QCD in hard exclusive processes
Selected results on generalized parton distributions

M. Diehl
Deutsches Elektronen-Synchroton DESY

24 September 2005
1. Introduction

2. Quarks and gluons: some lessons from data

3. $t$ dependence and impact parameter

4. Spin and the Pauli form factors

5. Conclusions
Generalized parton distributions in a nutshell

- GPDs ↔ matrix elements $\langle p' | O | p \rangle$
  $O$ = non-local operator with quark/gluon fields

- $p \neq p'$ depend on two longitud. momentum fractions $x, \xi$
  and on $t = (p - p')^2$

- for unpolarized quarks two dist's:
  - $H^q$ conserves proton helicity
  - $E^q$ responsible for proton helicity flip

- if $p = p'$ ↔ ordinary parton densities

$$H^q(x, 0, 0) = \begin{cases} 
q(x) & \text{for } x > 0 \\
-\bar{q}(x) & \text{for } x < 0
\end{cases}$$
Generalized parton distributions in a nutshell

- GPDs ↔ matrix elements \( \langle p'|O|p \rangle \)

  \( O \) = non-local operator with quark/gluon fields

- \( p \neq p' \) depend on two longitudinal momentum fractions \( x, \xi \)
  and on \( t = (p - p')^2 \)

- for unpolarized quarks two dist’s:
  - \( H^q \) conserves proton helicity
  - \( E^q \) responsible for proton helicity flip

- \( \int dx \, x^n \) \( \text{GPD}(x, \xi, t) \rightarrow \) local operators \( \rightarrow \) form factors

\[
\sum_{q} e_q \int_{-1}^{1} dx \, H^q(x, \xi, t) = F_1(t) \quad \text{Dirac}
\]

\[
\sum_{q} e_q \int_{-1}^{1} dx \, E^q(x, \xi, t) = F_2(t) \quad \text{Pauli}
\]
Processes

factorization theorems: GPDs appear in hard exclusive processes calculated to NLO in $\alpha_s$:

- DVCS $\gamma^* p \rightarrow \gamma p$ (including charm loop J. Noritzsch '03)
- light meson production $\gamma^* p \rightarrow \rho p, \pi p, \ldots$
  A. Belitsky and D. Müller '01, D. Ivanov et al. '04
- $\gamma p \rightarrow J/\Psi p$ D. Ivanov et al. '04

in meson production NLO corrections can be large
more detailed studies needed

![Diagram of DVCS process]

![Diagram of light meson production process]
Processes

factorization theorems: GPDs appear in hard exclusive processes calculated to NLO in $\alpha_s$:

- DVCS $\gamma^* p \rightarrow \gamma p$ (including charm loop J. Noritzsch '03)
- light meson production $\gamma^* p \rightarrow \rho p, \pi p, \ldots$
  A. Belitsky and D. Müller '01, D. Ivanov et al. '04
- $\gamma p \rightarrow J/\Psi p$ D. Ivanov et al. '04

in meson production NLO corrections can be large
more detailed studies needed
Evolution

- GPDs depend on resolution scale $\mu$
  $\sim$ large momentum in hard process
- evolution interpolates between DGLAP eqs. (parton densities) and ERBL eqs. (meson distribution amplitudes)
- known to NLO  A. Freund et al. ’99
Evolution

- GPDs depend on resolution scale $\mu$
  ~ large momentum in hard process
- evolution interpolates between DGLAP eqs. (parton densities) and ERBL eqs. (meson distribution amplitudes)
- known to NLO  A. Freund et al. ’99
- new: explicit solution of LO evolution  A. Manashov et al. ’05
  - usual parton densities: invert Mellin transform
    \[
    M^j(\mu) = \int dx \ x^{j-1} \ q(x, \mu) \quad \text{evolves multiplicatively}
    \]
    \[
    q(x, \mu) = -\frac{1}{2\pi i} \int_C dj \ x^{-j} \ M^j(\mu)
    \]
- GPDs: moments and inversion involve Legendre functions
- → fast numeric implementation
  analytic approximations
- vector meson production:
  quark and gluon GPDs at same $O(\alpha_s)$
- schematically:
  \[
  A_{\rho^0} \propto \frac{1}{\sqrt{2}} \left[ \frac{2}{3}(u + \bar{u}) + \frac{1}{3}(d + \bar{d}) + \frac{3}{4}g \right] \\
  A_{\phi} \propto \frac{1}{3}(s + \bar{s}) + \frac{1}{4}g
  \]
- vector meson production:
  - quark and gluon GPDs at same $O(\alpha_s)$
  
