QCD in hard exclusive processes Selected results on generalized parton distributions

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24 September 2005



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1. Introduction

- 2. Quarks and gluons: some lessons from data
- 3. t dependence and impact parameter
- 4. Spin and the Pauli form factors
- 5. Conclusions

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Generalized parton distributions in a nutshell

• GPDs \leftrightarrow matrix elements $\langle p' | \mathcal{O} | p \rangle$ $\mathcal{O} =$ non-local operator with quark/gluon fields



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- $\blacktriangleright \ p \neq p' \rightsquigarrow$ depend on two longitud. momentum fractions $x, \ \xi$ and on $t = (p-p')^2$
- for unpolarized quarks two dist's:
 - H^q conserves proton helicity
 - E^q responsible for proton helicity flip
- \blacktriangleright if $p=p' \leadsto$ ordinary parton densities

$$H^q(x,0,0) = \left\{ \begin{array}{ll} q(x) & \mbox{for } x>0 \\ -\bar{q}(x) & \mbox{for } x<0 \end{array} \right.$$

Generalized parton distributions in a nutshell

• GPDs \leftrightarrow matrix elements $\langle p' | \mathcal{O} | p \rangle$ $\mathcal{O} =$ non-local operator with quark/gluon fields



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- $\blacktriangleright \ p \neq p' \rightsquigarrow$ depend on two longitud. momentum fractions $x, \ \xi$ and on $t = (p-p')^2$
- for unpolarized quarks two dist's:
 - H^q conserves proton helicity
 - E^q responsible for proton helicity flip
- $\int dx \, x^n \operatorname{GPD}(x,\xi,t) \to \text{local operators} \to \text{form factors}$

$$\sum_{q} e_q \int_{-1}^{1} dx H^q(x,\xi,t) = F_1(t) \quad \text{Dirac}$$
$$\sum_{q} e_q \int_{-1}^{1} dx E^q(x,\xi,t) = F_2(t) \quad \text{Pauli}$$

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Processes

factorization theorems: GPDs appear in hard exclusive processes

calculated to NLO in α_s :

- ▶ DVCS $\gamma^* p \rightarrow \gamma p$ (including charm loop J. Noritzsch '03)
- ▶ light meson production $\gamma^* p \rightarrow \rho p$, πp , ... A. Belitsky and D. Müller '01, D. Ivanov et al. '04
- $\gamma p
 ightarrow J\!/\Psi p$ D. Ivanov et al. '04

in meson production NLO corrections can be large more detailed studies needed



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Evolution

- GPDs depend on resolution scale μ
 ~ large momentum in hard process
- evolution interpolates between DGLAP eqs. (parton densities) and ERBL eqs. (meson distribution amplitudes)
- known to NLO A. Freund et al. '99

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Evolution

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- evolution interpolates between DGLAP eqs. (parton densities) and ERBL eqs. (meson distribution amplitudes)
- known to NLO A. Freund et al. '99
- new: explicit solution of LO evolution A. Manashov et al. '05
 - usual parton densities: invert Mellin transform

 $M^{j}(\mu) = \int dx \; x^{j-1} \, q(x,\mu)$ evolves multiplicatively

 $q(x,\mu) = -\frac{1}{2\pi i} \int\limits_C dj \ x^{-j} \ M^j(\mu)$

- ► GPDs: moments and inversion involve Legendre functions
- $\label{eq:rescaled} \begin{tabular}{lll} \begin{tabular}{lll} \bullet & \to \end{tabular} \end{tabular} fast numeric implementation analytic approximations \end{tabular}$



quark and gluon GPDs at same $O(lpha_s)$

schematically:

$$\mathcal{A}_{\rho^0} \propto \frac{1}{\sqrt{2}} \left[\frac{2}{3} (u + \bar{u}) + \frac{1}{3} (d + \bar{d}) + \frac{3}{4} g \right]$$
$$\mathcal{A}_{\phi} \propto \frac{1}{3} (s + \bar{s}) + \frac{1}{4} g$$



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- vector meson production:
 quark and gluon GPDs at same O(α_s)
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CTEQ6L at $\mu=2~{\rm GeV}$

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- vector meson production:
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CTEQ6L at $\mu = 2 \text{ GeV}$



