Hadronic Structure from the Lattice

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Hadronic Structure - Introduction

- What are hadrons made of?
  Is a meson made of $\bar{q}q$, $\bar{q}\bar{q}qq$, meson-meson,..?
- A state that can decay strongly (resonance) necessarily has a meson-meson component - is this important? Are unstable particles different from stable ones?
- What is the nature of light-light scalar mesons? Where is the glueball? Are there hybrid mesons,....?

Lattice QCD is a first-principles method of attack. But we use unphysical (too heavy) quark masses, we have Euclidean time,... What can we learn?

Hadronic Decays - Introduction

- Relatively few hadronic states are stable (under QCD with degenerate u and d quarks). Mesons:
  Stable: $\pi\ K\ \eta\ D\ D_s\ B\ B_s\ B_c\ D_s^*\ B^*\ B_s^*\ D_s(0^+)\ B_s(0^+)$
  $\Gamma < 1$ MeV: $\eta'\ D^*\ \psi(1S)\ \psi(2S)\ \chi_1\ \chi_2\ \Upsilon(1S)\ \Upsilon(2S)\ \Upsilon(3S)$
  $\Gamma < 10$ MeV: $\omega\ \phi\ \chi_0\ X(3872)$
  $\Gamma > 10$ MeV: $\rho\ f_0\ a_0\ h_1\ b_1\ a_1\ f_2\ f_1\ a_2$, etc., inc η_c.
- Masses of unstable states defined as 90^0 phase shift still seem to fit patterns well:
  $\rho(776);\ \omega(783)$ are close in mass despite having widths of $150;\ 8$ MeV resp.
  $\Delta(1232);\ \Sigma(1385);\ \Xi(1530);\ \Omega(1672)$ are roughly equally spaced in mass despite having widths of $120;\ 37;\ 9;\ 0$ MeV resp. mass defined as real part of pole fits less well.

eg. $\Delta(1232)$ mass 22 MeV less   CM66

Decays in Euclidean Time on a Lattice

NO GO. At large spatial volume, two-body continuum masks resonance state. Extraction of spectral function from correlator $C(t)$ is ill-posed unless a model is made. (Since the low energy continuum dominates at large t). CM89, Maiani Testa 90

GO. For finite spatial volume (L^3), two-body continuum is discrete and Lüscher showed how to use the small energy shifts with L of these two-body levels to extract elastic scattering phase shifts. The phase shifts then determine the resonance mass and width.
The ρ appears as a distortion of the $\pi_n \pi_{-n}$ energy levels where $q = 2\pi n/L$. Lüscher 91

Lattice evaluation of $\rho \to \pi\pi$

As a check of this approach to unstable particles on a lattice, the coupling of ρ to $\pi\pi$ has been determined from first principles

McNeile CM 02

Method is to arrange ρ and $\pi\pi$ state (with definite relative momentum) to be approximately degenerate in energy on a lattice. Then several independent methods allow to determine the transtion amplitude x.

Lattice evaluation: $\rho \longrightarrow \pi\pi$

Determine coupling constant from lattice (where decay does not proceed) and compare with experiment:

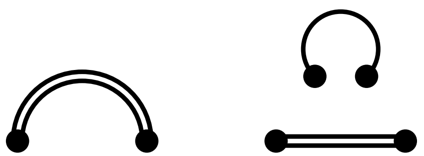
method	m_{val}	m_{sea}	$\bar{g}$
Lattice xt	s	s	1.40^{+47}_{-23}
Lattice ρ shift	s	s	1.56^{+21}_{-13}
$\phi \rightarrow K\bar{K}$	s	$u,\ d$	1.5
$K^* \rightarrow K\pi$	$u,\ d/s$	$u,\ d$	1.44
$\rho \rightarrow \pi\pi$	$u,\ d$	$u,\ d$	1.39