  - schematically:
    \[ A_{\rho^0} \propto \frac{1}{\sqrt{2}} \left[ \frac{2}{3} (u + \bar{u}) + \frac{1}{3} (d + \bar{d}) + \frac{3}{4} g \right] \]
    \[ A_{\phi} \propto \frac{1}{3} (s + \bar{s}) + \frac{1}{4} g \]

CTEQ6L at $\mu = 2$ GeV
vector meson production:
quark and gluon GPDs at same $O(\alpha_s)$

schematically:

\[
\mathcal{A}_\rho^0 \propto \frac{1}{\sqrt{2}} \left[ \frac{2}{3}(u + \bar{u}) + \frac{1}{3}(d + \bar{d}) + \frac{3}{4}g \right]
\]

\[
\mathcal{A}_\phi \propto \frac{1}{3}(s + \bar{s}) + \frac{1}{4}g
\]
vector meson production:
quark and gluon GPDs at same $O(\alpha_s)$
schematically:

$$A_{\rho^0} \propto \frac{1}{\sqrt{2}} \left[ \frac{2}{3} (u + \bar{u}) + \frac{1}{3} (d + \bar{d}) + \frac{3}{4} g \right]$$

$$A_\phi \propto \frac{1}{3} (s + \bar{s}) + \frac{1}{4} g$$

prelim. HERMES data ⇒ substantial gluon contrib’n
in $\rho^0$ production at $x_B \sim 0.1$

M.D. and A. Vinnikov, '04


- leading twist LO calculation
  ('conventional' double distribution model for GPDs)

- gluons may be non-negligible even in JLAB kinematics
- substantial uncertainties on conventional gluon densities
- warning: should do NLO evaluation
leading twist LO calculation
('conventional' double distribution model for GPDs)

M.D. et al. '05

- gluons may be non-negligible even in JLAB kinematics
- substantial uncertainties on conventional gluon densities
- warning: should do NLO evaluation
- calculated $\sigma(\rho^0)/\sigma(\phi)$ too large
  but expect extra suppression for $\phi$ (strange quark mass)
- leading-twist calculations for vector meson production overshoot data, factors of several at $Q^2 \lesssim 5 \text{ GeV}^2$
- strong suppression from meson $k_T$ in hard scattering
  
  L. Frankfurt et al. '95; M. Vanderhaeghen et al. '99
- new analysis for small $x_B$ (gluons only)
  
  P. Kroll, S. Goloskokov '05

\[ \begin{align*}
  Q^2 \text{ [GeV}^2\text{]} & \quad \sigma(\gamma^* p \rightarrow p) [\text{nb}] \\
  4 & \quad 10^3 \\
  6 & \quad 10^2 \\
  8 & \quad 10^1 \\
  10 & \quad 10^0 \\
  20 & \quad 10^0 \\
  40 & \quad 10^0 \\
  \end{align*} \]

hep-ph/0501242, CTEQ5M gluon, double distribution model
▶ leading-twist calculations for vector meson production overshoot data
factors of several at $Q^2 \lesssim 5$ GeV$^2$

▶ strong suppression from meson $k_T$ in hard scattering

▶ new analysis for small $x_B$ (gluons only)
P. Kroll, S. Goloskokov ’05

hep-ph/0501242, CTEQ5M gluon
- directly sensitive to gluon distribution at small $x$
  unlike inclusive struct. funct. $F_2(x, Q^2)$
- despite uncertainties in modelling $g(x) \rightsquigarrow \text{GPD}$
  $J/\Psi$ data cast doubt on some gluon distrib’s