▶ prelim. HERMES data \Rightarrow substantial gluon contrib'n in ρ^0 production at $x_B \sim 0.1$ M.D. and A. Vinnikov, '04

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leading twist LO calculation M.D. et al. '05 ('conventional' double distribution model for GPDs)



- gluons may be non-negligible even in JLAB kinematics
- substantial uncertainties on conventional gluon densities
- warning: should do NLO evaluation

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- gluons may be non-negligible even in JLAB kinematics
- substantial uncertainties on conventional gluon densities
- warning: should do NLO evaluation
- ► calculated σ(ρ⁰)/σ(φ) too large but expect extra suppression for φ (strange quark mass)

Quarks and gluons

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Conclusions

► leading-twist calculations for vector meson production overshoot data factors of several at Q² ≤ 5 GeV²

- strong suppression from meson k_T in hard scattering
 - L. Frankfurt et al. '95; M. Vanderhaeghen et al. '99
- new analysis for small x_B (gluons only)



Feg.





hep-ph/0501242, CTEQ5M gluon, double distribution model

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CTEQ5M gluon

P. Kroll, S. Goloskokov '05



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Conclusions

- ► directly sensitive to gluon distribution at small x unlike inclusive struct. funct. F₂(x, Q²)
- ► despite uncertainties in modelling g(x) ~→ GPD J/Ψ data cast doubt on some gluon distrib's



T. Teubner, DIS 2005, plots by P. Fleischmann (H1)

► alternative model for g(x), q(x) ~→ GPD (not based on double distrib's)

V. Guzey and M. Polyakov '05, based on M. Polyakov and A. Shuvaev '02

- satisfies Lorentz invariance (polynomiality) relations
- good description of DVCS data from HERA (LO calculation)



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The t dependence

 \blacktriangleright at small x and small t parametrize

 $d\sigma/dt \propto e^{-B|t|}$

▶ ρ and ϕ : "pointlike" $\gamma^* \rightarrow q\bar{q}$ for large Q^2 J/Ψ : "pointlike" $\gamma \rightarrow c\bar{c}$ even for $Q^2 = 0$





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H1 Coll. '05

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Small x

neglect interplay of x and ξ simple ansatz: GPD $\sim \left(\frac{1}{x}\right)^{\alpha+\alpha' t} = x^{-\alpha} e^{t\alpha' \log(1/x)}$

- exclusive J/Ψ production (gluons)
 - photoproduction

H1 preliminary (DIS 05)

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 $\alpha = 1.224 \pm 0.010 \pm 0.012$

 $\alpha' = 0.164 \pm 0.028 \pm 0.030 \text{ GeV}^{-2}$

- similar in electroproduction
- values very different in soft processes γp → ρp, pp → pp, ... for α is well-known from inclusive γ*p → X vs. γp → X

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Small x

neglect interplay of x and ξ simple ansatz: GPD $\sim \left(\frac{1}{x}\right)^{\alpha+\alpha' t} = x^{-\alpha} e^{t\alpha' \log(1/x)}$

- ▶ in nonsinglet sector (quarks only, no gluons) $\alpha \sim 0.4...0.5$ in parton distrib's at low scale similar to soft processes (meson trajectories)
- ▶ α' in partonic regime? ... wait a few slides

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Impact parameter

 states with definite light-cone momentum p⁺ and transverse position (impact parameter):

$$|p^+, \mathbf{b}\rangle = \int d^2 \boldsymbol{p} \, e^{-i\boldsymbol{b} \, \boldsymbol{p}} |p^+, \boldsymbol{p}\rangle$$

formal: eigenstates of 2 dim. position operator

- can exactly localize proton in 2 dimensions no limitation by Compton wavelength
- ► and stay in frame where proton moves fast → parton interpretation

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Impact parameter

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$$|p^+, \boldsymbol{b}\rangle = \int d^2 \boldsymbol{p} \, e^{-i\boldsymbol{b} \, \boldsymbol{p}} \, |p^+, \boldsymbol{p}\rangle$$

formal: eigenstates of 2 dim. position operator
b is center of momentum of the partons in proton

$$\boldsymbol{b} \underbrace{ \begin{array}{c} \boldsymbol{b} \\ \hline \boldsymbol{p}_i^+, \boldsymbol{b}_i \end{array}}_{p_i^+, \boldsymbol{b}_i} \qquad \boldsymbol{b} = \frac{\sum_i p_i^+ \boldsymbol{b}_i}{\sum_i p_i^+} \qquad (i = q, \bar{q}, g)$$

consequence of Lorentz invariance nonrelativistic analog: Galilei invariance \Rightarrow center of mass

