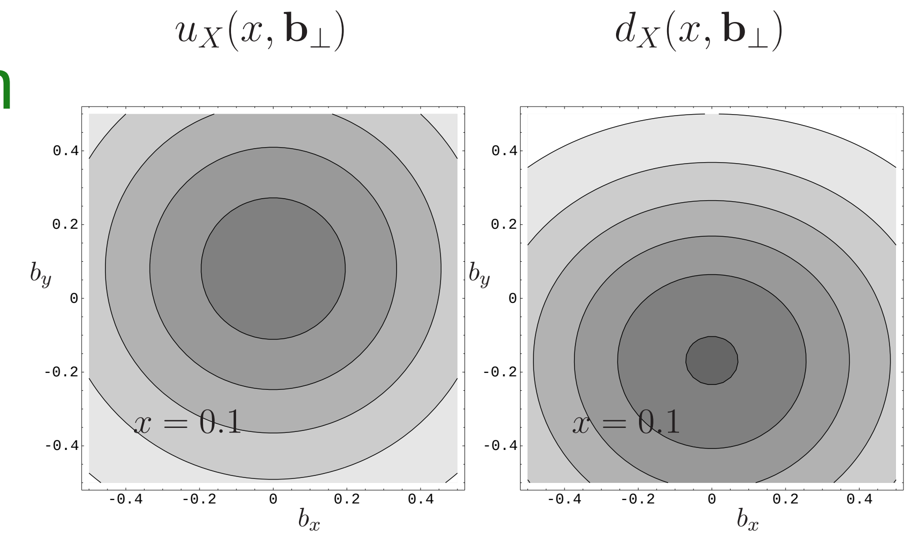
(obtained using anom. magn. moments & impact parameter dependent PDFs)



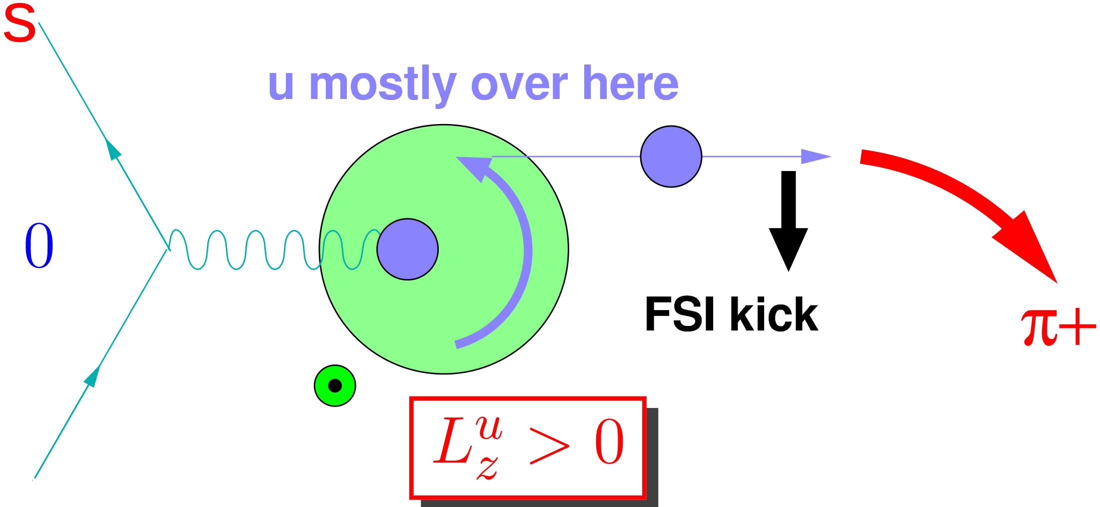
+ attractive QCD potential

(gluon exchange)