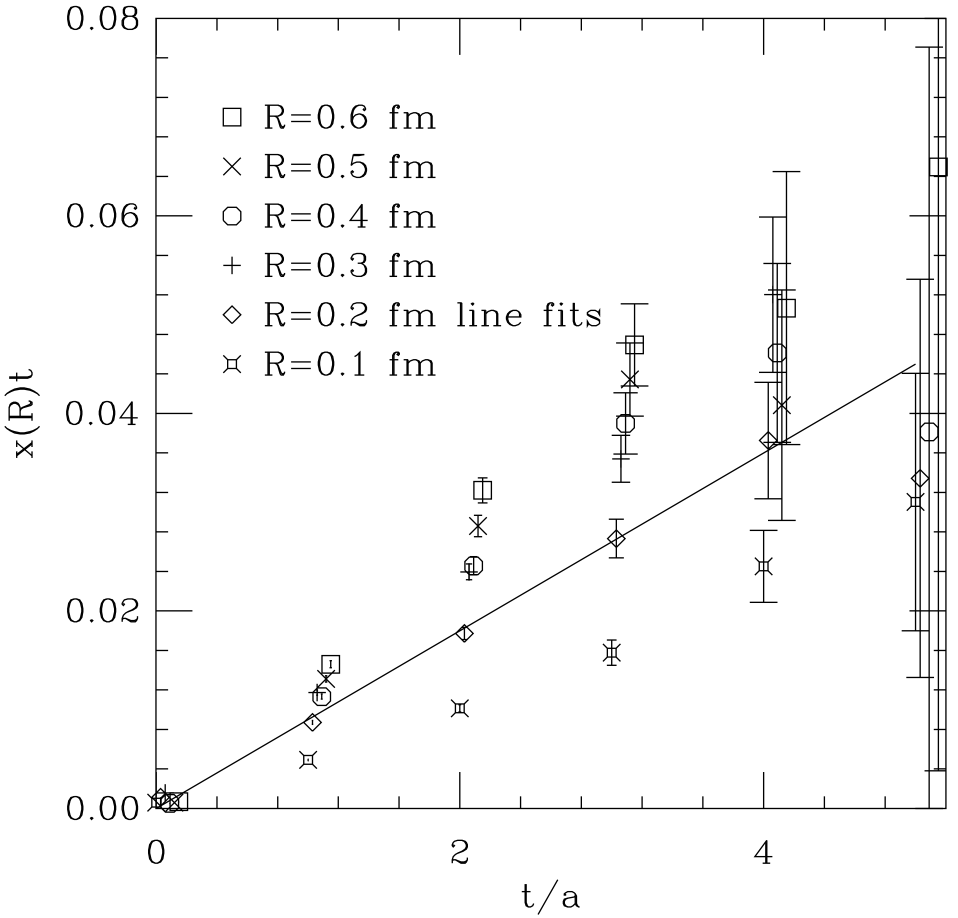
Note lattice has heavier sea quarks than experiment

Hybrid meson decay

For heavy quarks, dominant decay of H_b is string de-excitation to $\chi_b f_0$ CM CM PP 02

near on-shell for $R \approx 0.2$ fm, width predicted around 80 MeV





String breaking

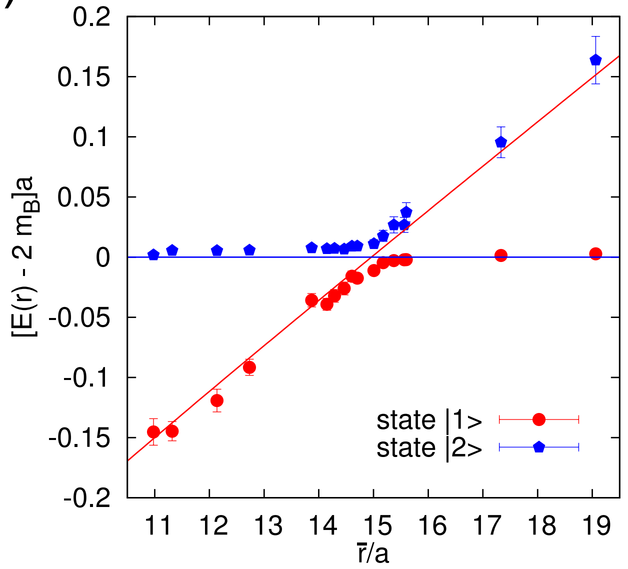
$Q\bar{Q} \to Q\bar{q}\; q\bar{Q}$

For static quarks at separation R, there will be a level crossing and associated mixing of $V(R)$ and $2m(B)$. This mixing is the measure of string breaking CM 92 Pennanen CM 00