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Impact parameter GPDs

in following specialize to $\xi=0$

impact parameter distribution

$$q(x, \mathbf{b}^2) = (2\pi)^{-2} \int d^2 \mathbf{\Delta} e^{-i\mathbf{\Delta} \cdot \mathbf{b}} H^q(x, \xi = 0, t = -\mathbf{\Delta}^2)$$

gives distribution of quarks with

- longitudinal momentum fraction \boldsymbol{x}
- transverse distance b from proton center M. Burkardt '00
- average impact parameter

$$\langle b^2 \rangle_x = \frac{\int d^2 b \ b^2 \ q(x, b^2)}{\int d^2 b \ q(x, b^2)} = 4 \frac{\partial}{\partial t} \log H^q(x, \xi, t) \Big|_{t=0}$$

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d = *b*/(1 − *x*) = distance of selected parton from spectator system gives lower bound on overall size of proton

 \blacktriangleright finite size of configurations with $x \to 1$ implies

$$\langle b^2 \rangle_x \sim (1-x)^2$$

M. Burkardt, '02, '04

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$$\begin{array}{ll} \mbox{Small x:} \\ H(x,t) \sim e^{tB + \alpha' \log(1/x)} & \rightsquigarrow & \langle b^2 \rangle_x \sim B + \alpha' \log(1/x) \end{array}$$

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Evolution

▶ q(x, b²) fulfils usual DGLAP evolution equation for non-singlet (e.g. q_{NS} = q - q̄ or q_{NS} = u - d):

$$\mu^{2} \frac{d}{d\mu^{2}} q_{\rm NS}(x, b^{2}, \mu^{2}) = \int_{x}^{1} \frac{dz}{z} \left[P\left(\frac{x}{z}\right) \right]_{+} q_{\rm NS}(z, b^{2}, \mu^{2})$$

evolution local in b (let $1/\mu \ll b$ to be safe)

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evolution local in b (let $1/\mu \ll b$ to be safe)

average

$$\langle b^2 \rangle_x = \frac{\int d^2 b \ b^2 \ q_{\rm NS}(x, b^2)}{\int d^2 b \ q_{\rm NS}(x, b^2)}$$

evolves as

$$\mu^2 \frac{d}{d\mu^2} \langle b^2 \rangle_x = -\frac{1}{q_{\rm NS}(x)} \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}\right) q_{\rm NS}(z) \left[\langle b^2 \rangle_x - \langle b^2 \rangle_z \right]$$

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Information from electromagnetic form factors

 \blacktriangleright ff's constrain interplay of x and b dependence

M.D. et al. '04, M. Guidal et al. '04

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• e.m. current \rightsquigarrow only $q - \bar{q}$ $H^q_v(x,t) = H^q(x,t) - H^{\bar{q}}(x,t)$

$$\begin{array}{lll} F_1^p(t) &=& \int_0^1 dx \left[\frac{2}{3} H_v^u(x,t) - \frac{1}{3} H_v^d(x,t) \right] \\ F_1^n(t) &=& \int_0^1 dx \left[\frac{2}{3} H_v^d(x,t) - \frac{1}{3} H_v^u(x,t) \right] \end{array}$$

ansatz: H^q_v(x,t) = q_v(x) exp[tf_q(x)] (b²)^q_x = 4f_q(x)
ansatz for f_q(x) interpolates between
f_q(x) → α' log(1/x) for x → 0
f_q(x) ~ (1-x)² for x → 1

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M.D. et al. '04, M. Guidal et al. '04

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• e.m. current \rightsquigarrow only $q - \bar{q}$

$$H_v^q(x,t) = H^q(x,t) - H^q(x,t)$$

$$\begin{aligned} F_1^p(t) &= \int_0^1 dx \left[\frac{2}{3} H_v^u(x,t) - \frac{1}{3} H_v^d(x,t) \right] \\ F_1^n(t) &= \int_0^1 dx \left[\frac{2}{3} H_v^d(x,t) - \frac{1}{3} H_v^u(x,t) \right] \end{aligned}$$