T. Teubner, DIS 2005, plots by P. Fleischmann (H1)
alternative model for $g(x), q(x) \sim \text{GPD}$
(not based on double distrib’s)
V. Guzey and M. Polyakov ’05, based on M. Polyakov and A. Shuvaev ’02
satisfies Lorentz invariance (polynomiality) relations
good description of DVCS data from HERA (LO calculation)

-hep-ph/0507183
The $t$ dependence

- at small $x$ and small $t$ parametrize

$$d\sigma/dt \propto e^{-B|t|}$$

- $\rho$ and $\phi$: “pointlike” $\gamma^* \rightarrow q\bar{q}$ for large $Q^2$
- $J/\Psi$: “pointlike” $\gamma \rightarrow c\bar{c}$ even for $Q^2 = 0$
The $t$ dependence

- at small $x$ and small $t$ parametrize

$$d\sigma/dt \propto e^{-B|t|}$$

- $\rho$ and $\phi$: “pointlike” $\gamma^* \rightarrow q\bar{q}$ for large $Q^2$
- $J/\Psi$: “pointlike” $\gamma \rightarrow c\bar{c}$ even for $Q^2 = 0$
The $t$ dependence

- at small $x$ and small $t$ parametrize
  
  \[ \frac{d\sigma}{dt} \propto e^{-B|t|} \]

- $\rho$ and $\phi$: “pointlike” $\gamma^* \rightarrow q\bar{q}$ for large $Q^2$
  
  $J/\Psi$: “pointlike” $\gamma \rightarrow c\bar{c}$ even for $Q^2 = 0$

- first measurement for DVCS H1 Coll. ’05

sea quarks and gluons
Small $x$

neglect interplay of $x$ and $\xi$

simple ansatz: $\text{GPD} \sim \left( \frac{1}{x} \right)^{\alpha+\alpha' t} = x^{-\alpha} e^{t\alpha' \log(1/x)}$

- exclusive $J/\Psi$ production (gluons)
  - photoproduction
    - $\alpha = 1.224 \pm 0.010 \pm 0.012$
    - $\alpha' = 0.164 \pm 0.028 \pm 0.030$GeV$^{-2}$
  - similar in electroproduction

- values very different in soft processes $\gamma p \rightarrow \rho p$, $pp \rightarrow pp$, ... for $\alpha$ is well-known from inclusive $\gamma^* p \rightarrow X$ vs. $\gamma p \rightarrow X$
Small $x$

neglect interplay of $x$ and $\xi$

simple ansatz: \[ \text{GPD} \sim \left( \frac{1}{x} \right)^{\alpha + \alpha' t} = x^{-\alpha} e^{t\alpha' \log(1/x)} \]

- in nonsinglet sector (quarks only, no gluons)
  \[ \alpha \sim 0.4 \ldots 0.5 \] in parton distrib's at low scale
  similar to soft processes (meson trajectories)

- $\alpha'$ in partonic regime? \[ \ldots \text{wait a few slides} \]
Impact parameter

- states with definite light-cone momentum $p^+$ and transverse position (impact parameter):

$$|p^+, b\rangle = \int d^2 p \, e^{-ib \cdot p} |p^+, p\rangle$$

formal: eigenstates of 2 dim. position operator

- can exactly localize proton in 2 dimensions
  no limitation by Compton wavelength
- and stay in frame where proton moves fast
  $\rightsquigarrow$ parton interpretation
Impact parameter