Here the energy shift for static Q is independent of L

This energy shift (mixing amplitude) has been determined at 51(3) MeV SESAM 05

This can be related to amplitudes for excited Υ decay to $B\bar{B}$



Scalar Mesons

$u\bar{u} + d\bar{d}$, $s\bar{s}$, glueball, and meson-meson components are possible for flavour-singlet scalars

decays to $\pi\pi$ or $\eta\pi$ are allowed in dynamical lattice studies.

- 0^{++} Glueball $\to \pi\pi$. Sexton Vaccarino Weingarten 96(Quenched)
- Glueball mixing with $q\bar{q}$ meson

  hadronic transition

  Weingarten Lee 98,00(Quenched); McNeile CM 01
- Full study needed but disconnected diagram for $f_0 \to \pi\pi$ is very noisy: start on flavour non-singlets...

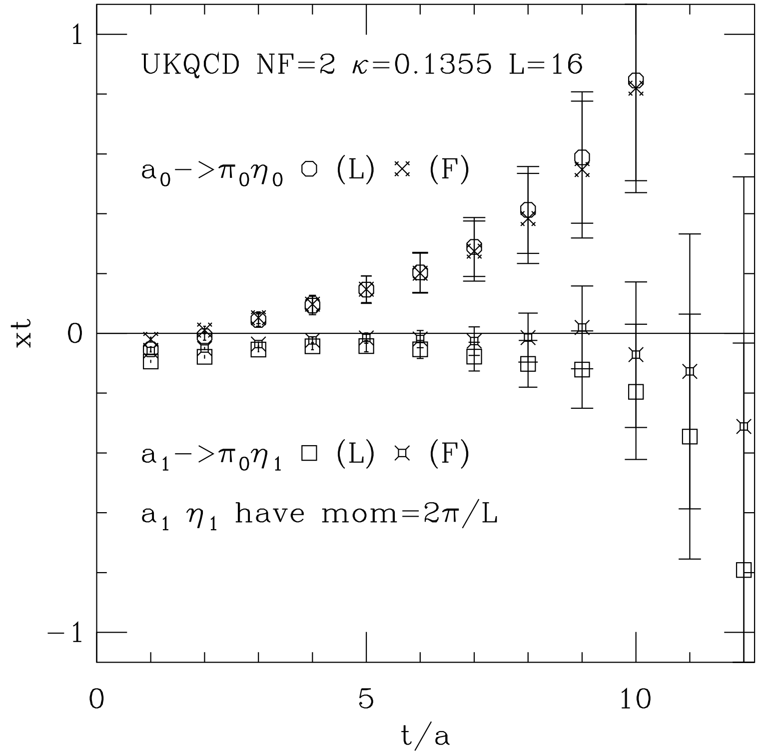
Scalar Mesons

Explore flavour non-singlet light scalars (a_0)

Quenched has ghost effects in $a_0 \to \eta\pi$ so use $N_F = 2$

Additional disconnected diagram:

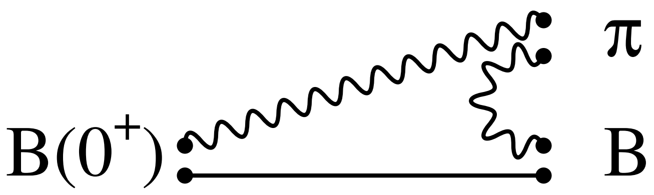
on-shell transition McNeile CM - in prep