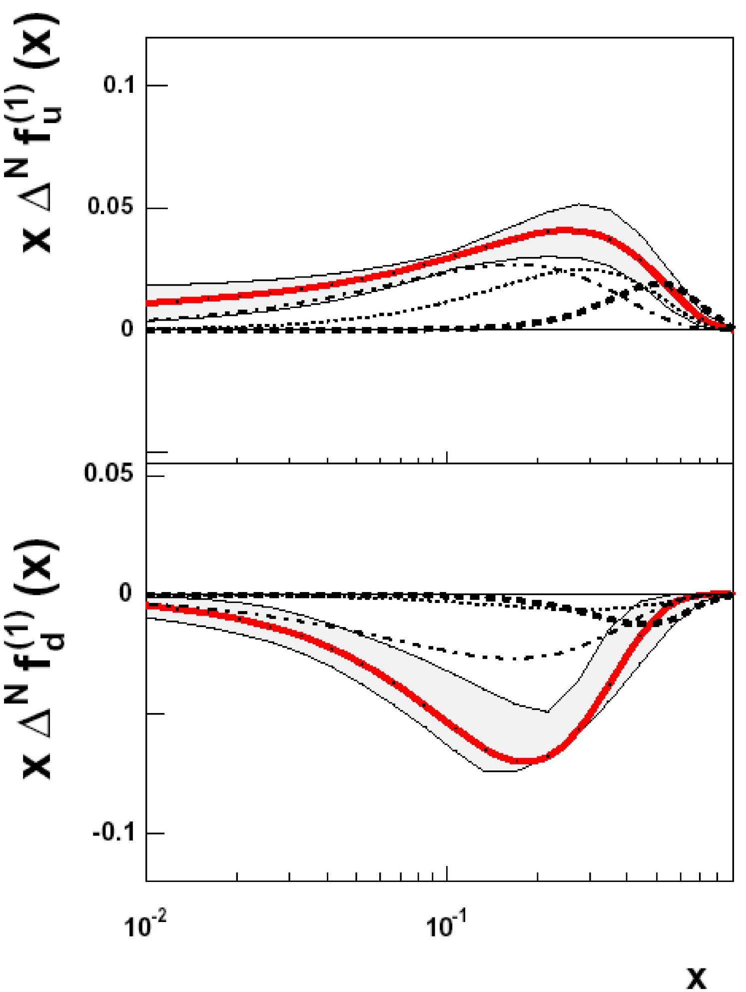
$\Rightarrow$ transverse asymmetries

$$\left.\begin{array}{l}\phi_S = \pi/2 \\ \phi \;=\; \pi\end{array}\right\} \sin(\phi - \phi_S) > 0$$

✓



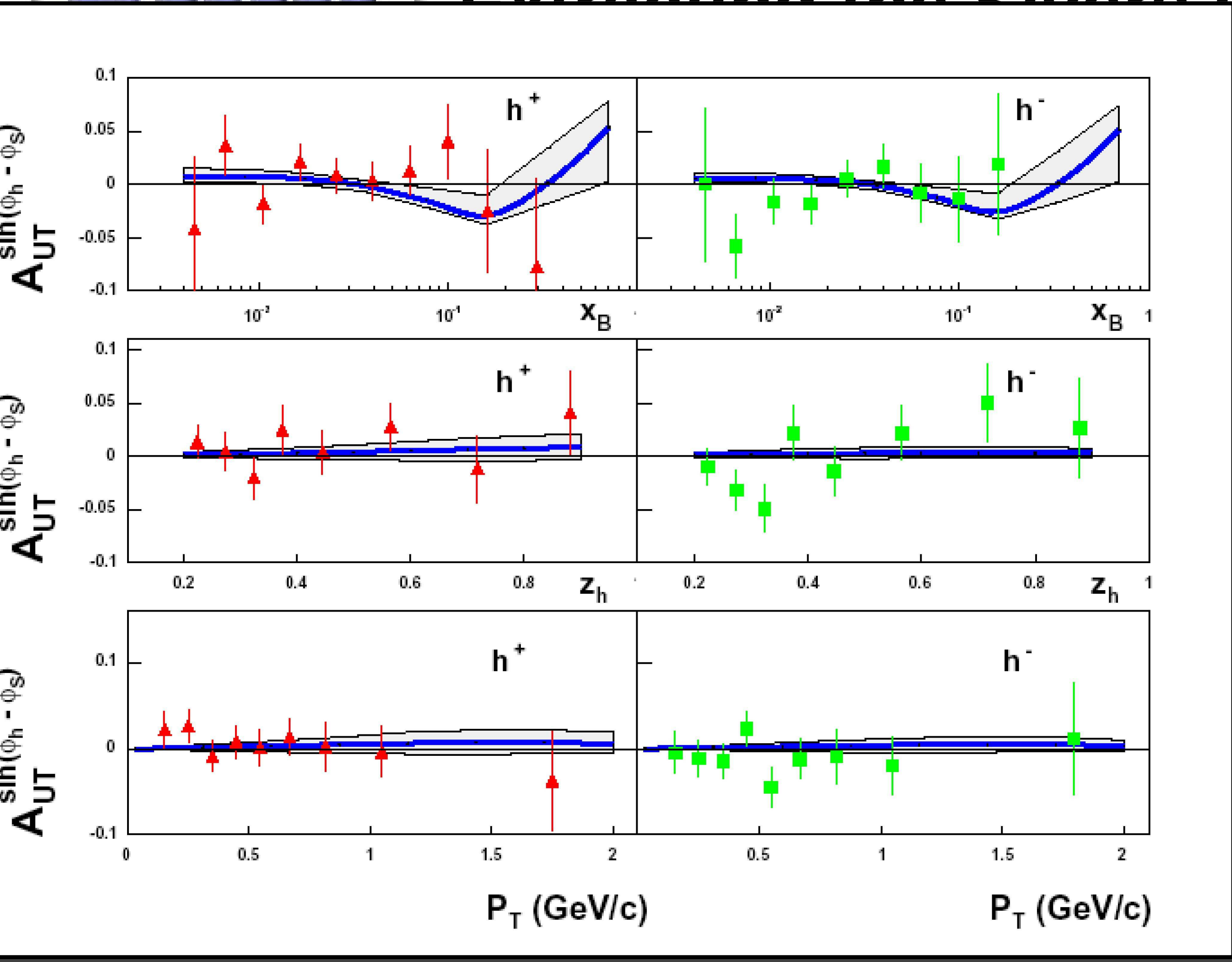
Extracting the Sivers Function from Data (M. Anselmino et al)