- states with definite light-cone momentum $p^+$ and transverse position (impact parameter):

$$|p^+, b\rangle = \int d^2p \ e^{-i b \cdot p} |p^+, p\rangle$$

formal: eigenstates of 2 dim. position operator

- $b$ is center of momentum of the partons in proton

$$b = \frac{\sum_i p_i^+ b_i}{\sum_i p_i^+} \quad (i = q, \bar{q}, g)$$

consequence of Lorentz invariance

nonrelativistic analog: Galilei invariance $\Rightarrow$ center of mass
Impact parameter GPDs

in following specialize to $\xi = 0$

- impact parameter distribution

$$q(x, b^2) = (2\pi)^{-2} \int d^2 \Delta e^{-i\Delta \cdot b} H^q(x, \xi = 0, t = -\Delta^2)$$

gives distribution of quarks with
- longitudinal momentum fraction $x$
- transverse distance $b$ from proton center

M. Burkardt '00

- average impact parameter

$$\langle b^2 \rangle_x = \frac{\int d^2 b \ b^2 \ q(x, b^2)}{\int d^2 b \ q(x, b^2)} = 4 \frac{\partial}{\partial t} \log H^q(x, \xi, t) \bigg|_{t=0}$$
**Large** $x$:

- $d = b/(1 - x)$
  - distance of selected parton from spectator system
  - gives lower bound on overall size of proton
- finite size of configurations with $x \to 1$ implies
  - $\langle b^2 \rangle_x \sim (1 - x)^2$

  "for $x \to 1$ get $b \to 0$
  nonrel. analog: center of mass of atom"

  $\leftrightarrow t$ dependence becomes flat

**Small** $x$:

- $H(x, t) \sim e^{tB + \alpha' \log(1/x)} \sim \langle b^2 \rangle_x \sim B + \alpha' \log(1/x)$

M. Burkardt, '02, '04

M. Diehl
Evolution

\( q(x, b^2) \) fulfils usual DGLAP evolution equation

for non-singlet (e.g. \( q_{NS} = q - \bar{q} \) or \( q_{NS} = u - d \)):

\[
\mu^2 \frac{d}{d\mu^2} q_{NS}(x, b^2, \mu^2) = \int_x^1 \frac{dz}{z} \left[ P \left( \frac{x}{z} \right) \right] q_{NS}(z, b^2, \mu^2)
\]

evolution local in \( b \) (let \( 1/\mu \ll b \) to be safe)
Evolution

- $q(x, b^2)$ fulfils usual DGLAP evolution equation for non-singlet (e.g. $q_{NS} = q - \bar{q}$ or $q_{NS} = u - d$):

$$
\mu^2 \frac{d}{d\mu^2} q_{NS}(x, b^2, \mu^2) = \int_{1/x}^1 \frac{dz}{z} \left[ P \left( \frac{x}{z} \right) \right] q_{NS}(z, b^2, \mu^2)
$$

evolution local in $b$ (let $1/\mu \ll b$ to be safe)

- average

$$
\langle b^2 \rangle_x = \frac{\int d^2b b^2 q_{NS}(x, b^2)}{\int d^2b q_{NS}(x, b^2)}
$$

evolves as

$$
\mu^2 \frac{d}{d\mu^2} \langle b^2 \rangle_x = - \frac{1}{q_{NS}(x)} \int_{1/x}^1 \frac{dz}{z} P \left( \frac{x}{z} \right) q_{NS}(z) \left[ \langle b^2 \rangle_x - \langle b^2 \rangle_z \right]
$$

M.D. et al. '04

M. Diehl
Information from electromagnetic form factors

- ff’s constrain interplay of $x$ and $b$ dependence
  M.D. et al. '04, M. Guidal et al. '04

- e.m. current $\leadsto$ only $q - \bar{q}$
  \[
  H^q_v(x, t) = H^q(x, t) - H^{\bar{q}}(x, t)
  \]

\[
F^p_1(t) = \int_0^1 dx \left[ \frac{2}{3} H^u_v(x, t) - \frac{1}{3} H^d_v(x, t) \right]
\]

\[
F^n_1(t) = \int_0^1 dx \left[ \frac{2}{3} H^d_v(x, t) - \frac{1}{3} H^u_v(x, t) \right]
\]