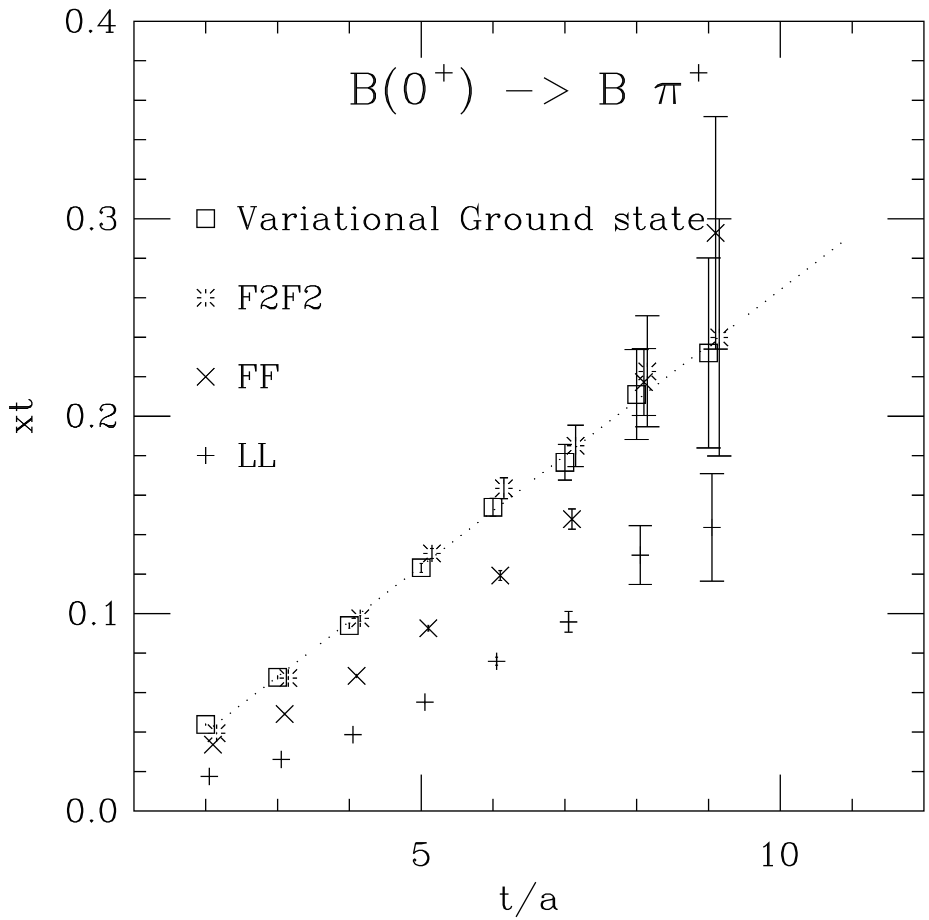


Results suggest strong dependence on momentum

Heavy-light scalar meson decay

$B(0^+) \to B(0^-)\pi$ CM CM GT 04 width predicted as 162(30) MeV
(expt results for $D(0^+)$ have 270 ± 50 MeV)





Do decays matter?

The above analysis shows that $q\bar{q}$ states mix with two-body states with the same quantum numbers. In the real world, the two-body states are a continuum and nearby states have a predominant influence, especially for S-wave thresholds.

For bound states there is an influence of nearby two-body states (eg $N\pi$ on N) which mix to reduce the mass - this is the province of low energy effective theories, especially Chiral perturbation theory.

For unstable states (resonances) the influence of the two-body continuum is less simple. For quenched QCD, however, where these two-body states are not coupled (or have the wrong sign as in $a_0 \to \eta\pi$); then the unstable states will be distorted. For instance the ρ will be too heavy since it is not repelled by the heavier $\pi\pi$ states. (with DF we saw the ρ mass decrease)

Molecular states?

Can lattice QCD provide evidence about possible molecular states: hadrons made predominantly of two hadrons?
The prototype is the deuteron: $n\ p$ bound by π exchange.
There are states close to two-body thresholds:

$f_0(980);\ a_0(980) \leftrightarrow K\bar{K}$	$D_s(0^+) \leftrightarrow D(0^-)K$
$B_s(0^+) \leftrightarrow B(0^-)K$	$X(3872) \leftrightarrow D^*\bar{D}$
$\Lambda(1405) \leftrightarrow \bar{K}N$	$N(1535) \leftrightarrow \eta N$

Some of these cases have been studied for 40 years
Isopsin breaking is enhanced by mass splittings in thresholds (eg $\bar{K^0}K^0$ compared to K^+K^- is 8 MeV higher)

Molecular states?