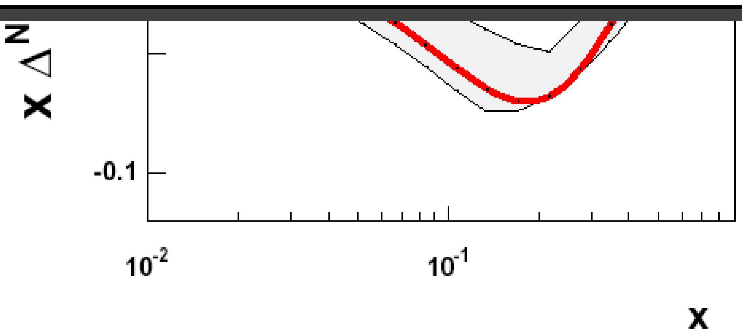


- Sivers function for u- and d- quarks with opposite sign and about same magnitude
- Burkardt Sum Rule ($\sum_{q,g} f_{1T}^{\perp} = 0$) almost satisfied already
- cancellations in deuteron target explain vanishing Sivers asymmetry at COMPASS

Extracting the Sivers Function from Data

M. Anselmino et al)





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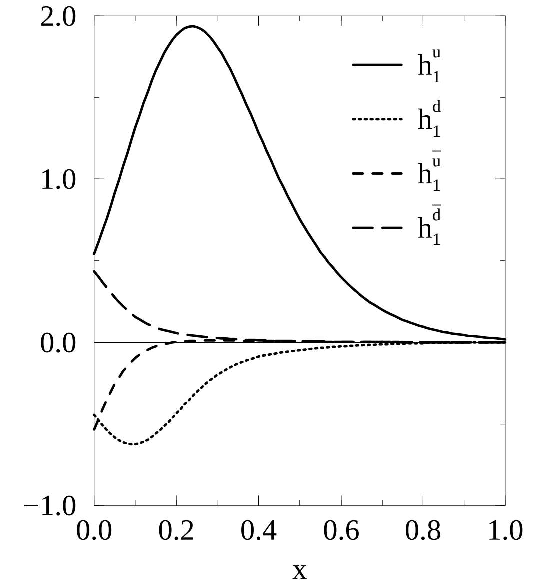
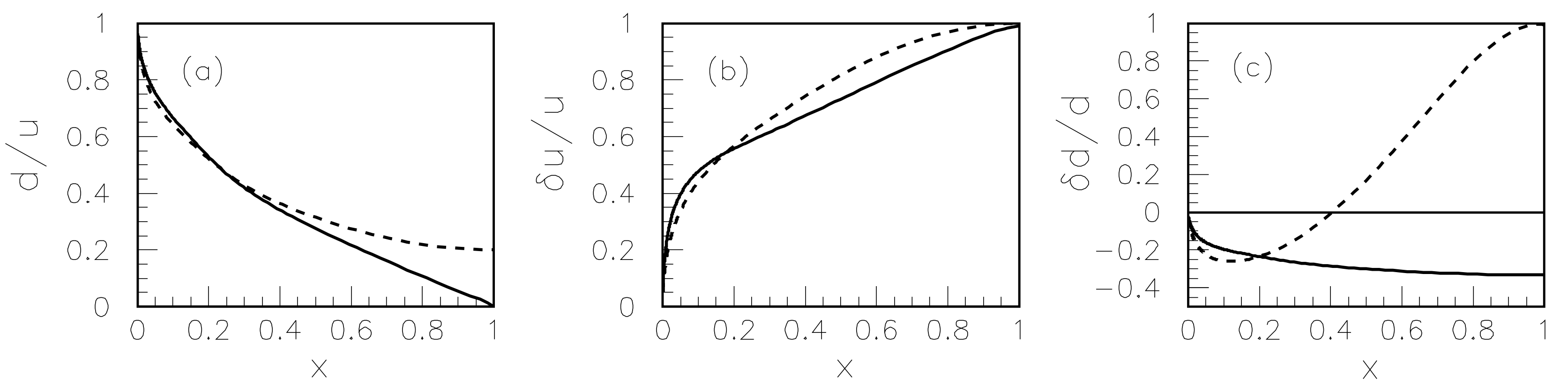
Backup Slides