▶ ansatz: $H^q_v(x,t) = q_v(x) \exp[tf_q(x)] \quad \langle b^2 \rangle^q_x = 4f_q(x)$

► ansatz for $f_q(x)$ interpolates between $f_q(x) \rightarrow \alpha' \log(1/x)$ for $x \rightarrow 0$ $f_q(x) \sim (1-x)^2$ for $x \rightarrow 1$

▶ good description of data with $\alpha' = 0.9$ to 1 GeV⁻²



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Lessons from the fit



▶ clear drop with x of average distance d = b/(1 - x) \leftrightarrow strong correlation of x and t dependence

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Lessons from the fit



- ▶ clear drop with x of average distance d = b/(1-x) \leftrightarrow strong correlation of x and t dependence
- region x ≥ 0.8 contributes less than 5% to form factors → data cannot fix asymptotic behavior of d_q(x) for x → 1



 d quark distribution less well determined improvement with better data for F₁ⁿ

► to describe both F_1^p and F_1^n well fit wants $d_d(x) > d_u(x)$ for moderate to large x $\leftrightarrow d$ quarks more "spread out" than u quarks

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Compare with lattice results

matrix elements of local operators \leftrightarrow form factors calculate in lattice QCD









- main systematic uncertainties from
 - omission of "disconnected" diagrams but: cancel in difference of u and d
 - extrapolation to physical pion mass

figure: J. Negele, hep-lat/0211022

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Compare with lattice results

matrix elements of local operators \leftrightarrow form factors calculate in lattice QCD



- J. Negele et al., hep-lat/0404005
 - Wilson fermions
 - ► $m_{\pi} = 870 \text{ MeV}$

• typical x in $\int dx \, x^n q(x, b)$ estimated as

$$\langle x \rangle = \frac{\int dx \, x^{n+1} q(x)}{\int dx \, x^n q(x)}$$

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Large t and the Feynman mechanism



 \blacktriangleright if impose that spectators have virtualities $\sim \Lambda^2$ then

$$1 - x \sim \Lambda / \sqrt{-t}$$

- ▶ large-t asymptotics with our ansatz: $\langle 1-x \rangle_t \sim 1/\sqrt{-t}$ numerically seen for $-t\gtrsim 5~{\rm GeV}^2$
- ▶ get Drell-Yan relation $F_1^q(t) \sim |t|^{-(1+\beta_q)}$ if $q(x) \sim (1-x)^{\beta_q}$ at large xCTEQ6M distributions at $\mu = 2$ GeV: $\beta_u \sim 3.4$ and $\beta_d \sim 5.0$ (for 0.5 < x < 0.9)

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• very different u(x) and d(x) for large x

 \leftrightarrow very different u and d moments at large t

▶ hope to test with experimental data on Fⁿ₁(t) and F^p₁(t) and Ittice calculations of higher moments



$$\begin{split} h^q_{n,0}(t) = \\ \int_0^1 \! dx \, x^{n-1} \, H^q_v(x,t) \end{split}$$

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Spin and the Pauli form factors

► $E \leftrightarrow$ nucleon helicity flip $\langle \downarrow | \mathcal{O} | \uparrow \rangle$ \leftrightarrow transverse pol. difference $|X\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)$

 $\langle X + |\mathcal{O}|X + \rangle - \langle X - |\mathcal{O}|X - \rangle = \langle \uparrow |\mathcal{O}| \downarrow \rangle + \langle \downarrow |\mathcal{O}| \uparrow \rangle$

 \blacktriangleright quark density in proton state $|X\!+\rangle$

$$q_v^X(x,\mathbf{b}) = q_v(x,b) - \frac{b^y}{m} \frac{\partial}{\partial b^2} e_v^q(x,b)$$

shifted in y direction $e_v^q(x,b)$ is Fourier transform of $E_v^q(x,t)$

M. Burkardt '02

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- ► $\int dx E^u(x,0) = \kappa^u \approx 1.67$ and $\int dx E^d(x,0) = \kappa^d \approx -2.03$ \rightarrow large spin-orbit correlations
- relation with transverse momentum dependent densities
 → Sivers function
 M. Burkardt et al. '04
- similar for generalized transversity distributions