- ansatz: $H^q_v(x, t) = q_v(x) \exp[tf_q(x)]$ \quad $\langle b^2 \rangle^q_x = 4f_q(x)$

- ansatz for $f_q(x)$ interpolates between
  \[
  f_q(x) \rightarrow \alpha' \log(1/x) \quad \text{for } x \rightarrow 0
  \]
  \[
  f_q(x) \sim (1 - x)^2 \quad \text{for } x \rightarrow 1
  \]
Information from electromagnetic form factors

- ff’s constrain interplay of $x$ and $b$ dependence
  M.D. et al. ’04, M. Guidal et al. ’04

- e.m. current $\sim$ only $q - \bar{q}$
  
  $$H^q_v(x, t) = H^q(x, t) - H^\bar{q}(x, t)$$

  $$F^p_1(t) = \int_0^1 dx \left[ \frac{2}{3} H^u_v(x, t) - \frac{1}{3} H^d_v(x, t) \right]$$

  $$F^n_1(t) = \int_0^1 dx \left[ \frac{2}{3} H^d_v(x, t) - \frac{1}{3} H^u_v(x, t) \right]$$

- ansatz: $H^q_v(x, t) = q_v(x) \exp[ t f_q(x) ]$  $\langle b^2 \rangle^q_x = 4 f_q(x)$

- ansatz for $f_q(x)$ interpolates between
  
  $$f_q(x) \rightarrow \alpha' \log(1/x) \quad \text{for } x \rightarrow 0$$

  $$f_q(x) \sim (1 - x)^2 \quad \text{for } x \rightarrow 1$$

- good description of data with $\alpha' = 0.9$ to 1 GeV$^{-2}$
$t^2 F_1^p$

$\chi^2 / \text{d.o.f.} = 1.93$

pull $F_1^p = \text{data/fit} - 1$

pull $F_1^m$
Lessons from the fit

- clear drop with $x$ of average distance $d = b/(1 - x)$
- strong correlation of $x$ and $t$ dependence
clear drop with $x$ of average distance $d = b / (1 - x)$

$\leftrightarrow$ strong correlation of $x$ and $t$ dependence

region $x \gtrsim 0.8$ contributes less than 5% to form factors

$\rightarrow$ data cannot fix asymptotic behavior of $d_q(x)$ for $x \to 1$
Lessons from the fit

- $d$ quark distribution less well determined
  improvement with better data for $F_{1n}^n$

- to describe both $F_{1p}^p$ and $F_{1n}^n$ well
  fit wants $d_d(x) > d_u(x)$ for moderate to large $x$
  $\leftrightarrow$ $d$ quarks more “spread out” than $u$ quarks
Compare with lattice results

matrix elements of local operators $\leftrightarrow$ form factors
calculate in lattice QCD

- main systematic uncertainties from
  - omission of “disconnected” diagrams
    but: cancel in difference of $u$ and $d$
  - extrapolation to physical pion mass

figure: J. Negele, hep-lat/0211022
Compare with lattice results

matrix elements of local operators ↔ form factors
calculate in lattice QCD

J. Negele et al., hep-lat/0404005

- Wilson fermions
- $m_\pi = 870 \text{ MeV}$
- typical $x$ in $\int dx \, x^n q(x, b)$ estimated as

$$\langle x \rangle = \frac{\int dx \, x^{n+1} q(x)}{\int dx \, x^n q(x)}$$
Large $t$ and the Feynman mechanism

- if impose that spectators have virtualities $\sim \Lambda^2$ then
  \[ 1 - x \sim \frac{\Lambda}{\sqrt{-t}} \]

- large-$t$ asymptotics with our ansatz: $\langle 1 - x \rangle_t \sim 1/\sqrt{-t}$
  numerically seen for $-t \gtrsim 5 \text{ GeV}^2$

- get Drell-Yan relation $F_1^q(t) \sim |t|^{-(1+\beta_q)}$
  if $q(x) \sim (1 - x)^{\beta_q}$ at large $x$