Transversity Phenomenology

- ∃ a number of model calculation (facing a lack of experimental data)
- h_1 must satisfy Soffer inequality
- in common: h_1 behaves more valence-like

χQSM (A.V. Efremov et al)





Quark-Diquark (solid), pQCD based model (dashed) (B.Q. Ma et al.)

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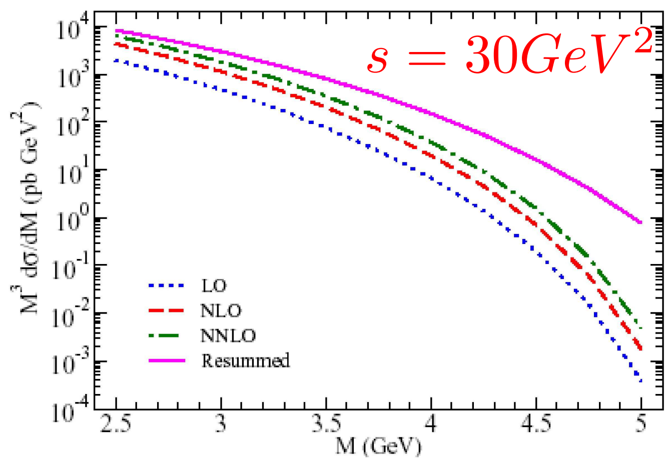
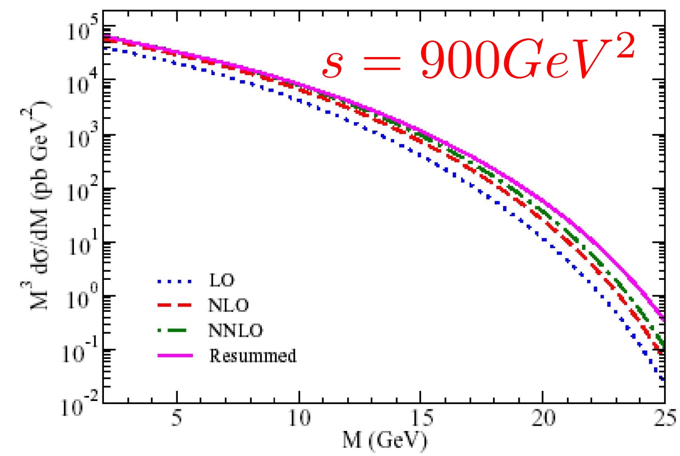
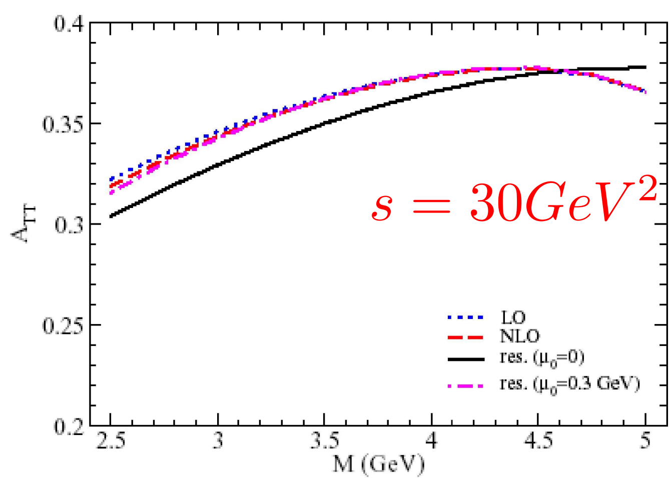
NLO Corrections to Polarized DY