M.D. and P. Hägler '05, M. Burkardt '05

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density representation

$$q_v^X(x, \mathbf{b}) = q_v(x, b) - \frac{b^y}{m} \frac{\partial}{\partial b^2} e_v^q(x, b)$$

gives positivity bound

M. Burkardt '03

$$\begin{split} \left[E^q(x,t=0) \right]^2 &\leq m^2 \Big[q(x) + \Delta q(x) \Big] \left[q(x) - \Delta q(x) \right] \\ &\times 4 \, \frac{\partial}{\partial t} \ln \left[H^q(x,t) \pm \tilde{H}^q(x,t) \right]_{t=0} \end{split}$$

 \Rightarrow E^q must fall faster than H^q at large x



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- ▶ sum rules: Pauli ff's $\leftrightarrow E^q_v(x,t) = E^q(x,t) E^{\bar{q}}(x,t)$
- ▶ ansatz $E_v^q(x,t) = e_v(x) \exp[t g_q(x)]$ $g_q(x)$ same form as $f_q(x)$ in ansatz for H_v^q
- ▶ shape of forward limit $e_v^q(x)$ not known → ansatz

$$e_v^q = \mathcal{N}_q \ x^{-\alpha} (1-x)^{\beta_q}$$

 \mathcal{N}_q determined by p and n magnetic moments

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$$e_v^q = \mathcal{N}_q \ x^{-\alpha} (1-x)^{\beta_q}$$

 \mathcal{N}_q determined by p and n magnetic moments

- obtain good fit of $F_2^p(t)$ and $F_2^n(t)$
 - $\alpha' = 0.9 \ {\rm GeV}^{-2}$ and $\alpha = 0.55$

ok with Regge phenomenology

large allowed regions of β_q and parameters in g_q(x)
 but positivity constraints seriously limit parameter space in particular β_d ≥ 5 and β_u ≥ 3.5





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orbital angular momentum carried by valence quarks





individual u and d quite well determined

- $2\langle J_v^u \rangle = 2\langle L_v^u \rangle + 0.93$ and $2\langle J_v^d \rangle = 2\langle L_v^d \rangle 0.34$
- ▶ $2\langle L_v^u L_v^d \rangle = -(0.77 \div 0.92)$ at $\mu = 2$ GeV

strong cancellations in $2\langle L_v^u + L_v^d \rangle = -(0.11 \div 0.22)$

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orbital angular momentum carried by valence quarks

 $\langle L_v^q \rangle = \frac{1}{2} \int dx \left[x e_v^q(x) + x q_v(x) - \Delta q_v(x) \right]$



▶ $2\langle L_v^u - L_v^d \rangle = -(0.77 \div 0.92)$ at $\mu = 2 \text{ GeV}$

lattice results:

QCDSF: $2\langle L_v^u - L_v^d \rangle = -0.9 \pm 0.12$ G. Schierholz, LC 2005 LHPC: $2\langle L_v^u - L_v^d \rangle = -0.25 \pm 0.05$ for $m_{\pi} = 897$ MeV from hep-lat/0410017

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calculation of $E^u + E^d$ in chiral soliton model J. Ossmann et al. '05



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calculation of $E^u + E^d$ in chiral soliton model J. Ossmann et al. '05



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Conclusions

- vector meson prodution: very sensitive to gluon distrib'n even in fixed-target kinematics higher twist corrections essential for realistic cross sections
- impact parameter picture: naturally implemented at a rigorous level
- \blacktriangleright $F_1(t)$ data and lattice $\, \rightsquigarrow \,$ strong decrease of $\langle {m b}^2 \rangle$ with x
- $F_1(t)$ data consistent with Feynman mechanism
 - $\rightsquigarrow \quad \text{Drell-Yan relation}$
 - \rightsquigarrow striking effects for d quark part of form factors
- ► $E \rightarrow$ physics of transverse spin and \rightarrow orbital angular momentum
- ▶ attempts for quantitative understanding of E "valence" contributions: L^{u-d} big and L^{u+d} small need direct measurements to learn more

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