CTEQ6M distributions at $\mu = 2 \text{ GeV}$:
  $\beta_u \sim 3.4$ and $\beta_d \sim 5.0$ (for $0.5 < x < 0.9$)
very different $u(x)$ and $d(x)$ for large $x$

$\leftrightarrow$ very different $u$ and $d$ moments at large $t$

hope to test with experimental data on $F_1^m(t)$ and $F_1^p(t)$

and lattice calculations of higher moments
Spin and the Pauli form factors

▶ $E \leftrightarrow$ nucleon helicity flip $\langle \downarrow | \mathcal{O} | \uparrow \rangle$

$\leftrightarrow$ transverse pol. difference $|X\pm\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle \pm |\downarrow\rangle)$

$\langle X^+ | \mathcal{O} | X^+ \rangle - \langle X^- | \mathcal{O} | X^- \rangle = \langle \uparrow | \mathcal{O} | \downarrow \rangle + \langle \downarrow | \mathcal{O} | \uparrow \rangle$

▶ quark density in proton state $|X^+\rangle$

$q^X_v (x, b) = q_v (x, b) - \frac{b^y}{m} \frac{\partial}{\partial b^2} e^q_v (x, b)$

shifted in $y$ direction

$e^q_v (x, b)$ is Fourier transform of $E^q_v (x, t)$

M. Burkardt '02
quark density in proton state $|X^+\rangle$

$$q_v^X(x, b) = q_v(x, b) - \frac{b^y}{m} \frac{\partial}{\partial b^2} e^q_v(x, b)$$

is shifted

$$(d - \bar{d}) \text{ density in transverse plane} \quad \text{M.D. et al. '04}$$

$$\int dx \, E^u(x, 0) = \kappa^u \approx 1.67 \quad \text{and} \quad \int dx \, E^d(x, 0) = \kappa^d \approx -2.03$$

$\rightarrow$ large spin-orbit correlations

$\Rightarrow$ relation with transverse momentum dependent densities

$\rightarrow$ Sivers function

$\Rightarrow$ similar for generalized transversity distributions

M.D. and P. Hägler '05, M. Burkardt '05
density representation

\[ q^X_v(x, b) = q_v(x, b) - \frac{b^y}{m \partial b^2} e^q_v(x, b) \]

gives positivity bound

\[
\left[ E^q(x, t = 0) \right]^2 \leq m^2 \left[ q(x) + \Delta q(x) \right] \left[ q(x) - \Delta q(x) \right] \\
\times 4 \frac{\partial}{\partial t} \ln \left[ H^q(x, t) \pm \tilde{H}^q(x, t) \right]_{t=0}
\]

\[ \Rightarrow \ E^q \text{ must fall faster than } H^q \text{ at large } x \]

\[ E^q \leftrightarrow \text{orbital angular momentum} \]

\[ \Rightarrow \text{ carried by partons with smaller } x \]
M.D. et al. '04, M. Guidal et al. '04

- sum rules: Pauli ff’s ↔ $E_{v}^{q}(x, t) = E^{q}(x, t) - E^{\bar{q}}(x, t)$
- ansatz $E_{v}^{q}(x, t) = e_{v}(x) \exp[t g_{q}(x)]$
  $g_{q}(x)$ same form as $f_{q}(x)$ in ansatz for $H_{v}^{q}$
- shape of forward limit $e_{v}^{q}(x)$ not known → ansatz

$$e_{v}^{q} = N_{q} x^{-\alpha} (1 - x)^{\beta_{q}}$$

$N_{q}$ determined by $p$ and $n$ magnetic moments
sum rules: Pauli ff’s $\leftrightarrow E^q_v(x,t) = E^q(x,t) - E^\bar{q}(x,t)$