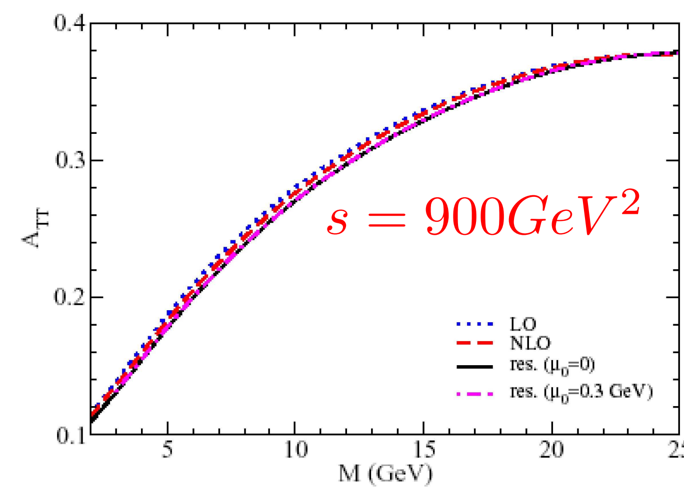
Large Corrections to Cross Sections





Smaller Corrections to Asymmetries





[hep-ph/0503270]

SIDIS Cross Section
(up to subleading order in 1/Q)

$$d\sigma = d\sigma^0_{UU} + \cos 2\phi \, d\sigma^1_{UU} + \frac{1}{Q} \cos\phi \, d\sigma^2_{UU} + \lambda_e \frac{1}{Q} \sin\phi \, d\sigma^3_{LU}$$

$$+ S_L \left\{ \sin 2\phi \, d\sigma^4_{UL} + \frac{1}{Q} \sin\phi \, d\sigma^5_{UL} + \lambda_e \left[d\sigma^6_{LL} + \frac{1}{Q} \cos\phi \, d\sigma^7_{LL} \right] \right\}$$

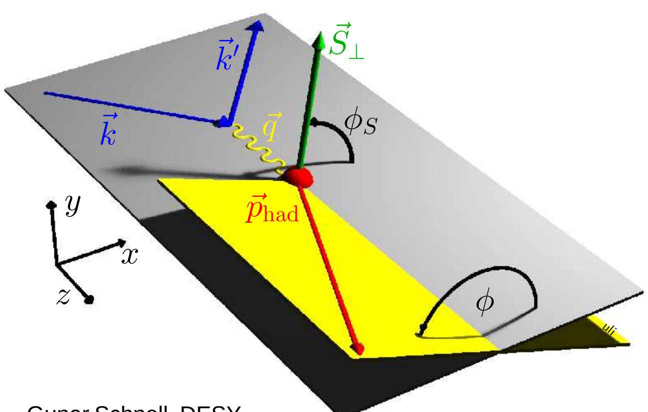
$$+ S_T \left\{ \sin(\phi - \phi_S) \, d\sigma^8_{UT} + \sin(\phi + \phi_S) \, d\sigma^9_{UT} + \sin(3\phi - \phi_S) \, d\sigma^{10}_{UT} \right.$$

$$+ \frac{1}{Q} \left(\sin(2\phi - \phi_S) \, d\sigma^{11}_{UT} + \sin\phi_S \, d\sigma^{12}_{UT} \right)$$

$$\left. + \lambda_e \left[\cos(\phi - \phi_S) \, d\sigma^{13}_{LT} + \frac{1}{Q} \left(\cos\phi_S \, d\sigma^{14}_{LT} + \cos(2\phi - \phi_S) \, d\sigma^{15}_{LT} \right) \right] \right\}$$

σ_{XY}

Beam Target
Polarization



Mulders and Tangermann, Nucl. Phys. B 461 (1996) 197

Boer and Mulders, Phys. Rev. D 57 (1998) 5780

Bacchetta et al., Phys. Lett. B 595 (2004) 309

“Trento Conventions”, Phys. Rev. D 70 (2004) 117504

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σ_{XY}: X → **Beam**, Y → **Target** **Polarization**

Terms with 1/Q are 'subleading twist'

(Factorization for SIDIS (including transverse momentum) not yet proven)

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SIDIS Cross Section (up to subleading order in 1/Q)

$$d\sigma = d\sigma^0_{UU} + \cos 2\phi\, d\sigma^1_{UU} + \frac{1}{Q}\cos\phi\, d\sigma^2_{UU} + \lambda_e \frac{1}{Q}\sin\phi\, d\sigma^3_{LU}$$

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σ_{XY}: X → Beam, Y → Target Polarization

This talk:

$\sin(\phi-\phi_S)\, d\sigma^8_{UT}$ ... Sivers Effect

$\sin(\phi+\phi_S)\, d\sigma^9_{UT}$ ... Collins Effect

A Closer Look at Collins Asymmetries I

rewrite asymmetries in terms of favored and disfavored fragmentation:

- neglect strange quarks
- assume Gaussian k_T dependence of Collins FF $\rightarrow$ can resolve convolution
- employ isospin symmetry among fragmentation functions, i.e.