State near threshold <-> attractive interaction
but chicken or egg?

- Vary $m(q_1)$, $m(q_2)$: does mass of state track two-body threshold?
- Explore wavefunction: is it long ranged?
- Explore coupling of state to two-body channel

Molecule (rather than $qq\bar{q}\bar{q}$) needs spatial separation to preserve hadronic constituents. Only π exchange has long range.
Examples: deuteron, also BB bound states. CM PP 99
Mesons with one heavy quark are an ideal laboratory for this study.

$B_s(0^+)$

$\bar{b}s$ cannot couple (in isospin limit) to $\bar{b}s\ \pi$. So B_s^* cannot decay to $B_s\pi$.
The lightest open channel is then BK - reached by $\bar{s}s$ pair production.
Is $B_s(0^+)$ stable?

The corresponding $\bar{c}s$ scalar meson is lighter than DK and hence stable.

Theorists have questioned whether the experimental state is not mainly a DK molecule, since it is lighter than quark model expectation.

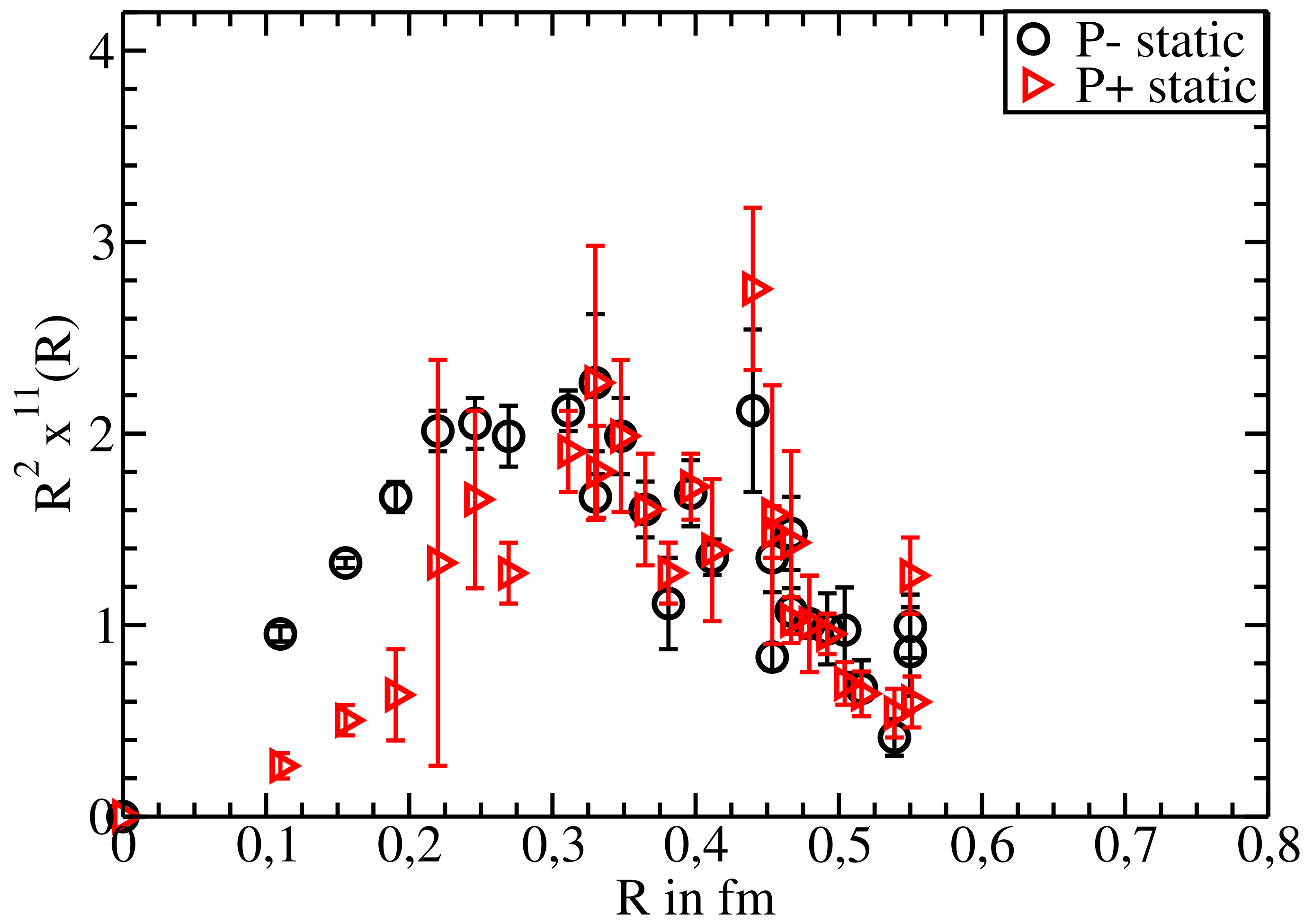
Are $D_s(0^+)$, $B_s(0^+)$ molecules?