ansatz $E^q_v(x,t) = e_v(x) \exp[t g_q(x)]$

$g_q(x)$ same form as $f_q(x)$ in ansatz for $H^q_v$

shape of forward limit $e^q_v(x)$ not known $\to$ ansatz

$$e^q_v = N_q x^{-\alpha} (1 - x)^{\beta_q}$$

$N_q$ determined by $p$ and $n$ magnetic moments

obtain good fit of $F^p_2(t)$ and $F^n_2(t)$

$\alpha' = 0.9$ GeV$^{-2}$ and $\alpha = 0.55$

ok with Regge phenomenology

large allowed regions of $\beta_q$ and parameters in $g_q(x)$

but positivity constraints seriously limit parameter space

in particular $\beta_d \geq 5$ and $\beta_u \geq 3.5$
Quarks and gluons

Impact parameter

Spin

Conclusions

**Introduction**

**Quarks and gluons**

**Impact parameter**

**Spin**

**Conclusions**

![Graph](image-url)

\[ t^2 F_2^p \]

\[ t^2 F_2^n \]

\[ \chi^2 / \text{d.o.f.} = 1.31 \]

\[ \text{pull} = \frac{\text{data}}{\text{fit}} - 1 \]

---

M. Diehl

QCD in hard exclusive processes
\[ \chi^2 / \text{d.o.f.} = 1.31 \]

pull = data/fit – 1
orbital angular momentum carried by valence quarks

\[
\langle L^q_v \rangle = \frac{1}{2} \int dx \left[ x e^q_v(x) + xq_v(x) - \Delta q_v(x) \right]
\]
orbital angular momentum carried by valence quarks

\[\langle L_v^q \rangle = \frac{1}{2} \int dx \left[ xe_v^q(x) + xq_v(x) - \Delta q_v(x) \right] \]

- 2\langle L_v^u - L_v^d \rangle = -\left(0.77 \div 0.92\right) \text{ at } \mu = 2 \text{ GeV}

- lattice results:
  QCDSF: \( 2\langle L_v^u - L_v^d \rangle = -0.9 \pm 0.12 \) \hspace{1em} G. Schierholz, LC 2005
  LHPC: \( 2\langle L_v^u - L_v^d \rangle = -0.25 \pm 0.05 \) \hspace{1em} for \( m_\pi = 897 \text{ MeV} \)

\[\text{from hep-lat/0410017}\]
calculation of $E^u + E^d$ in **chiral soliton model**  

**J. Ossmann et al. ’05**

\[ x(E^u + E^d)(x,0,0) \]

- **$\beta_u = 4$**  
- **$\beta_d = 5.6$**

- **hep-ph/0411172, $\mu^2 = 5$ GeV$^2$**  
- **$E^q - E^{\bar{q}}, \mu^2 = 4$ GeV$^2$**

▶ **shape** differs from our fit (with central values of par’s)
calculation of $E^u + E^d$ in chiral soliton model

\[ x(E^u + E^d)(x,0,0) \]

\[ \beta_u = \beta_d = 5 \]

hep-ph/0411172, $\mu^2 = 5 \text{ GeV}^2$

- shape differs from our fit (with central values of par’s)
- but fit has large parameter uncertainties for $E^u + E^d$
- normalization: chiral soliton model: $\kappa_u + \kappa_d = 0.35$
  experiment: $\kappa_u + \kappa_d = 0.12$

$E^q - \bar{E}^\bar{q}, \mu^2 = 4 \text{ GeV}^2$
Conclusions

- Vector meson production:
  very sensitive to gluon distribution even in fixed-target kinematics
  higher twist corrections essential for realistic cross sections
- Impact parameter picture:
  naturally implemented at a rigorous level
- \( F_1(t) \) data and lattice \( \sim \) strong decrease of \( \langle b^2 \rangle \) with \( x \)
- \( F_1(t) \) data consistent with Feynman mechanism
  \( \sim \) Drell-Yan relation
  \( \sim \) striking effects for \( d \) quark part of form factors
- \( E \rightarrow \) physics of transverse spin
  and \( \rightarrow \) orbital angular momentum
- Attempts for quantitative understanding of \( E \)
  “valence” contributions: \( L^{u-d} \) big and \( L^{u+d} \) small
  need direct measurements to learn more