$$\begin{aligned}
D_f &\equiv D(u \rightarrow \pi^+) \simeq D(d \rightarrow \pi^-) \simeq D(\bar{d} \rightarrow \pi^+) \simeq D(\bar{u} \rightarrow \pi^-) \\
D_d &\equiv D(d \rightarrow \pi^+) \simeq D(u \rightarrow \pi^-) \simeq D(\bar{u} \rightarrow \pi^+) \simeq D(\bar{d} \rightarrow \pi^-) \\
\frac{1}{2}(D_f + D_d) &\simeq D(u \rightarrow \pi^0) \simeq D(d \rightarrow \pi^0) \simeq D(\bar{d} \rightarrow \pi^0) \simeq D(\bar{u} \rightarrow \pi^0)
\end{aligned}$$

$$\begin{aligned}
\hookrightarrow \tilde{A}_C^{\pi^+/\pi^-}(x,z) &\propto \frac{(4\delta u + \delta \bar{d}) H_{f/d} + (4\delta \bar{u} + \delta d) H_{d/f}}{(4u + \bar{d}) D_{f/d} + (4\bar{u} + d) D_{d/f}} \\
\tilde{A}_C^{\pi^0}(x,z) &\propto \frac{\left[4(\delta u + \delta \bar{u}) + \delta d + \delta \bar{d}\right] (H_f + H_d)}{\left[4(u + \bar{u}) + d + \bar{d}\right] (D_f + D_d)}
\end{aligned}$$

A Closer Look at Collins Asymmetries II

express asymmetries in terms of flavor ratios:

$$\tilde{A}_C^{\pi^+} = \mathcal{K}(x,z)\frac{4+\delta r\mathcal{H}}{4+r\mathcal{D}}$$

$$\tilde{A}_C^{\pi^-} = \mathcal{K}(x,z)\frac{4\mathcal{H}+\delta r}{4\mathcal{D}+r}$$

$$\tilde{A}_C^{\pi^0} = \mathcal{K}(x,z)\frac{(4+\delta r)(1+\mathcal{H})}{(4+r)(1+\mathcal{D})}$$

Polarized Objects

$$\mathcal{H} = \frac{H_d}{H_f}$$

$$\delta r = \frac{\delta d + 4\delta\bar{u}}{\delta u + \frac{1}{4}\delta\bar{d}}$$

Unpolarized Objects

$$\mathcal{D} = \frac{D_d}{D_f}$$

$$r = \frac{d+4\bar{u}}{u+\frac{1}{4}\bar{d}}$$

e.g., CTEQ6,R1990 and Kretzer et al.

Mixed

$$\mathcal{K} = \frac{(\delta u+\frac{1}{4}\delta\bar{d})zH_f}{(u+\frac{1}{4}\bar{d})D_f}$$

$\Rightarrow$ 3 constraints and 3 unknowns!

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$$\tilde{A}_C^{\pi^0} = \mathcal{K}(x,z)\frac{(4+\delta r)(1+\mathcal{H})}{(4+r)(1+\mathcal{D})}$$

The three asymmetries are not independent ($C(x,z) \equiv \frac{r(x)+4\mathcal{D}(z)}{r(x)\mathcal{D}(z)+4}$):

$$\tilde{A}_C^{\pi^+}(x,z) + C(x,z)\tilde{A}_C^{\pi^-}(x,z) - (1+C(x,z))\tilde{A}_C^{\pi^0}(x,z) = 0$$

e.g., CTEQ6,R1990 and Kretzer et al.

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e.g., CTEQ6,R1990 and Kretzer et al.

$\Rightarrow$ ~~3~~ 2 constraints and 3 unknowns!

A Closer Look at Collins Asymmetries III

eliminate $\mathcal{K}$ and relate $\mathcal{H}$ to δr

$\Rightarrow$ scan solution space for $\mathcal{H}$ and δr by sampling set of $(\tilde{A}_C^{\pi^+}, \tilde{A}_C^{\pi^-}, \tilde{A}_C^{\pi^0})$

(around measured values according to statistical uncertainty)