Lattice: $B_s(0^+)$

- Lattice results for the mass indicate that both $\bar{c}s$ and $\bar{b}s$ scalar mesons are lighter than the corresponding thresholds(DK, BK) - so stable. Dougall et al 03; Green et al. 03
- Moreover the wavefunction (actually charge distribution) of the $\bar{b}s$ scalar meson has been measured - and it looks just like that for other $\bar{b}s$ mesons which are not considered to have any molecular content. Green et al. 04 see fig
- The hadronic transition to BK has also been determined (see above) and this has a similar coupling to other (non-molecular) scalar mesons.

Evidence points to $B_s(0^+)$ being predominantly $\bar{b}s$ rather than BK. Green et al. 05.

Wavefunctions B_s: $J^P = 0^+, 2^+$



BB states

An ideal testing ground for 4-quark mesons is the BB system.
$\bar{b}\bar{b}qq$ has no quark-antiquark coupling.
Treating b as infinitely heavy (ie static) then the separation R between the two B mesons is adjustable on the lattice: one can measure the energy versus R for each light quark total spin (0 or 1) and isospin (0 or 1).
Results show several different scenarios

CM PP 99

- long range attractive force (from π exchange) giving binding - as a BB molecule. (for I=0, S=1)
- short range binding (two light quarks arranged as in the Λ_b and Σ_b baryons) as a $\bar{b}\bar{b}qq$ state with the two anti-b quarks forming an anti-triplet. (for I=0, S=0 and I=1, S=1)
- some unbound channels (I=1, S=0)

Multi-quark states?

I told you so: a narrow pentaquark above KN threshold is not possible in QCD. I lived through the split A_2 and baryonium.
Lattice exploration of pentaquark,etc. An attractive phase shift in some KN channel is not sufficient to resolve the issue. The width needs to be evaluated.

- 2+1 flavours of sea quark, with light u,d.
- Operators to create multi-quark states and two-body states
- Vary spatial size to determine phase shift of two-body interaction.

Multi-quark states?

In practice: quenched studies (OK as an exploratory step if decay width is expected to be small). Also a narrow resonance will show up as a stable state (except near the avoided-level crossing when it is degenerate with a discrete two-body energy).

Mathur et. al (Kentucky et al.) 04 conclude no evidence for pentaquark

some groups claim evidence for a signal: Alexandrou Tsapalis 05

some said yes Csikor et al. 03 then later no Csikor et al. 05

Conclusions

Lattice can address hadronic structure:

- form factors (eg. charge wavefunctions) can be evaluated
- decay transitions (and mixing transitions) can be evaluated
- structure function moments can be evaluated (steady but slow progress here)
- hadronic matrix elements are needed to interpret experiment (eg f_B relates B meson to b quark, etc..)

Hadronic physics involves unstable states. Lattice techniques are being developed to study these. There is a lot to be learnt beyond mass spectra.