A Closer Look at Collins Asymmetries III

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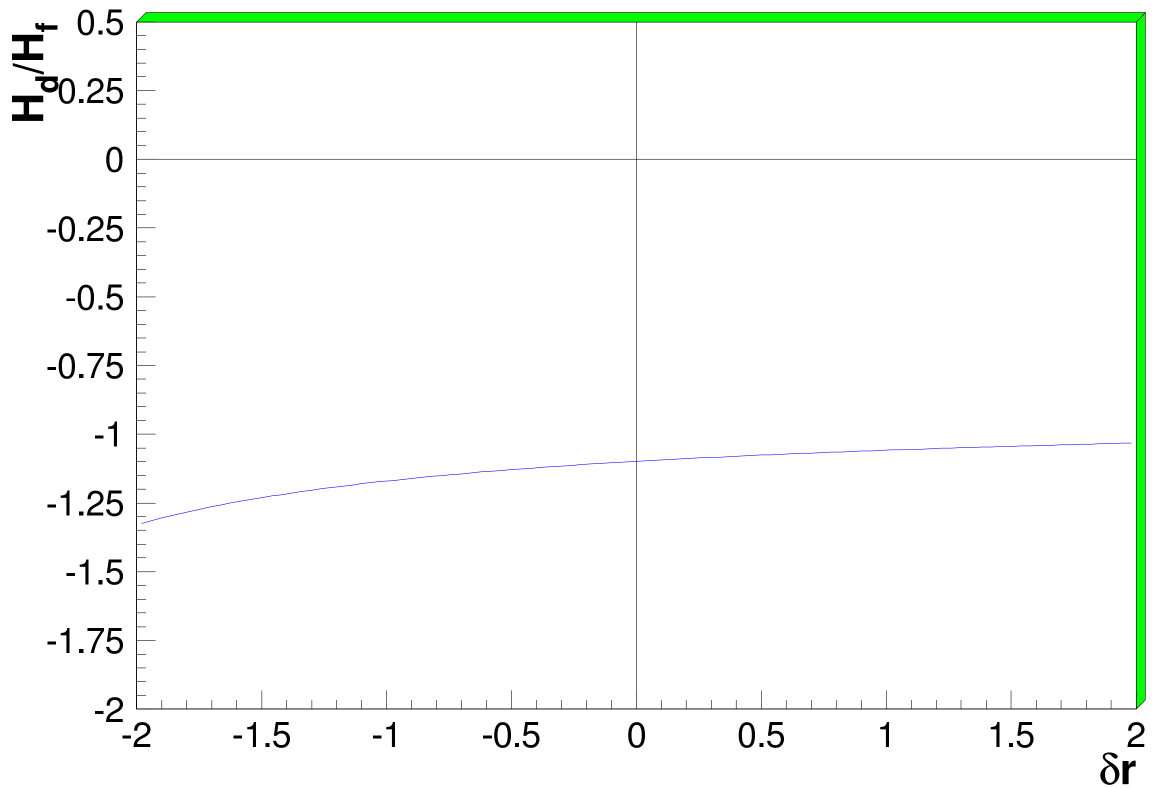

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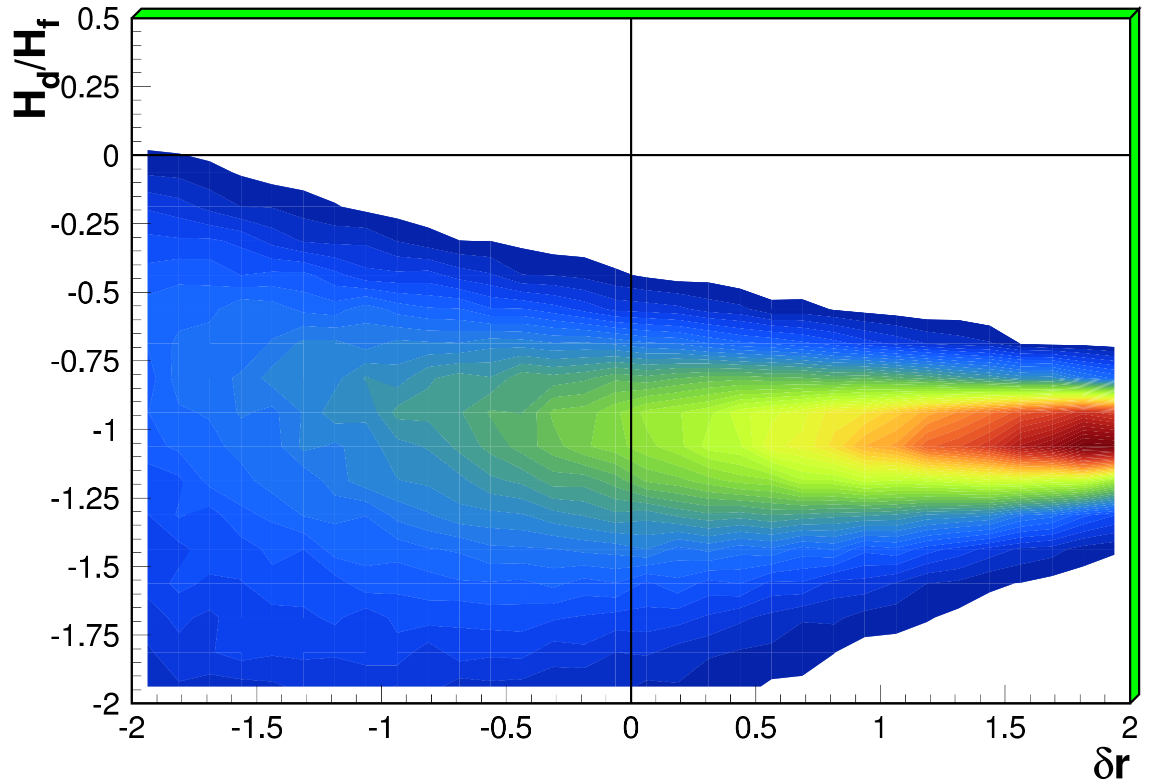
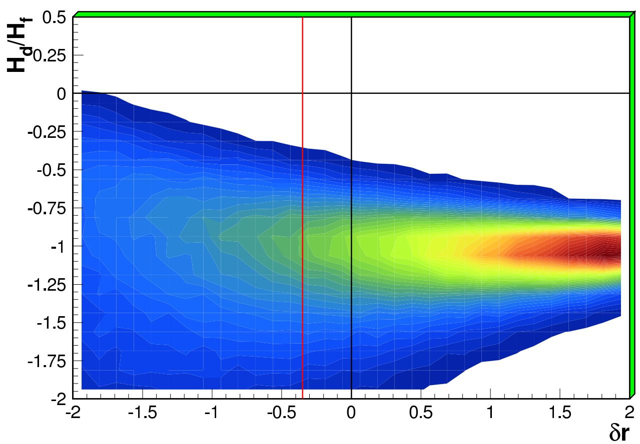
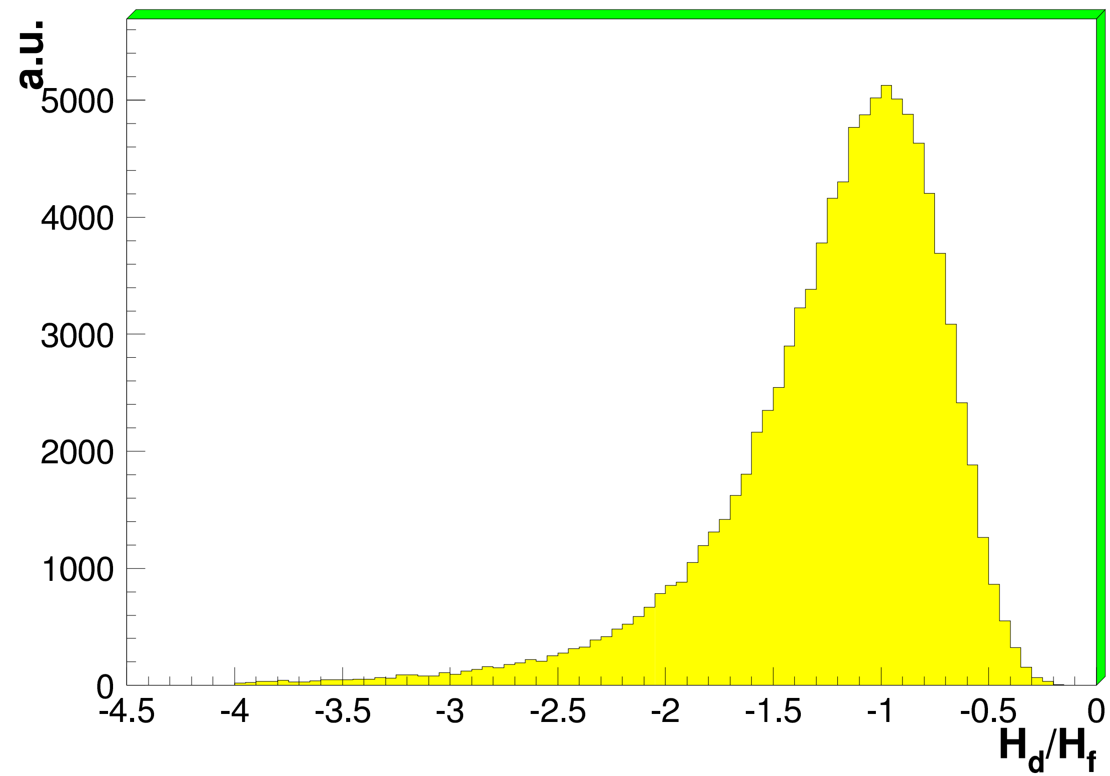


Limits on Transversity and Collins FF

$\delta r \approx \delta d/\delta u$ from χQSM



look at slice of distribution:



strong hint for H_d/H_f negative